

# On hyperholomorphic polynomials for the Cauchy-Riemann and the Dirac operators of Clifford analysis.

Michael Shapiro

*Instituto Politécnico Nacional, Mexico City, Mexico*

E-mail: shapiro@esfm.ipn.mx

*Clifford Analysis, hyperholomorphic polynomials, Taylor Series.*

Let  $\mathcal{D} := \sum_{k=0}^n e_k \frac{\partial}{\partial x_k}$  be the Cauchy-Riemann (CR) operator of Clifford analysis and let  $\tilde{\mathcal{D}} := \sum_{k=1}^n e_k \frac{\partial}{\partial x_k}$  be the corresponding Dirac operator. As was shown first by Fueter for quaternionic analysis, the functions  $\zeta_\ell(x) := x_\ell - x_0 e_\ell$ ,  $\ell \in \{1, \dots, n\}$ , form a basis of CR-hyperholomorphic homogeneous polynomials of degree one, and they generate a basis of all CR-hyperholomorphic polynomials thus having consequences in constructing the Taylor series and rational functions for the Cauchy-Riemann operator.

The functions  $\zeta_\ell$  cannot serve for Dirac-hyperholomorphic functions but if one introduces the new functions  $\tilde{\zeta}_k := x_1 e_1 e_k + x_k$ ,  $k \in \{2, \dots, n\}$ , they form already a basis of Dirac-hyperholomorphic functions and all the statements about arbitrary CR-hyperholomorphic polynomials, Taylor series and rational functions, have their counterparts in the context of Dirac-hyperholomorphy.

Since any Dirac-hyperholomorphic function can be identified with a CR-hyperholomorphic one which does not depend on  $x_0$ , we can expect that all the objects considered in the Dirac theory are particular cases of their CR-analogs which is not so even on the level of the functions  $\zeta_\ell$  and  $\tilde{\zeta}_k$ . Thus the question arises whether it is possible to construct a similar theory for the CR-operator but in such a way that for CR-hyperholomorphic functions which do not depend on  $x_0$ , it would turn directly into the theory for the Dirac operator.

The aim of the talk is to expose a positive answer and to explain a number of new phenomena arising here. The talk is based on joint results with D. Alpay, F.M. Correa-Romero and M.E. Luna-Elizarrarás.

The author was partially supported by CONACYT projects as well as by Instituto Politécnico Nacional in the framework of COFAA and SIP programs.