MOVEMENT DETECTION AND RECOGNITION WITH QUATERNION WAVELETS

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ABSTRACT

We show how a Morlet type quaternion wavelet may be tuned to a pair of stereoscopic films in order to detect moving objects and calculate its velocity shape volume and other characteristics. We show some practical examples of simple objects taken with normal pocket avi cameras.

Introduction

hen we talk about movements normally authomatically, one is prone to think in trajectories forgeting that movements are a lot more, which is precisely expressed by quaternionic functions. In the same way when we think in wavelets think in wavelets we have the tendency to associate them to its most common uses like signal processing or image compression. That is the reason why most attempts to use quaternion wavelets since I invented them back to 1993 (and published about it in 1995 Traversoni[1]) were related to image processing.

We will try to be focused in which kind of data we will be dealing with, from where it comes and which are the preproccessing procedures that have to be performed before its use.

If we are speaking about a movement to describe it we need the trajetory the velocity the acceleration that is we need equations able to tell us in any instant where is the moving object, which is its line velocity its angular velocity and its acceleration. Describe a random movement as the composition of several simpler movements is a very good tool to understand and predict such a movement, remeber for example the description of the movements of the planets keystone of the modern science.

The data however, needs always some preprocessing because may be for example a series of photographs of the moving object (a movie) or periodical signals like radar signals. The preprocessing consists then in determining the set of quaternions or quaternion functions that will form our database starting with that data that consists basically in positions at given times.

Locating an object in 3D

We assume that some observations from several cameras scanners or other devices (at least two)are given of the object. From them we will deduce the position of the object in the space, in the following section we show how to do it using a quaternionic interpretation of the main operations of projective geometry as well as a quaternionic definition of simple figures like point line and plane, useful to achieve our goal.

We define a transformation f_p mapping vectors of E^3 in vectors of R^4 its action over the base vectors γ_i is given by:

$$
\begin{aligned}
f_p(\gamma_1) &= \alpha_1\gamma_1 + \alpha_2\gamma_2 + \alpha_3\gamma_3 + \overline{\alpha}\gamma_4 \\
f_p(\gamma_2) &= \beta_1\gamma_1 + \beta_2\gamma_2 + \beta_3\gamma_3 + \overline{\beta}\gamma_4 \\
f_p(\gamma_1) &= \delta_1\gamma_1 + \delta_2\gamma_2 + \delta_3\gamma_3 + \overline{\delta}\gamma_4 \\
f_p(\gamma_1) &= \epsilon_1\gamma_1 + \epsilon_2\gamma_2 + \epsilon_3\gamma_3 + \overline{\epsilon}\gamma_4
\end{aligned}
$$

When we use homogeneous coordinates a general point P in E^3 given by $\mathbf{x} = x\sigma_1 + y\sigma_2 + z\sigma_3$ is transformed in $\mathbf{X} = X\gamma_1 + Y\gamma_2 + Z\gamma_3 + W\gamma_4$ in R^4 where $x = X/W$, $y = Y/W$, $z = Z/W$, now using f_p the linear mapping of $\mathbf{X}$ in $\mathbf{X}'$ is given by:

$$\mathbf{X}' = \sum_{i=1}^{3}\{(\alpha_i X + \beta_i Y + \delta_i Z + \epsilon_i W)\gamma_i\} + (\overline{\alpha}X + \overline{\beta}Y + \overline{\delta}W)\gamma_4$$

The coordinates of the vector $x' = x'\sigma_1 + y'\sigma_2 + z'\sigma_3$ in E^3 corresponding to $\mathbf{X}'$ are given by:

$$x' = \frac{\alpha_1 X + \beta_1 Y + \delta_1 Z + \epsilon_1 W}{\overline{\alpha}X + \overline{\beta}Y + \overline{\delta}Z + \overline{\epsilon}W} = \frac{\alpha_1 x + \beta_1 y + \delta_1 z + \epsilon_1}{\overline{\alpha}x + \overline{\beta}y + \overline{\delta}z + \overline{\epsilon}}$$

similarly:

$$y' = \frac{\alpha_2 x + \beta_2 y + \delta_2 z + \epsilon_2}{\overline{\alpha}x + \overline{\beta}y + \overline{\delta}z + \overline{\epsilon}}$$

$$z' = \frac{\alpha_3 x + \beta_3 y + \delta_3 z + \epsilon_3}{\overline{\alpha} x + \overline{\beta} y + \overline{\delta} z + \overline{\epsilon}}$$

All this represents the projection of the real world in the plane of the image of a camera so we require to bear in mind the focal distance of the camera:

$$\alpha_3 = f\overline{\alpha}, \ \ \beta_3 = f\overline{\beta}$$

we can define $z' = f$ focal distance independently from the chosen point.

In this way any object may be defined as a set of points projected on the plane of the "photo" of the camera. We will show now how, having the pictures of the same object taken from several or at least two cameras we can have the 3D position of the object.

The projective split

It is done to relate the even algebra G_{n+1} with the geometric algebra of one dimension less G_n.

We can define a mapping between the spaces choosing a preferent direction, taking the geometric product of a vector $\underline{X} \in G_{n+1}$:

$$\mathbf{X}_{\gamma_{n+1}} = \mathbf{X} \cdot \gamma_{n+1} + \mathbf{X} \wedge \gamma_{n+1} = \mathbf{X} \cdot \gamma_{n+1} \left(1 + \frac{\mathbf{X} \wedge \gamma_{n+1}}{\mathbf{x} \cdot \gamma_{n+1}} \right)$$

In this way we associate the vector $x \in G_n$ with thebivector $\frac{\mathbf{X} \wedge \gamma_{n+1}}{\mathbf{x} \cdot \gamma_{n+1}} \in G_{n+1}$

The result is projectively interpreted as the pencil of lines through the point γ_{n+1} in Physics this relates the space time G_4 with Minkowski metric with the space G_3 with euclidean metric.

The incidence Algebra

Duality

We define a dual A^* of a vector A like:

$$A^* = AI^{-1}$$

Note then that the dual of a vector r is a vector $(n - r)$ for example in the space 3D the dual of a vector $(r = 1)$ is a plane or bivector $n - r = 3 - 1 = 2$

Using the idea of duality we are able to link for example the internal product to incidence operators:

$$A \cdot B^* = (A \wedge B)^*$$

We call Bracket of a pseudoscalar to its magnitude so

$$A \cdot B^* = [A \wedge B]$$

We define projection and section as:

$$A \wedge B$$

y

$$A^* \wedge B^*$$

Detecting a movement in 2D

All the existing approaches for quaternion wavelets are via quaternion multirresollution analysis that is defined as follows (using the ideas from Mitrea[2]): A multirresollution analysis of $L^2(R,H)$ consist in a sequence of embedded closed subspaces $\{V_m\}_{m\in Z}$ that have the following properties: 1).... $\subset V_{-2} \subset V_{-1} \subset V_0 \subset V_1 \subset V_2$ 2)$\bigcup_{j\in Z} V_j = L^2(R,H), \bigcap_{j\in Z} V_j = \{0\}$ 3) There exists a function $\phi \in V_0$ such that $\{\phi_{0,n}(x) = \phi(x-n) : n \in Z\}$ is an orthonormal basis for V_0 that is for all $f \in V_0$:

$$||f||^2 = \int_{-\infty}^{+\infty} |f(x)|^2 dx = \sum_{n\in Z} |(f, \phi_{0,n})|^2$$

ϕ is the father wavelet To complete our picture on how we can build our wavelets we have that:

$$\phi(x) = \sqrt{2} \sum_{n\in Z} h_n \phi(2x-n)$$

$$\psi(x) = \sqrt{2} \sum_{n\in Z} g_n \phi(2x-n)$$

Where h and g are respectively the quaternion valued high and low pass filters which can be found as:

$$H(\xi) = \frac{1}{\sqrt{2}} \sum_{n=0}^{2N+1} h_n e^{-im\xi} =$$

$$\frac{1}{\sqrt{2}}\left(\sum_{n=0}^{N} h_n e^{-in\xi} + \sum_{k=N+1}^{2N+1} h_{2N+1-k} e^{-ik\xi}\right)$$

solving the system of equations we have the coordinates of the quaternion valued functions searched.

The physical meaning

Now the functions describe movements so how can be filters be interpreted in terms of that? That is how can be such embedded subspaces be built in terms of movements?

The idea is to try to decompose the movement in traslations and rotations from a very general to the detailed. For example the traslational movements may be schematized in some "polygonal movements" and the rotations depending upon its frequency.

In order to divide the movement in several "elementary " ones that make physical sense for each case we will use Barault[1] approach in order to tune our wavelets to some kind of movement and also to do the whole process using wavelets.

Tunning the wavelet in some ranges ov velocities for example may distinguish an object moving at that velocity and the motion parameters will be extracted from the wavelet parameters. In that direction, but to detect the movement some work has been made by Barault[3].

The first step is then to suppose that the mother wavelet support is concentrated in a velocity plane:

$$\omega = -k\vec{v}_0$$

then we define a spatial and a temporal translation $T^{(\vec{b},\tau)}$

$$\left[T^{(\vec{b},\tau)}\psi\right](\vec{x},t) = \psi(\vec{x}-\vec{b}, t-\tau)$$

$$\left[T^{(\vec{b},\tau)}\hat{\psi}\right](\vec{k},\omega) = e^{-j(\vec{k},\vec{b}+\omega\tau)} \cdot \hat{\psi}(\vec{k},\omega)$$

We can also define a rotation transformation R^{θ}:

$$\left[R^{\theta}\psi\right](\vec{x},t) = \psi(r^{-\theta}\vec{x},t)$$

$$\left[R^{\theta}\hat{\psi}\right](\vec{k},\omega) = \hat{\psi}(r^{-\theta}\vec{k},\omega)$$

where:

$$r^{-\theta} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

finally we can define a scale changing:

$$\left[D^{\alpha}\psi\right](\vec{x},t) = \alpha^{-3/2}\psi\left(\frac{\vec{x}}{\alpha},\frac{t}{\alpha}\right)$$

$$\left[D^{\alpha}\hat{\psi}\right](\vec{k},\omega)=\alpha^{-3/2}\hat{\psi}(\alpha\vec{k},\alpha\omega)$$

Our data as we said before is in some kind of filmation so we must know the frame rate (usually it is known depending upon the camera) the frame size and the object dimension with all that using the following we can have speed tuning:

$$\left[\wedge^{c}\psi\right](\vec{x},t)=\psi(c^{-1/3}\vec{x},c^{2/3}t)$$

$$\left[\wedge^{c}\hat{\psi}\right](\vec{k},\omega)=\hat{\psi}(c^{+1/3}\vec{k},c^{-2/3}\omega)$$

we can take for example values of 1,3,10 pixels/frame for c and have the object real time speed. Now we are ready to choose the wavelet to be used or the type of wavelet to be used, Barault[1] uses a Morlet one that we will show but a B spline as the one shown above may be better because it doesn't wiggle so much and it is more linked to the experiment. This will be:

$$\psi(\vec{x},t)=\left(e^{-1/2|\vec{x}|^{2}}\cdot e^{i\vec{k}_{0}\vec{x}}\right)\times\left(e^{-1/2t^{2}}\cdot e^{i\omega_{0}t}\right)$$

$$\hat{\psi}(\vec{k},\omega)=\left(e^{-1/2|\vec{k}-\vec{k}_{0}|^{2}}\right)\times\left(e^{-1/2(\omega-\omega_{0})^{2}}\right)$$

If we apply to such wavelets all the transformations defined above we have a composite transform that will be a wavelet tuned to motion we call Ω_g:

$$[\Omega_{g}\psi](\vec{x},t)=\left[T^{\vec{b},\tau}R^{\theta}\wedge^{c}D^{\alpha}\psi\right](\vec{x},t)$$

and our wavelet may be Morlet or the other.

With this procedure (trying with diferent c,α, θ, $\vec{b}$ and τ we can determin e from a film the characteristics of some movements.

The problem now that we have determined a series of recognizable movements and their characteristics how we express them in terms of a QMRT.

Obtaining the characteristics of the 3D movement

With this operations defined, having two images taken from two known cameras we can project the image of the camera from its focus and intersect the corresponding lines with the others coming from the other focus to obtain the 3D position of the object.

We have then series of movements $\{m_0, m_1, ...m_n\}$ we can interpolate them using the quaternionic Bsplines as follows:

First of all we will introduce briefly the dual quaternions:

A dual quaternion:

$$Q=Q^{0}+\epsilon Q^{\epsilon}=(q^{0}+\vec{q}^{0})+\epsilon(q^{\epsilon}+\vec{q}^{\epsilon})$$

Consists in a real part $Q^0 = Re\,Q \in \mathcal{H}$ and a dual part $Q^\epsilon = DuQ \in \mathcal{H}$

The dual quaternion :

$$2v_0 + \epsilon\vec{v} \quad (v_0 \in R, \vec{v} \in R^3)$$

corresponds with a traslation with a displacement vector:

$$\frac{1}{v_0}\vec{v} \;\; v_0 \neq 0$$

The quaternion:

$$D = d_0 + \vec{d} \in \mathcal{H} \;\; D \neq 0$$

describes a rotation around the origin

We will call a spatial displacement to the composition of a traslation and a rotation represented by the dual quaternion:

$$Q = Q^0 + Q^\epsilon = (2v_0 + \epsilon\vec{v}) * (d_0 + \vec{d})$$

satisfying the Plücker condition:

$$Du(Q * \overline{Q}) = q^0 q^\epsilon + \vec{q}^0 \cdot \vec{q}^\epsilon = 0$$

If two quaternions are different only by a factor, that is they are proportional they describe the same movement. Multiplication between dual quaternions is equal to composition of its corresponding movements

A movement then may be considered a curve on the quadric hypersurface of the real projective 7-space

Consider now the polynomial :

$$Q(t) = \sum_{i=0}^{n} b_i^n(t) B_i \quad t \in R$$

where

$$b_i^n(t) = \binom{n}{i} t^i (1-t)^{n-i}$$

are the Bernstein polynomials and the coefficients $B_i \in H$

Let now be $m+1$ spatial displacements named P_i so that:

$$P_i = (2 + \epsilon\vec{s}_i) * (r_{i,0} + \vec{r}_i)$$

The first parentheses is the traslational and the second the rotational

$$\vec{s}_i \;,\; \vec{r}_i \in R^3 \;;\; r_{i,0} \in R$$

with parameters $t_i \in R$, $0 = t_0 < t_1 < ... < t_m = 1$

The $m+1$ positions will be interpolated to obtain a Q-motion of degree n:

$$Q(t_i) = \lambda_i P_i \quad (i = 0, ..., m)$$

The traslational spline is then:

$$Q_{tras}^{(i)} = 2 + \epsilon \sum_{j=0}^{3} d_j^3 \left(\frac{t - t_i}{t_{i+1} - t_i} \right) \vec{p_j^{(i)}}$$

and the rotational:

$$Q_{rot}^{(i)} = 2 + \epsilon \sum_{j=0}^{3} d_j^3 \left(\frac{t - t_i}{t_{i+1} - t_i} \right) C_j^{(i)}$$

We will have also the characteristic of such movements in a way that we can define (by composition of the planar defined ones) appropiate filters to decompose and compose the movement using the true meaning of wavelets not over images but over movement itself. For example we can apply the projection and section operations to the pair of Ω_g let's call them $\Omega_g^{f_1}$ and $\Omega_g^{f_2}$ composing them at the point where the lines projecting the moving point meet, and composing them as quaternions.

Bibliography

[1] Traversoni Leonardo Quaternions On wavelet problems, Approximation theory VIII Vol 2, College Station, Texas, 1995 World Scientific. [2] Mitrea Marius Clifford Wavelets, singular integrals and Hardy spaces. Lecture notes in Mathematics 1575, Springer Verlag.

[3]Barault Pierre Motion estimation and video compression with spacio-temporal motion tuned wavelets. WSEAS Transactions on mathematics V2,2003