17th International Conference on the Applications of Computer
Science and Mathematics in Architecture and Civil Engineering
K. Gürlebeck and C. Könke (eds.)
Weimar, Germany, 12–14 July 2006

APPLICATION OF THE PHASE TRAJECTORIES MAPPING TO IDENTIFICATION OF DISSIPATIVE CHARACTERISTICS

V. Volkova*

* *Dnepropetrovsk National University of the Railway Transport, Dnepropetrovsk, Ukraine*
Televizionnaya str.12, apt.48, Dnepropetrovsk, 49005, Ukraines
E-mail: drvev@mail.ru

Keywords: structural friction, phase trajectories, identification.
Abstract. *The reduction of oscillation amplitudes of structural elements is necessary not only for maintenance of their durability and longevity but also for elimination of a harmful effect of oscillations on people and technology operations. The dampers are widely applied for this purpose. One of the most widespread models of structural friction forces having piecewise linear relation to displacement was analysed. T The author suggests the application of phase trajectories mapping in plane "acceleration – displacement". Unlike the trajectories mapping in a plane "velocity – displacement", they don't require large number of geometrical constructions for identification of the characteristics of dynamic systems. It promotes improving the accuracy. The analytical assumptions had been verified by numerical modeling. The results show good enough coincide between numerical and analytical estimation of dissipative characteristic.*

1 INTRODUCTION

The decreasing of oscillation amplitudes of structural elements is necessary not only for maintenance of their durability and longevity, but also for elimination of an ill effect of oscillations on the people and operations. The dampers are widely applied for this purpose [2]. The urgency of a given investigation is caused by a diversification of the types of dampers, conditions of their exploitation and types of outer excitements.

There a lot of studies devoted of influence of forces of structural friction on dynamic behaviour of separate structural elements [2,5,11,12]. The analysis of activities, accessible to familiarization, indicates to necessity of further development of methods of prediction of dynamic behaviour of dynamic systems with structural friction.

The purpose of the given paper is the detection of features of phase trajectories and mappings on planes "acceleration – displacement" and "acceleration – speed" for systems with piecewise linear structural friction.

2 PHASE TRAJECTORIES IN THE ANALYSIS OF NON-LINEAR DYNAMIC SYSTEMS

The qualitative investigation of behaviour of a dynamic system is reduced to analysis of behaviour of trajectories in a phase space. A fundamentals of the qualitative theory of investigation of dynamic processes were built up by A. Poincare. The exclusive role in development of qualitative method of dynamic systems investigation belongs to A.A. Andronov [1]. E.A. Leontovich, I.I. Gordon, A.M. Lyapunov. Primary goal of the classic theory of qualitative investigation is the definition of dynamic properties of systems without obtaining the selfcontained analytical solution. With this purpose the phase trajectories on a plane $(y, \dot{y})$ were widely used.

The branch of application of these methods did not limit by problems of autonomous oscillations. Forshing in [9], the phase trajectories in a plane $(y, \dot{y})$ for finding of the aeroelastic oscillations period were used by Forshing in [9]. It is known the attempts to apply the trajectories in plane $(y, \dot{y})$ in the solution of a reverse problem of mechanics - identification. So, in [6] the author had received numerical estimations of the dissipative characteristics in isolated points of phase spaces using Shafer's method. As against the mentioned above studies, the purpose of the given paper is the obtaining not of numerical estimations of dissipative characteristics parameters, and their generalized graphic image, which one is more convinient for after treatment.

The method of dot mappings has appeared simultaneously with the classic qualitative theory of differential equations in basic studies of A.Pounacre. Further development the method has received in activities N.V. Butenin, J.I. Nejmark, Shil'nikov and others. The mappings become the form of the description of dynamic systems, friend for concrete numerical of investigations.

In the given paper the procedure of identification of the non-linear dissipative characteristics by mappings of phase trajectories on planes "acceleration – displacement" and "acceleration – velocity" are shown.

There are two in principal different approaches in construction of mathematical models of real mechanical systems of mechanics exist.

The first approach realized on a design stage of flexible structural elements by idealized compiling of mathematical models, founded on usage of the fundamental laws of mechanics of discrete and continuum systems expressed by analytical relations.

The second approach consists in usage of relations of known mathematical form, but with unknowns of parameters and characteristics. These unknown parameters are determined by processing on special algorithms of dynamic tests results of structure or their physical analogs made with observance of principles of a theory of similarity and dimension in mechanics. This approach also has received a title of identification of real objects.

The first and most responsible phase of identification is the detection, classification and localization of non-linearity. Despite of a significant amount of methods of qualitative identification, the unified approach misses. The majority of these methods are based on usage of the special types of an outer excitement for a wide range of frequencies. For example, as a symmetrical monoharmonic excitement and square wave. Those types of a excitement can not be always realized in mechanical systems. The methods basing on a Fourier transform, don't allow to classified and to localize non-linearity [8] and they are not applicable to investigation of stochastic processes [3,7]. Let's note also, that the application of Wiener and Hilbert series for identification of burst types of non-linearities is not correct. [8,10].

3 METHODS OF SIMULATION OF DYNAMIC BEHAVIOR OF MECHANICAL SYSTEMS

There are two radically different approaches to construction of mathematical models of real mechanical systems of mechanics: design&theoretical and theoretical&experimental ones.

The first approach is being realized on a design stage of flexible structural elements by theoretical compiling of mathematical models based upon the usage of the fundamental laws of mechanics of discrete and continuum systems expressed by analytical relations.

The second one consists in the usage of relations of known mathematical structure but with unknown parameters and characteristics. These unknown parameters are determined by processing according to special algorithms of dynamic tests results of structures themselves or their physical analogues manufactured with observance of principles of a similarity-and-dimension theory in mechanics. This approach also had received a title of "identification of real objects".

The first and most responsible phase of identification is the detection, classification and localization of non-linearity. Despite the significant amount of methods of qualitative identification, the unified approach is absent. The majority of these methods are based on usage of the special types of an outer excitation for a wide range of frequencies, for example, such as a symmetrical monoharmonic excitation and square wave. These excitation types can not be always realized in mechanical systems. The Fourier-transform-based methods don't allow classifying and localizing non-linearity [8] and they are not applicable to investigation of stochastic processes [3, 4]. Let's also note that the application of Wiener and Hilbert series for identification of disruption types of non-linearities is not correct [5, 6].

It is customary to revel the presence of structural friction forces by the form envelope of damped oscillations process is used. The amplitudes of oscillation is decreasing under the law of arithmetic regression for systems with viscous friction and by the law of geometrical regression for the systems with structural friction. Let's note that in some cases such indicator is not illustrative.

4 DIFFERENTIAL EQUATION OF FORCED VIBRATIONS OF A SYSTEM WITH STRUCTURAL FRICTION

Let's consider forced vibrations of a mechanical system with structural friction. We studied system are described by a differential equation of a view

$$m\,\ddot{y} + h(y,\,\dot{y}) + r(y) = 0\,. \tag{1}$$

where m is mass; $y, \dot{y}, \ddot{y}$ are generalized co-ordinates, velocity and acceleration; $r(y)$ is elastic force; $h(y,\,\dot{y})$ - force of structural friction.

One of the most widespread models of structural friction forces having piecewise linear relation to displacement were analyzed. The graphs of a time processes of forces of structural are adduced for the most widespread models of structural are given in [5].

Let's note, that the phase space of dynamic systems is multi-dimensional. Diverse selection of parameters of phase planes is also possible. For the first time attempt to apply phase trajectories on planes $(y,\ddot{y})$ and $(\dot{y},\ddot{y})$ to investigation of dynamic systems was made in the monograph [4]. As follows from the obtained results, the phase trajectories on a plane $(y,\ddot{y})$ can be rather effectively utilized for identification of dynamic systems. In the monograph [4] the results of qualitative investigation of oscillations of conservative systems having the non-linear dissipative and elastic characteristics of different types are shown.

The most simplest model of structural friction is Coulomb model $h(y,\,\dot{y}) = H_1\;sign\,\dot{y}$. Most prime model of dry friction. It was taken up in the analysis of dynamic behavior of structures under seismic loading.

The relation $h(y,\,\dot{y}) = H_1\,(1 + y/y_c)\,sign\;\dot{y}$ is used for the mechanical systems with initial static displacement. So the given model describes behavior of sheet spring plates and ground basis.

5 TECHNIQUE OF A NUMERICAL MODELING

For obtaining parametric relations $y(t)$. $\dot{y}(t)$. $\ddot{y}(t)$ the software was built. In its basis the method of Runge - Kutta of the fourth order utilized. The integration step was adopted from a stability condition of a procedure of a numerical integration at different parameters of an equation (1). He has compounded $\Delta t = T/600$, where T - a period of free oscillation, that provided stability. It is known, that the free oscillations of non-linear systems are not monoharmonic. Estimation of influence of separate harmonics in the software the unit of spectral analysis utilized. It realized on the basis of algorithm of a fast Fourier transform.

6 ANALYSIS OF THE OBTAINED RESULTS

The correlation of three models of the dissipative characteristics was performed: 1) viscous friction model $h(y, \dot{y}) = H_1 \, \dot{y}$, 2) Coulomb model $h(y, \dot{y}) = H_1 \; sign \; \dot{y}$ and 3) structural friction $h(y, \dot{y}) = H_1 \left(1 + y/y_c\right) sign \; \dot{y}$.

The outer asymmetrical periodic excitation of the following form was applied to a system (1)

$$P(t) = P_1 \cos \omega t \quad . \tag{2}$$

The numerical modeling had been executed for the following parameters of a system (1): $m = 1$; $H_1 = 1$; $y_c = 0.5$. Amplitudes of harmonic excitement had taken the values $P_1 = 0.1; 0.5; 1$. The results of numerical modelling are also shown in Figs. 1- 2.

As follows, from given relations the presence of structural friction not only results in amplitudes of resonance oscillations decreasing, but also to a considerable decrease of resonance frequencies. So, system with viscous friction has the resonance frequency $\omega_{r,1} = 0.69 \; rad/s$, and system with Coulomb friction - $\omega_{r,2} = 0.12 \; rad/s$; for a system with structural friction $\omega_{r,3} = 0.16 \; rad/s$.

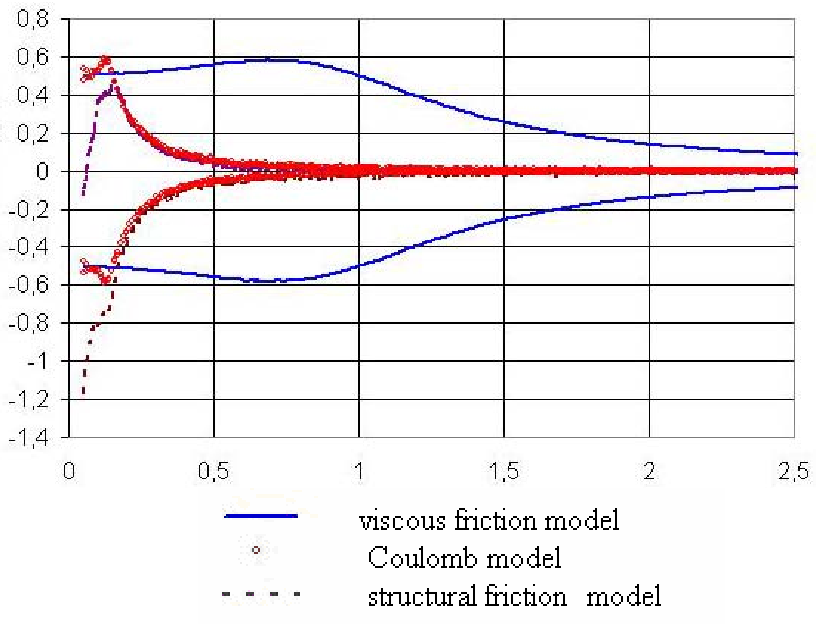

Figure 1. Amplitude-frequencies characteristics of dissipative systems

Analysing the presented relations in a Fig. 2 for transient processes of systems with Coulomb and structural friction, one may mark following to regularity. Both processes (Fig. 2.a, b) represent monoharmonic damped oscillations close by the form to oscillations of systems with viscous friction. The phase trajectories on planes $\left(y, \dot{y}\right)$ look like untwisting spirals.

In contrast to systems with viscous friction, the time processes $\ddot{y}\left(t\right)$ of both systems (Fig. 2) it is possible to note instantaneous decreasing of acceleration in points of their maximum values. However, value of jumps for positive and negative values of accelerations equal and constants for systems with Coulomb friction. While for systems with structural friction, values of jumps a different. It depends, both on the sign of acceleration $\ddot{y}(t)$, and value of displacement $y(t)$.

The hysteretic loops on a phase plane $(y, \ddot{y})$ for systems with Coulomb friction back are symmetric concerning origin of the coordinate. It changes length with a constant step, distance between their orbits are constant.

The phase trajectories on a plane $(\dot{y}, \ddot{y})$ represent members of elliptical arcs. It displaced from each other in the left and right half-planes.

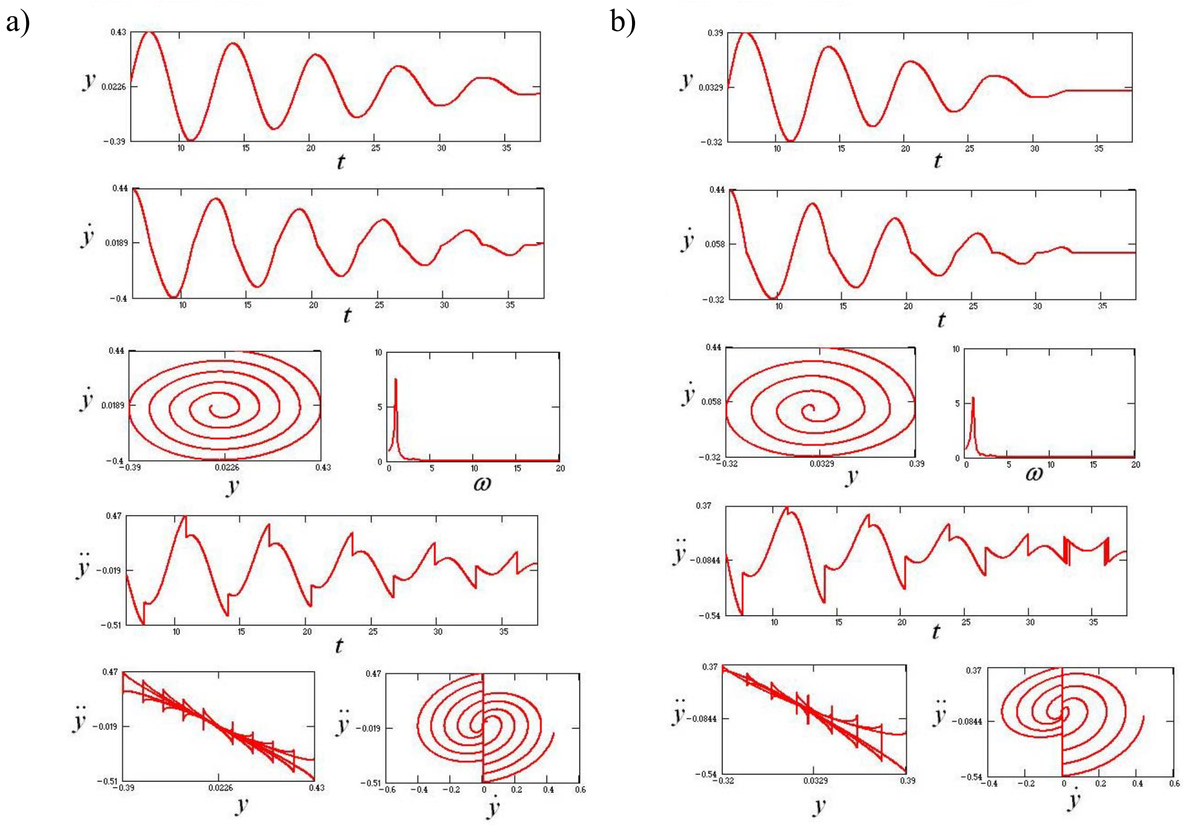

Figure 2. Time processes, phase trajectories and spectral characteristics of dissipative systems: a) a system with Coulomb friction; b) a system with structural friction.

The phase trajectories on a plane $(y, \ddot{y})$ for systems with structural friction are nonsimmetric concerning an axis y.

7 CONCLUSIONS

The purpose of the given paper is obtaining a generalized geometrical presentation of the dissipative characteristics of a dynamic system. The result is that for the solution of the given problem the usage of a phase plane "acceleration – displacement" is more effective than a phase plane "velocity – displacement". E. g., for an estimation of parameters of the characteristics the necessity of geometrical constructions is eliminated that leads to improving the accuracy.

REFERENCE

[1] Andronov A.A., E. A. Leontovich, I. I. Gordon, A. G. Mayer, *The Qualitative Theory of Dynamic Systems of the Second Order*.Main publishing house. State publishing house of the physical and mathematical literature. Moscow,1966. (in Russian)

[2] *Vibrations in Engineering: the Reference Book in 6 vols. ed. Blekhman I. I.* Mashinostroeniye Publ. House. - Moscow: V. 2 “Oscillations of Non-linear Mechanical Systems”. 1979.. (in Russian)

[3] Groop, I*dentification of Mechanical Systems*. “Mir” Publ. House. Moscow, 1979.

[4] Kazakevitch M. I., Volkova V. E., *Phase Trajectories of Non-linear Dynamic Systems. The atlas.* Nauka i Obrazovaniye Publ. House, Dnepropetrovsk, 2002. (in Russian)

[5] Kulyabko V. V., About Non-linear Characteristics and Properties of Some Positional Slip Dampers. Mashinovedeniye (Mashine Sciences), **3**, 29-32,1980.(in Russian).

[6] Plakhtiyenko M. P., On Determination of Characteristic of Non-linear Oscillating System upon Phase Trajectory Analysis. *Dopovidi AN URSR. Ser. A (Transactions of the Academy of Sciences of the Ukrainian Soviet Socialist Republic. Part A)*, **4**,336-338, 1976(in Ukrainian).

[7] Plakhtiyenko M. P., Methods of Identification of Mechanical Oscillating Systems. *Prikladnaya mekhanika (Applied Mechanics)*. **36**,38-68, 2000.(in Russian).

[8] Adams D. E., Allemang R. J., Survey of Nonlinear Detection and Identification Techniques for Experimental Vibrations.I*SMA23*, 1998.

[9] Forshing H., Manea V., Zur analytischen Behandlung des nichtlinearen aroelastischen Galloping Problems. *Ing. Archiv Bd*, **42**, 178-193, 1973.

[10] Ghanem R., Romeo F., A Wavelet-based Approach for Model and Parameter Identification of Non-linear Systems. *Int. J. of Non-Linear Mechanics*, **36**, 835-859, 2001.

[11] .Kalman, Peleg, Dynamic Restoring and Dissipative Parameters of Non-linear Systems. *J. of Mechanical Scienc*, **23**,No. 10, 595-605, 1981

[12] Weber B., Damping of Vibrating Footbridges. *Footbridge-2002*, Paris,2002.

[13] K.-J. Bathe, *Finite Element Procedures*. Prentice-Hall Inc., New Jersey, 1996.

[14] C. V. Thompson, Grain Growth in Thin Films. *Annual Review of Materials Science*, **20**, 245-268, 1990.

[15] A. R. Ingraffea, E. Iesulauro, K. Dodhia and P. A. Wawrzynek, A Multiscale Modeling Approach to Crack Initiation in Aluminum Polycrystals. H. A. Mang, F. G. Rammerstorfer and J. Eberhardsteiner eds. *Fifth World Congress on Computational Mechanics ,* WCCM V, Vienna, Austria, 2002.