

Soft Sensor Design for Restricted Variable Sampling Time ^{*}

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Abstract: Difficult-to-obtain variables in industrial applications have led to the rise of soft sensors, which use prior system information and measurements to estimate these difficult-to-obtain variables. In real systems, the measurements that need to be estimated by a soft sensor are often infrequently measured or delayed. Sometimes, these delays and sampling time are variable in time. Though there are papers considering soft sensors in the presence of time delays and different sampling times, the variation of those parameters has not been considered when evaluating the adequacy of the soft sensors. Therefore, this paper will evaluate the impact of such variations for a data-driven soft sensor and propose modifications of the soft sensor that increase its robustness. The reliability of its estimate will be shown using the Bauer-Premaratne-Durán Theorem. Furthermore, the soft sensor will be simulated applying it to a continuous stirred tank reactor. Simulation showed that the modified soft sensor gives good estimates, whereas the traditional soft sensor gives an unstable estimate.

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Keywords: Soft Sensors, Variable Sampling, Variable Time Delay

1. INTRODUCTION

Control and monitoring of a plant requires updated information, mostly obtained by measurements of system variables. In the presence of high sampling times or time delays, the measured information may not suffice for control purposes. Since it can be difficult and expensive to install better hard sensors, soft sensors can be used to provide estimates of variables without further expenditure on hard sensors. Soft sensors use the available easily measured information to provide estimates of the harder-to-measure system variables. As well, if accurate, slowly-sampled system measurements can be obtained, then they can be combined into a bias update term to improve the estimate provided by the system model.

The choice of the system model is essential for a soft sensor. Depending on what model is used, soft sensors are divided into data-driven soft sensors and model-driven soft sensors (Fortuna et al., 2007; Kadlec et al., 2009). Model-driven soft sensors use a first principle model of the system to estimate the desired system variables. The model is constructed based on the prior knowledge of the system. Data-driven methods analyse the behaviour of signals to provide estimates. Typical applications use principal component analysis (PCA) (Flynn et al., 2005),

partial least squares (PLS) (Flynn et al., 2005; Zamprogna et al., 2005; Sharmin et al., 2006), or artificial neural networks (ANN) (Shang et al., 2014) to provide estimates of system variables.

A soft sensor providing reliable estimates is said to provide good tracking. A soft sensor provides good tracking if and only if the error of its estimate is bounded for a bounded disturbance and the estimate converges to the actual system value (Shardt and Huang, 2012a).

Assuming that some, slowly sampled measurements of the desired system variables are available, adding a bias update term to the soft sensor may significantly improve the estimate. However, such a term may, especially in the presence of time delay and high sampling times, lead to an unreliable estimate. This issue was pointed out in Shardt and Huang (2012a,b). Thus, a method to tune the bias update term in a way that yields reliable estimates was proposed. The method was shown to lead to good estimates given exact, constant values of the time delay and sampling time. In real application, the time delay or sampling time may be variable. Variation of time delay was only considered in Yang et al. (2019). However, variable sampling time was not considered in previous publications.

Therefore, the first objective of the paper is to develop a soft sensor system that can handle variable time delays and sampling times. Then, its properties for the open-loop case are analysed mathematically. The second objective is to test the performance of the newly developed soft

^{*} The authors would like to thank the National Natural Science Foundation of China under grant #61673053, the Fundamental Research Funds for the Central Universities under Grant #FRF-BD-19-002A, and the National Key Research and Development Programme of China under grant #2017YFB0306403 for funding.

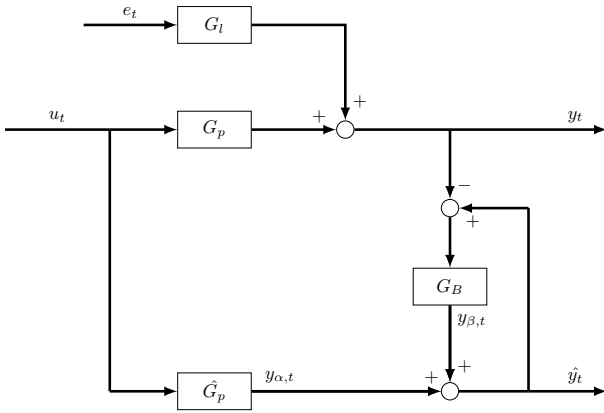


Fig. 1. Open-loop soft sensor with bias update term sensor in simulation using a continuous stirred tank reactor (CSTR).

2. BACKGROUND THEORY

Figure 1 shows the soft sensor system considered in this paper. The system is given by a transfer function G_p . The system input u_t can be assumed to be known. The unknown disturbance is given by the error transfer function G_l and a disturbance e_t , being white noise with mean μ and variance σ^2 . The variable to be estimated is y_t . A system model \hat{G}_p is used and simulated with the same input u_t . Ideally, $\hat{G}_p = G_p$ holds; however in real systems, this rarely occurs. Thus, the error term $y_t \neq y_{\alpha,t}$. Therefore, the bias update term $y_{\beta,t}$ is used to correct any estimation errors. It uses measurements of the variable y_t and compares them with the current estimate \hat{y}_t . This estimation error is used as the input to the bias update transfer function G_B . Note that infrequent or delayed measurement of y_t will be considered in the bias update transfer function G_B .

Some mathematical relationships follow from Figure 1:

$$\begin{aligned} y_t &= G_p u_t + G_l e_t \\ y_{\alpha,t} &= \hat{G}_p u_t \\ y_{\beta,t} &= G_B (\hat{y}_t - y_t) \\ \hat{y}_t &= y_{\alpha,t} + y_{\beta,t} \end{aligned} \quad (1)$$

Using basic algebraic operations, it follows that

$$\hat{y}_t = \frac{\hat{G}_p - G_B G_p}{1 - G_B} u_t + \frac{-G_B G_l}{1 - G_B} e_t. \quad (2)$$

2.1 Form of the bias update term

In simulation, the bias update transfer function is used to implement two aspects of the measurements y_t .

- (1) **Sampling Time:** If the measurements of y_t can only be taken infrequently, this can be interpreted as sampling y_t with a sampling time of N time steps.
- (2) **Time Delays:** Time delays in the measurement can be implemented as a time delay in the bias update term.

For an adequate comparison between y_t and \hat{y}_t , the restrictions on the availability of y_t are imposed on \hat{y}_t , as

well. Therefore, the time delay and sampling on y_t apply to $\hat{y}_t - y_t$ in the same way. Therefore, G_B can be decomposed as shown in Figure 2.

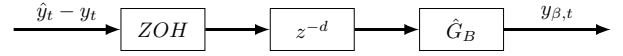


Fig. 2. G_B (after (Shardt and Huang, 2012a))

Here, the time delay is a simple backshift operator z^{-d} and sampling with a sampling time N is a zero-order hold ZOH.

2.2 Mathematical analysis of performance

The most important property of a soft sensor is its tracking capability. A soft sensor provides good tracking if and only if the error of its estimate is bounded for a bounded disturbance and the estimate converges to the actual system value.

Convergence will be determined by analysing the behaviour of the soft sensor as time approaches infinity. The error will be shown to be bounded by analysing the stability of the transfer functions of the soft sensor. Assuming the stability of \hat{G}_p , the output \hat{y} is bounded for a bounded input. Thus, the estimate is bounded, if the output of the bias update term G_B is bounded.

The bias update term G_B consists of the time delay z^{-d} , the sampling modelled by a zero order hold, and the bias update transfer function \hat{G}_B . Since the time delay and sampling time cannot be chosen in the construction of the soft sensor, the relevant choice is \hat{G}_B . The performance of the soft sensor in case of a constant bias update term was discussed in Shardt and Huang (2012a). The following structure of \hat{G}_B was proposed,

$$\hat{G}_B = \frac{\sum_{i=0}^{n_a} \alpha_i z^{-i}}{\sum_{i=0}^{n_b} \beta_i z^{-i}} \quad (3)$$

This transfer function must be chosen in a way that the estimate \hat{y}_t is stable and approaches y_t .

The behaviour approaching steady state is described by evaluating Equation (2).

Lemma 1. (Shardt and Huang, 2012a). The limit value of the estimate of the soft sensor \hat{y}_t is equal to the limit value of the system variable y_t if the following two conditions hold:

$$\begin{aligned} \lim_{z^{-1} \rightarrow 1} \frac{\hat{G}_B z^{-d}}{1 - \hat{G}_B z^{-N}} &= -1 \\ \lim_{z^{-1} \rightarrow 1} \frac{1}{1 - \hat{G}_B z^{-N}} &= 0 \end{aligned} \quad (4)$$

The following structure for the bias update term was proposed for a delay d in measurements, but no infrequent sampling

$$\hat{G}_B = -\frac{1}{1 - z^{-d}} \quad (5)$$

In the case of infrequent sampling with sampling time N , and a delay $d < N$, the soft sensor is chosen as

$$\hat{G}_B = -\frac{1}{1 - z^{-N}} \quad (6)$$

The soft sensors were shown to provide good tracking.

Proof. see Shardt and Huang (2012a) \square

2.3 Properties under assumption of variable parameters

This paper will give a soft sensor providing good estimates in the presence of both time delay and infrequent measurements. It is assumed that the time delay is smaller than the sampling time at all times. The behaviour of the estimate \hat{y} is analysed using the mathematical relationships from Figure 1. Simple algebraic manipulations are omitted in the re-organisation of the formulae. First, using $\hat{y}_t = \hat{G}_p u_t + \hat{G}_B e_t$ and $e_t = \hat{y}_t - y_t$, we get

$$\hat{y}_t = \hat{G}_p u_t + G_B(\hat{y}_t - y_t) \quad (7)$$

Since G_B includes a time delay, the values $(\hat{y}_t - y_t)$ are delayed by d steps. Thus, Equation (7) becomes

$$\hat{y}_t = \hat{G}_p u_t + \hat{G}_B(\hat{y}_{t-d} - y_{t-d}). \quad (8)$$

Note that $\hat{y}_t = y_{\alpha,t} + y_{\beta,t}$ and the value $y_{\beta,t}$ is part of the feedback loop. Hence, it is sampled with a zero-order hold. Since $d < N$ at any point of time, sampling the output every N steps implies that $y_{\beta,t-N} = y_{\beta,t-N+1} = \dots = y_{\beta,t-1}$, therefore, $y_{\beta,t-d} = y_{\beta,t-N}$. Thus, Equation (8) becomes

$$y_{\alpha,t} + y_{\beta,t} = \hat{G}_p u_t + \hat{G}_B(y_{\alpha,t-d} + y_{\beta,t-N} - y_{t-d}). \quad (9)$$

Using $y_t = G_p u_t + G_l e_t$ and $u_{t-d} = u_t z^{-d}$ gives

$$y_{\beta,t} = \frac{\hat{G}_p - \hat{G}_B G_p z^{-d}}{1 - \hat{G}_B z^{-N}} u_t - \frac{\hat{G}_B G_l z^{-d}}{1 - \hat{G}_B z^{-N}} e_t + \frac{\hat{G}_B z^{-d} - 1}{1 - \hat{G}_B z^{-N}} y_{\alpha,t}. \quad (10)$$

Note that Equation (2), and thus, Lemma 1 still hold. Furthermore, Equation (10) can be used for the proof of stability.

Both stability and convergence will be proven for a general case, where both the sampling time and the time delay are variable in time. The time delay is an arbitrary $d \in [d_\alpha, d_\Omega]$ and the sampling time is an arbitrary $N \in [N_\alpha, N_\Omega]$ in each time instance. To enforce $d < N$, the smallest possible sampling time is chosen to be bigger than the largest possible time delay $d_\Omega < N_\alpha$. Lemma 1, equations (2), (10), and consecutive statements hold for given d and N . This includes a constant and variable time delay and sampling time, which can be described as $d_\alpha = d_\Omega \wedge N_\alpha = N_\Omega$.

The bias update transfer function proposed is

$$\hat{G}_B = -\frac{1}{N_\Omega - N_\alpha + 1 - \sum_{i=N_\alpha}^{N_\Omega} z^{-i}}, \quad (11)$$

where $N_\Omega - N_\alpha + 1$ is the number of possible sampling times.

The structure of the state-of-the-art soft sensors was given. For infrequently sampled systems with delayed measurements, the soft sensor provides good tracking (Shardt and Huang, 2012a). This was proven assuming a constant time delay and constant sampling. Since those parameters are possibly time-variant, a modification of said soft sensor was presented. This modification will

be shown to maintain the properties of the state-of-the-art method. Furthermore, it will be shown to provide good tracking for an system with an infrequent, variable sampling and a variable delay on the measurements.

3. THEORETICAL RESULTS FOR THE TRACKING OF THE SOFT SENSOR

To prove the tracking properties of the proposed soft sensor design, the limit value and stability of the estimate will be shown.

To show that the limit value of the estimate is the same as the limit value of the output, Lemma 1 is used. Lemma 2 shows that Lemma 1 applies.

Lemma 2. For the bias update term G_B taken from (11) two equations hold:

$$\lim_{z^{-1} \rightarrow 1} \frac{\hat{G}_B z^{-d}}{1 - \hat{G}_B z^{-N}} = -1 \quad (12)$$

$$\lim_{z^{-1} \rightarrow 1} \frac{1}{1 - \hat{G}_B z^{-N}} = 0 \quad (13)$$

Proof.

To prove this, the bias update transfer function (11) is inserted into Equation (12). Then, the equation is simplified, leading to the desired result.

$$\begin{aligned} \lim_{z^{-1} \rightarrow 1} \left(\frac{-\frac{z^{-d}}{N_\Omega - N_\alpha + 1 - \sum_{i=N_\alpha}^{N_\Omega} z^{-i}}}{1 + \frac{z^{-N}}{N_\Omega - N_\alpha + 1 - \sum_{i=N_\alpha}^{N_\Omega} z^{-i}}} \right) \\ = \frac{-1}{N_\Omega - N_\alpha + 1 - N_\Omega + N_\alpha - 1 + 1} = -1 \quad (14) \end{aligned}$$

The same simplification is done to show Equation (13).

$$\begin{aligned} \lim_{z^{-1} \rightarrow 1} \left(\frac{1}{1 + \frac{z^{-N}}{N_\Omega - N_\alpha + 1 - \sum_{i=N_\alpha}^{N_\Omega} z^{-i}}} \right) \\ = \frac{0}{N_\Omega - N_\alpha + 1 - N_\Omega - N_\alpha + 1 + 1} = 0 \quad \square \quad (15) \end{aligned}$$

To show the stability of the soft sensor, the Bauer-Premaratne-Duran Theorem will be used (Lin and Antsaklis, 2009; Bauer et al., 1993).

Theorem 3. (Bauer-Premaratne-Durán Theorem).

A switched linear system $x_{k+1} = A_{\sigma(k)} x_k$, where $A_{\sigma(k)} \in \{A_1, A_2, \dots, A_M\}$, is asymptotically stable under arbitrary switching if and only if there exists a finite integer n such that

$$\|A_{i_1} A_{i_2} \dots A_{i_n}\|_\infty < 1 \quad (16)$$

for all n -tuple $A_{i_j} \in \{A_1, A_2, \dots, A_M\}$, where $j = 1, \dots, n$.

The Bauer-Premaratne-Durán Theorem uses the maximum norm (Bauer et al., 1993). The maximum norm of a matrix $A = (a_{ij})$ is defined as $\|A\|_\infty = \max_i(\max_j(a_{ij}))$.

To show that the Bauer-Premaratne-Durán Theorem applies to the system, a state-space realisation of either the transfer function $\frac{\hat{G}_B z^{-d}}{1 - \hat{G}_B z^{-N}}$ or $\frac{1}{1 - \hat{G}_B z^{-N}}$ must be found, since they can be described by the same state-space matrix. Therefore, it is sufficient to analyse $\frac{\hat{G}_B z^{-d}}{1 - \hat{G}_B z^{-N}}$. The sampling time at the current sampling step is N_0 .

To build a state-space representation of the bias update transfer function, the following equation is defined:

$$\frac{\hat{G}_B z^{-d}}{1 - \hat{G}_B z^{-N_0}} = - \frac{z^{-d}}{N_\Omega - N_\alpha + 1 - \sum_{i=N_\alpha}^{N_\Omega} z^{-i} + z^{-N_0}} = \frac{y_B}{u_B} \quad (17)$$

which can be re-arranged to give

$$y_B = \sum_{i=N_\alpha}^{N_0-1} \frac{z^{-i}}{N_\Omega - N_\alpha + 1} y_B + \sum_{i=N_0}^{N_\Omega} \frac{z^{-i}}{N_\Omega - N_\alpha + 1} y_B - z^{-d} u_B \quad (18)$$

The bias update term is constructed using information from the discrete sampling times. For any choice of N_Ω and N_α , $N_\alpha < N_\Omega \leq 2N_\alpha$ holds. If the current sampling $N_0 < N_\omega$, the current and the last measurement are used for the estimate. Equation (18) is re-organised to take this into account.

$$y_B = \sum_{i=N_\alpha}^{N_0-1} \frac{z^{-i}}{|S|} y_B + \sum_{i=N_0}^{N_\Omega} \frac{z^{-i}}{N_\Omega - N_\alpha + 1} y_B - z^{-d} u_B$$

$$= (N_0 - N_\alpha) \frac{z^{-N_0}}{N_\Omega - N_\alpha + 1} y_B + (N_\Omega - N_0) \frac{z^{-N_0-N_1}}{N_\Omega - N_\alpha + 1} y_B - z^{-d} u_B. \quad (19)$$

Given the state $\mathbf{x}^T = (y_B y_B z^{-1} \dots y_B z^{-(2N_\Omega-1)})$, a state-space realisation is

$$\mathbf{x}_{m+1} = A \mathbf{x}_m + B u_m, \quad (20)$$

where A is the state-space matrix and $B = -(0 \dots 0 \ 1 \ 0 \dots 0)^T$ the input matrix, with 1 in the d^{th} row. The matrices C and D are irrelevant for stability. $A \in \mathbb{R}^{(2N_\Omega-1) \times (2N_\Omega-1)}$ is constructed as

$$A = \Lambda_i + M_0 = \begin{pmatrix} 0_{N_0-1} & \frac{N_0 - N_\alpha}{N_\Omega - N_\alpha + 1} & 0_{N_\Omega-1} & \frac{N_\Omega - N_0}{N_\Omega - N_\alpha + 1} & 0 & \dots & 0 \\ 0 & & \dots & & 0 & & 0 \\ \vdots & & \ddots & & \vdots & & \vdots \\ 0 & & \dots & & 0 & & 0 \end{pmatrix} + \begin{pmatrix} 0 & \dots & 0 \\ \vdots & & \vdots \\ I & & 0 \end{pmatrix}, \quad (21)$$

where I is the identity matrix and 0_k a row vector of length k with all entries equal zero. Note that Λ_i only has entries in the first row which are all positive and add up to less than one. M_0 is the subdiagonal shifting matrix. Lemma 4 gives that the left multiplication of Λ_i with another matrix gives a matrix with a smaller largest entry.

Lemma 4. Given Λ_i as above. For any matrix $X \in \mathbb{R}^{(2N_\Omega-1) \times (2N_\Omega-1)}$ the following inequality holds

$$\|\Lambda_i X\|_\infty < \|X\|_\infty. \quad (22)$$

Proof.

The Lemma is proven by converting the general form of $\|\Lambda_i X\|_\infty$ to a form that is smaller than $\|X\|_\infty$.

First, the matrices Λ_i and X are multiplied. The entries of X are $x_{i,j}$, where i is the number of the row of the entry and j the number of the column.

$$\Lambda_i X = \begin{pmatrix} \dots & \frac{N_0 - N_\alpha}{N_\Omega - N_\alpha + 1} x_{N_0,j} + \frac{N_\Omega - N_0}{N_\Omega - N_\alpha + 1} x_{(N_1+N_2),j} & \dots \\ 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{pmatrix} \quad (23)$$

Now, the norm of this matrix is calculated. It is the maximum of the absolute value of all entries. Since all rows except the first are zero,

$$\|\Lambda_i X\|_\infty = \max_j \left(\left| \frac{N_0 - N_\alpha}{N_\Omega - N_\alpha + 1} x_{N_0,j} + \frac{(N_\Omega - N_0)}{N_\Omega - N_\alpha + 1} x_{(N_0+N_1),j} \right| \right). \quad (24)$$

By splitting the equation, the right hand side is greater than or equal to $\|\Lambda_i X\|_\infty$. Then, all constant values are put outside the maximum.

$$\|\Lambda_i X\|_\infty \leq \frac{N_0 - N_\alpha}{N_\Omega - N_\alpha + 1} \max_j (|x_{N_0,j}|) + \frac{N_\Omega - N_0}{N_\Omega - N_\alpha + 1} \max_j (|x_{(N_1+N_2),j}|) \quad (25)$$

Since all entries in X are smaller than its maximum norm, the right hand side of the inequality can be replaced by a term greater than or equal to the previous term, that is

$$\|\Lambda_i X\|_\infty \leq \left(\frac{N_0 - N_\alpha}{N_\Omega - N_\alpha + 1} + \frac{N_\Omega - N_0}{N_\Omega - N_\alpha + 1} \right) \|X\|_\infty. \quad (26)$$

The sum of the constant is calculated to give

$$\|\Lambda_i X\|_\infty \leq \frac{N_\Omega - N_\alpha}{N_\Omega - N_\alpha + 1} \|X\|_\infty < \|X\|_\infty \quad \square \quad (27)$$

Using Lemma 4, Theorem 5 shows the stability of the estimate.

Theorem 5. The switched system with sampling time

$$N \in \mathbb{S}, d_\alpha \leq d_\Omega < N_\alpha \leq N_\Omega < 2N_\alpha$$

$$\hat{G}_B = - \frac{1}{N_\Omega - N_\alpha + 1 - \sum_{i=N_\alpha}^{N_\Omega} z^{-i}} \quad (28)$$

is asymptotically stable for arbitrary switching N .

Proof.

Asymptotic stability of the system is shown using Theorem 3. To apply it, it is shown using Lemma 4, that the considered system has state matrices that have the property $\|\prod_{i=1}^{2N_\Omega-1} A_{ij}\|_\infty < 1$ for any choice of A_{ij} .

Given $A_{ij}, j = 1, \dots, n$ being n matrices in the form described by Equation (21). Choose $n = 2N_\Omega - 1$. It is shown by induction that the first k lines of the matrix product $\prod_{i=1}^k A_{ij}$ only consist of entries less than one and all other values are less or equal to one.

$k = 1$: Holds per definition of A_{ij} .

$k > 1$:

$$\prod_{i=1}^k A_{ik} = A_{i1} \prod_{i=2}^k A_{ik}$$

Since the matrices are arbitrary, new indices can be chosen.

$$\begin{aligned} A_{i0} \prod_{i=1}^{k-1} A_{ik} &= (M_0 + \Lambda_0) \prod_{i=1}^{k-1} A_{ik} \\ &= M_0 \prod_{i=1}^{k-1} A_{ik} + \Lambda_0 \prod_{i=1}^{k-1} A_{ik}. \end{aligned} \quad (29)$$

Any matrix multiplied with the shifting matrix from the left is shifted one row down. Thus, due to the induction hypothesis, $M_0 \prod_{i=1}^{k-1} A_{ik}$ has only values less than one in the first k rows and values less or equal to one in the other rows. Due to Lemma 4,

$$\|\Lambda_0 \prod_{i=1}^{k-1} A_{ik}\|_\infty < \|\prod_{i=1}^{k-1} A_{ik}\|_\infty \leq 1. \quad (30)$$

Note that Λ_0 only has entries in the first row and M_0 no entries in the first row. Hence, the induction hypothesis holds.

By choosing $k = n$, it is shown that $\|\prod_{i=1}^k A_{ik}\|_\infty < 1$. Thus, the Bauer-Premaratne-Durán Theorem can be used and the system is asymptotically stable for arbitrary switching. \square

This theorem is a generalisation of the results in (Shardt and Huang, 2012a). This is shown by Corollary 6.

Corollary 6. The system with constant sampling time and constant time delay

$$\begin{aligned} N \in \mathbb{N}, d \in \mathbb{N} \\ \hat{G}_B = -\frac{1}{1-z^{-N}} \end{aligned} \quad (31)$$

is asymptotically stable.

Proof. Choose $\mathbb{S} := N \implies N_\alpha = N_\Omega = d, d_\alpha = d_\Omega = d$. By applying Theorem 3, $\hat{G}_B = -\frac{1}{N_\Omega - N_\alpha + 1 - \sum_{i=N_\alpha}^{N_\Omega} z^{-i}}$ reduces to $\hat{G}_B = -\frac{1}{1-z^{-N}}$. Asymptotic stability for arbitrary switching implies asymptotic stability. \square

A soft sensor using a bias update transfer function as given by Equation (11) was analysed with regard to the convergence and stability of its estimates. The tracking properties were shown for a system with an infrequent, variable sampling and a variable delay on the measurements. It was shown that the soft sensor estimate converges to the correct estimate by calculating its limit value and by showing that the estimation error approaches zero asymptotically. These statements provide mathematical proof for the good tracking of the soft sensor. To confirm the quality of the tracking, the soft sensor is tested on a simulated CSTR example.

4. SIMULATION

We test the generalised soft sensor to verify the results in simulation and application on the continuous stirred tank reactor (CSTR) proposed in Morningred et al. (1990).

The CSTR is a nonlinear system, modelling an irreversible, exothermic reaction. There is a substrate A reacting to a single product B . The following differential equations model the reactor:

$$\begin{aligned} \dot{C}_A &= \frac{\dot{V}}{V} (C_{A0} - C_A) - k_0 C_A \exp\left(-\frac{E}{RT}\right) \\ \dot{T} &= \frac{\dot{V}}{V} (T_0 - T) - \frac{(-\Delta H)k_0 C_A}{\rho c_p} \exp\left(-\frac{E}{RT}\right) + \\ &\quad \frac{\rho_c c_{pc}}{\rho c_p V} \dot{V}_c \left(1 - \exp\left(-\frac{hA}{\dot{V}_c \rho_c c_{pc}}\right)\right) (T_{C0} - T), \end{aligned} \quad (32)$$

where the nominal values are given in Table 1.

Table 1. Nominal CSTR parameter values

product concentration	C_A	0.1 mol/l
reactor temperature	T	438.54 K
coolant flow rate	\dot{V}_c	103.41 l/min
process flow rate	\dot{V}	100 l/min
feed concentration	C_{A0}	1 mol/l
feed temperature	T_0	350 K
inlet coolant temperature	T_{C0}	350 K
CSTR volume	V	100 l
heat transfer term	hA	7×10^5 cal/min/K
reaction rate constant	k_0	$7.2 \times 10^{10} \text{ min}^{-1}$
activation energy term	E/R	1×10^4 K
heat of reaction	ΔH	-2×10^5 cal/mol
liquid densities	ρ, ρ_c	1×10^3 g/l
specific heats	c_p, c_{pc}	1 cal/g/K

The system is implemented in MATLAB/SIMULINK and simulated. The value estimated with the soft sensor is the substrate concentration C_A . The coolant flow rate \dot{V}_c is considered the input of the system and the inlet flow rate \dot{V} is considered a disturbance. The input is a step, the disturbance is coloured noise. The soft sensor proposed with the bias update transfer function from Equation (6) is used to estimate the concentration in the reactor. For this purpose, an exact model of the reactor is simulated, providing the nonlinear behaviour of C_A . The behaviour of this exact model will represent the real system. The simulation is done over 100 steps with each step representing one minute. The time delay in the measurement varies between 1 minute and 2 minutes and the sampling time between 3 minutes and 5 minutes.

The soft sensor uses two transfer functions. First, a linear model of the CSTR to calculate a prior estimate of the concentration C_A . The transfer function is the discretised model from Shardt and Huang (2012a)

$$\hat{G}_p = -\frac{0.112z^{-1}}{1-0.8724z^{-1}} \frac{0.001708z^{-1}}{1-0.3578z^{-1}}, \quad (33)$$

Second, the bias update transfer function is used to correct the estimate. It is $\hat{G}_B = -\frac{1}{1-z^{-4}}$, as described by Equation (6).

The behaviour of the concentration in the CSTR and its estimate provided by the soft sensor is shown in Figure 3.

The estimate oscillates with increasing magnitude. Hence, the soft sensor does not provide good tracking. Similar behaviour is observed for $\hat{G}_B = -\frac{1}{1-z^{-5}}$ and $\hat{G}_B =$

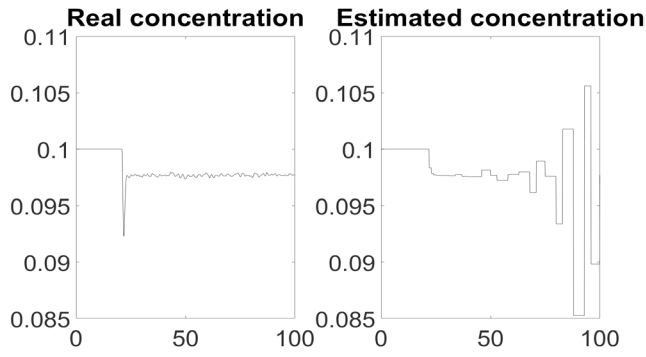


Fig. 3. (left) actual and (right) estimated concentration of the CSTR as a function of time using the bias update transfer function $\hat{G}_B = -\frac{1}{1-z^{-N}}$

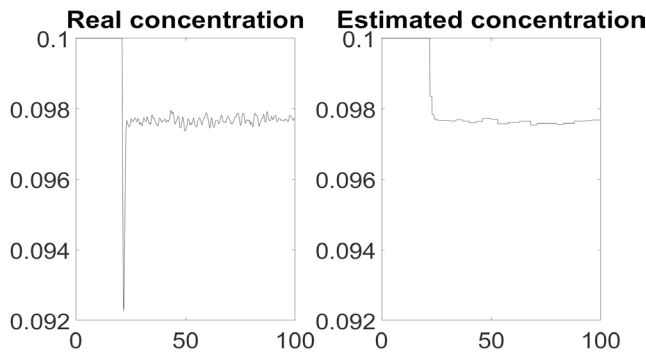


Fig. 4. (left) actual and (right) estimated concentration of the CSTR as a function of time using the bias update transfer function $\hat{G}_B = -\frac{1}{N_\Omega - N_\alpha + 1 - \sum_{i=N_\alpha}^{N_\Omega} z^{-i}}$

$-\frac{1}{1-z^{-3}}$. This instability is related to the poor estimate of the sampling time. The sampling time and thus the exact time of the measurement is unclear. Therefore, the soft sensor frequently compares the estimate of the soft sensor with the state of a different time. Using this erroneous comparison, the bias update term can cause instable estimates.

To provide a stable estimate of C_A , the soft sensor is implemented using $\hat{G}_B = -\frac{1}{3 - \sum_{i=3}^5 z^{-i}}$, as described by Equation (11).

Figure 4 shows the actual concentration C_A and the estimate of C_A using this soft sensor.

The soft sensor provides good tracking of the measured variable. The peak visible in the measurement is caused by the initialisation.

5. CONCLUSION

A soft sensor design was proposed to estimate variables in the presence of variable time delay d and variable sampling time N . It was shown that the proposed soft sensor converges to the correct limit and that it has asymptotically stable behaviour. Furthermore, its performance was shown in application and compared to a state-of-the-art soft sensors.

It was proven that a soft sensor using a bias update transfer function as $\hat{G}_B = -\frac{1}{N_\Omega - N_\alpha + 1 - \sum_{i=N_\alpha}^{N_\Omega} z^{-i}}$ provides good tracking in the presence of variable time delay and variable sampling time, as long as the delay does not exceed the sampling time, and as long as the minimal sampling time is at least half as large as the maximal sampling time. If the delay and the sampling time are constant, and if the delay is an integer multiple of the sampling time, Shardt and Huang (2012a) stated that $\hat{G}_B = -\frac{1}{1-z^{-d}}$ provides good tracking. However, the general case for $d > N$ is still an open problem. Furthermore, the case of arbitrarily switching sampling times can be considered in future work. Finally, the proposed soft sensor system should be tested in closed-loop applications.

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