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MATHEMATICAL DESCRIPTION OF SELF-EXCITED VIBRATIONS IN SILO WALLS

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Abstract. *The paper contains a description of dynamic effects in the silo wall during the outflow of a stored material. The work allows for determining the danger of construction damage due to resonant vibrations and is of practical importance by determining the influence of cyclic pressures and vibro –creeping during prolonged use of a silo. The paper was devised as a result of tests on silo walls in semi-technical scale. The model is generally applicable and allows for identification of parameters in real- size silos as well.*

1 INTRODUCTION

The design norms and the dimensioning tradition of silo walls and funnels assume that the constructional elements (walls and funnels) are subjected to static load. The pressure value is influenced by many factors, such as stored material's bulk density, silo's slenderness ratio, mass velocity, etc. The dynamic effects of the silo emptying processes are taken into account by increasing factor. Its value is determined experimentally by measuring the pressure on silo walls during the flow of a given material. This mode of calculating the load capacity of a construction and utilization conditions proves correct, if there is no danger of resonant vibrations.

This paper presents the method of describing the vibrations of the silo walls during the flow of the material. The source of the vibrations is the friction. The method is based on differential equations taken from the theory of structural analysis and dynamics of buildings. The paper is limited to the presentation of the assumptions and the research method on the chosen example models. The current research concentrates on detailed solutions leading to compatibility with tests on objects.

Niniejszy referat pokazuje jak opisać drgania ściany silosu podczas przepływu materiału. Źródłem drgań jest zjawisko suchego tarcia (). Jako bazę przyjęto równania różniczkowe z teorii statyki i dynamiki budowli. Referat ograniczono do przedstawienia założeń i pokazania metody badawczej na wybranych przykładach modeli. Zagadnienia dotyczące szczegółowych rozwiązań w celu uzyskania zgodności z wynikami badań na obiektach są przedmiotem aktualnych badań.

2 RESEARCH RESULTS

The friction of the flowing material against the silo walls causes self-excited vibrations. The characteristic feature of the self-excited vibrations is the permanent source of complemented energy, which is dispersed due to friction.

There are two kinds of self-excited vibrations of a silo during the outflow of a loose material. One kind appears in a silo with deformable walls and a relatively stiff medium (e.g. monofractional sand). In the other case the material is deformable. In the other case the material is deformable whereas the walls may be rigid.

The vibrations appearing in the silos with deformable walls create cyclic acceleration and deceleration of the outflow. The changing pressure on the walls consists of two basic waves:

a) a main wave, characterized by a non-uniform outflow

b) a wave of much higher frequency due to the motion of the wall itself.

Depending on the speed of the outflow, the dynamic effects of the pressure on the silo walls may even be very heavy, surpassing other effects. Unless the designer anticipated such effects of self-excited vibrations, damage to the silo construction may even occur.

In the case of silos with rigid walls and a deformable material, the vibrations will also create pulsation of the outflow speed and pulsation of pressure on walls.

The self-excited pulsation effect, known from real object observations as well as laboratory tests, has not yet been described in the form of an algebraic calculation model allowing for a quantitative analysis of the effect.

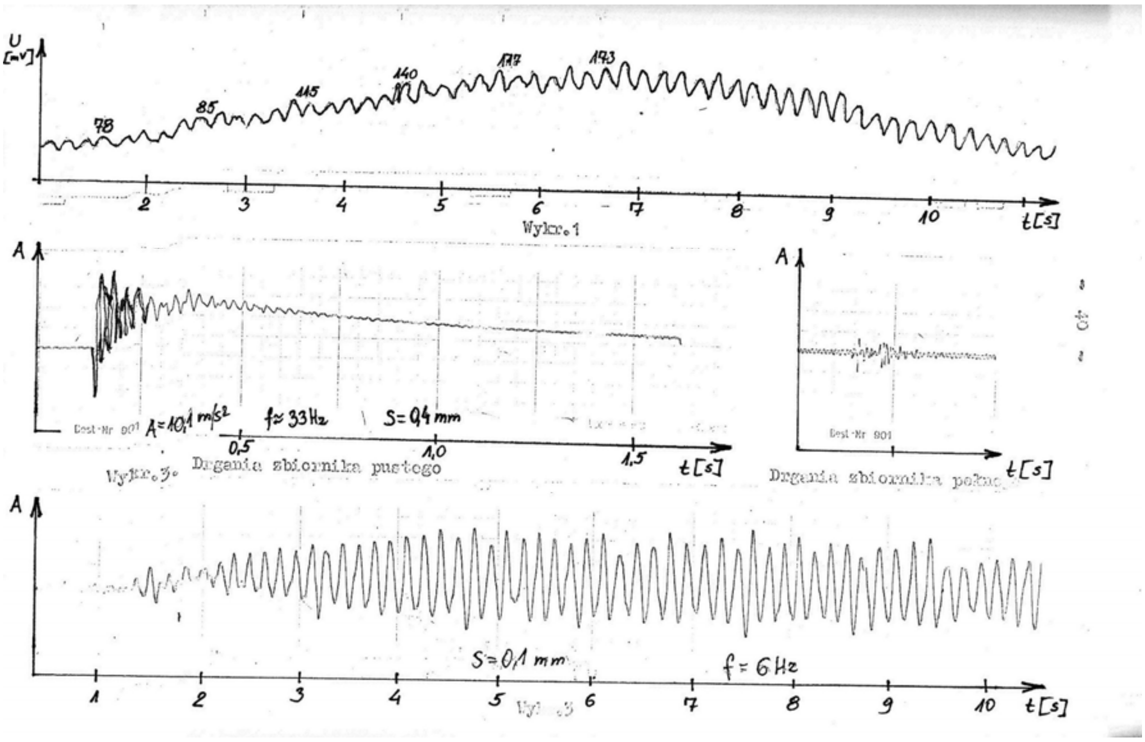

Fig. 1. Example test result. X-axis – time[s]:

a) Pressure on the silo wall during the outflow of material
b) Proper vibration of an empty silo
c) Proper vibration of a full silo
d) Amplitude function during the outflow of material

3 ALALYSIS OF SELF-EXCITED VIBRATIONS INDUCED BY FRICTION

3.1 Analysis assumptions

The starting point is the equations of motion for one-dimensional continuous systems together with assumptions made when deriving them in the classic dynamics. This model is justified by geometry of silos, in which there is a significant polarization of the deformation and stress patterns.

The characteristic assumption made for this problem is the proportionality of the deflections perpendicular to the silo wall to the corresponding pressure values.

The formulation of the equation’s right side, characteristic to self-excited vibrations, marks an important point in the analysis. The exciting force may be proportional to the wall deflection, their speed and acceleration. Introducing numerous parameters allows for a fuller experimental analysis and a more precise description of the effect. However, in many cases a large amount of parameters may, make obtaining a closed/particular solution impossible.

The formulated models were presented for the axial-symmetrical problem.

3.2 Models of a self-excited system with a deformable wall

There are several possibilities of formulating a model of a self-excited system. It can be assumed, that the effect is due to the force acting in the form of concentrated masses (granulation of masses). The structure of mass field can be left uniform in sections, and the state of deflections can be described in a limited base of parameters. These issues have been dealt with by Boroch and Langer [4]. Another important problem of formulating the model is the wall's rigidity. When determining the dynamic parameters (frequency and amplitude), a board of certain flexural rigidity or as a membrane, whose flexural rigidity is disregarded, can be used as a model of a wall. In the axial-symmetrical problem, it is possible to use a beam or a string, respectively.

A membrane of a unitary width (a string), mounted on two opposite ends, was accepted as a model. According to the author, it is the simplest model for describing a wall in a silo with circular cross-section, yet one that allows for a quite pictorial presentation of hte essence of self-excited vibrations induced by dry/Coulomb friction. Fig.1 shows a sector of a wall subjected to both horizontal pressure $p_h(x)$ and quasi-harmonic pressure $p_h(x,t)$. Furthermore, $N(x,t)$, $N(x+\Delta x,t)$ express local longitudinal force and $\alpha(x,t)$, $\alpha(x+\Delta x,t)$ – deflection. The string load $p_h(x,t)$ is a shearing force acting on a length unit. The balance condition ΣX (vertical axis), incorporating the movement forces and inertial forces (d'Alembert's principle), yields:

$$N(x+\Delta x,t)\sin\alpha(x+\Delta x,t) - N(x,t)\sin\alpha(x,t) = \Delta x\left[m\frac{d^2w(x,t)}{dt^2} - p_h(x,t)\right] \quad (1)$$

where: m – mass on length unit; $\ddot{w}$ –acceleration

The expression $m\ddot{w}$ is the inertial force, resulting from Newton's second principle of dynamics.

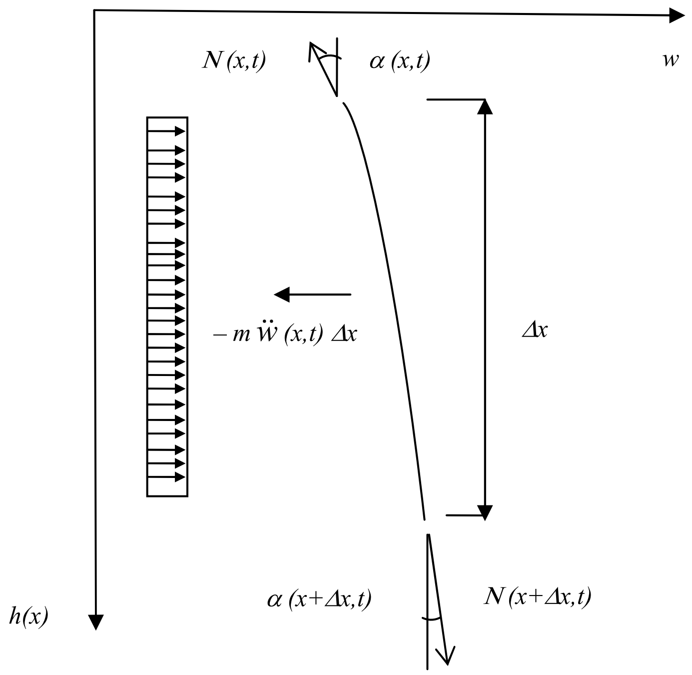

Fig. 2. The string section and the distribution of forces

If $\Delta x \rightarrow 0$, $\sin \alpha \rightarrow dw/dx$ and for a constant on the segment Δx the value of the shearing force N equation (1) takes the form:

$$Nw''(x,t) - m\ddot{w} = -p_h(x,t) \tag{2}$$

This equation has a general character. In order for the exciting force to induce self-excited vibrations it has to depend on deflection or its derivatives; in this case - on the wall deflection, its speed and acceleration or n the speed of the material's motion inside the silo. Some authors (e.g Ryczek [ ]) accept the static friction force f_s as an auxiliary parameter. However, it is necessary to separate the acting of forces in a self-excited system from the static or quasi-static pressure that appears during the storage, filling or emptying a silo. Hence, the general form of the non-heterogeneous equation can be formulated as follows:

$$Nw''(x,t) - m\ddot{w} = -p_{h1}(x,t) - p_{h2}(x, \frac{dx}{dt}, \frac{d^2x}{dt^2}, f_s, t) \tag{3}$$

3.3 Models of a self-excited system with a rigid wall

The rigidity of a non-deformable wall has to be taken into account. Similarly to the case of a deformable wall, a section of unitary width is analysed. Basing on general relations form the mechanics of materials, geometrical relations and taking into account physical conditions, the equation for small deflections looks as follows:

$$M(x,t) = -EI(x)\,\frac{d^2w(x,t)}{dx^2} \tag{4}$$

The condition of the balance of forces (fig.3) yields:

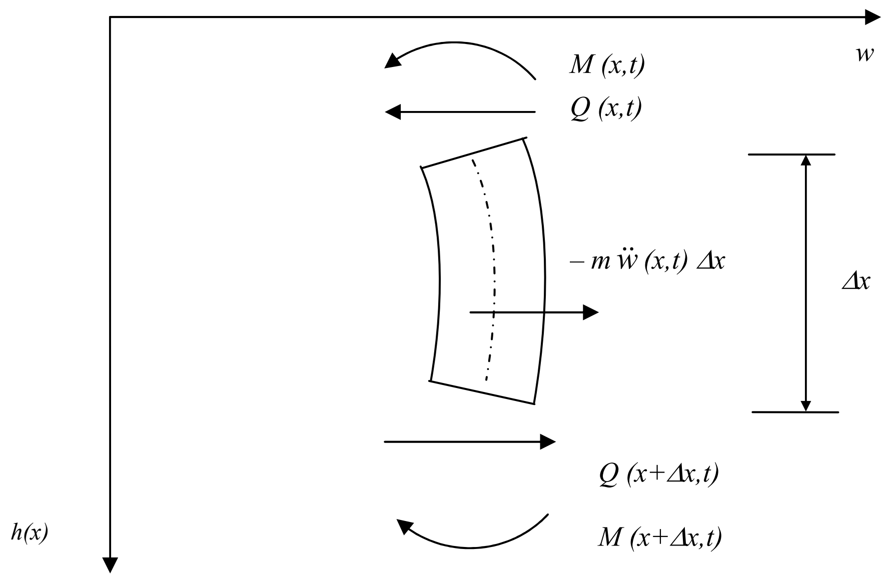

Fig. 3. Segment of wall and distribution of forces

$$- m\,\ddot{w}\,(x,t)\,\Delta x + Q\,(x+\Delta x,t) - Q\,(x,t) + p_h(x,t)\,\Delta x = 0 \qquad (5)$$

Similarly to equation (2), by $\Delta\xi \rightarrow 0$ the outcome is:

$$Q'(x,t) = m\,\ddot{w}\,(x,t) - p_h(x,t) \qquad (6)$$

Equation (4) yields:

$$Q(x,t) = \frac{d}{dx} M(x,t) = \frac{d}{dx} \left[- EI\,(x)\, \frac{d^2 w(x,t)}{dx^2} \right] \qquad (7)$$

And after inserting (7) to (6) the outcome is:

$$\frac{d^2}{dx^2} M(x,t) = \frac{d^2}{dx^2} \left[- EI\,(x)\, \frac{d^2 w(x,t)}{dx^2} \right] = m\,\ddot{w}\,(x,t) - p_h(x,t) \qquad (8)$$

In silos the rigidity of wall on a height is constant in segments, which allows for accepting the expression for rigidity EI (x) = const. and simplify the equation (8):

$$EI\,(x)\, \frac{d^4 w(x,t)}{dx^4} + m\,\ddot{w}\,(x,t) = p_h(x,t) \qquad (9)$$

Similarily to the previous case, it is possible to separate the part of the excitement caused by vibrations in the self-excited system:

$$EI\,(x)\, \frac{d^4 w(x,t)}{dx^4} + m\,\ddot{w}\,(x,t) = p_{h1}(x,t) + p_{h2}\left(x, \frac{dx}{dt}, \frac{d^2 x}{dt^2}, f_s, t\right) \qquad (10)$$

In practice, the model is a good approximation for most silos with circular cross-section.

4 SUMMARY

The models presented in the paper apply to silos with axial-symmetrical cross-section. The most important application of the equations is the utilization for calculating the vibrations in silos. The outcome of tests on actual scale silos will allow for adjusting the parameter values so that the solution describes the real conditions. This should enable designers to prevent unwelcome and difficult-to-predict effects, such as resonant vibrations. It will also facilitate predicting the effects of fatigue more precisely.

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