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## ON SHORT CYCLES THROUGH PRESCRIBED VERTICES OF A POLYHEDRAL GRAPH

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### Abstract

Guaranteed upper bounds on the length of a shortest cycle through  $k \leq 5$  prescribed vertices of a polyhedral graph or plane triangulation are proved.

**Keywords:** polyhedral graph, triangulation, short cycle, prescribed vertices.

**2000 Mathematics Subject Classification:** 05C38.

## 1. Introduction and Results

G.A. Dirac [2] proved that for a given integer  $c \geq 2$  any  $k$  ( $1 \leq k \leq c$ ) prescribed vertices of a  $c$ -connected graph belong to a common cycle. However, the complete bipartite graph  $K_{c,c+1}$  shows that this is not true for  $c+1$  prescribed vertices. In [3] we investigated the length of short cycles through  $k$  prescribed vertices with  $1 \leq k \leq \min\{c, 3\}$  in a  $c$ -connected graph  $G$ . From A.K. Kelmans and M.V. Lomonosov [6] we know that any five vertices of a polyhedral graph (that is a planar and 3-connected graph) belong to a common cycle which is best possible.

For given integers  $k, l$  with  $1 \leq k \leq 5$ ,  $3 \leq l$  and  $k \leq l$  let  $n_k(l)$  denote the minimum number  $n$  such that there exists a polyhedral graph  $G$  of order  $n$  having a subset of  $k$  vertices with the property that the length of every cycle containing those  $k$  vertices is at least  $l$ . In [3] we proved

- (i)  $n_1(l) = 3l - 5$  for  $l \geq 3$ ,

- (ii)  $n_2(l) = \lfloor \frac{3l-1}{2} \rfloor$  for  $l \geq 3$ ,  
 (iii)  $n_3(l) = \lfloor \frac{3l-1}{2} \rfloor$  for  $l \geq 5$ ,

and the following results which will be proven here is a continuation of the investigation [3] of short cycles through prescribed vertices for a polyhedral graph.

**Theorem 1.**

$$n_4(l) = \begin{cases} l & \text{if } l \in \{4, 8\}, \\ l+1 & \text{if } l \in \{5, 6, 7, 9, 10\}, \\ l+2 & \text{if } l \in \{11, 12\}, \\ \lceil \frac{4l-5}{3} \rceil & \text{if } l \geq 13. \end{cases}$$

**Theorem 2.**

$$n_5(l) = \begin{cases} l & \text{if } l = 5 \text{ or } l \geq 8, \\ l+1 & \text{if } l = 6 \text{ or } 7. \end{cases}$$

For integers  $k, l$  with  $2 \leq k \leq 5$ ,  $3 \leq l$  and  $k \leq l$  denote by  $t_k(l)$  the minimum number  $n$  such that there exists a plane triangulation  $T$  of order  $n$  with certain  $k$  vertices such that the length of every cycle containing them is at least  $l$ . Then we have  $n_k(l) \leq t_k(l)$  since every plane triangulation is 3-connected and thus a polyhedral graph. Notice that even  $n_k(l) = t_k(l)$  holds in every considered case. If, namely,  $G$  is any one of the here or in [3], respectively, constructed graphs to prove an upper bound for  $n_k(l)$  with certain  $k$  and  $l$ , then we were able to construct a plane triangulation  $T$  from  $G$  by adding edges only such that the length of a shortest cycle containing the prescribed  $k$  vertices is at least  $l$ .

## 2. Proofs

For terminology and notation not defined here we refer to [5]. Let  $G$  be a graph and  $A, B \subseteq V(G)$ . A path  $P$  of  $G$  with one end-vertex in  $A$  and  $B$ , respectively, and with  $|V(P) \cap A| = |V(P) \cap B| = 1$  is called an  $A$ - $B$ -path. If  $A$  or  $B$  consists of a single vertex  $x$  we write  $x$  instead of  $\{x\}$ . We use  $[x, y]$  to denote an  $x$ - $y$ -path and, moreover,  $[x, y)$  or  $(x, y)$  to denote the segments obtained from  $[x, y]$  by removing  $y$  or both end-vertices, respectively. A path

system is a set of internally disjoint paths. For a path system  $\mathcal{P}$  let  $[\mathcal{P}]$  and  $EV(\mathcal{P})$  denote the union of all paths and the set of all end-vertices of paths of  $\mathcal{P}$ , respectively. For some  $a \in V(G)$  and  $B \subseteq V(G) \setminus \{a\}$  a path system  $\mathcal{P}$  of  $a$ - $B$ -paths is called an  $a$ - $B$ -fan if  $P \cap Q = \{a\}$  for different  $P, Q \in \mathcal{P}$ .

We need the following lemma which is proved in [3] in more general form.

**Lemma 1.** *Let  $G$  be a  $c$ -connected graph with  $a \in V(G)$ ,  $B \subseteq V(G) \setminus \{a\}$  and a path system  $\mathcal{P}$  of  $c - 1$   $a$ - $B$ -paths. Let  $B' = B \setminus EV(\mathcal{P})$  if this is not empty, and  $B'$  be an arbitrary nonempty subset of  $B$  otherwise. Then there is a vertex  $b \in B'$  and a path system  $\mathcal{Q}$  of  $c$   $a$ - $B$ -paths such that  $EV(\mathcal{Q}) = EV(\mathcal{P}) \cup \{b\}$ , all vertices of  $B \setminus \{b\}$  are end-vertices of as many paths of  $\mathcal{P}$  as of  $\mathcal{Q}$ , and  $\mathcal{Q}$  has one more path with end-vertex  $b$  than does  $\mathcal{P}$ .*

We define five polyhedral graphs containing the vertices of a prescribed 4-element set  $X$  as follows. Let  $F_1$  be the complete graph  $K_4$  on  $X$ . Let  $F_2$  denote the graph which is obtained from a 4-cycle  $C$  on  $X$  by connecting an additional vertex  $a \notin X$  with all vertices of  $C$ . Let  $F_3$  denote the graph which results from  $C$  and two adjacent vertices  $a, b \notin X$  by connecting two adjacent vertices of  $C$  with  $a$  and the remaining two vertices of  $C$  with  $b$ . The graph  $F_4$  is obtained if two non-adjacent vertices  $a, b \notin X$  are connected with three vertices of a 4-path  $P$  on  $X$ , respectively, such that every vertex of  $X$  becomes degree 3. Eventually, let  $F_5$  denote the cube graph containing the vertices of  $X$  such that no two vertices of  $X$  are adjacent.

**Lemma 2.** *Every polyhedral graph  $G$  with  $X = \{x_1, x_2, x_3, x_4\} \subseteq V(G)$  has a subdivision  $H$  which is a subdivision of some  $F_i$  with  $1 \leq i \leq 5$ .*

**Proof of Lemma 2.** Lemma 1 implies that  $G$  has an  $x_1$ - $x_2$ -path system  $\{P_1, P_2, P_3\}$  which contains  $x_3$  by planarity of  $G$ , i.e., we may assume that  $x_3 \in V(P_1)$ . Moreover, Lemma 1 yields an  $x_3$ - $V(P_2 \cup P_3)$ -fan  $\mathcal{Q} = \{[x_1, x_3], [x_2, x_3], [a, x_3]\}$ , where we may assume that  $a \in V(P_2)$ . Thus,  $G$  has a path system  $\mathcal{P} = \{[x_1, x_2], [x_1, x_3], [x_2, x_3], [a, x_1], [a, x_2], [a, x_3]\}$ .

Suppose first, that  $x_4$  is contained in  $[\mathcal{P}]$ . Considering symmetries we have to examine three different cases.

*Case 1.*  $x_4 = a$ .

Then  $[\mathcal{P}]$  is a subdivision of  $F_1$ .

*Case 2.*  $x_4 \in (x_1, x_2)$ .

By Lemma 1 there is an  $x_4$ - $V([\mathcal{P}] \setminus (x_1, x_2))$ -fan  $\mathcal{Q} = \{[x_1, x_4], [x_2, x_4], [b, x_4]\}$  where  $b \in V([\mathcal{P}] \setminus (x_1, x_2))$ . Let  $H$  denote the subgraph  $[\mathcal{P} \cup \mathcal{Q}] \setminus (x_1, x_2)$  of  $G$ , then by symmetries there are following subcases. If  $b = x_3$  or  $b = a$  then  $H$  is a subdivision of  $F_1$  or  $F_2$ , respectively. If  $b \in (x_1, x_3)$  or  $b \in (a, x_1)$  then  $H$  is a subdivision of  $F_4$  or  $F_3$ , respectively.

*Case 3.*  $x_4 \in (a, x_1)$ .

Applying Lemma 1 again there is an  $x_4$ - $V([\mathcal{P}] \setminus (a, x_1))$ -fan  $\mathcal{Q} = \{[x_1, x_4], [a, x_4], [b, x_4]\}$  where  $b \in V([\mathcal{P}] \setminus (a, x_1))$ . Let  $H$  denote the subgraph  $[\mathcal{P} \cup \mathcal{Q}] \setminus (a, x_1)$  of  $G$ . Considering symmetries we have: If  $b \in (x_1, x_2)$  or  $b \in [x_2, a)$  then  $H$  is a subdivision of  $F_4$  or  $F_1$ , respectively.

Suppose now, that  $x_4$  is not contained in  $[\mathcal{P}]$  and in any other such path system of  $G$ . Applying Lemma 1 we obtain an  $x_4$ - $V([\mathcal{P}])$ -fan  $\mathcal{Q} = \{[b, x_4], [c, x_4], [d, x_4]\}$  such that each path of  $\mathcal{P}$  contains at most one vertex of  $EV(\mathcal{Q})$  and that at most one path of  $\mathcal{P}$  with end vertex  $a$  contains a vertex of  $EV(\mathcal{Q})$ . Thereby and since  $G$  is planar we may assume that  $b \in (x_1, x_2)$ ,  $c \in (x_2, x_3)$  and  $d \in (x_1, x_3)$  which implies that  $[\mathcal{P} \cup \mathcal{Q}]$  is a subdivision of  $F_5$ . ■

Figure 1 contains further three polyhedral graphs which contain the vertices of  $X = \{x_1, x_2, x_3, x_4\}$  and which are needed to prove Theorem 1.

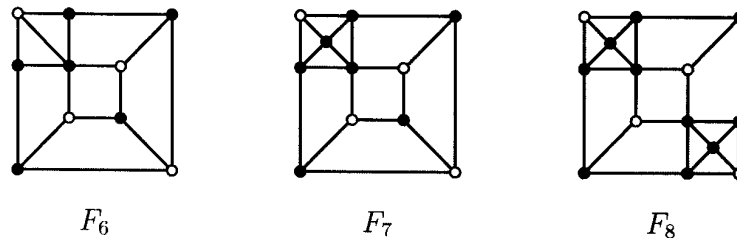


Figure 1

**Proof of Theorem 1.** For  $l = 6, 7, 11$  and  $l \geq 13$  connect a vertex  $a$  with each vertex of a 4-cycle  $C = x_1x_2x_3x_4x_1$ . Put  $\alpha = \lfloor \frac{l-5}{3} \rfloor$  and suppose  $l \equiv r \pmod{3}$  where  $r \in \{0, 1, 2\}$ . Subdivide every edge  $e$  of  $C$  with respect to  $r$  by the number of new vertices given in Table 1. Connect every new vertex with  $a$  and denote the so constructed polyhedral graph by  $G$ .

Table 1

$r \setminus e$	$x_1x_2$	$x_2x_3$	$x_3x_4$	$x_4x_1$
0	$\alpha + 1$	$\alpha + 1$	$\alpha$	$\alpha$
1	$\alpha + 1$	$\alpha + 1$	$\alpha + 1$	$\alpha$
2	$\alpha$	$\alpha$	$\alpha$	$\alpha$

A simple calculation shows that the length of a shortest cycle in  $G$  containing  $X = \{x_1, x_2, x_3, x_4\}$  is  $l$  and that the order of  $G$  is  $\lceil \frac{4l-5}{3} \rceil$ , in every case.

For  $l = 4, 5, 8, 9, 10, 12$  let  $G$  be  $F_1, F_4, F_5, F_6, F_7, F_8$ , respectively, with  $X \subseteq V(G)$ . In these special cases it is not hard to see that the length of a shortest cycle of  $G$  containing  $X$  is  $l$ . That together with  $n_4(l) \leq |G|$  completes the proof of the upper bound.

Suppose, now, that  $G$  is a polyhedral graph of order  $n$  with a 4-element subset  $X = \{x_1, x_2, x_3, x_4\}$  of  $V(G)$  such that the length of a shortest cycle containing  $X$  is at least  $l$ . Because of Lemma 2 it is sufficient to estimate for  $i = 1, \dots, 5$  the order of a subgraph  $H$  of  $G$  which is a subdivision of  $F_i$  with  $X \subseteq V(F_i)$  and to deduce a lower bound for  $n_4(l)$ .

$i = 1$ :  $H$  has three different cycles  $C_1, C_2, C_3$  passing each vertex of  $F_1$ . Every vertex of  $V(H) \setminus V(F_1)$  occurs in precisely two of these three cycles. Thus,  $2|H| + 4 \geq |C_1| + |C_2| + |C_3| \geq 3l$  and, consequently,  $|H| \geq \lceil \frac{3l-4}{2} \rceil$ .

$i = 2$ :  $H$  has four cycles  $C_1, \dots, C_4$  containing all vertices of  $F_2$  and one cycle  $C_5$  containing  $X$  but no other vertex of  $F_2$ . Every vertex of  $V(H) \setminus V(F_2) \setminus V(C_5)$  occurs in precisely two and every vertex of  $V(C_5) \setminus V(F_2)$  in precisely three of the cycles  $C_1, \dots, C_4$ . Thus,  $2|H| + |C_5| + 4 \cdot 1 + 2 \geq |C_1| + \dots + |C_4| \geq 4l$  and, thereby,  $2|H| + |C_5| + 6 \geq 4l$ . From  $|C_5| \leq |H| - 1$  we further obtain  $|H| \geq \lceil \frac{4l-5}{3} \rceil$ .

$i = 3, 4$ :  $H$  has three different cycles  $C_1, C_2, C_3$  passing each vertex of  $F_i$ . Every vertex of  $V(H) \setminus V(F_i)$  occurs in precisely two of these three cycles. Thus,  $2|H| + 6 \geq |C_1| + |C_2| + |C_3| \geq 3l$  and, consequently,  $|H| \geq \lceil \frac{3l-6}{2} \rceil$ .

$i = 5$ :  $H$  has six different cycles  $C_1, \dots, C_6$  passing each vertex of  $F_5$ . Every vertex of  $V(H) \setminus V(F_5)$  occurs in precisely four of these six cycles. Thus,  $4|H| + 2 \cdot 8 \geq |C_1| + \dots + |C_6| \geq 6l$  and, consequently,  $|H| \geq \lceil \frac{3l-8}{2} \rceil$ . Because of  $|G| \geq \min\{|H_i| : 1 \leq i \leq 5\}$  and  $|G| \geq l$  we obtain

$$n_4(l) \geq \begin{cases} l & \text{if } l \in \{4, 5, 6, 8\}, \\ l + 1 & \text{if } l \in \{7, 9, 10\}, \\ l + 2 & \text{if } l \in \{11, 12\}, \\ \lceil \frac{4l-5}{3} \rceil & \text{if } l \geq 13. \end{cases}$$

In the special cases  $l = 5, 6$  one can observe that since  $G$  has a subgraph  $H$  which is a subdivision of  $F_i$  for some  $i \in \{1, \dots, 5\}$  the order of  $G$  can not be smaller than 6 or 7, respectively. That proves the lower bound. ■

**Proof of Theorem 2.** For  $l = 5, 6, 7, 8, 9$  let  $G_l$  be the polyhedral graphs with  $X = \{x_1, \dots, x_5\} \subseteq V(G_l)$  given in Figure 2.

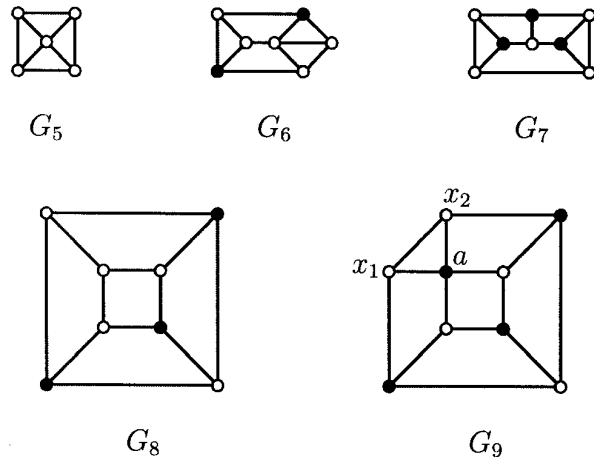


Figure 2

For  $l > 9$  let  $G_l$  be the polyhedral graph which results from  $G_9$  by subdividing  $x_1x_2$  by  $l - 9$  new vertices and connecting each of them with  $a \notin X$ . Notice that  $|G_l| = l$  if  $l = 5$  or  $l \geq 8$  and  $|G_l| = l + 1$  if  $l = 6$  or  $7$ . It is not hard to see that for every  $l \geq 5$  the length of any cycle of  $G_l$  passing all the vertices of  $X$  is at least  $l$ .

So, it remains to prove  $n_5(l) > l$  for  $l = 6, 7$ . Let  $l = 6$  and suppose that there exists a polyhedral graph  $G$  of order 6 with  $V(G) = X \cup \{a\}$  such that every cycle which contains the vertices of  $X$  is a hamiltonian one. Let  $\mathcal{C}(G)$  denote the set of all cycles of  $G$ . Then we may suppose that  $x_1x_2x_3x_4x_5ax_1 \in \mathcal{C}(G)$ . Clearly,  $x_1x_5 \notin E(G)$  which implies that  $x_1x_3$

or  $x_1x_4 \in E(G)$ . If  $x_1x_3 \in E(G)$  then  $x_2x_5 \notin E(G)$  because otherwise  $x_1x_2x_5x_4x_3x_1 \in \mathcal{C}(G)$ . Thus,  $x_3x_5 \in E(G)$  and also  $x_1x_4, x_2x_4 \notin E(G)$  because otherwise  $x_1x_2x_3x_5x_4x_1$  or  $x_1x_2x_4x_5x_3x_1 \in \mathcal{C}(G)$ , respectively. Thereby,  $x_2$  and  $x_4$  are connected with  $a$  which yields that  $\{x_3, a\}$  is a cutset, a contradiction. So, we have that  $x_1x_3 \notin E(G)$  and  $x_1x_4 \in E(G)$  which implies that  $x_3x_5 \notin E(G)$  because otherwise  $x_1x_2x_3x_5x_4x_1 \in \mathcal{C}(G)$ . That implies  $x_2x_5 \in E(G)$  and thereby  $d_G(x_3) = 2$ , a contradiction.

Now, let  $l = 7$  and suppose that there exists a polyhedral graph  $G$  of order 7 with  $V(G) = X \cup \{a, b\}$  such that every cycle which contains the vertices of  $X$  is a hamiltonian one. We may assume that  $\mathcal{C}(G)$  contains one of the cycles  $C_1 = x_1x_2x_3x_4x_5abx_1$ ,  $C_2 = x_1x_2x_3x_4ax_5bx_1$ ,  $C_3 = x_1x_2x_3ax_4x_5bx_1$ .

*Case 1.  $C_1 \in \mathcal{C}(G)$ .*

Clearly,  $x_1x_5, x_1a, x_5b \notin E(G)$ . If  $x_1x_3 \in E(G)$  then  $x_2x_5, x_2a \notin E(G)$  because otherwise  $x_1x_2x_5x_4x_3x_1$  or  $x_1x_2ax_5x_4x_3x_1 \in \mathcal{C}(G)$ , respectively. Thus,  $x_3x_5 \in E(G)$  which yields  $x_1x_4, x_2x_4, x_4b \notin E(G)$  because otherwise  $x_1x_2x_3x_5x_4x_1$  or  $x_1x_2x_4x_5x_3x_1$  or  $x_1x_2x_3x_5x_4bx_1 \in \mathcal{C}(G)$ , respectively. That implies  $x_2b, x_4a \in E(G)$  which means that  $\{x_3, a\}$  or  $\{x_3, b\}$  would be a cutset of  $G$ , a contradiction. If  $x_1x_3 \notin E(G)$  we have  $x_1x_4 \in E(G)$  and  $x_3x_5, x_3a \notin E(G)$  because otherwise  $x_1x_2x_3x_5x_4x_1$  or  $x_1x_2x_3ax_5x_4x_1 \in \mathcal{C}(G)$ , respectively. That implies  $x_2x_5 \in E(G)$  which means by planarity that  $x_3b \notin E(G)$ . Thus,  $d_G(x_3) = 2$ , a contradiction.

*Case 2.  $C_2 \in \mathcal{C}(G)$ .*

Clearly,  $x_1x_5, x_4x_5 \notin E(G)$ . Suppose, first,  $x_1x_3 \in E(G)$  then  $x_2x_5 \notin E(G)$  because otherwise  $x_1x_2x_5ax_4x_3x_1 \in \mathcal{C}(G)$ . Thereby,  $x_3x_5 \in E(G)$  which implies that  $x_1x_4, x_2x_4 \notin E(G)$  because otherwise  $x_1x_2x_3x_5ax_4x_1$  or  $x_1x_2x_4ax_5x_3x_1 \in \mathcal{C}(G)$ , respectively. Thus,  $x_4b \in E(G)$  which yields by planarity  $x_1a, x_2a \notin E(G)$ , i.e.,  $\{x_3, b\}$  would be a cutset of  $G$ , a contradiction. Suppose, now,  $x_1x_3 \notin E(G)$  and  $x_1x_4 \in E(G)$ . Then  $x_2x_5, x_3x_5 \notin E(G)$  because otherwise  $x_1x_4x_3x_2x_5bx_1$  or  $x_1x_2x_3x_5ax_4x_1 \in \mathcal{C}(G)$ , respectively. That yields  $d_G(x_5) = 2$ , a contradiction. Suppose  $x_1x_3, x_1x_4 \notin E(G)$  then  $x_1a \in E(G)$ . If, here,  $x_2x_5 \in E(G)$  then  $x_3x_5 \notin E(G)$  because otherwise  $x_1x_2x_5x_3x_4ax_1 \in \mathcal{C}(G)$ . By planarity,  $x_3b, x_4b \notin E(G)$  which means that  $\{x_2, a\}$  would be a cutset of  $G$ , a contradiction. If  $x_2x_5 \notin E(G)$  then  $x_3x_5 \in E(G)$  and, consequently,  $x_2x_4 \notin E(G)$  because otherwise  $x_1x_2x_4x_3x_5ax_1 \in \mathcal{C}(G)$ . Planarity implies  $x_4b \notin E(G)$  and, hence,  $d_G(x_4) = 2$ , a contradiction.

*Case 3.*  $C_3 \in \mathcal{C}(G)$ .

Clearly,  $x_1x_5, x_3x_4 \notin E(G)$ . Suppose, first,  $x_1x_3 \in E(G)$  then  $x_2x_4, x_2x_5 \notin E(G)$  because otherwise  $x_1x_3x_2x_4x_5bx_1$  or  $x_1x_3ax_4x_5x_2x_1 \in \mathcal{C}(G)$ , respectively. That implies  $x_1x_4$  or  $x_4b \in E(G)$ . If  $x_1x_4 \in E(G)$  then  $x_2b \notin E(G)$  because otherwise  $x_1x_3x_2bx_5x_4x_1 \in \mathcal{C}(G)$ . Thereby,  $x_2a \in E(G)$  which implies  $x_3x_5, x_3b \notin E(G)$  because otherwise  $x_1x_2x_3x_5x_4x_1$  or  $x_1x_2x_3bx_5x_4x_1 \in \mathcal{C}(G)$ , respectively. That gives  $d_G(x_3) = 2$ , a contradiction. If  $x_1x_4 \notin E(G)$  then  $x_4b \in E(G)$  which yields  $x_3x_5 \notin E(G)$  because otherwise  $x_1x_2x_3x_5x_4bx_1 \in \mathcal{C}(G)$ . Thus,  $x_5a \in E(G)$  and  $\{a, b\}$  would be a cutset of  $G$ , a contradiction.

Suppose, now,  $x_1x_3 \notin E(G)$  and  $x_1x_4 \in E(G)$ . Then  $x_3x_5, x_3b \notin E(G)$  because otherwise  $x_1x_2x_3x_5x_4x_1$  or  $x_1x_2x_3bx_5x_4x_1 \in \mathcal{C}(G)$ , respectively. That implies  $d_G(x_3) = 2$ , a contradiction.

Suppose, eventually,  $x_1x_3, x_1x_4 \notin E(G)$  then  $x_1a \in E(G)$ . That implies  $x_3x_5 \notin E(G)$  because otherwise  $x_1x_2x_3x_5x_4ax_1 \in \mathcal{C}(G)$ . Thereby,  $x_3b \in E(G)$  and by planarity  $x_2x_4, x_2x_5 \notin E(G)$  which means that  $\{a, b\}$  would be a cutset of  $G$ , a contradiction, and the proof is complete. ■

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