Available online at www.sciencedirect.com

ScienceDirect

IFAC PapersOnLine 54-7 (2021) 268–273

Modulating Function Based Fault Diagnosis Using the Parity Space Method

Luis Enciso *, **Matti Noack** **, **Johann Reger** **, **Gustavo Pérez-Zuñiga** *

* *Department of Engineering, Pontifical Catholic University of Peru, Peru (e-mail: {lenciso, gustavo.perez}@pucp.pe).*
** *Control Engineering Group, Technische Universität Ilmenau, Germany (e-mail: {matti.noack, johann.reger}@tu-ilmenau.de)*

Abstract: A model-based method for the detection and estimation of faults in dynamic systems is proposed. The method is based on the combination of the parity space approach and the modulating function framework for estimation. The parity space method is employed as an efficient geometric procedure determining null subspaces for annihilating unknown terms and formulating residuals. With the modulating functions technique the dynamic relation from output differentiation is reformulated as an algebraic expression. This substantially reduces the noise sensitivity of the output derivatives required. The design allows for the robust fault detection and isolation also for some nonlinear systems. The robustness of the approach is demonstrated on a nonlinear model of a four-tank process.

Keywords: Modulating Functions Approach, Fault Detection, Parity Space Method.

1. INTRODUCTION

Recognizing the faults in dynamic systems is a major task in supervision architectures aiming at avoiding accidents and incorporating maintenance activities. Generally, methods for fault diagnosis are classified as model-based, signal-based, and data-based. Among those the first is widely studied, underscored by numerous contributions dedicated to this matter, see (Blanke et al., 2006) for a collection. In diagnosis, the main tasks range from fault detection, which recognizes the occurrence of faults, over fault isolation, which determines the set of faults, to a precise identification of the fault signal. Estimating the faults is decisive for assessing the magnitude and trend of faults, and thus is crucial for designing fault tolerant systems (Gao, 2015).

In fault estimation, various techniques have been developed for robust design (Zhaohui and Noura, 2013), (Maiying Zhong et al., 2008) and for descriptor observers (Jiang et al., 2004), (Gao, 2015) with the objective of increasing the fault-tolerance. A particular model-based technique for fault detection is the generation of parity space relations. Past research has shown that this method can be difficult to implement in the continuous-time setting. Despite the straight-forward design, which only depends on linear algebra, it has some drawbacks such as sensitivity to noise, as observed by Höfling and Pfeufer (1994), Sun et al. (2019), and also Xue et al. (2018) where the parity space generation is based on a stationary wavelet transform. The approach requires time-derivatives of the output, prone to measurement noise. For non-Lipschitz dynamical systems, problems occur due to state differentiation that may lead to singularities. These challenges impose an obstacle for the practical realization in the diagnosis scheme with low cost sensors.

In this context, the modulating function approach may play a central role when generating and robustly solving a system of residual equations. The approach was originally proposed by Shinbrot (1957) for the identification of linear SISO systems. It has recently been extended to a simultaneous parameter and state estimation procedure (Jouffroy and Reger, 2015). In general, the approach admits a non-asymptotic estimation and has successfully been applied to several tasks in parameter, state, and input estimation problems (Noack et al., 2018), (Liu et al., 2014). The modulation of a dynamic input-output equation is performed by applying an integral transform with respect to a specified modulation kernel which allows to reformulate and avoid time derivatives of measured signals, leading to algebraic relations only. The smoothing property of the integral as well as the avoidance of numerical differentiation mitigate noise and disturbance effects. Early concepts of algebraic residual generation were introduced by Fliess et al. (2004). Lately in Li et al. (2018), other kernel based estimators are applied for the purpose of fault detection and isolation, examining also the influence of external disturbances.

Here the proposed technique uses modulating functions for generating a set of algebraic equations that do not involve time differentiation of the measurement signals. Resorting to different modulation kernels a larger set of equations is generated when compared to just considering a certain number of output derivatives. Based on this, parity space calculations are performed and respective null spaces are generated. This allows to purge the residual from undesired influences, for example, from external disturbances. After that, the fault signals are reconstructed, directly using the residual by modulation based estimation of the unknown input. Furthermore, the approach is extended to a certain class of nonlinear systems, which due to modulation allows to avoid differentiation of non-Lipschitz expressions to maintain a continuous-time perspective, and thus to circumvent discrete filtering. In view of model-based preprocessing, the real-time implementation of our algorithm is efficient and shows significant robustness against sensor noise.

The paper is structured as follows: After presenting theoretical prerequisites for the modulating function approach in Section II, the parity space formulation for diagnosis applications is introduced in Section III. In Section IV the methodologies are

10.1016/j.ifacol.2021.08.370

combined to generate a residual with the appropriate null space conditions, marking the major result of this paper. Section V provides simulations of the proposed fault detection and estimation on a nonlinear model of a four-tank system, including parameter calibration. Conclusions are drawn in Section VI.

2. MODULATING FUNCTIONS APPROACH

The modulating function approach is based on a general integral transform, leading to a algebraic reformulation of a dynamic system, thus, to non-asymptotic estimation algorithms.

2.1 Parameter estimation

The modulating function framework formulated in (Jouffroy and Reger, 2015) is devised for parameter and state estimation. The central notion of the approach is an integration over a moving horizon with respect to a predefined modulation kernel. In order to formulate a regressor form for identifying constant parameters, we define modulating functions as follows.

Definition 1. A sufficiently smooth function $\varphi \in \mathcal{C}^k([0,T],\mathbb{R})$ is called total modulating function of order $k \in \mathbb{N}$ if

$$\varphi^{(i)}(0) = \varphi^{(i)}(T) = 0 \quad \forall i = 0,1,...,k-1\,, \tag{1}$$

where $T > 0$ is a fixed time-horizon.

When dealing with dynamic input-output equations, time-derivatives of measurement signals occur which cannot be computed directly due to sensor noise, biases, and other perturbations. However, for some diagnosis approaches, as in Section 3, several differentiations of the output signal need to be performed for obtaining additional information. Here we may benefit from a key property of a modulating signal $f \in \mathcal{C}^i(\mathbb{R}_0^+,\mathbb{R})$ and its derivatives with a total modulating function from Definition 1 as kernel. Due to (1), integration by parts yields

$$\int_0^T \varphi(\tau) f^{(i)}(\tau)d\tau = (-1)^i \int_0^T \varphi^{(i)}(\tau) f(\tau)d\tau\,, \tag{2}$$

i.e., the derivative of a (measured) signal is shifted to a known kernel function. For brevity, we shall use the modulating function operator $L : \mathcal{C}^i \to \mathbb{R}$ from (Noack et al., 2018) with

$$L^i[f] = \int_{t-T}^t (-1)^i \varphi^{(i)}(\tau - t + T) f(\tau)d\tau. \tag{3}$$

The integral is considered on the receding horizon $[t-T,t]$. No virtual dynamic loop is introduced due to the non-asymptotic, fixed-time algorithm, making control and fault detection easier.

2.2 Unknown input estimation

For reconstructing unknown external signals such as faults, a signal-expansion based modulation approach is applied as introduced by Liu et al. (2014) and applied by Noack et al. (2018). Consider the following weighted scalar product with $\phi_1, \phi_2 \in \mathcal{L}_2([0,1])$

$$\langle \phi_1, \phi_2 \rangle_w := \int_0^1 w(\tau)\phi_1(\tau)\phi_2(\tau)d\tau, \tag{4}$$

taken with respect to the weighting function $w : [0,1] \to \mathbb{R}_0^+$. Based on this, an orthonormal basis is chosen as $\Phi_{[0,1]} = \text{span}\ \{\phi_0, \phi_1, \phi_2, \ldots\}$ for expanding the unknown signal $g \in \Phi_{[0,1]} \subset \mathcal{L}_2([0,1])$ over the considered time horizon as per

$$g(\tau) \simeq \sum_{i=0}^{\infty} \lambda_i \phi_i(\tau) = \sum_{i=0}^{\infty} \langle g, \phi_i \rangle_w\, \phi_i(\tau)\,. \tag{5}$$

In view of the series formulation, a modulating function is selected as $\varphi_k(\tau) := w(\tau)\phi_k(\tau)$ with the resulting operator

$$L_k^j[f] := \frac{(-1)^j}{h^j} \int_0^1 \varphi_k^{(j)}(\tau)\, f(t+(\tau-1)h)d\tau, \tag{6}$$

with horizon window length $h > 0$. The weighting function w is chosen with respect to the desired modulation characteristics. Here, the boundary conditions (1) are realized. Modulating the function representation (5) of a shifted signal f leads to

$$L_k^0[f] = \langle \phi_k, f \rangle_w = \lambda_{t,k}\,, \tag{7}$$

where the respective modal coefficient is isolated. This can be applied for continuous signal identification by

$$f(t+(\tau-1)h) \approx \sum_{i=0}^{N} \underbrace{\langle \phi_k, f(t+(\tau-1)h) \rangle_w}_{=:\hat{\lambda}_{t,k}} \phi_i(\tau) \tag{8}$$

where $N \in \mathbb{N}$ is the approximation order. This is applied to dynamic input-output equations by receding horizon integration. The application to systems with output derivatives is sketched in (Noack et al., 2018). Liu et al. (2014), advocate Jacobi Polynomials (JP) due to their robustness against sensor noise. For selected $\alpha, \beta > 0$ the basis functions are defined as $\phi_k = P_k^{(\alpha,\beta)}$ with

$$P_k^{(\alpha,\beta)}(\tau) := \sum_{j=0}^{k} \binom{k+\alpha}{j} \binom{k+\beta}{k-j} (\tau-1)^{k-j}\tau^j\,. \tag{9}$$

The associated weighting function is $w(\tau) = \tau^\alpha(1-\tau)^\beta$ where α and β are chosen according to the desired modulating function boundary conditions. Having implemented (6) for a given dynamic system, an unknown signal may be reconstructed via

$$\hat{f}(t+(\tau_0-1)h) = \sum_{i=0}^{N} \hat{\lambda}_{t,k} P_k^{(\alpha,\beta)}(\tau_0) \tag{10}$$

for selected $\tau_0 \in [0,1)$. A delay $\delta = (1-\tau_0)h$ is imposed which is tuned for mitigating measurement noise. Order N is chosen with respect to the expected signal behavior over the moving time horizon.

3. PARITY SPACE METHOD

The basic theory of the parity space approach is introduced along the lines in (Blanke et al., 2006). Consider the LTI system

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + E_x d(t) + F_x f(t) \\ y(t) = Cx(t) + Du(t) + E_y d(t) + F_y f(t) \end{cases} \tag{11}$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ the input, $d(t) \in \mathbb{R}^{n_d}$ the disturbance, $y(t) \in \mathbb{R}^p$ the output, and $f(t) \in \mathbb{R}^{n_f}$ the fault. By calculating $q \in \mathbb{N}$ successive derivatives of output y, $q+1$ relations can be obtained:

$$\begin{aligned} y(t) &= Cx(t) + Du(t) + E_y d(t) + F_y f(t)\,, \\ \dot{y}(t) &= CAx(t) + CBu(t) + D\dot{u}(t) + CE_x d(t) + \\ &\quad E_y \dot{d}(t) + CF_x f(t) + F_y \dot{f}(t)\,, \\ &\ \vdots \\ y^{(q)}(t) &= CA^q x(t) + CA^{q-1}Bu(t) + \ldots + Du^{(q)}(t) + \\ &\quad CA^{q-1}E_x d(t) + \ldots + E_y d^{(q)}(t) + \\ &\quad CA^{q-1}F_x f(t) + \ldots + F_y f^{(q)}(t)\,. \end{aligned} \tag{12}$$

These equations may be written in matrix form

$$\bar{y} = Ox + T_u\bar{u} + T_d\bar{d} + T_f\bar{f}\,, \tag{13}$$

where $\bar{y} = \left[y^\top, \dot{y}^\top, \ldots, y^{(q)}\right]^\top$ for the signals y, and for u, d, and f correspondingly. The resulting matrices $O \in \mathbb{R}^{(q+1)p\times n}$, $T_d \in \mathbb{R}^{(q+1)p\times(q+1)n_d}$, $T_u \in \mathbb{R}^{(q+1)p\times(q+1)m}$ and $T_f \in \mathbb{R}^{(q+1)p\times(q+1)n_f}$ are given as

$$O = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^q \end{bmatrix}, \tag{14}$$

$$T_d = \begin{bmatrix} E_y & 0 & \ldots & 0 \\ CE_x & E_y & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{q-1}E_x & CA^{q-2}E_x & \ldots & E_y \end{bmatrix}, \tag{15}$$

$$T_u = \begin{bmatrix} D & 0 & \ldots & 0 \\ CB & D & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{q-1}B & CA^{q-2}B & \ldots & D \end{bmatrix}, \tag{16}$$

$$T_f = \begin{bmatrix} F_y & 0 & \ldots & 0 \\ CE_x & F_y & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{q-1}F_x & CA^{q-2}F_x & \ldots & F_y \end{bmatrix}. \tag{17}$$

Terms related to states and perturbations are not known and they shall not appear in the diagnosis criteria. For canceling them, the existence of a null-space related to the extended matrix $(O \quad T_d)$ is required. The condition to satisfy this requirement is expressed in terms of the complementary rank $n_W \in \mathbb{N}$, i.e.

$$n_W = (q+1)\,p - \operatorname{rank}(O \quad T_d) \overset{!}{>} 0\,. \tag{18}$$

Then the columns of annihilation matrix $W \in \mathbb{R}^{(q+1)p\times n_W}$ can be defined as basis spanning the left kernel related to (18) with

$$\begin{aligned} \ker(O^\top \quad T_d^\top)^\top &= \{w \in \mathbb{R}^{(q+1)p} \,|\, w^\top [O \quad T_d] = 0\} \\ &= \operatorname{span}\{w_1, \ldots, w_{n_W}\} \end{aligned}$$

$$\Rightarrow \qquad W = [w_1 \; \cdots \; w_{n_W}]\,. \tag{19}$$

As a consequence, left multiplication of (13) by $W^\top$ yields

$$W^\top \bar{y} - W^\top T_u \bar{u} = W^\top T_f \bar{f}\,. \tag{20}$$

The left-hand side of (20) is employed to define residuals that are zero whenever $f(t) = 0$ and non-zero once $f(t) \neq 0$. Yet, the implementation of residuals based on this approach requires the numerical computation of derivatives of the signals u and y which are usually subject to noise. There are workarounds utilizing discrete-time system approximations (Blanke et al., 2006), but these show significant sensitivity to high frequency noise components. To attenuate this influence the parity space method is enhanced with the modulating functions approach.

4. MODULATING FUNCTIONS APPLIED TO THE PARITY SPACE RELATION

4.1 Main approach

For avoiding the time derivatives in the classical parity space approach, a total modulating function from Definition 1 is applied sequentially to (11) and its subsequent q derivatives in (12). Using the modulation operator notation (3) leads to

$$\begin{aligned} L^0[y] &= CL^0[x] + DL^0[u] + F_y L^0[f]\,, \\ L^1[y] &= CAL^0[x] + CBL^0[u] + DL^1[u] + CE_x L^0[d] + \\ &\quad CF_x L^0[f] + F_y L^1[f]\,, \\ &\;\;\vdots \\ L^q[y] &= CA^q L^0[x] + CA^{q-1}BL^0[u] + \ldots + DL^q[u] + \\ &\quad CA^{q-1}E_x L^0[d] + \ldots + E_y L^q[d] + \\ &\quad CA^{q-1}F_x L^0[f] + \ldots + F_y L^q[f]\,. \end{aligned}$$

Defining new substituted vectors in the form for $\bar{L}[y] = \left[L^0[y]^\top, L^1[y]^\top, \ldots, L^q[y]^\top\right]^\top$ for the signals y, and for u, d and f, respectively, the integral transformed analog to (13) is

$$\bar{L}[y] = OL^0[x] + T_u \bar{L}[u] + T_d \bar{L}[d] + T_f \bar{L}[f]. \tag{21}$$

The matrices are preserved due to linearity of the modulation operation. Considering condition (18) with respect to (21), a null-space matrix W is calculated similarly to (19).

However, inequality (18) can be unfeasible in many cases. A remedy proposed by Nguang et al. (2011) is the use of linear matrix inequalities (LMIs) to approximate the matrix W that minimizes $\|WO\|$ and $\|WT_d\|$ while maximizing $\|WT_f\|$:

$$Q_o - W^\top O\,(W^\top O)^\top > 0\,, \tag{22}$$

$$Q_d - W^\top T_d\,(W^\top T_d)^\top > 0\,, \tag{23}$$

$$Q_f - (I - W^\top T_f)(I - W^\top T_f)^\top > 0\,, \tag{24}$$

where Q_o and Q_d are diagonal matrices that can be conveniently selected for the optimization. Applying the Schur complement, the following LMIs are to be solved subsequently:

$$\begin{bmatrix} -Q_o & W^\top O \\ (W^\top O)^\top & -I \end{bmatrix} > 0\,, \tag{25}$$

$$\begin{bmatrix} -Q_d & W^\top T_d \\ (W^\top T_d)^\top & -I \end{bmatrix} > 0\,, \tag{26}$$

$$\begin{bmatrix} -Q_f & (I - W^\top T_f) \\ (I - W^\top T_f)^\top & -I \end{bmatrix} > 0\,. \tag{27}$$

Thus, with appropriate W the residuals are obtained via

$$r = W^\top \bar{L}(y(t)) - W^\top T_u \bar{L}(u(t)), \tag{28}$$

equal to the right-hand side of the fault detection equation, i.e.

$$r = W^\top T_f \bar{L}[f]\,. \tag{29}$$

To achieve exact fault identification, the function expansion from Section 2 is applied to the fault signal components of f. Hence, (5) with the use of modulation (6) for a set of $M \in \mathbb{N}$ polynomials and plugging this into the residual (29) leads to

$$r = W^\top T_f \begin{bmatrix} \sum_{i=0}^{M} \hat{\lambda}_{t,i} \int_0^1 \varphi_k(\tau)\phi_i(\tau)d\tau \\ \frac{1}{h}\sum_{i=0}^{M} \hat{\lambda}_{t,i} \int_0^1 \dot{\varphi}_k(\tau)\phi_i(\tau)d\tau \\ \vdots \\ \frac{1}{h^q}\sum_{i=0}^{M} \hat{\lambda}_{t,i} \int_0^1 \varphi_k^{(q)}(\tau)\phi_i(\tau)d\tau \end{bmatrix}. \tag{30}$$

By factorizing the coefficients that have to be estimated, i.e. $\hat{\Lambda}_t = (\lambda_{t,1}, \cdots, \lambda_{t,M})^\top$, the following system of equations for fault estimation is obtained:

$$r = W^\top T_f \underbrace{\begin{bmatrix} & e_k^\top & \\ \frac{1}{h}\langle \dot{\varphi}_k, \phi_1 \rangle & \cdots & \frac{1}{h}\langle \dot{\varphi}_k, \phi_M \rangle \\ \vdots & & \vdots \\ \frac{1}{h^q}\langle \varphi_k^{(q)}, \phi_1 \rangle & \cdots & \frac{1}{h^q}\langle \varphi_k^{(q)}, \phi_M \rangle \end{bmatrix}}_{=:\mathcal{A}} \hat{\Lambda}_t \,. \quad (31)$$

This result for matrix $\mathcal{A}$ in (31) considers one fault, but can easily be extended to a multiple fault case by treating each component individually, then matrix $\mathcal{A}^*$ is obtained. The coefficient vector $\hat{\Lambda}_t^* = (\hat{\Lambda}_{1,t}, \cdots, \hat{\Lambda}_{n_f,t})^\top$ follows from inverting

$$r = W^\top T_f \mathcal{A}^* \hat{\Lambda}_t^* \,, \quad (32)$$

which parameterizes the estimated fault signal $\hat{f}$.

4.2 Nonlinear Case

Furthermore note that some nonlinear systems can be handled naturally by this approach. For demonstrating the idea, assume

$$\begin{cases} \dot{x}(t) = Ax(t) + \hat{A}g(x(t)) + Bu(t) + E_x d(t) + F_x f(t) \\ y(t) = Cx(t) + Du(t) + E_y d(t) + F_y f(t) \end{cases}$$

and apply the same procedure to successive $q \in \mathbb{N}$ derivatives. Then a result similar to (21) is established:

$$\begin{aligned} \bar{L}[y] = OL^0[x] + T_{g(x)}\bar{L}(g(x)) + T_u\bar{L}[u] \\ + T_d\bar{L}[d] + T_f\bar{L}[f] \end{aligned} \quad (33)$$

where

$$T_{g(x)} = \begin{bmatrix} 0 & 0 & \dots & 0 \\ C\hat{A} & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{q-1}\hat{A} & CA^{q-2}\hat{A} & \dots & 0 \end{bmatrix} \quad (34)$$

Equation (33) shows that nonlinear terms can be treated independently from linear terms due to the modulating operator which avoids partial state-derivatives. Rearranging yields

$$\bar{L}[y] = \bar{O}L^0[x] + T_u\bar{L}[u] + T_d\bar{L}(\bar{d}) + T_f\bar{L}[f] \quad (35)$$

with $\bar{O} = [O \quad T_{g(x)}]$ and $\bar{x} = [x^\top \quad g(x)^\top]^\top$. Thus, (19) may be used to obtain the respective annihilator W. In cases where such condition is too restrictive, one may resort to (25), (26) and (27). Residuals are obtained using (28). For further fault identification and estimation steps, (32) is employed.

4.3 Sensibility and threshold selection

The system sensibility to faults is governed by relation (29), thus it partly depends in the annihilator matrix W which is a design parameter and the matrix T_f that is derived from the model. The third factor are the modulation functions applied to the unknown signals f. In this article we aim at detection of faults with magnitude larger than 1. Thus, thresholds J can be calculated from

$$J = W^\top T_f \bar{L}[H(t_0)], \quad (36)$$

where $H(t_0)$ is the unit step with respect to time t_0, i.e.

$$H(\tau, t_0) = \begin{cases} 0, \text{ for } \tau < t_0 \\ 1, \text{ for } \tau \geqslant t_0 \end{cases} \quad (37)$$

with t_0 selected close to t. We recommended to choose $t_0 \in \left(t - \frac{T}{k}, t\right)$. For the corresponding modulation we get

$$L^i[H(t, t_0)] = \int_{t-t_0}^{t} (-1)^i \varphi^{(i)}(\tau - t + T) d\tau. \quad (38)$$

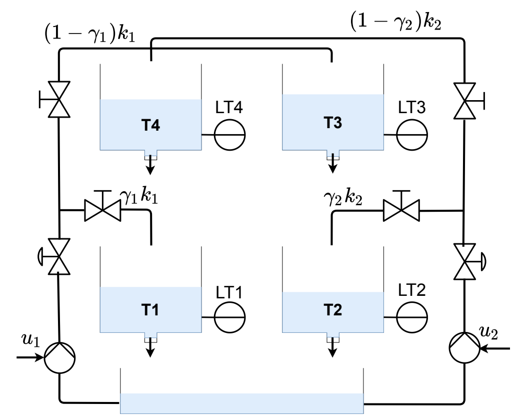

Fig. 1. Sketch of four-tank system.

Then, for each residual r_i in (29) the threshold is drawn from

$$J_i = \mathrm{col}_i \left(W^\top T_f\right)^\top \bar{L}[H(t, t_0)]. \quad (39)$$

Moreover, for better optimization results using (25), (26) and (27), for the corresponding r_i an approximate threshold is

$$J_i \approx L^i[H(t, t_0)]. \quad (40)$$

5. APPLICATION EXAMPLE

The approach is applied to a non-linear model of a four-tank system (Fig. 1). The four-tank system is a MIMO benchmark model to evaluate FDI algorithms, first proposed by Johansson (2000). It provides opportunities to test sensor and actuator faults simultaneously and to implement fault-tolerant solutions. It has been used for structural fault diagnosis for faults in the order of 10 cm in magnitude (Sánchez-Zurita et al., 2019). In this article, we analyze faults in the order of 1 cm of magnitude. The four-tank system is described by

$$\dot{h}_1(t) = \frac{-a_1\sqrt{2gh_1(t)}}{A_1} + \frac{a_4\sqrt{2gh_4(t)}}{A_1} + \frac{\gamma_1 k_1 u_1}{A_1}$$
$$\dot{h}_2(t) = \frac{-a_2\sqrt{2gh_2(t)}}{A_2} + \frac{a_3\sqrt{2gh_3(t)}}{A_2} + \frac{\gamma_2 k_2 u_2(t)}{A_2}$$
$$\dot{h}_3(t) = \frac{-a_3\sqrt{2gh_3(t)}}{A_3} + \frac{(1-\gamma_1)k_1 u_1(t)}{A_3}$$
$$\dot{h}_4(t) = \frac{-a_4\sqrt{2gh_4(t)}}{A_4} + \frac{(1-\gamma_2)k_2 u_2(t)}{A_4}$$
$$y(t) = h(t) + \nu(t)\,, \quad x(t) = h(t)$$

with tank levels h_i for $i = 1, 2, 3, 4$. The measurement y is corrupted by noise ν. Parameters and operating points for the plant in our laboratory are given in Table 1.

Table 1. Process Parameters and Operating Points.

Parameter	Value	Units
Bottom area, A_i, for $i = 1, 2, 3, 4$	706.85	cm^2
Outlet pipe cross section, a_i, for $i = 1, 2$	5.39	cm
Outlet pipe cross section, a_i, for $i = 3, 4$	5.39	cm
Gravity constant, g	981	cm^2/s
Pump constants, k_i, for $i = 1, 2$	1	$cm^3/(Vs)$
Main valve positions, γ_i for $i = 1, 2$	0.7	
Heights 1 and 2, h_{1o}, h_{2o}	18.3	cm
Heights 3 and 4, h_{3o}, h_{4o}	25	cm
Inputs 1 and 2, u_{1o}, u_{2o}	1193.73	cm^3/s

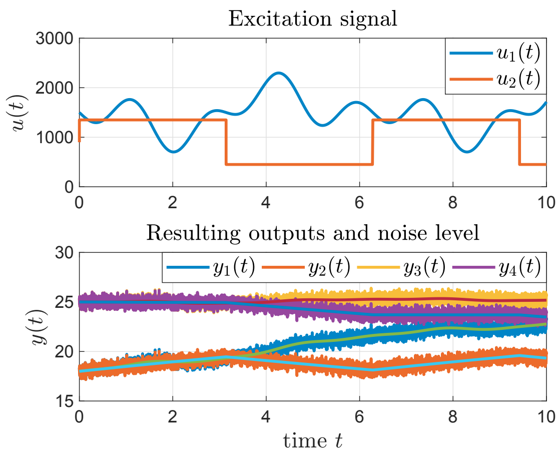

Fig. 2. Excitation signal and measurements for calibration.

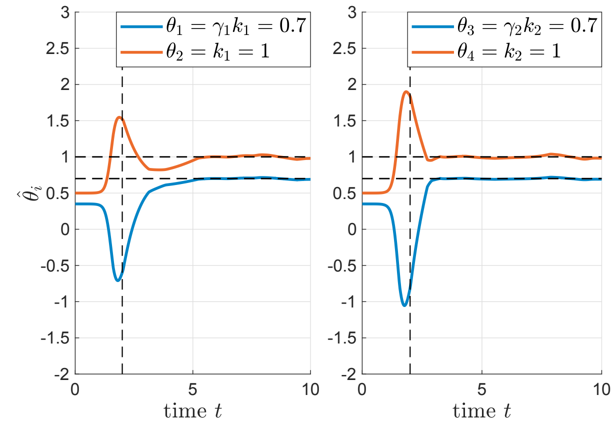

Fig. 3. Parameter identification based on the data from Fig. 2 with the marked moving horizon length $T = 2$.

5.1 Calibration step

A challenging task is to continuously identify the pump constants k_1 and k_2, and the main valve positions γ_1 and γ_2. To this end, we use the parameter identification proposed in (Jouffroy and Reger, 2015) with a total modulation function from Definition 1. We obtain

$$
\begin{aligned}
L^1[y_1] &= \hat{a}_1 L^0[\sqrt{y_1}] + \hat{a}_2 L^0[\sqrt{y_4}] + \hat{b}_1 \lambda_1 k_1 L^0[u_1] \\
L^1[y_2] &= \hat{a}_3 L^0[\sqrt{y_2}] + \hat{a}_4 L^0[\sqrt{y_3}] + \hat{b}_2 \lambda_2 k_2 L^0[u_2] \\
L^1[y_3] &= \hat{a}_5 L^0[\sqrt{y_3}] + \hat{b}_3 (1-\lambda_1) k_1 L^0[u_1] \\
L^1[y_4] &= \hat{a}_6 L^0[\sqrt{y_4}] + \hat{b}_4 (1-\lambda_2) k_2 L^0[u_2].
\end{aligned} \tag{41}
$$

Regrouping terms to estimate the pair k_1 and γ_1 we have the regressor form

$$
\begin{bmatrix} L^1[y_1] - \hat{a}_1 L^0[\sqrt{y_1}] - \hat{a}_2 L^0[\sqrt{y_4}] \\ L^1[y_3] - \hat{a}_5 L^0[\sqrt{y_3}] \end{bmatrix} = \begin{bmatrix} \hat{b}_1 L^0[u_1] & 0 \\ -\hat{b}_3 L^0[u_1] & \hat{b}_3 L^0[u_1] \end{bmatrix} \begin{bmatrix} k_1 \gamma_1 \\ k_1 \end{bmatrix}. \tag{42}
$$

Similarly for the pair k_2 and γ_2 we get

$$
\begin{bmatrix} L^1[y_2] - \hat{a}_3 L^0[\sqrt{y_2}] - \hat{a}_4 L^0[\sqrt{y_3}] \\ L^1[y_4] - \hat{a}_6 L^0[\sqrt{y_4}] \end{bmatrix} = \begin{bmatrix} \hat{b}_2 L^0[u_2] & 0 \\ -\hat{b}_4 L^0[u_2] & \hat{b}_4 L^0[u_2] \end{bmatrix} \begin{bmatrix} k_2 \gamma_2 \\ k_2 \end{bmatrix}. \tag{43}
$$

Excitation signals and measurements subject to noise are depicted in Fig. 2, corresponding results are shown in Fig. 3.

5.2 Fault diagnosis

The system is affected by additive faults, i.e. one process fault $f_1(t)$ in the first state and two sensor faults $f_2(t)$ and $f_3(t)$ in measurements 2 and 3, respectively. In Fig. 4 the testing conditions of the simulation are shown, including the input signals, and the noisy / perturbed measurement with standard deviation of 0.1 in the process. The loop is closed with a linear state-feedback with gain $K = [1100, 1000, 0, 0; 1000, 1000, 0, 0]$. Note that fault f_1 is active for $t \in [3.5, 5]$ s, fault f_2 for $t \in [10, 12]$ s, and fault f_3 for $t \in [13.5, 15]$ s.

Generating the residuals until order $q = 2$, with $A = 0$ we get

$$
O = \begin{bmatrix} C^\top & 0 & 0 \end{bmatrix}^\top. \tag{44}
$$

Considering $C = I_4$ and $E_x = [1, 0, 1, 0]^\top$, the matrices related to the perturbations and nonlinearities are

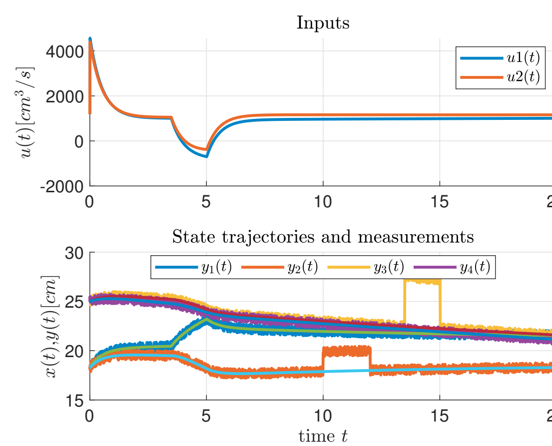

Fig. 4. Simulation of the fault-scenario with inputs and measurement outputs, affected by faults illustrated in Fig. 6.

$$
T_d = \begin{bmatrix} 0 & 0 & 0 \\ CE_x & 0 & 0 \\ CAE_x & CE_x & 0 \end{bmatrix}, T_{\sqrt{x}} = \begin{bmatrix} 0 & 0 & 0 \\ C\hat{A} & 0 & 0 \\ CA\hat{A} & C\hat{A} & 0 \end{bmatrix}. \tag{45}
$$

In this case, using (25), (26), (27) and (28), nine residuals are obtained. These residuals correspond to different orders of the applied modulating functions, as presented in Fig. 5 with the corresponding detection threshold. For the first and second-order of modulating functions, three residuals are obtained, respectively. Residuals of zero-order are less effective for this specific case and are not presented. First-order residuals are adequate for process faults, whereas, for sensor-fault detection, it is more sensible to use second-order residuals.

For fault identification, we use a set of Jacobi Polynomials with $\alpha = 3$, $\beta = 3$ and approximation order $N = 2$ as introduced in Section 2.2. The modulating function has a moving horizon length of $T = 1$ s and shift $\tau_0 = 0.5$ s from (10). Fault isolation is an inherent part of fault identification but is used in conjunction with fault detection; thus, estimation is only performed when a fault is detected. The fault estimation results obtained from (32) are demonstrated in Fig. 6. It can be observed that the faults are tracked relatively smoothly under noisy measurement conditions for this nonlinear plant. A time delay can be seen as expected from Section 2.

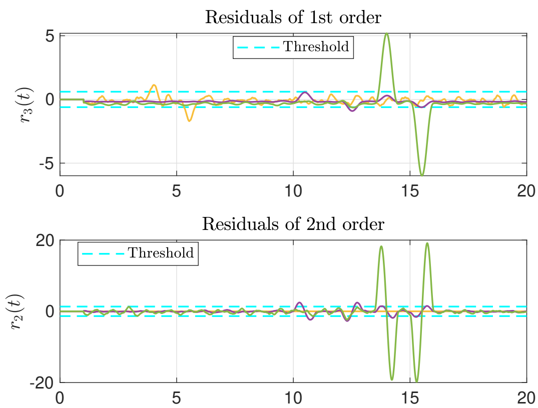

Fig. 5. Residual components generated from (28) with respective detection threshold caused by faults shown in Fig. 6.

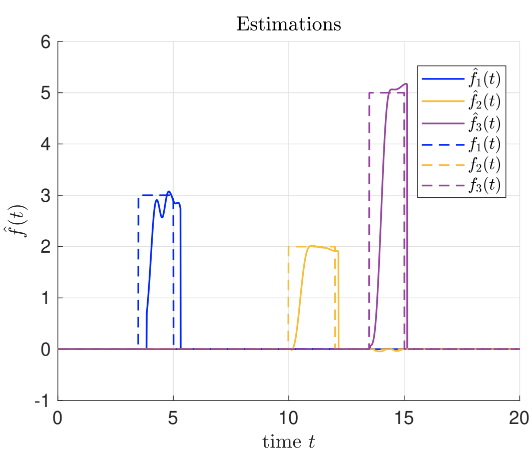

Fig. 6. Real and isolated faults, estimated with (32).

6. CONCLUSION

We present a fault diagnosis scheme based on the modulation of a series of output derivatives, yielding a set of algebraic equations only. For this purpose, the parity space method is rearranged accordingly. The residual generation is improved, and especially the sensitivity to signal noise is reduced. A robust implementation procedure is sketched that is efficient to compute and easy to tune by mainly adjusting the moving horizon length. The continuous-time perspective allows for a filtering setup, also for the nonlinear case. Singularities in the state are avoided in the output relation due to early shifting the time differentiation operation. Thus, the modified parity space method is adjusted in a way that renders it feasible to reconstruct time-varying fault trajectories directly.

The extension towards nonlinear systems will be pushed forward to a broader class of problems. Also, state estimation can be included in the framework for constructing additional residual and parity constellations. Different modulating functions may also be utilized for distinct output signal projections that result in a larger variety of residual information to be fused.

ACKNOWLEDGMENT

This project has received funding from the European Union Horizon 2020 research and innovation program under Marie Skłodowska-Curie grant agreement No. 824046.

REFERENCES

Blanke, M., Kinnaert, M., Lunze, J., Staroswiecki, M., and Schröder, J. (2006). *Diagnosis and Fault-Tolerant Control*. Springer.

Fliess, M., Join, C., and Sira-Ramírez, H. (2004). Robust residual generation for linear fault diagnosis: An algebraic setting with examples. *International Journal of Control*, 77(9), 1223–1242.

Gao, Z. (2015). Fault estimation and fault-tolerant control for discrete-time dynamic systems. *IEEE Transactions on Industrial Electronics*, 62(6), 3874–3884.

Höfling, T. and Pfeufer, T. (1994). Detection of additive and multiplicative faults - parity space vs. parameter estimation. In *IFAC Symposium on Fault Detection, Supervision and Safety for Technical Processes*, 515–520.

Jiang, B., Staroswiecki, M., and Cocquempot, V. (2004). Fault estimation in nonlinear uncertain systems using robust/sliding-mode observers. *Control Theory and Applications*, 151(1), 29–37.

Johansson, K.H. (2000). The quadruple-tank process: a multivariable laboratory process with an adjustable zero. *IEEE Transactions on Control Systems Technology*, 8(3), 456–465.

Jouffroy, J. and Reger, J. (2015). Finite-time simultaneous parameter and state estimation using modulating functions. In *IEEE Conference on Control Applications*, 394–399.

Li, P., Boem, F., Pin, G., and Parisini, T. (2018). Fast-convergent fault detection and isolation in an uncertain scenario. In *IEEE Conference on Decision and Control*, 5544–5549.

Liu, D.Y., Laleg-Kirati, T.M., Perruquetti, W., and Gibaru, O. (2014). Non-asymptotic state estimation for a class of linear time-varying systems with unknown inputs. In *IFAC World Congress*, 3732–3738.

Maiying Zhong, Yunxia Liu, and Zeyun Huo (2008). H_∞ fault estimation and accommodation for LTI systems. In *World Congress on Intelligent Control and Automation*, 1806–1809.

Nguang, S.K., Zhang, P., and Ding, S. (2011). Parity relation based fault estimation for nonlinear systems: An LMI approach. In *IFAC Symposium on Fault Detection, Supervision and Safety of Technical Processes*, 366–371.

Noack, M., Botha, T., Hamersma, H.A., Ivanov, V., Reger, J., and Els, S. (2018). Road profile estimation with modulation function based sensor fusion and series expansion for input reconstruction. In *IEEE International Workshop on Advanced Motion Control*, 547–552.

Sánchez-Zurita, V., Pérez-Zuñiga, G., and Sotomayor-Moriano, J. (2019). Reconfigurable Model Predictive Control applied to the Quadruple Tank Process. In *European Workshop on Advanced Control and Diagnosis*, 2067–2072.

Shinbrot, M. (1957). On the analysis of linear and nonlinear systems. *Trans. ASME*, 79(3), 547–552.

Sun, B., Wang, J., He, Z., Qin, Y., Wang, D., and Zhou, H. (2019). Fault detection for closed-loop control systems based on parity space transform. *IEEE Access*, 7, 75153–75165.

Xue, T., Zhong, M., Ding, S.X., and Ye, H. (2018). Stationary wavelet transform aided design of parity space vectors for fault detection in LDTV systems. *Control Theory and Applications*, 12, 857–864.

Zhaohui, C. and Noura, H. (2013). A composite fault tolerant control based on fault estimation for quadrotor uavs. In *IEEE Conference on Industrial Electronics and Applications*, 236–241.