Insight into hydrochemistry: a multi-catchment comparison using Horizontal Visibility Graphs

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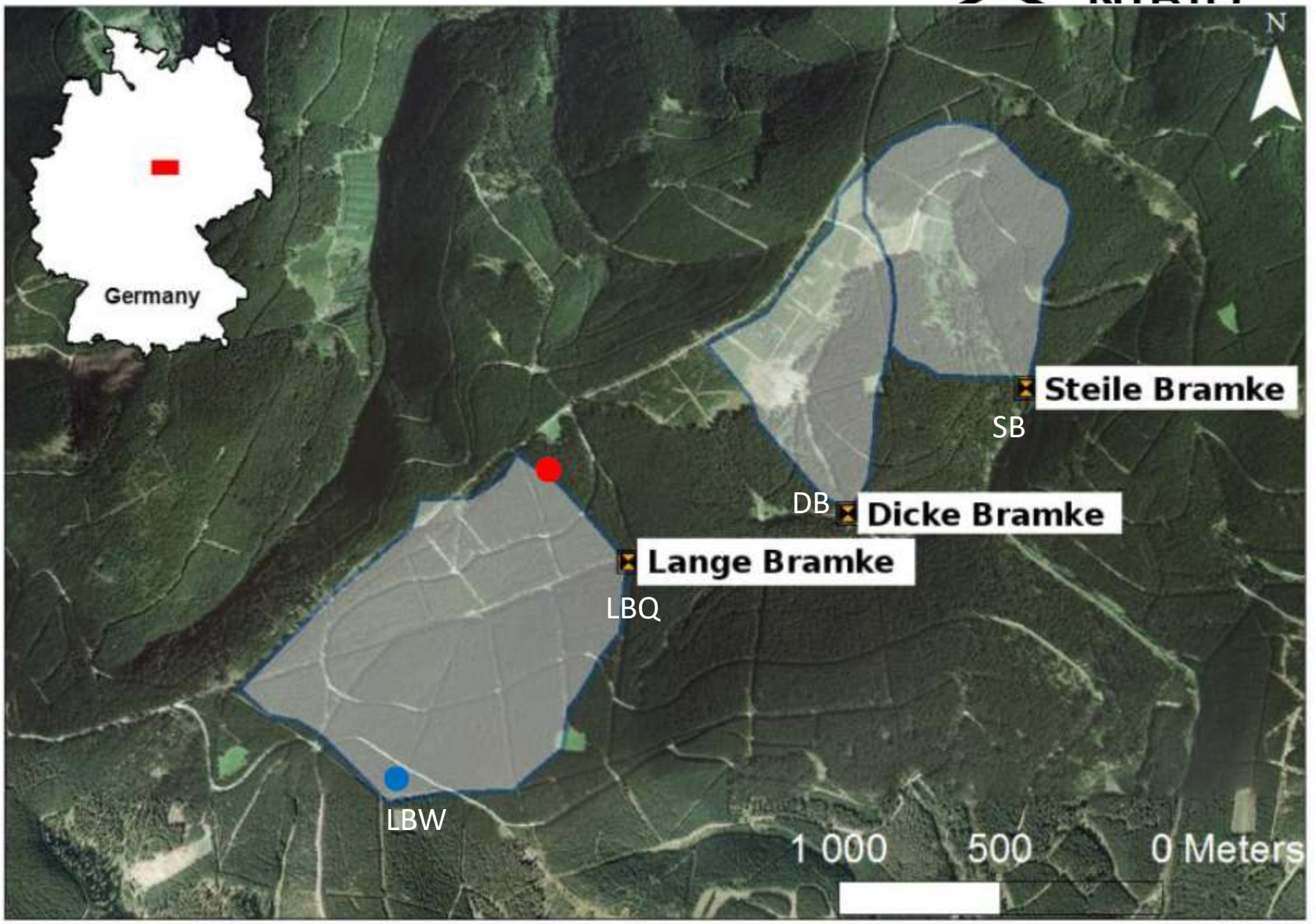
- The Bramke catchments – where, why, what?
- Horizontal Visibility Graphs (HVGs): theory
- HVGs and the Bramke catchments
- Stochastic or deterministic? Or neither?
- Conclusions

The Bramke catchments (Harz mountains, NE Germany)



The Bramke catchments

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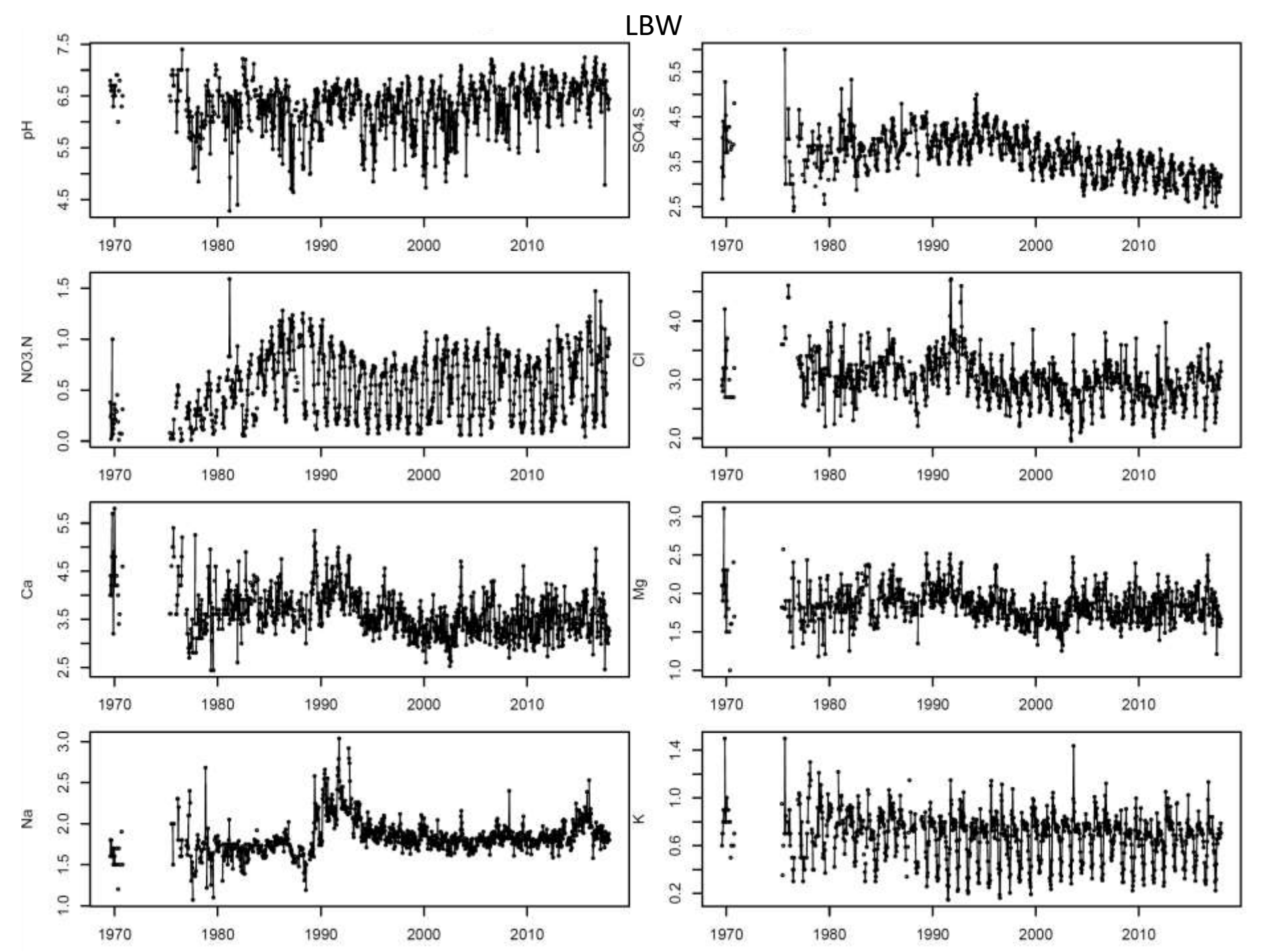
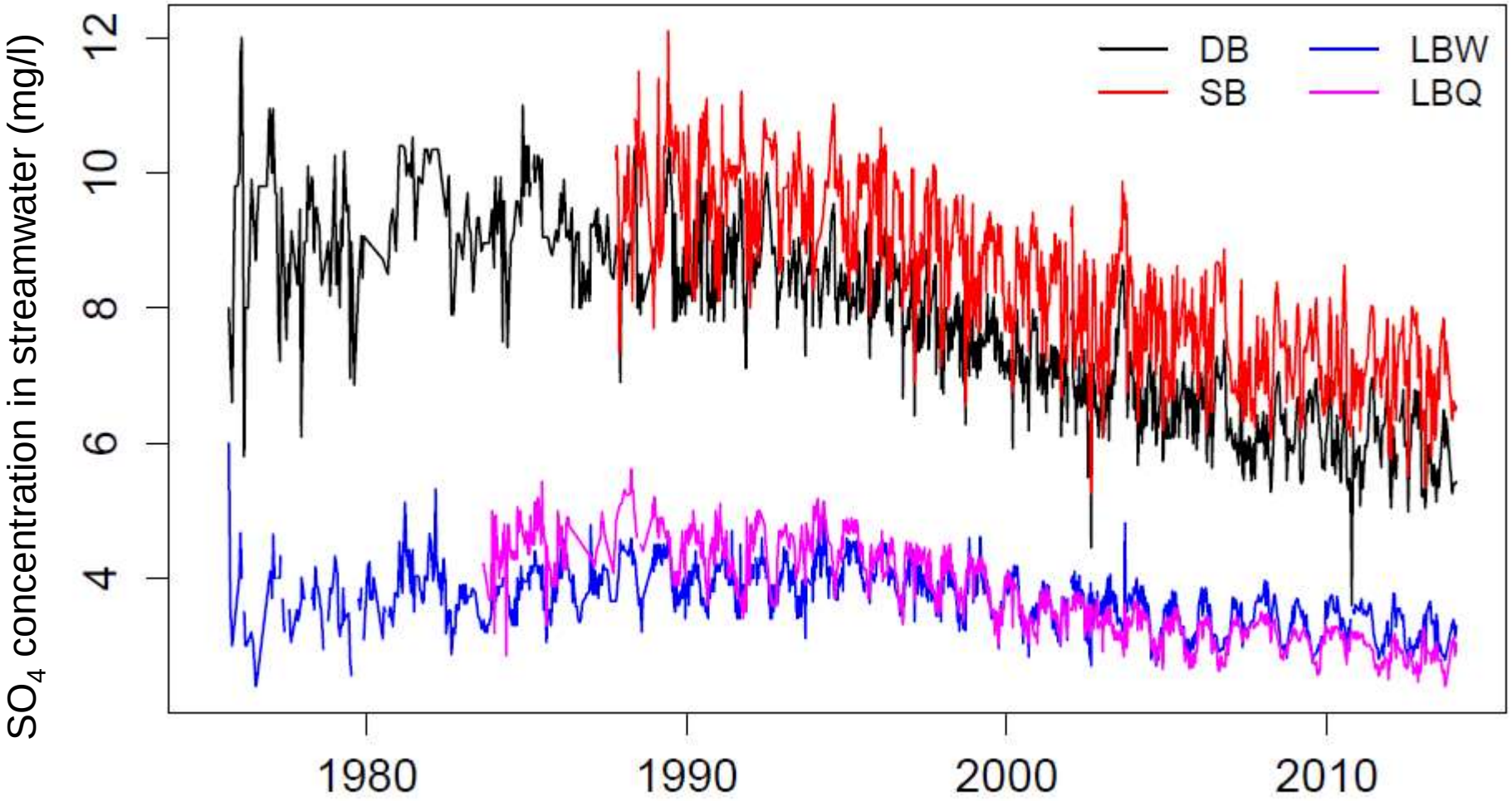
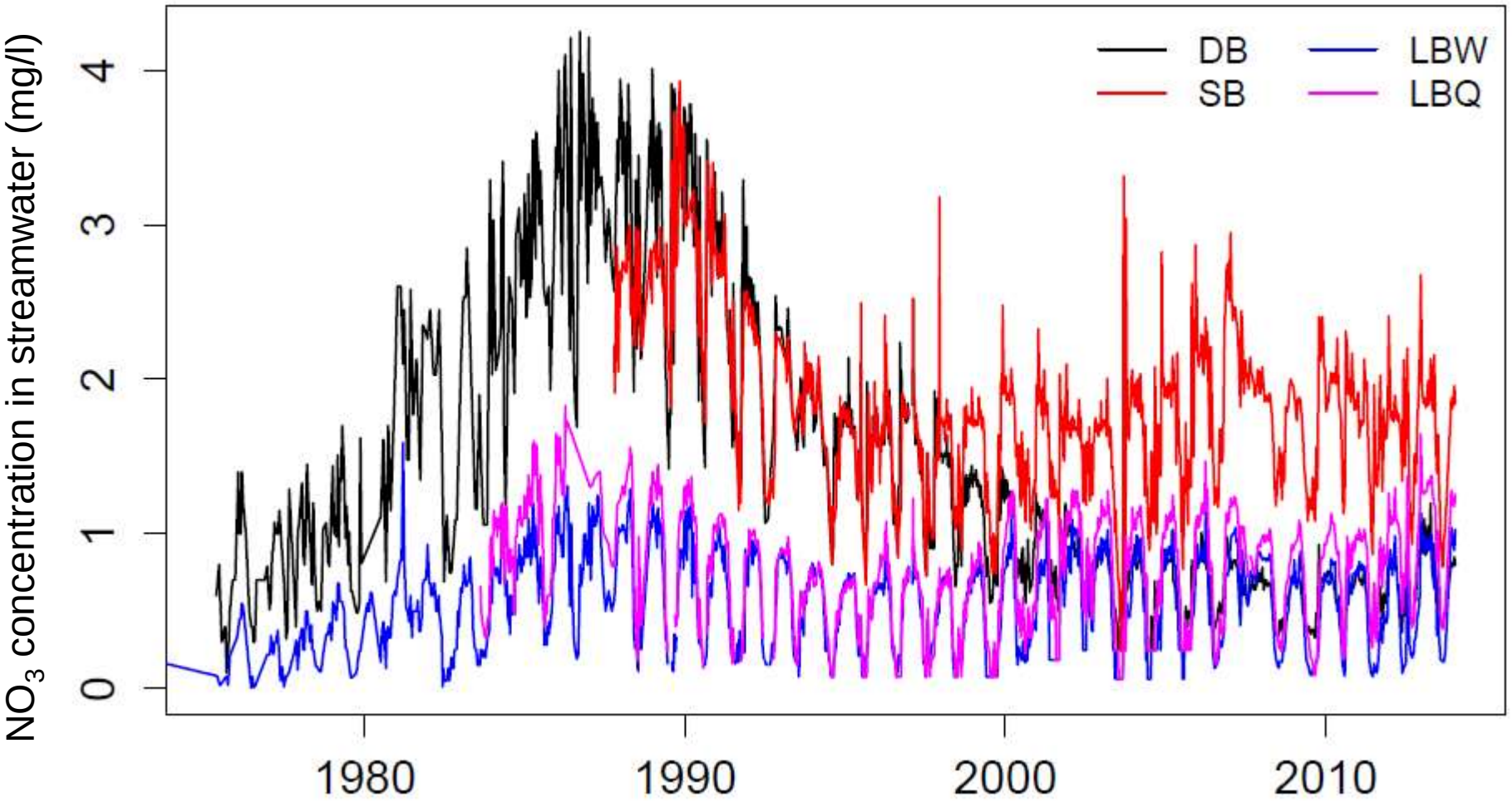
LBW



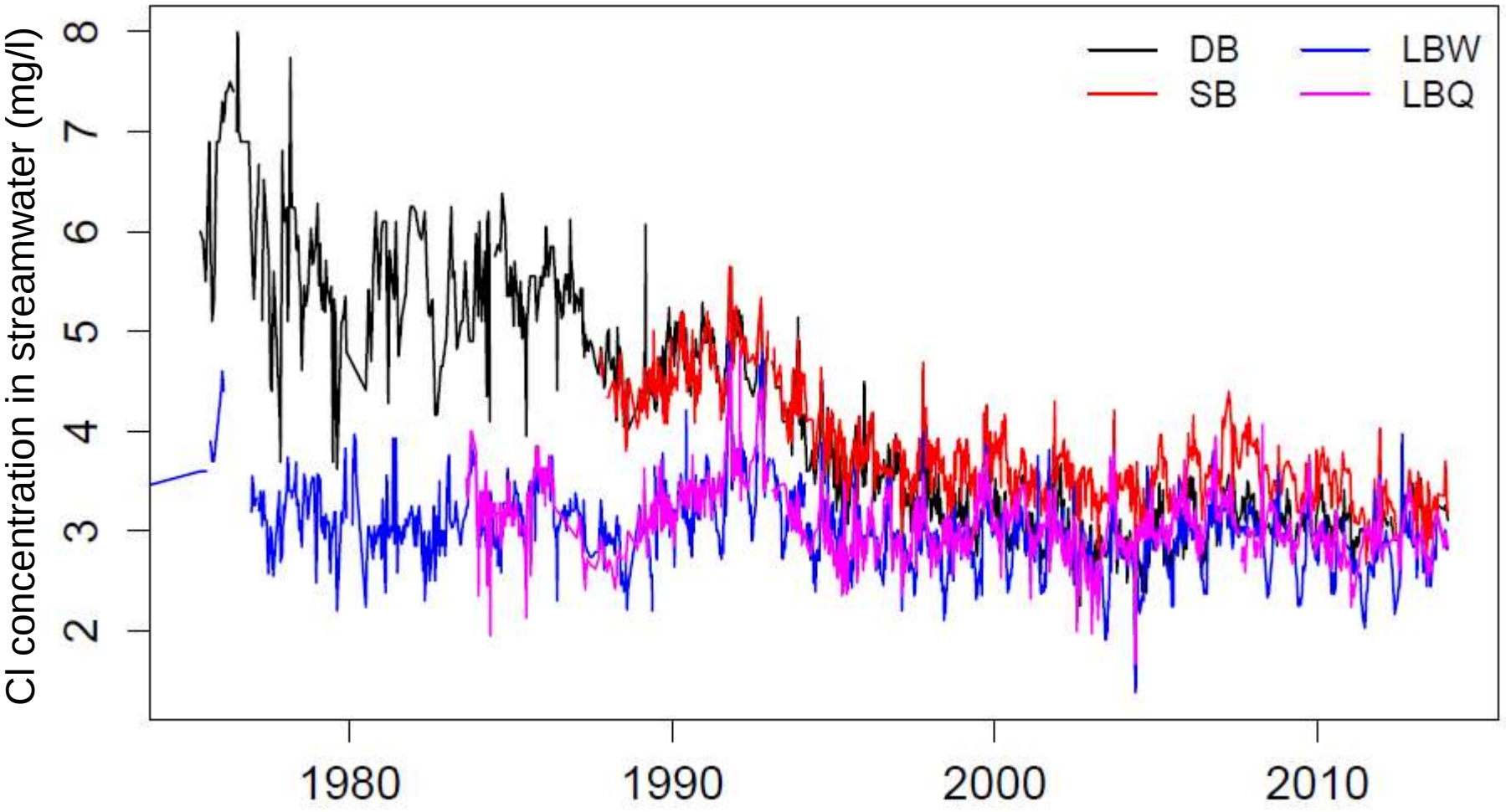
Long-term changes in a pristine system



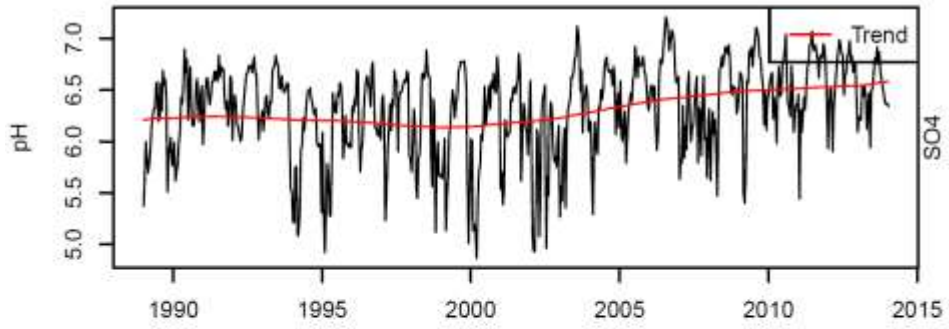
Long-term changes in a pristine system

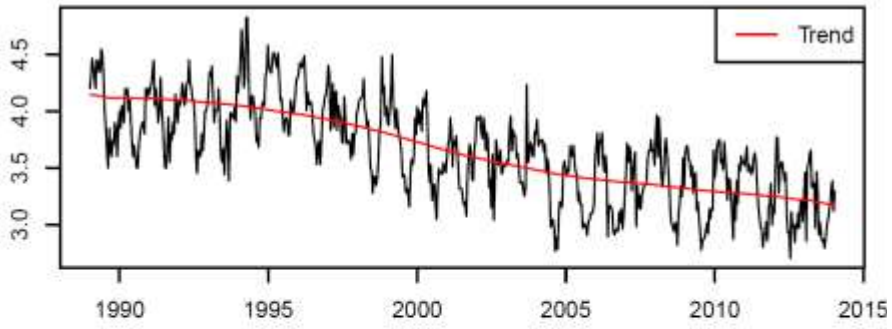


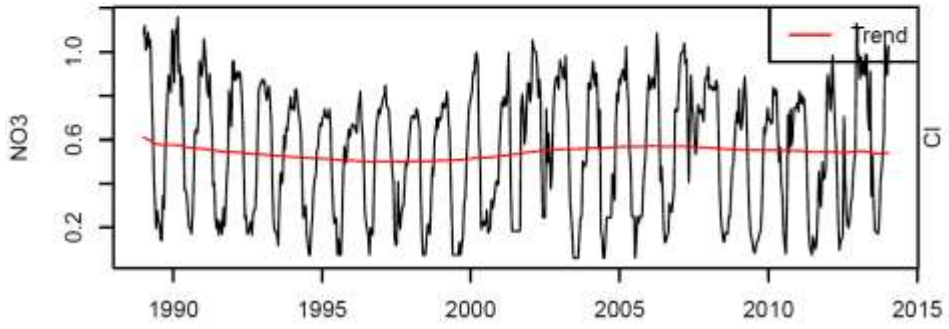
Long-term changes in a pristine system

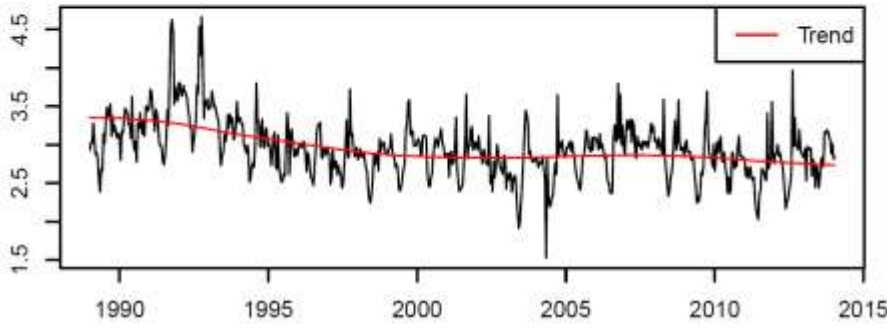


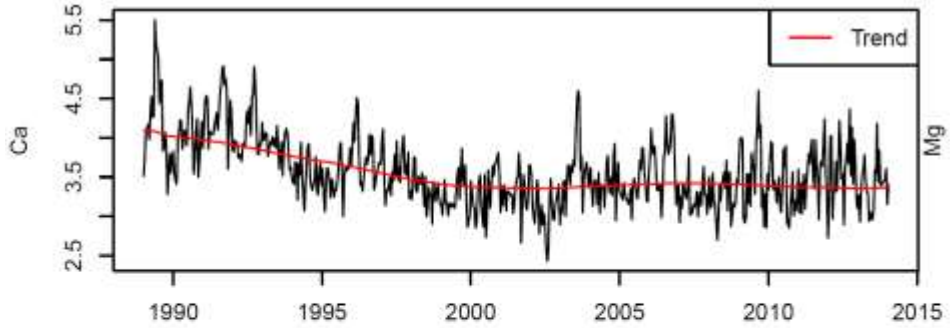
LBW Trends

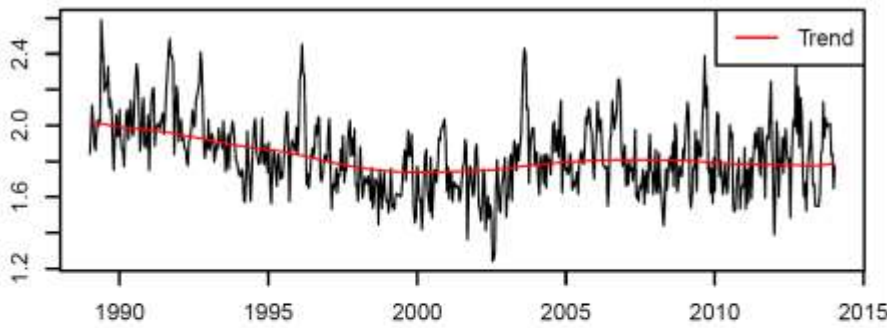

pH
Trend
5.0
5.5
6.0
6.5
7.0
1990
1995
2000
2005
2010
2015

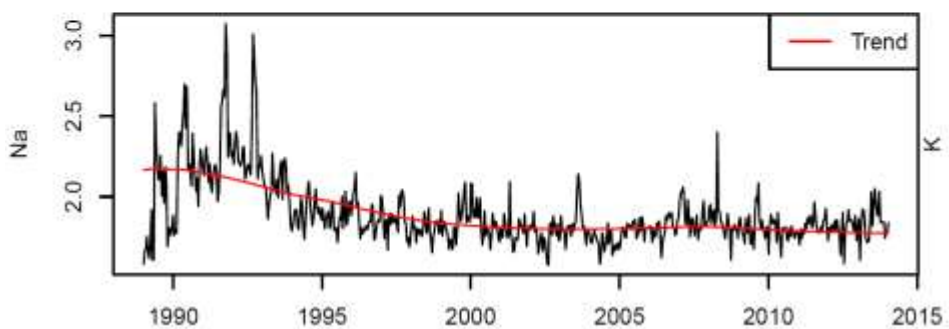

SO4
Trend
3.0
3.5
4.0
4.5
1990
1995
2000
2005
2010
2015

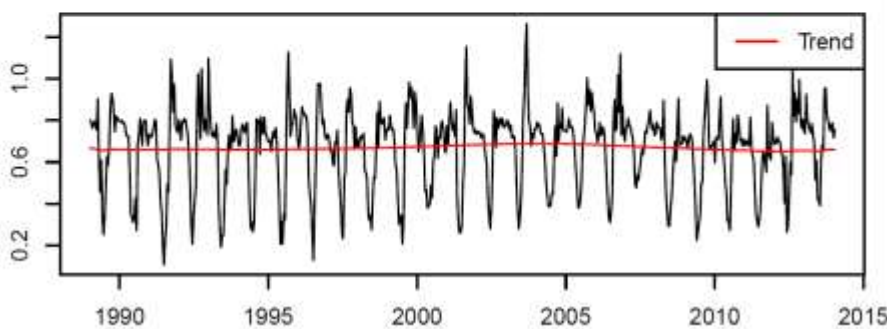

NO3
Trend
0.2
0.6
1.0
1990
1995
2000
2005
2010
2015


Cl
Trend
1.5
2.5
3.5
4.5
1990
1995
2000
2005
2010
2015


Ca
Trend
2.5
3.5
4.5
5.5
1990
1995
2000
2005
2010
2015


Mg
Trend
1.2
1.6
2.0
2.4
1990
1995
2000
2005
2010
2015


Na
Trend
2.0
2.5
3.0
1990
1995
2000
2005
2010
2015

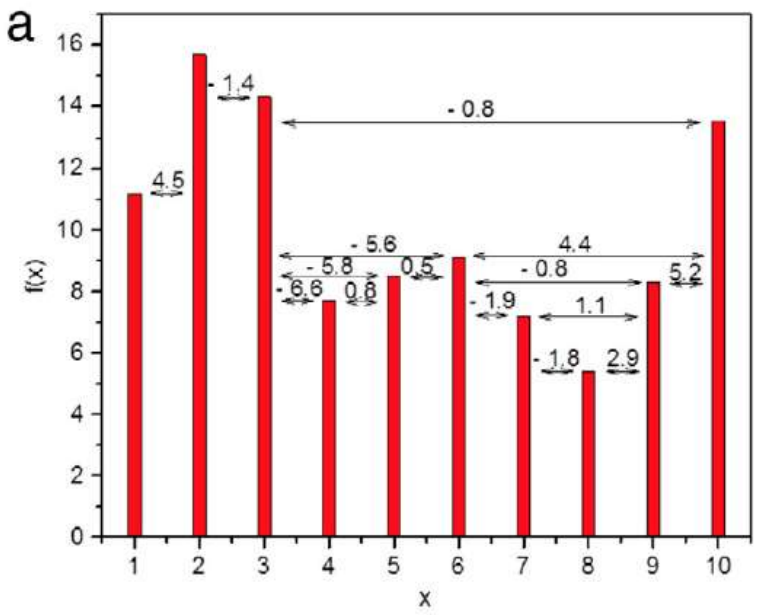

K
Trend
0.2
0.6
1.0
1990
1995
2000
2005
2010
2015

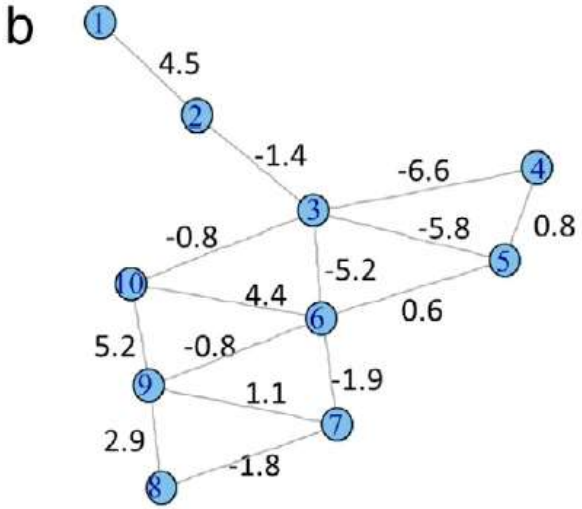
Horizontal Visibility Graphs (HVG)

- For a time series, a network is constructed
- Each observation in the time series is a node i
- Two nodes i and j are *linked* iff one can draw a horizontal line between them without intersecting any other observation:

$$i \leftrightarrow j \Leftrightarrow [f_t(i), f_t(j)] > f_t(l)\ \forall\ l | (i < l < j)$$

- The number of links of a given node is its **degree**





Gonçalves et al. (2016)

Theory of HVG-degree distributions

- For (infinite) uncorrelated white noise, $P_d(k)$ is *exponential*

$$P_d(k) = Aexp(-\lambda_c k)$$

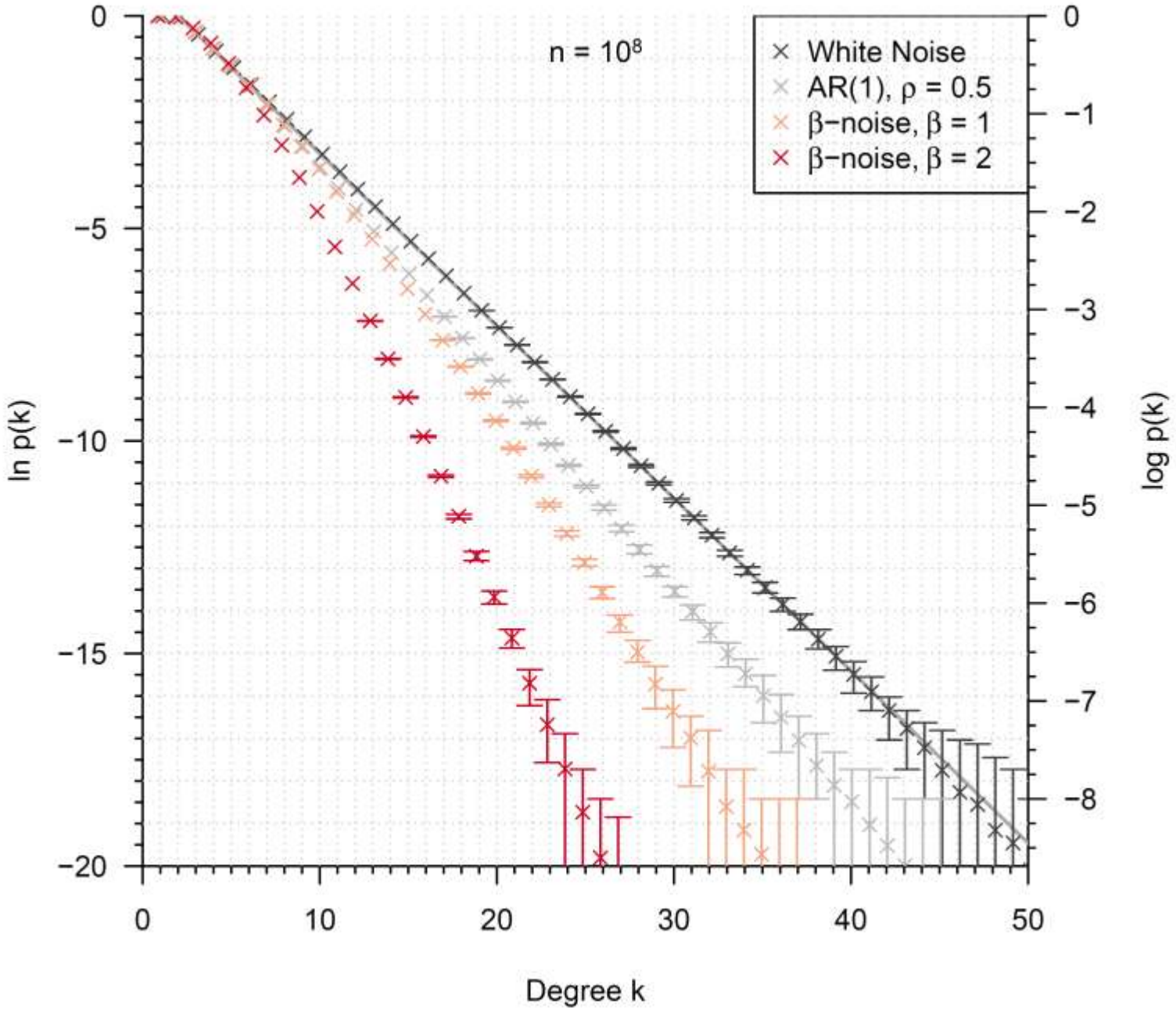
with $\lambda_c = \ln\left(\frac{3}{2}\right)$ (Lacasa & Toral 2010)

These authors claim:

The HVG-degree distributions are *always* exponential, with $\lambda < \lambda_c$ for deterministic-chaotic and $\lambda > \lambda_c$ for correlated stochastic processes

- No analytical proof available
- Proven to be numerically wrong by Ravetti et al. 2014

- For white noise, mean degree = 4; for correlated noise, mean degree < 4

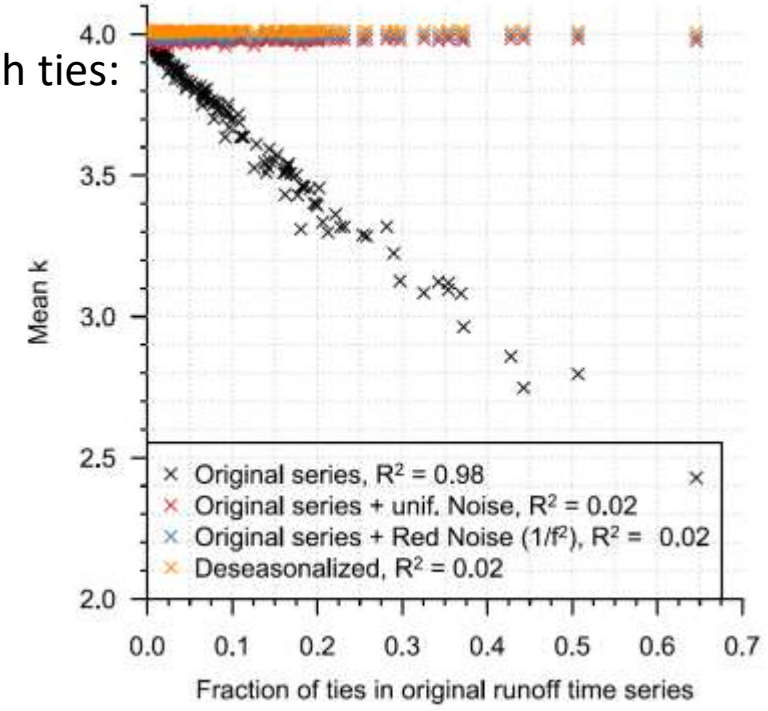


Lange et al. (2018)

Methodological issues

- The slope estimation method has to be chosen with care
- Trends impact on the λ estimates
- (Seasonal) periodicities impact on the λ estimates
- Ties in the time series corrupt the network
- -> A careful pretreatment of the time series might be necessary

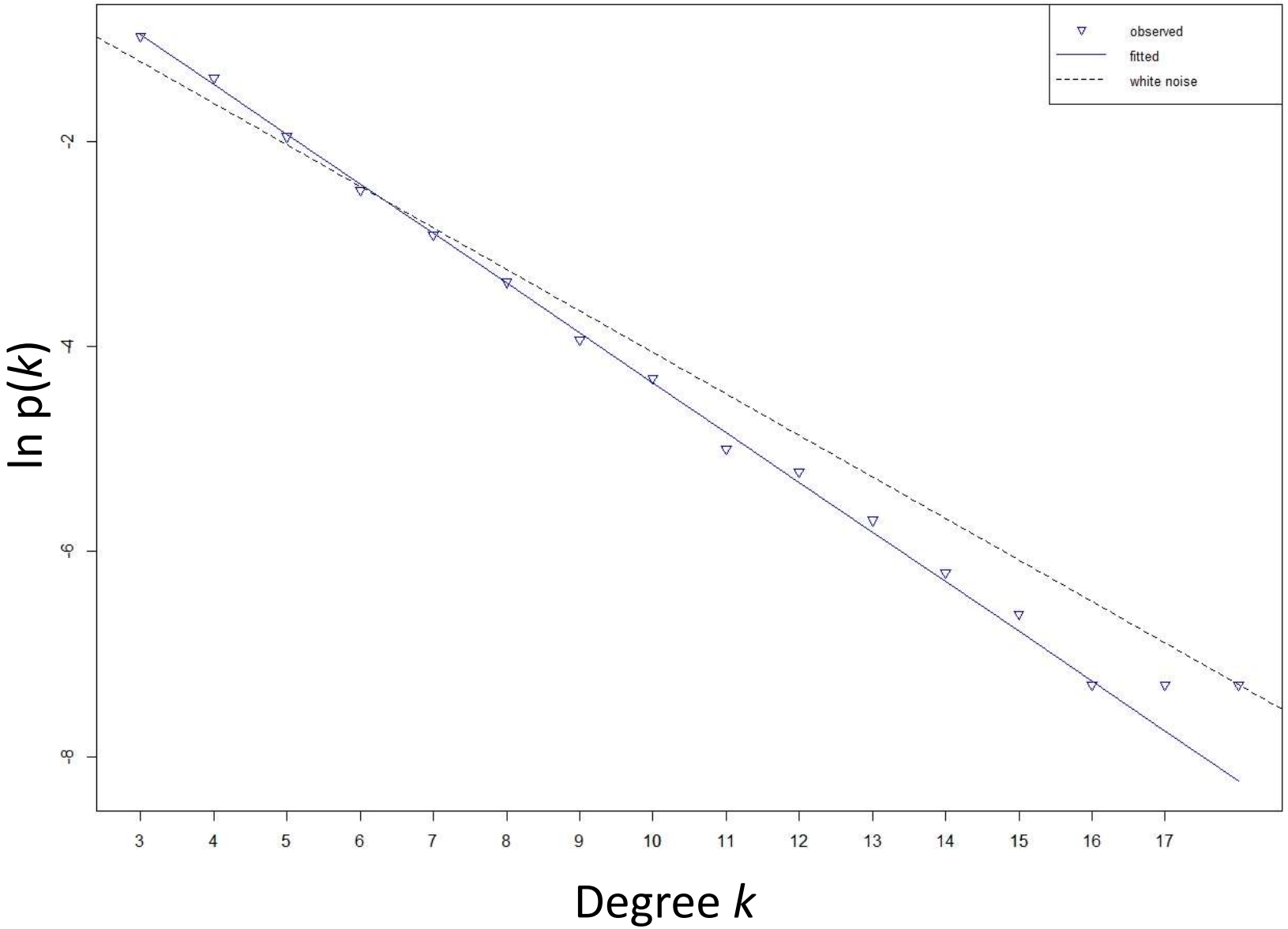
The problem with ties:



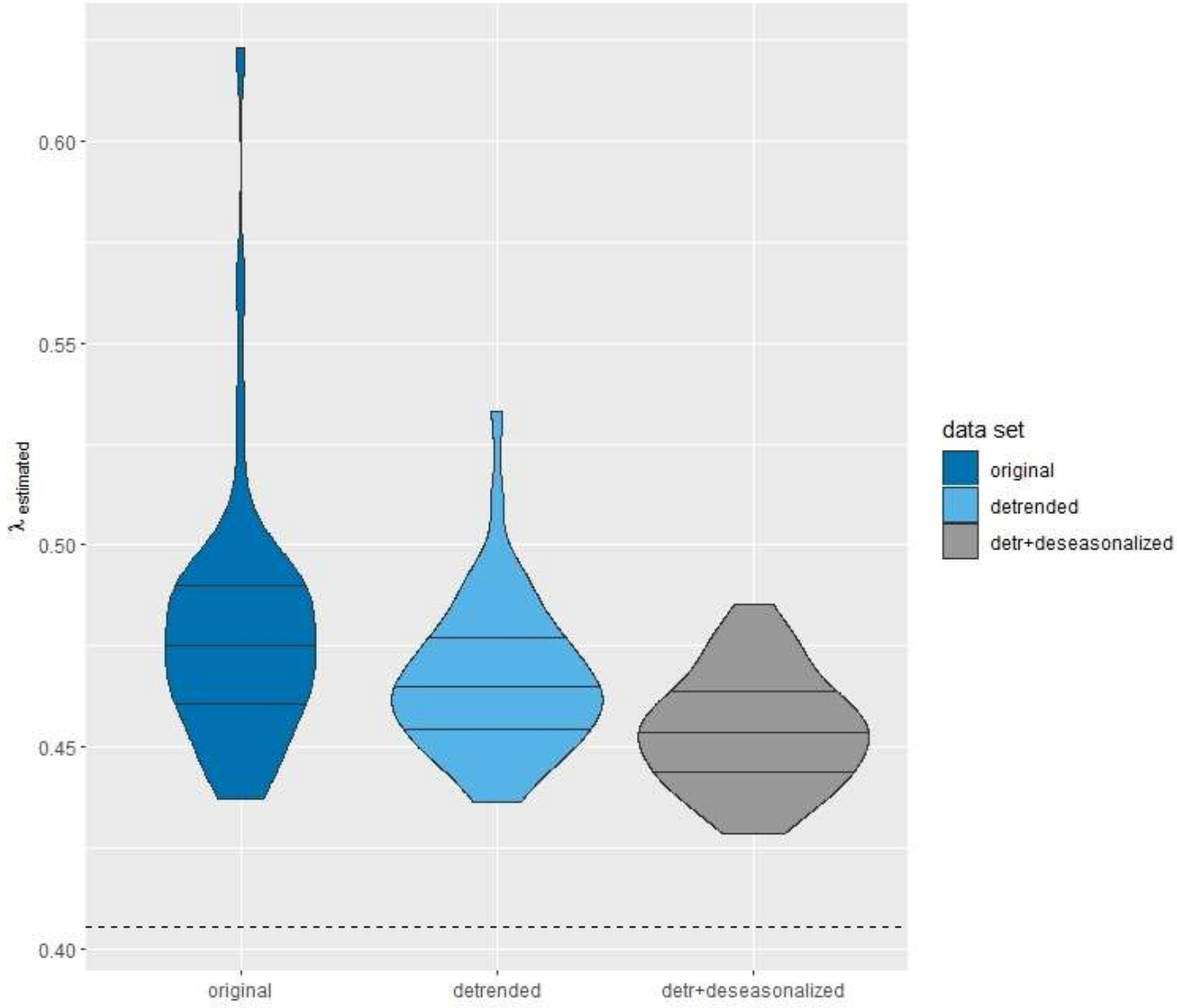
Lange et al. (2018)

How to estimate λ: one example

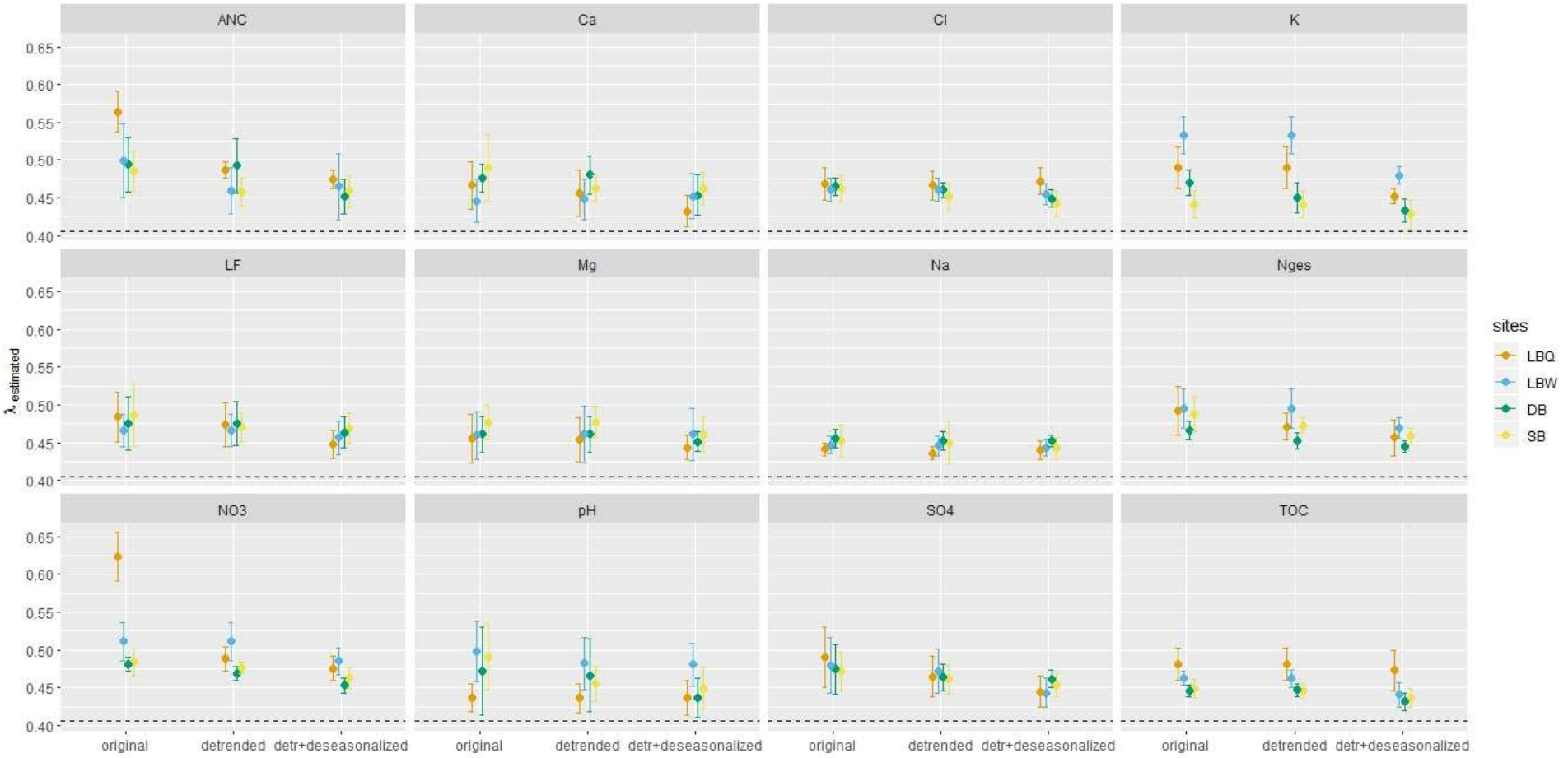
NO_3 detr. + des. at LBW



λ estimates for the Bramke data



Results for the λ estimates

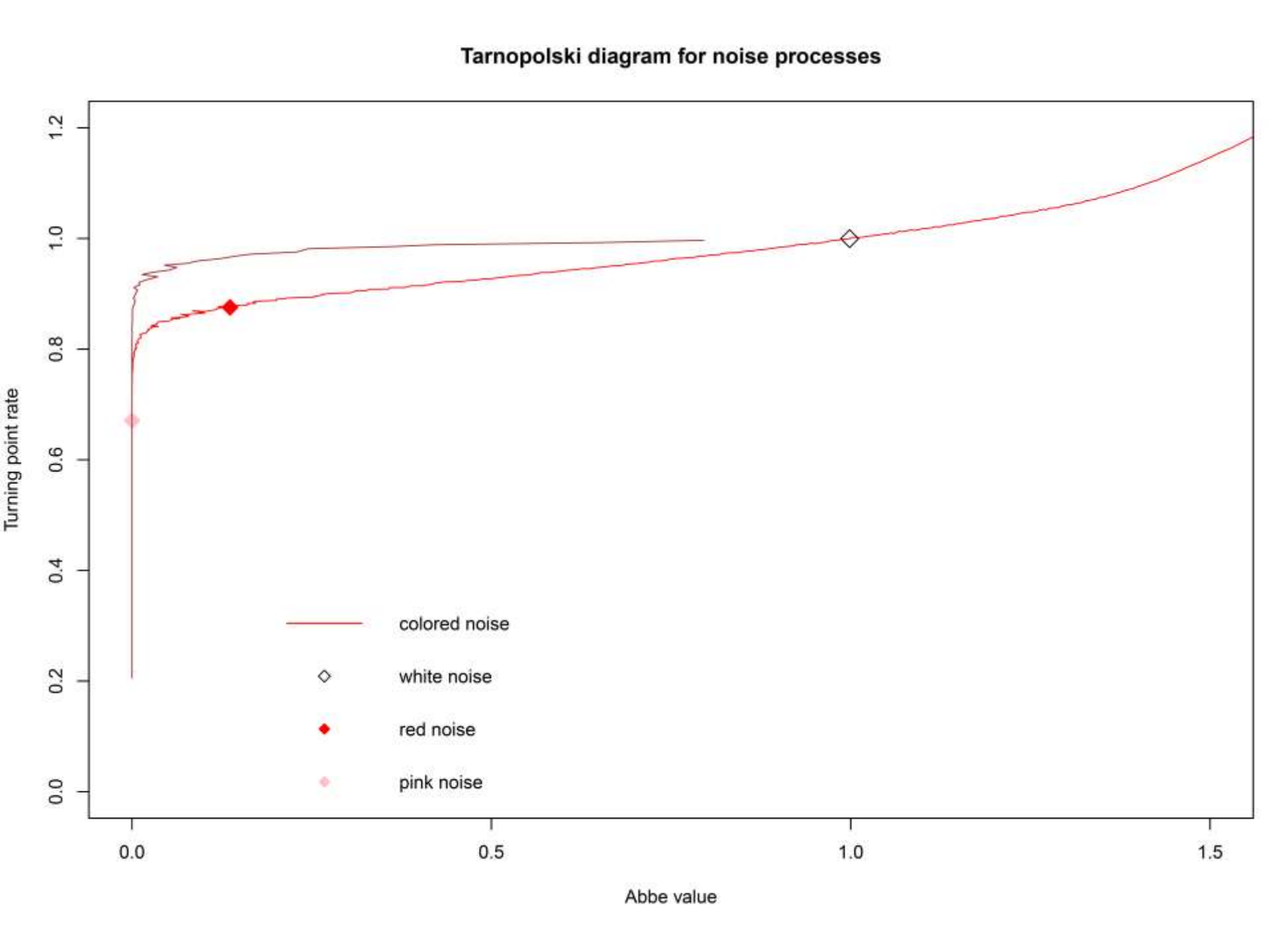


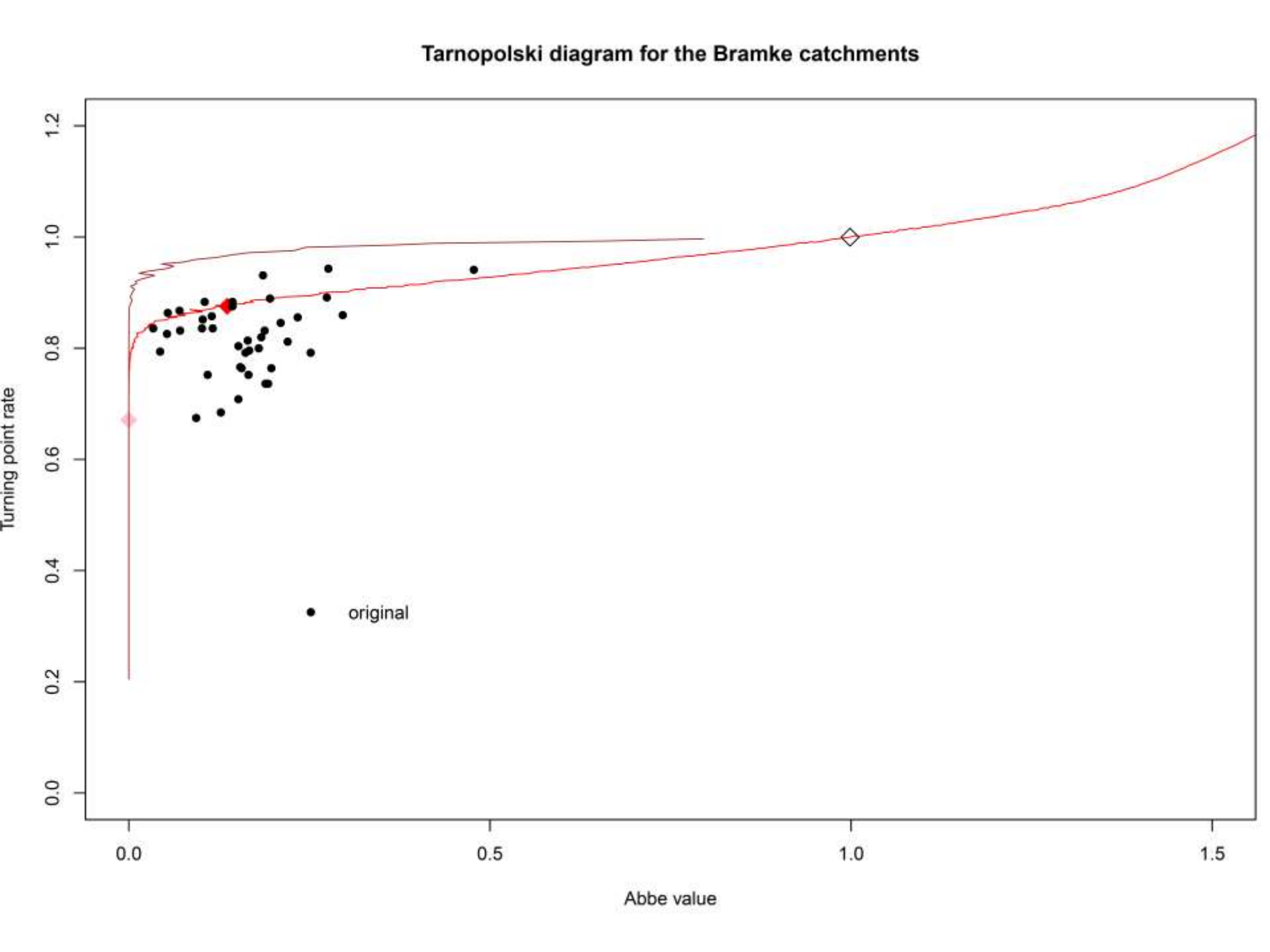
Digging deeper into the noise: Tarnopolski analysis

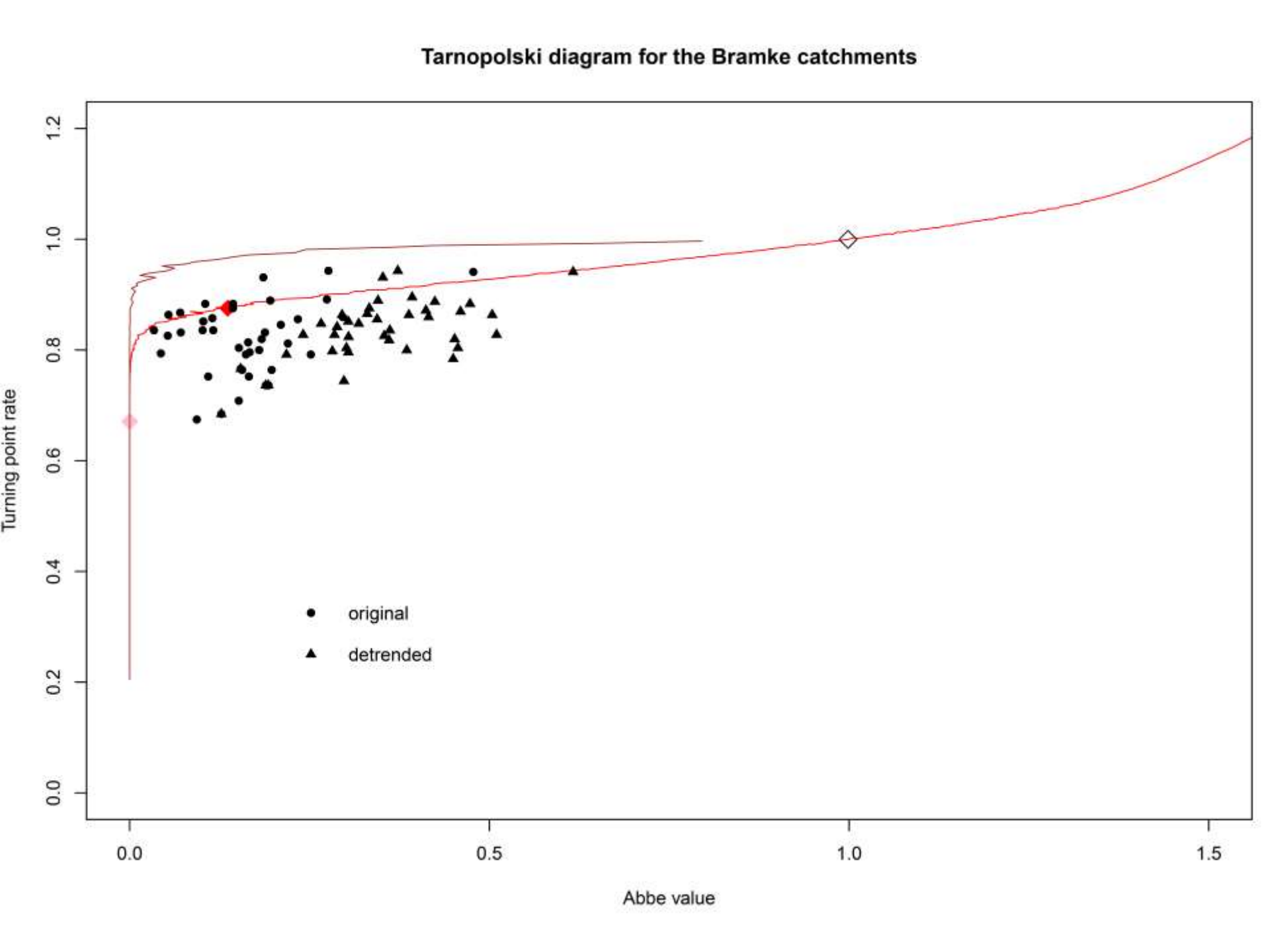
For a given time series, calculate two quantities:

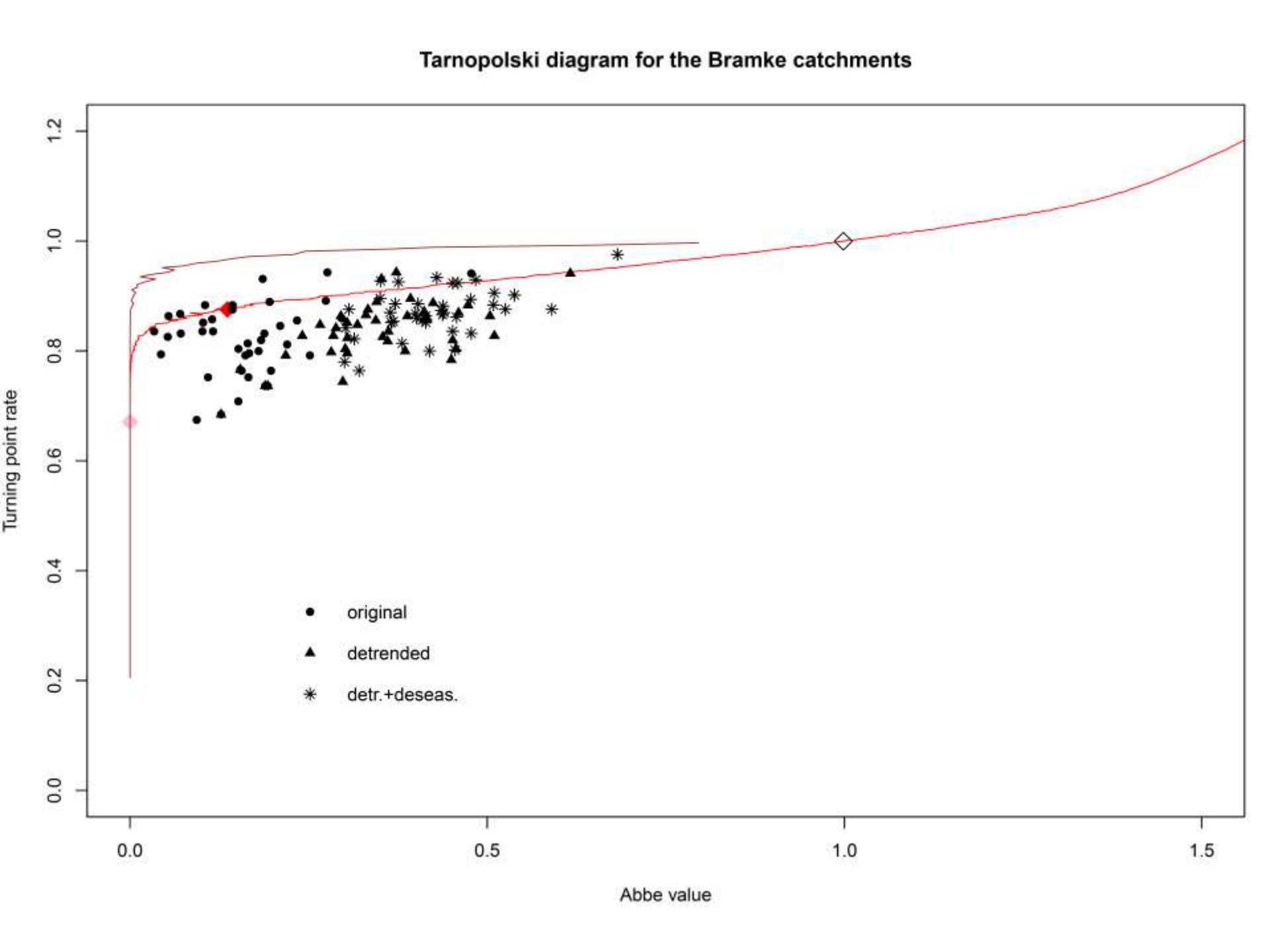
- The Abbe value $\mathbb{A}$: ratio of the mean square successive difference to the variance
- The number of turning points $\mathbb{T}$: count the local «extrema»
- Normalize such that for white noise, ($\mathbb{A}$, $\mathbb{T}$) = (1,1)
- Plot in a 2D diagram
- This easily distinguishes different types of noise

M. Tarnopolski, Physica A 416 (2016), 662-673


Tarnopolski diagram for noise processes
Turning point rate
0.0
0.2
0.4
0.6
0.8
1.0
1.2
0.0
0.5
1.0
1.5
Abbe value
colored noise
white noise
red noise
pink noise


Tarnopolski diagram for the Bramke catchments
Turning point rate
0.0
0.2
0.4
0.6
0.8
1.0
1.2
0.0
0.5
1.0
1.5
Abbe value
original


Tarnopolski diagram for the Bramke catchments
Turning point rate
0.0
0.2
0.4
0.6
0.8
1.0
1.2
0.0
0.5
1.0
1.5
Abbe value
original
detrended


Tarnopolski diagram for the Bramke catchments
Turning point rate
0.0
0.2
0.4
0.6
0.8
1.0
1.2
Abbe value
0.0
0.5
1.0
1.5
original
detrended
detr.+deseas.

Conclusions, Questions and Outlook

- The Bramke data are a very rich source of information on catchment dynamics
- HVG is a sensitive tool to investigate temporal dynamics
- Hydrochemical time series are on the «correlated noise side» of degree distributions, independent of pretreatment
- But: are degree distributions from the Bramke exponentials?
- Are the time series correlated noise, or something else?
- Outlook:
  - Other network properties (distance distribution, clustering coefficient, network transitivity, …)
  - extensions of HVG (limited penetrable, non-horizontal, multivariate, …)