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STRUCTURAL BEHAVIOUR OF MASONRY VAULTS

J. P. Szołomicki

Wrocław University of Technology
Wybrzeże St. Wyspiańskiego 27, 50-370 Wrocław, POLAND
E-mail: Jerzy.Szolomicki @ pwr.wroc.pl

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Abstract. *This paper deals with the modelling and the analysis of masonry vaults. Numerical FEM analyses are performed using LUSAS code. Two vault typologies are analysed (barrel and cross-ribbed vaults) parametrically varying geometrical proportions and constraints. The proposed model and the developed numerical procedure are implemented in a computer analysis. Numerical applications are developed to assess the model effectiveness and the efficiency of the numerical procedure. The proposed computer model is validated by a comparison with experimental results available in the literature.*

1 INTRODUCTION

The increasing interest in historic architectural heritage and the need for conservation of historical structures has led to the continuous development of many methods for the analysis of masonry vaults. However some type of vaults have not been thoroughly analysed, mainly because of the problem of applying simplified theories to their complicated shapes. The major simplification that is usually taken is supposed to reduce the vault to a series of adjacent arches, without transversal connection. So this model is not able to properly simulate the three-dimensional effects in the vaults. This limitation and the need for a flexible method to study the different types of vaults could be solved by using the connection conception of three-dimensional laser scanning and FEM. Three-dimensional laser scanning is essential to get the accurate geometry of the vault. However FEM allows to carry out static-strength analysis.

The main object of the present paper is the development of a computational procedure which allows to define 3D structural behaviour of masonry vaults.

2 STRUCTURAL ASSUMPTION FOR MASONRY VAULTS

The building technique of masonry vaults continued throughout the ages, influencing the history of architecture and initiating a large amount of the typology and shapes (barrel vault, cross vault, fun vault, etc.). Diverse factors influence their structural behaviour, such as geometry, stiffness and mass distribution, chronological succession of the building works and more further reconstructions. For example, different masonry pattern can be adopted on equal terms of geometry. This aspect plays a fundamental role in a structural system, where the equilibrium is ensured by the mutual thrust of the blocks making up the vault. As a consequence, this aspect represents a characteristic building feature, strongly affecting the overall behaviour: paying attention on the barrel vault, varying the brickwork bond, its structural response may be interpreted as that of a three-dimensional shell. The brick (block) bond influences not only for the static behaviour, but also for the damage pattern. This latter is obviously affected by the potential sliding planes, identified by the scheme of bed and head joints. The presence of loose fill, structural backing and constraint boundary conditions may be considered further factors affecting the vault behaviour. The constraint conditions to be modelled as the function of the interlocking provided by the walls in correspondence of the vault springing points. The constraint degree may derive by a precise constructive purpose or by cracking or degradation situations, which the structure may have undergone during the ages.

The structural behaviour of masonry vaults and their collapse mechanisms depends on the material property forming them. Present theory is supported on the assumption of large compressive strength for the blocks, no tension transmitted across the joints and finite friction. The last hypothesis reflects a more realistic masonry mechanical behaviour because after the deterioration of the contact surfaces or of a the binding material, the original friction coefficient could be substantially reduced. Therefore the shear strength at blocks interfaces is determined by the cohesion and the internal friction angle having assumed the Mohr-Coulomb criterion. Modelling a vault of any shape as a three-dimensional discrete system of rigid blocks, along meridians and parallels, it is possible to determine, by use of the general shell theory, the meridian stresses, the hoop stresses and the shear stresses. It is evident that the discretization scale could not be representative of each single brick or block constituting the vault. Each portion identified by the intersection of 2 parallels with two adjacent meridians can be considered as a macro element of homogenous masonry material. As masonry by hypothesis is generally not able to resist tensile stresses for most shapes of vaults the gravity load distribution will cause tensile stresses near the spring level that the material is not able to absorb and hence cracks will form.

3 MASONRY VAULTS

In historic buildings, the forms used to span between vertical supports and enclosing walls have great influence on overall structural behaviour and are important on determining what is required of their supports. The structural elements allowing masonry to have more free and open spaces are arches, vaults and domes. The dominant profile used for these elements are semicircle, parabola and elliptic. The load bearing capacity and deformation behaviour of the surfaces generated with these profiles and distribution of forces on such variety of geometry show different characteristics and it is important to understand the surface in order to predict possible weaknesses.

In historic masonry buildings, vaults are used as roof or floor to enclose space. The strength of a vault depends on how the units forming the vault are assembled. The construction of a vault may be of arch assemblies, each arch leaning back against the previous one or enchainment of masonry units making a continuous vault surface. Although the vault forms look similar, the surfaces that constitute a vault may have different characteristics. The behaviour of each surface is different whether the form is cylindrical or elliptic parabolic. These are singly or doubly curved surfaces.

A cylindrical surface is a translational surface where a curve profile, generetrix, moves parallel to itself, along a line, directrix. It is as if an arch extended laterally. Such a surface in masonry buildings is called barrel vault. The rising profile constituting the surface may be circular, parabolic, elliptic, pointed or derived from any other kind of a curve. In singly curved vault surfaces, the principle stresses along the curve will always be compressive and the inclined thrusts at the edge require enough mass of supporting system.

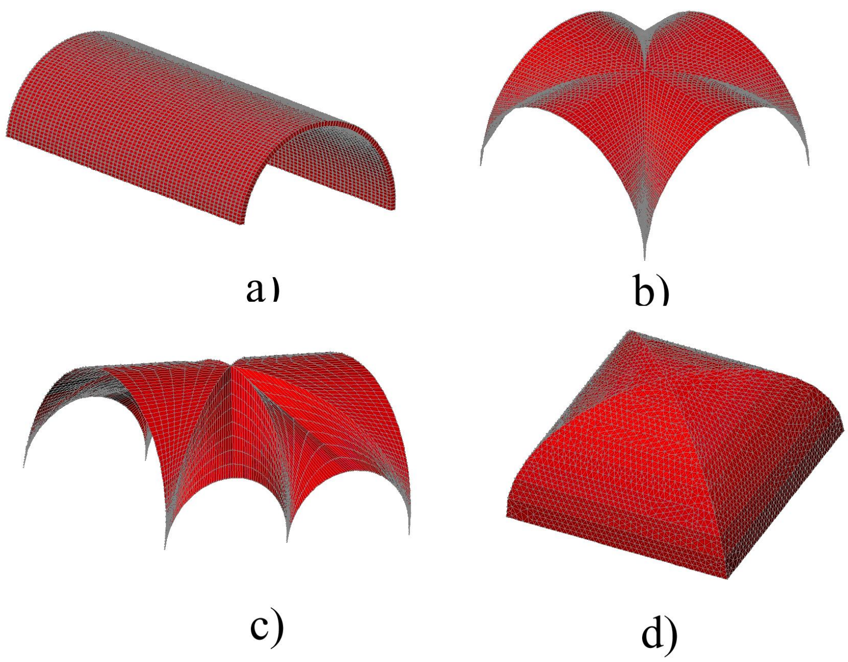


Figure 1 Different types of vaults: a) barrel vault, b) cross-ribbed vault, c) six-partite vault, d) cloister vault.

Doubly curved vaults may be an elliptic parabolic. Elliptic parabolic surface is a translational surface where a curve profile moves parallel to itself, over another curve that has the centre of curvature at the same side. Such vaults are used generally to cover square spaces. The principal stresses of the surface are compression on both of its principal curvature. Cracks appear due to the displacement of their supports.

To span over rectangular bays, groin vaults or cloister vaults were used in historical structures. Groin vault is obtained by the intersection of two or more cylindrical vaults forming diagonal arches over the space to be covered. The arch action of each barrel vault brings all the loads as compressive forces down to the springing with an outward thrust at the supports. Buttressing forces are required to stabilize these diagonal ribs. Cloister vault is formed by the intersection of two or more vaults forming a ridge at the intersection. Composition of a cloister vault may be of one of the vault surfaces or may be composed of different forms. Crack development in the parabolic surface may be due to the movement of the supports. When cloister vaults are built in series, one after the other, the components of the thrusts are cancelled out by equal thrusts in adjacent bays except the two ends of the series.

4 COMPUTER MODELLING – HOMOGENIZED LIMIT ANALYSIS APPROACH

Masonry is a composite material made by units bonded together with mortar joints. In most cases of building practice, units and mortar are periodically arranged, i.e. walls are constituted by the regular repetition of bricks bonded with joints. When a curved masonry surface X, identified at a point P by the two principal curvatures $1/q_s$ and $1/q_r$,(Figure 2) is considered, it is very straightforward to conclude that it is not always possible to rigorously consider X as a regular repetition of the elementary volume Y, thus precluding in principle the utilization of homogenization in the most general case. Nevertheless, a heuristic but technically suitable approach is to identify in any case a representative volume element, as depicted in Figure 3, which generates the double curvature shell by repetition. Obviously, such an approach should require a variable dimension of bricks, passing from a row to the neighbours.

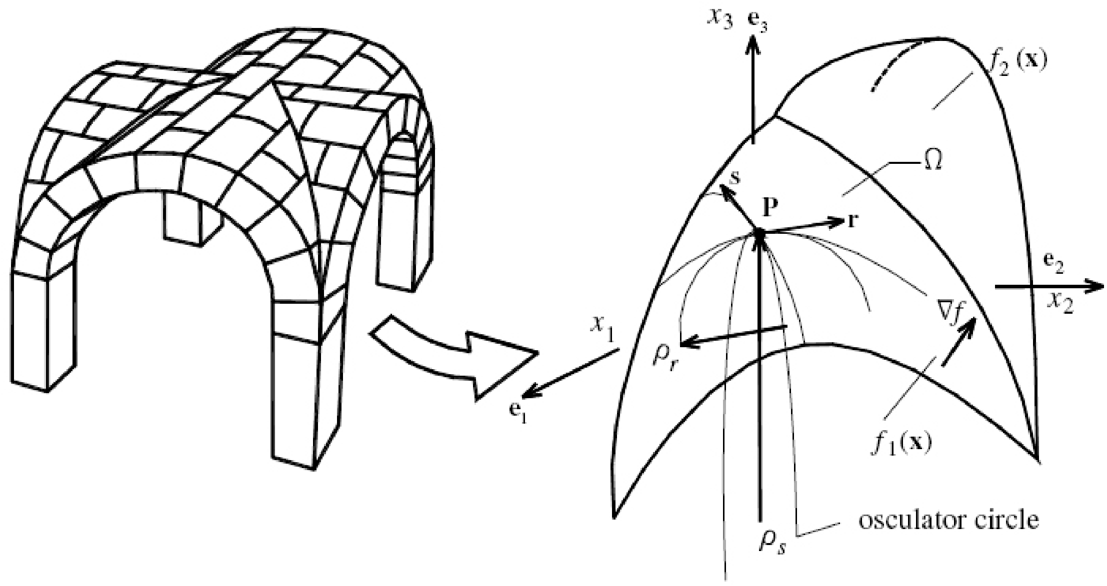


Figure 2 Double curvature shell structures Ω constituted by more that one infinitely differentiable surface (e.g. $f_1,\ldots,f_4$).

The basic idea of the homogenization procedure consists in introducing averaged quantities representing the macroscopic membrane actions and strain tensors (N and E) for in-plane actions, the macroscopic bending moment and curvature tensors for the out-of-plane problem (M and χ) and the out-of-plane sliding and shear (Γ_3 and T_3) defined as follows (direction is assumed perpendicular to the masonry middle plane):

$$\mathbf{E} = \langle \varepsilon \rangle = \frac{1}{V}\int_V \varepsilon(\mathbf{u}) dV , \tag{1}$$

$$\mathbf{N}/\bar{t} = \langle \sigma \rangle = \frac{1}{V}\int_V \sigma dV , \tag{2}$$

$$\chi = \left\langle \frac{\partial \varepsilon}{\partial \mathbf{y}} \right\rangle = \frac{1}{V}\int_V \frac{\partial \varepsilon(\mathbf{u})}{\partial \mathbf{y}} dV , \tag{3}$$

$$\mathbf{M} = \langle \sigma y_3 \rangle = \frac{1}{V}\int_V \sigma y_3 dV , \tag{4}$$

$$\mathbf{\Gamma_3} = \langle \gamma_3 \rangle = \langle [\partial u_3 / \partial y_1 + \partial u_1 / \partial y_3 ; \partial u_3 / \partial y_2 + \partial u_2 / \partial y_3] \rangle = \frac{1}{V}\int_V \boldsymbol{\gamma}_3 dV , \tag{5}$$

where V is the volume of the elementary cell, $\bar{t}$ the transverse thickness, **u** is the displacements vector (components u_i), $\boldsymbol{\varepsilon}$ and $\boldsymbol{\sigma}$ stand for the local quantities (stress and strain tensors).

Anti-periodicity and periodicity conditions are imposed, respectively, to the stress field $\boldsymbol{\sigma}$ and the displacement field **u**:

$$\begin{cases} \mathbf{u} = \tilde{\mathbf{E}}\mathbf{y} + \tilde{\chi}\mathbf{y} + \tilde{\mathbf{\Gamma}}\mathbf{y} + \mathbf{u}^{\mathbf{per}} & \mathbf{u}^{\mathbf{per}} \ \ \mathbf{on} \ \ \partial\mathbf{Y} \\ \sigma\mathbf{n} & \mathbf{anti-periodic} \ \ \mathbf{on} \ \ \partial\mathbf{Y} \end{cases} , \tag{6}$$

where:

- $\mathbf{u}^{per}$ stands for a periodic displacement field,
- ∂Y is the cell internal boundary,
- $\tilde{\mathbf{E}} = [\mathbf{E} \ \ \mathbf{O}; \ \mathbf{O}^\mathbf{T} \ \ 0]$,**O** is a 2 x 1 zero vector,
- $\tilde{\chi} = \left[y_3\chi \quad \mathbf{O} \quad \left(\mathbf{1/2}\chi[y_1 \ y_2]^T\right)^T \quad 0 \right]$,
- $\tilde{\mathbf{\Gamma}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \Gamma_3(1) & \Gamma_3(2) & 0 \end{bmatrix}$.

Let S^i, S^b, and S^h denote respectively, the strength domains of the interface between mortar and bricks, of the bricks and of the homogenized macroscopic material. S^h domain of the equivalent medium is defined in the space of the macroscopic stresses as follows:

$$
S^h \equiv (\mathbf{N} \quad \mathbf{T} \quad \mathbf{M}) \begin{cases}
\mathbf{N}/t = \langle \boldsymbol{\sigma} \rangle = \dfrac{1}{V} \displaystyle\int_V \boldsymbol{\sigma}\, dV & (7) \\
\mathbf{M}/t = \langle \boldsymbol{\sigma} y_3 \rangle = \dfrac{1}{V} \displaystyle\int_V \boldsymbol{\sigma} y_3\, dV & (8) \\
\mathbf{T_3}/t = \langle \tau_3 \rangle = \dfrac{1}{V} \displaystyle\int_V \tau_3\, dV & (9) \\
\operatorname{div} \boldsymbol{\sigma} = 0 & (10) \\
[\![\boldsymbol{\sigma}]\!]\mathbf{n}^{\text{int}} = 0 & (11) \\
\boldsymbol{\sigma}\mathbf{n} \ \text{- anti - periodic on } \partial\mathbf{Y} & (12) \\
\boldsymbol{\sigma}(\mathbf{y}) \in \mathbf{S}^{\mathbf{i}} \ \ \forall \mathbf{y} \in \mathbf{Y}^{\mathbf{i}}; \boldsymbol{\sigma}(\mathbf{y}) \in \mathbf{S}^{\mathbf{b}} \ \ \forall \mathbf{y} \in \mathbf{Y}^{\mathbf{b}} & (13)
\end{cases}
$$

Here, $[\![\sigma]\!]$ denotes the jump of micro-stresses across any discontinuity surface of normal $\mathbf{n}^{\text{int}}$. Conditions (7) are typical for homogenization, condition (12) is derived from anti-periodicity, condition (10) imposes the micro-equilibrium and condition (13) represents the yield criteria for the components (brick and mortar).

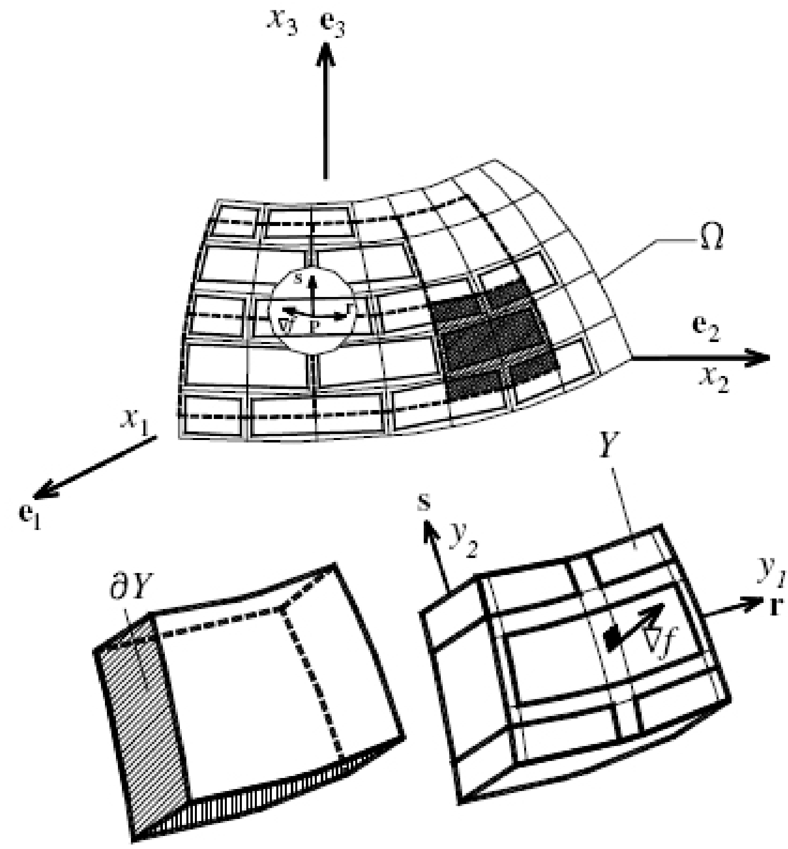


Figure 3 Homogenization procedure - Heuristic identification of the elementary cell for a double curvature masonry shell.

5 NUMERICAL ANALYSES

Numerical FEM analyses are performed using LUSAS code (release 14.3). Vaults are modelled by means of shell elements. Two vault typologies are analysed (barrel and cross-ribbed vault) parametrically varying geometrical proportions and constraints.

In the first step the elastic FEM simulations are performed with dead loads and the presence of fill and backfill is neglected. The proposed analysis lead in any case to stiffness estimation on the safe side. For each calculations, the homogenized limit analysis approach has been employed, assuming for the constituent materials experimentally determined mechanical properties.

The first analysis relies on the determination of distribution of principal stress σ_1 of a barrel rectangular vault. The vault is a circular arch with a span of 4 m, a width of 8 m and a thickness of 0,12 m.

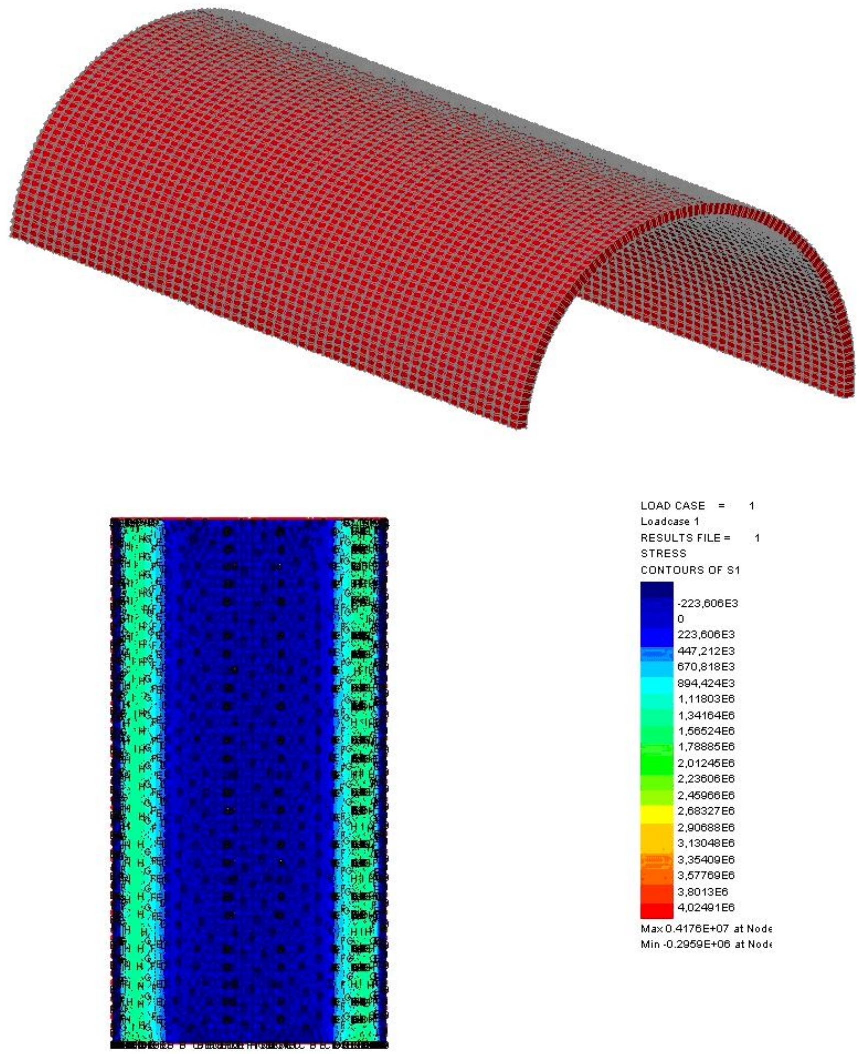


Figure 4 A diagram of principal stresses σ_1 for the barrel vault subjected to dead-weight loading.

The next non-linear analysis relies on the determination of the ultimate strength of a barrel vault. Mechanical properties assumed for joints and bricks are reported in Table 1, and have been taken in agreement with the experimental data available.

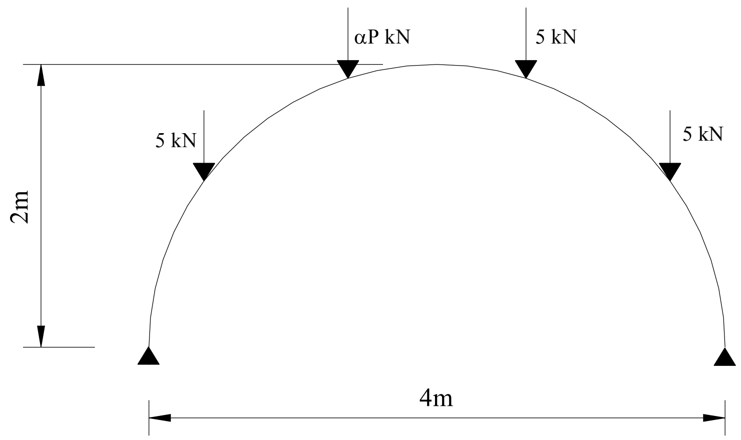


Figure 5 Geometry and loading condition of the barrel vault in non-linear analysis.

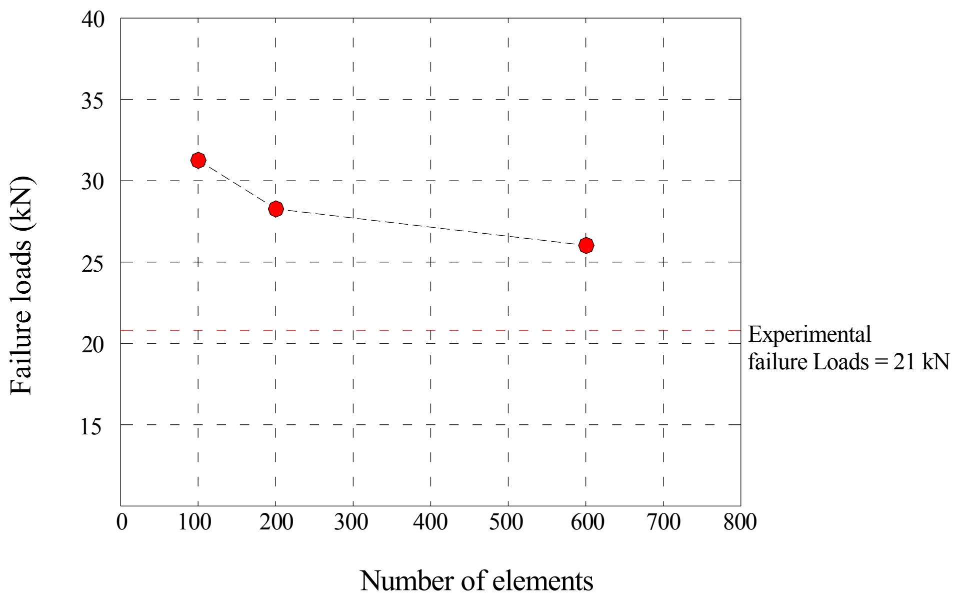


Figure 6 Mesh dependence study for the barrel vault (Failure loads vs. number of elements).

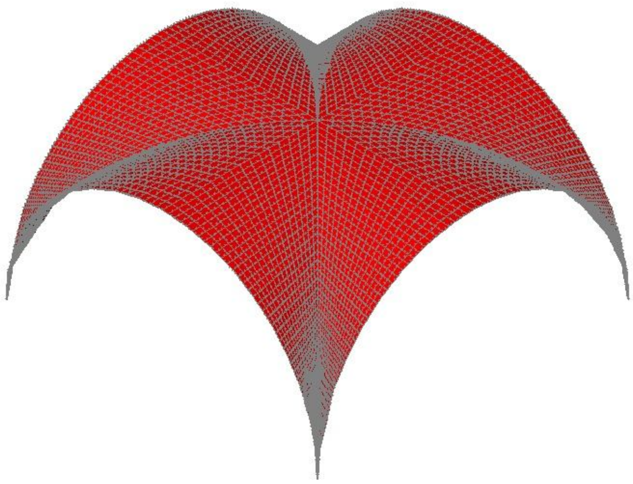
Joint	
Tensile strength	f_t (C) = 0,3
Compressive strength	f_c (N/mm^2) = 2,5
Cohesion	$c=1{,}2f_t$
Friction angle	$\Phi=20^o$
Angle of the linerized compressive cap	$\Phi_2=40^o$
Brick (Mohr–Coulomb failure criterion with compressive cutoff)	
Compressive strength	f_c (N/mm^2) = 30
Cohesion	c(N/mm^2) = 1
Friction angle	$\Phi=45^o$

Table 1 Mechanical characteristic assumed for joints and bricks in non-linear analyses of barrel rectangular vault tested by Vermeltfoort (2001).

Joint	
Tensile strength	f_t (N/mm^2) = 0,05
Compressive strength	f_c (N/mm^2) = 2,3
Cohesion	$c=1,2f_t$
Friction angle	$\Phi=25^o$
Angle of the linerized compressive cap	$\Phi_2=40^o$
Brick (compressive cut-off)	
Compressive strength	f_c (N/mm^2) = 30

Table 2 Mechanical characteristic assumed for joints and bricks in non-linear analyses of cross ribbed vault by Faccio et al. (1999).

A ribbed cross vault formed by the intersection of two barrels vaults with an external radius of 4 m, is considered as a next example. Mechanical properties assumed for joints and bricks are reported in Table 2.



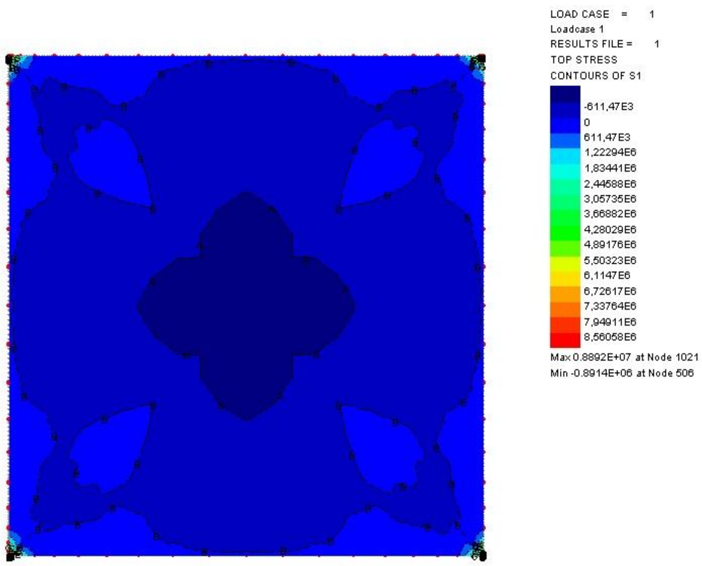


Figure 7 A diagram of principal stresses σ_1 for the cross-ribbed vault subjected to dead-weight loading.

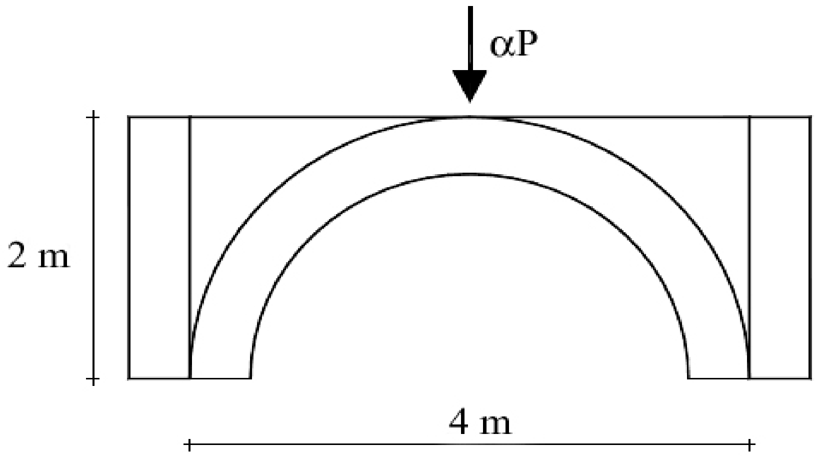


Figure 8 The cross-ribbed vault. Geometry and loading condition in non-linear analysis.

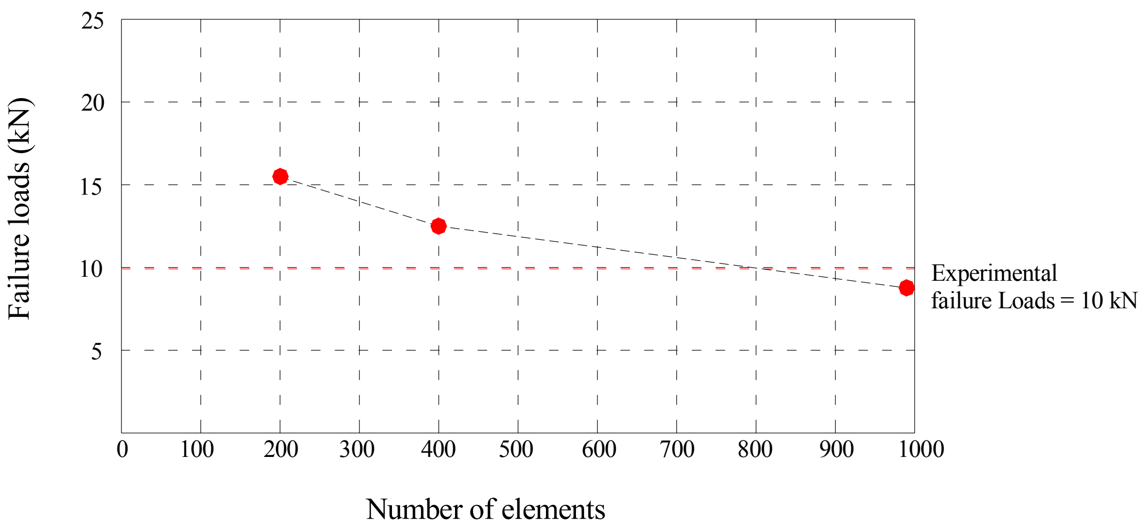


Figure 9 Mesh dependence study for the cross-ribbed vault (Failure loads vs. number of elements).

6 CONCLUSIONS

For each investigated example, the homogenized limit analysis approach has been employed to predict ultimate load and failure mechanisms. Finally, both a mesh dependence study and a sensitivity analysis are reported. Sensitivity analysis is conducted varying in a wide range mortar tensile strength and mortar friction angle with the aim of investigating the influence of the mechanical properties of joints on collapse load and failure mechanisms.

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