18th International Conference on the Application of Computer
Science and Mathematics in Architecture and Civil Engineering
K. Gürlebeck and C. Könke (eds.)
Weimar, Germany, 07–09 July 2009

QUALITY OPTIMIZATION USING LOCALLY REFINED METAMODELS

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Keywords: robust design optimization, reliability based design optimization, adaptive metamodeling, sample recycling, design for six sigma

Abstract. *Quality is one of the most important properties of a product. Providing the optimal quality can reduce costs for rework, scrap, recall or even legal actions while satisfying customers demand for reliability. The aim is to achieve "built-in" quality within product development process (PDP). The common approach therefore is the robust design optimization (RDO). It uses stochastic values as constraint and/or objective to obtain a robust and reliable optimal design. In classical approaches the effort required for stochastic analysis multiplies with the complexity of the optimization algorithm. The suggested approach shows that it is possible to reduce this effort enormously by using previously obtained data. Therefore the support point set of an underlying metamodel is filled iteravely during ongoing optimization in regions of interest if this is necessary. In a simple example, it will be shown that this is possible without significant loss of accuracy.*

1 INTRODUCTION

The cost of a product includes more than the development, production and management costs. There are also costs that are related to the quality of the product e.g. scrap, rework or recalling costs. Providing the corresponding quality is clearly seen as one of the most essential competition advantage in international business [Buzzell and Gale(1987)]. Hence products have to be designed to provide a optimal quality.

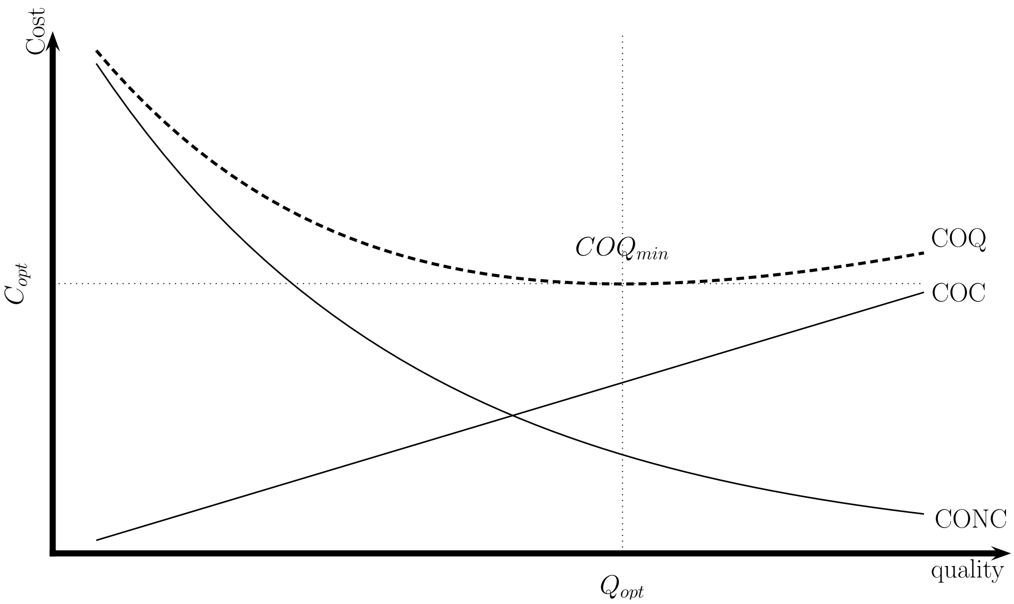

Figure 1: conformance model

In conformance models "cost of quality" (COQ) is the described as the sum of "cost of conformance" (COC) and "cost of non-conformance" ($CONC$).

$$COQ = COC + CONC \tag{1}$$

COC are costs for prevention and appraisal e.g. material or amount of work. $CONC$ are costs for failure and costs related to failures e.g. rework, recall, legal actions, less made sells due to bad costumers opinion and so on. A survey can be found in [Schiffauerova and Thomson(2006)]. The aim is to find the optimal quality or COQ_{min} respectively.

According to Boehm's *cost of change curve* (fig.2) the earlier one starts with improvements in production process, the lower the costs of these changes are. Drawing a conclusion, avoiding errors early in development phase is much more cheaper than repairing them later. It is necessary to spend every possible attention in the research and design phase. Otherwise, savings in early development phases may have to be payed 100 times in the production phase.

Numerical models are used to create and describe new products within the product development process (PDP). Additionally companies established Robust Design Optimization (RDO) to achieve "built-in quality". The aim of RDO is to consider uncertainties during optimization. Stochastic values are used as constraint and/or objective to obtain a robust and reliable optimal design of the new product. In classical approaches of RDO the effort required for stochastic analysis multiplies with the complexity of the optimization algorithm. Because of costly computation time for one single simulation, classical robust design optimization is not useable for most problems in real life.

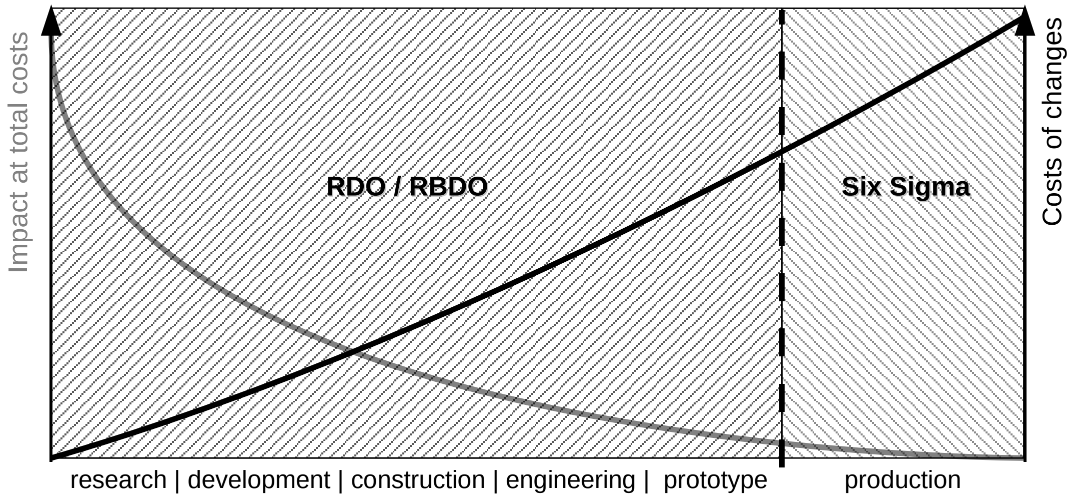

Figure 2: cost of change curve in production process

Using metamodels instead of large systems e.g. finite elements, is one possibility to solve problems more efficiently. But, due to creating a support point set has exponential complexity in respect to the problem size, in a lot of cases, especially in high dimensional problems, local effects or interactions have to be neglected. So the most important information for quality evaluation is lost.

An approach will be proposed which shows the possibility of reducing the effort enormously. Therefore the support point set of an underlying metamodel is filled iteratively during ongoing optimization in regions of interest if this is necessary. In a simple example, it will be shown that this is possible without any loss of accuracy.

2 QUANTIFYING QUALITY

Measuring robustness is a quantification of quality. To apply RDO it is necessary to measure the robustness of an optimization design. These values are used in constraints or in objectives. In most problems finding formulations for the conformance model (Eq.1) is not possible or to complicated. So the definition for quality is used to define a measurement. More or less suitable, definitions were given in the past:

- “fitness for use”(Juran, Joseph M.)
- “loss to society” or “uniformity around a target value”(Taguchi G.)
- “conformance to requirements”(Crosby P.)
- “degree to which a set of inherent characteristic fulfills requirements.” (ISO 9000)

Thus it is possible to formulate at least 5 groups of problems for quality measurements:

Weighting the outputs with a loss function $f(x)$
e.g. Taguchi Loss Functions [Byrne and Taguchi(1987), Phadke(1989)]
some applications can be found in [Fathi and Poonthanomsook(2007), Teeravaraprug(2002)]

Rating the probability of occurrence $p(x)$ or density function
e.g. Shannon entropy [Shannon(1948)]

Appraisal of an assumed function
e.g. PRESS-value for response surfaces or chi-square test for distributions

Parameters of the distribution function
e.g. mean, standard deviation

Exceeding or fall below limits or performance measure
e.g. reliability index, process capability indices, probability of failure

For later use we call the measurement of quality δ_q or ”robustness value”. Methods for obtaining these values differ depending on the probability level and the desired accuracy. An overview of some of these methods is given in [Bucher(2009)].

The choice of the applied δ_q belongs to the field of application. In [Thornton(2001)] suggestions which description to use are made. Combining all data in one cost of quality function may avoid constraints. So the only objective for the optimization would be the COQ. Thereby multiobjective problems can be reduced to an easier single objective function. For early PDP-stages this won’t be possible. In this case simpler loss functions (e.g. Taguchi’s quadratic loss function) or less complicated formulations are used. The most common is, in practical cases, the use of simple parameters of the distribution function(mean, standard deviation).

3 QUALITY OPTIMIZATION

The quality of a product is related to uncertainties in the production process, in component parts, loads and so on. Deterministic optimization does not consider these uncertainties. This may lead to designs which are not robust or reliable. The use of safety factors is the common approach to cope with this problem. The main weaknesses of the achieved results are:

1. Overdesign, (too safe, too expensive)
2. Underdesign, (not safe enough, more re-work and scrap)
3. Oversensitivity, (very sensitive with respect to small changes)
4. Cross independence, (interactions between inputs are neglected)

One widely known approach is Design for Six Sigma (DFSS). It has its roots in Crosby's approach on self paying quality "Zero Defects" [Crosby(1979)]. The goal is to achieve a probability of failure level of 6σ (4.5σ respectively). But this also leads to over- or underdesign. Thus "Zero" should mean "very low" and it should be used by developers that start collecting experiences in quality optimization. In this case DFSS has to be seen as a goal for a *Kaizen* (way to the better) - process.

The advanced approach is Robust Design Optimization. Therein quality measurements are used as constraint or objective. The user of this approach is abled to adjust directly the goals to his PDP. Therefore more information, and another definition for quality is needed (see sec.2).

Deterministic, random and total space

A RDO problem has variables that can be controlled, the deterministic design parameters

$$\mathbf{d} = [d_1, d_2, ...d_{n_d}] \tag{2}$$

They can be used for optimization.

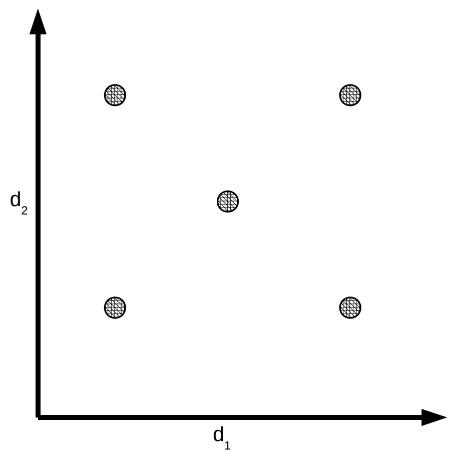

Figure 3: optimization space

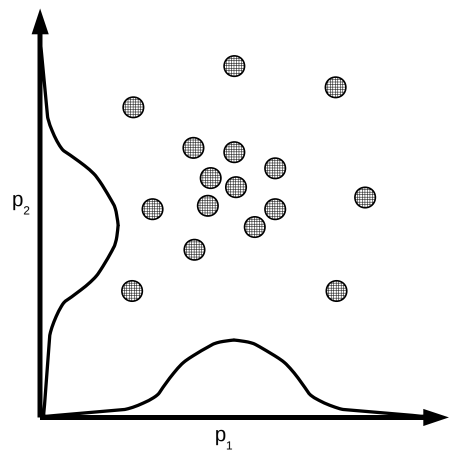

Figure 4: stochastic space

There also exist parameters that have inherent uncertainties, the random parameters.

$$\mathbf{p} = [p_1, p_2, \dots p_{n_r}] \tag{3}$$

Design parameters that scatter are called mixed variables, they can be found in $\mathbf{p}$ and $\mathbf{d}$. Now we can define a problem with two spaces. First there is the deterministic space which is defined by $\mathbf{d}$ (fig. 3). And second, there is the random space which is defined by $\mathbf{p}$ (fig. 4). Random and deterministic space are building the total space (fig. 5).

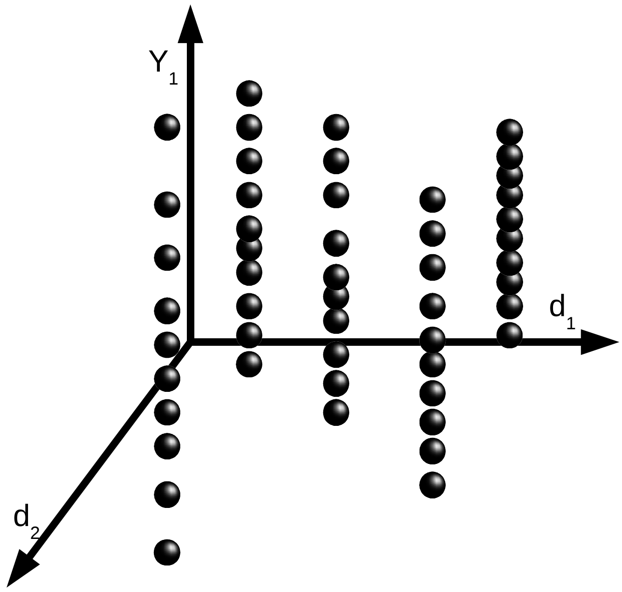

Figure 5: total space

Therein the responses

$$\mathbf{Y} = [y_1, y_2, \dots y_n] \tag{4}$$

are calculated as a function of all parameters.

$$y_i = f_i\,(\mathbf{p} \cup \mathbf{d}) \tag{5}$$

In real problems, this function can be a complex FE-model. Thus solving it can be a time-consuming process.

The quality measurements (δ_q) are used in RDO as objective

$$\begin{aligned} minimize: &\quad \delta_q(d,p) \\ subject \quad to: &\quad d_L \le d \le d_U \end{aligned} \tag{6}$$

or as constraint

$$\begin{aligned} minimize: &\quad Cost(d) \\ subject \quad to: &\quad \delta_q(d,p) \ge C_{\delta_q} \\ &\quad d_L \le d \le d_U \end{aligned} \tag{7}$$

for the outer loop optimization. Mostly the quality measurements are only estimates. The optimization has to cope with this fact. The appearing noise has to be known and treated. Through this it is not useful to have a convergence criterion that is more stringent than the error in the estimation.

Double loop approach Classical approaches solve the RDO-problem in a double loop method (fig.6). This means that the optimization takes place in deterministic space (outer loop) and for every deterministic design a value of robustness $\delta_q(d, p)$ is computed. Therefore robustness analysis (such as sampling, reliability) is executed in random space (inner loop). In fig. 5 it can be seen that in a projection of the results to the deterministic space a (hyper-)line appears for each optimization design (if there are no mixed variables). The number of evaluations for optimization N_{det} multiplies with that of stochastic analysis N_{sto}. So complexity is growing rapidly with the size of the problem. In most practical applications one evaluation (FE-solver run) can take a lot of time. For that reason methods are needed to reduce the effort.

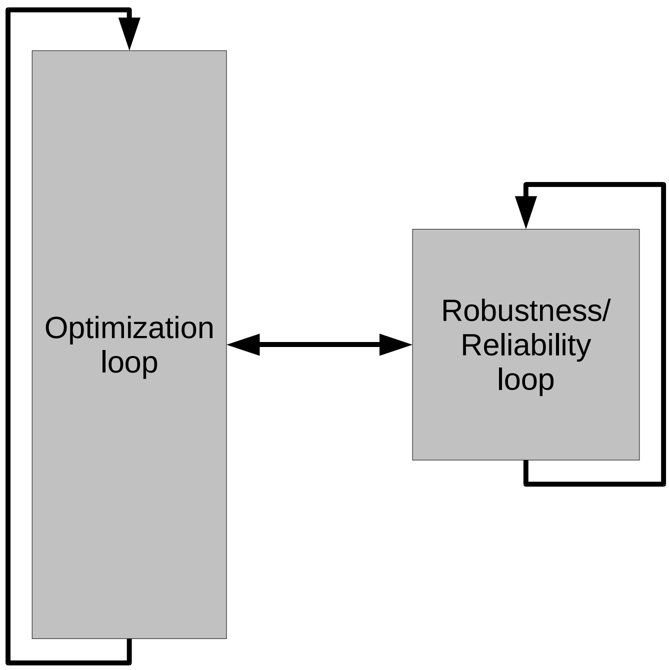

Figure 6: double loop

A lot of approaches exist to reduce this effort, some of them are shortly mentioned here. E.g. [Egorov et al.(2002)] describes a method which adapts the methods while progressing. This approach uses mathematical models with varying accuracy (from the lowest to the highest) during the solution process. The usage of meta models is another possibility to work more efficiently. Improving the classical approach by expanding the sampling strategy for stochastic dimensions according to the assumed probability level is suggested in [Youn and Choi(2004)]. A comparison of different approaches for obtaining reliability measures and their convergence in deterministic space can be found in [Youn et al.(2003)].

Decoupled loop approach Another option is to work iteratively. Optimization loop and stochastic loop are decoupled. Depending to the result of a stochastic analysis a shift in optimization space is accomplished. In [Chen et al.(2003)] a method is presented which shifts the design itself with respect to the expected distance to a security level.

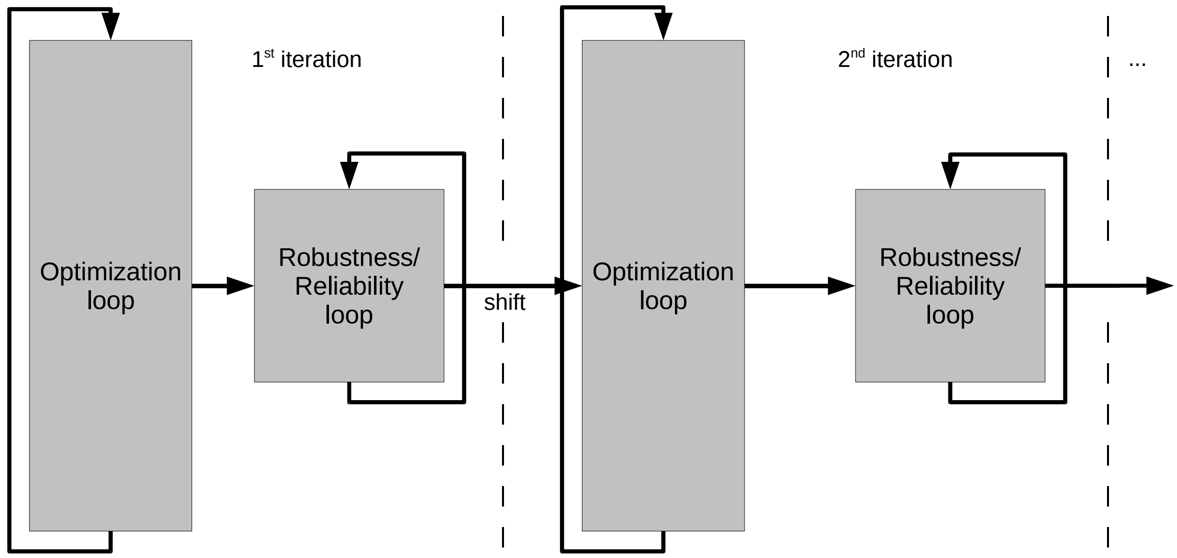

Figure 7: decoupled loop

Single loop approach Using the distance to the most probable point of failure (MPP) as part of the objective in optimization is presented in [Kharmanda et al.(2002)]. But this very efficient method can only be applied to small and smooth problems.

Orthogonal arrays and other DOE should be shortly mentioned here. They are widely used, but their use is in very controversial discussion. Reasons are the weaknesses of hard simulated samplings, neglecting interactions and so on. Advantages are a small effort of evaluations, an easy to implement method and a historically grown sympathy and understanding [Otto and Antonsson(1993)].

Criticism Most of the methods as mentioned are directly related to the kind of robustness value which is evaluated. Reliability methods often use FORM or other β/MPP based methods. Linearization and other (over-)simplifications inhibit their usage for large real problems with dynamic or nonlinear results. Additionally a classification into two scopes of interest, reliability based design optimization (RBDO) and (variance based) robust design optimization (RDO), inhibits coupled analysis of failure modes (small probability) with a description of the robustness close to the mean design (higher probability).

4 ADAPTIVE METAMODELING

The goal of using metamodels is to replace expensive simulations by a simple approximation.

$$y(x) = \hat{y}(x) + \epsilon \tag{8}$$

To create a metamodel information about the real model is needed. Therefore in most cases a set of support points is created by DOE-techniques. The responses of these points are derived from computer experiments. Afterwards the metamodel is filled with the obtained data. Every following evaluation of a response can be made on the surrogate (fig. 8). There exists various types of metamodels and the quality of the response relates on their choice and settings. But at this point it should be mentioned, that it is not part of this paper. A selection can be found in literature.

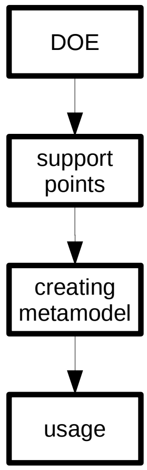

Figure 8: classical metamodeling

Obviously the choice of the underlying support point set also defines the quality of the results. So a lot of approaches exists to adapt the DOE iteratively. Some of them try to find an optimal support point set for a global approximation (I) [Queipo et al.(2005), Klotz et al.(2006)]. The goal of these approaches is to find a support point set that represents the whole model with a limited error. Once created metamodels can be used for all further explorations without the need of more complex simulations. The argue at these approaches is that good approximations in global space are not necessary for converging outer algorithms (e.g. optimization).

Some other approaches refine the support point set in a region of interest (II) e.g. obtained optimum [Jones et al.(1998), Etman et al.(1996), Roos and Bucher(2003), Abspoel et al.(1996), Toropov and Alvarez(1998), Stander and Craig(2002), Kurtaran et al.(2002)]. The problem of these is the convergence criteria as well as the n-th DOE settings. The main weakness of these methods is their limitation to one class of problems. A treatment for RDO can be found in [Jurecka(2007)]. But this is specialized for kriging models and a limited group of RDO-formulations.

In conclusion, evaluating the metamodel should be much cheaper than performing a complex computer simulation. This reduces the computational effort for a demanding, superior process. Additionally metamodels can be used to smooth noise that comes from the computer experiments.

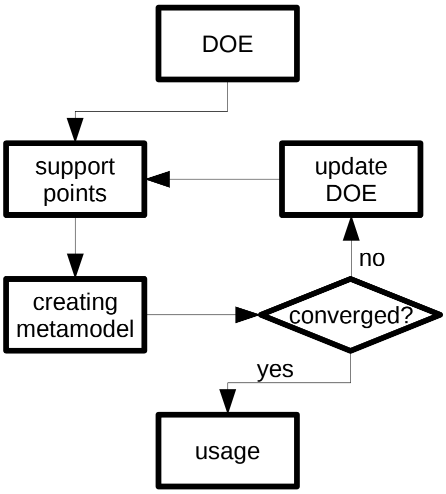

Figure 9: adaptive metamodeling (I)

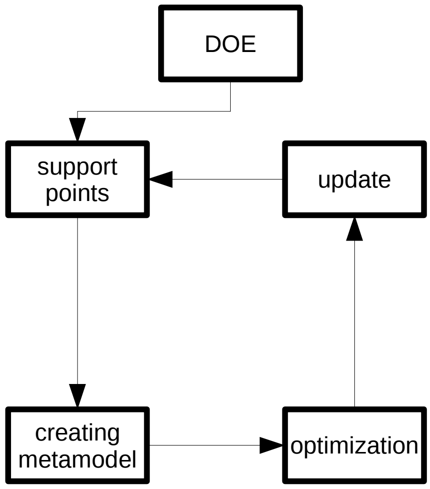

Figure 10: adaptive metamodeling (II)

new approach - sample recycling

As fig.5 points out, the more the optimization converges into one subspace the closer the hyperlines are getting. The basic idea of this recycling approach is to decide whether it is necessary to actually analyze a design or it is possible to use the previously analyzed designs that are nearby. Recycling of data is made in total space. The decision criterion whether recycling is possible (fig.11) or not is based (fig.12) on the quality of the meta model in the region of interest. There exist several methods to determine the quality of a response surface.

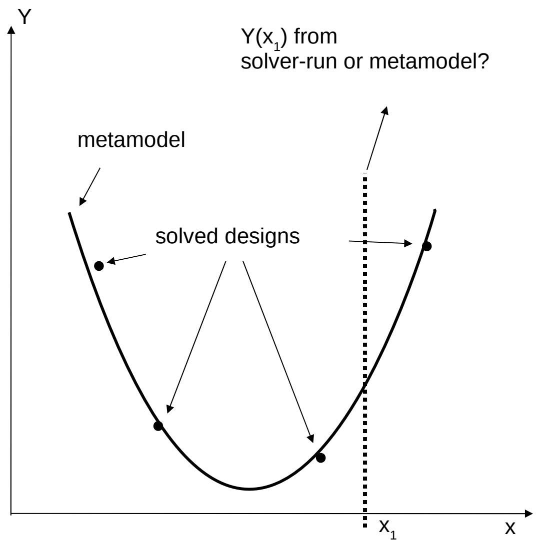

Figure 11: good approximation

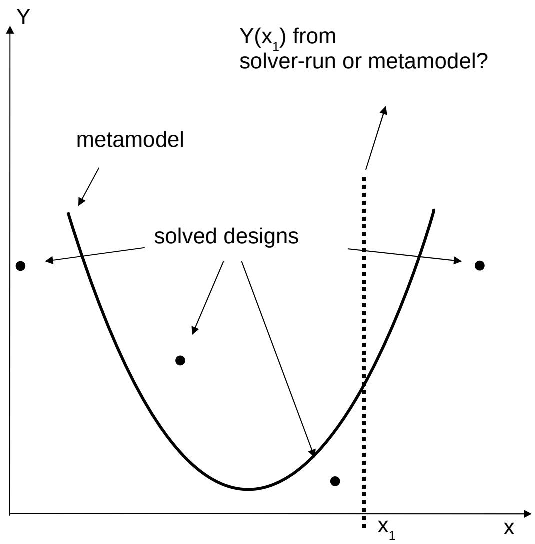

Figure 12: bad approximation

It can easily be defined offline e.g. by distances between the support points. Offline quality means that a calculation of the responses is not necessary. More complex methods use online quality therefore the responses are considered. Splitting the sample set in training and test data or cross-validation are examples for this kind [Queipo et al.(2005), Li(2007)]. With the help of these methods an error estimate $\hat{e}_P$ of the metamodel for every point P in total space can be obtained.

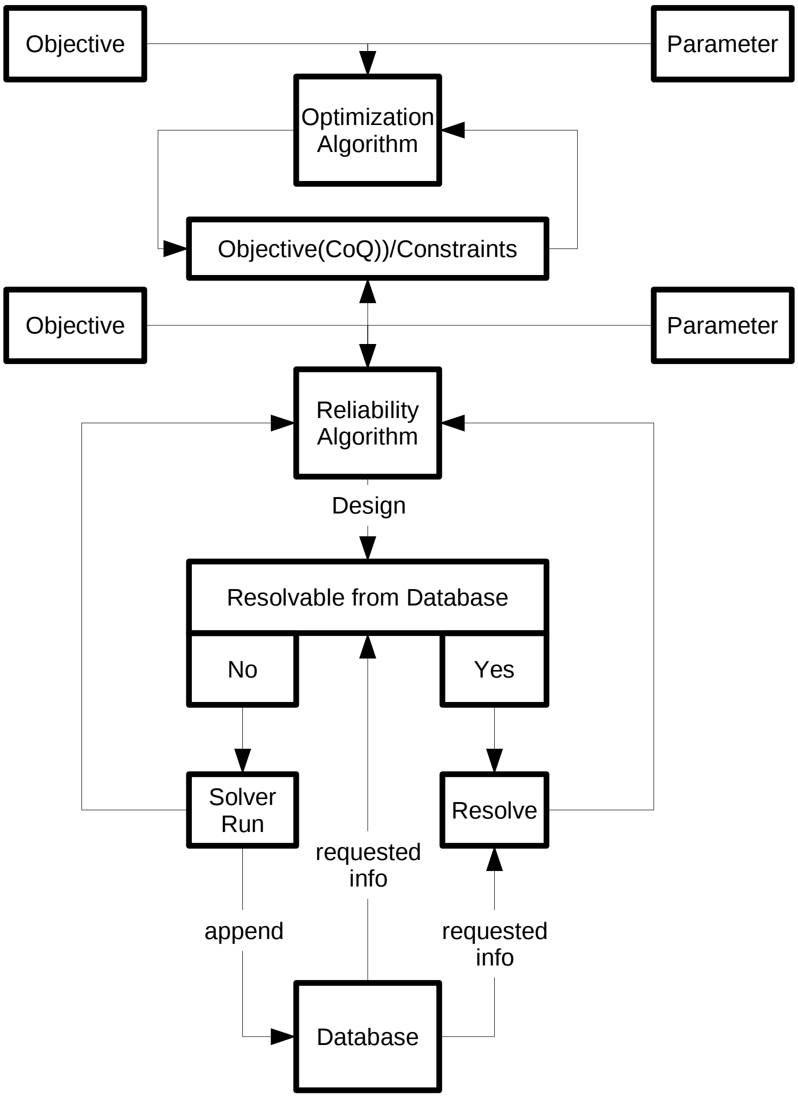

Figure 13: Algorithm for sample recycling

In the proposed method the computed error is used to calculate a measure of quality for every point ever demanded in space by the superior algorithm. Depending on its value, it is decided whether the result can be taken from the metamodel or a real calculation has to be done. So the calculation of new support points will be only necessary in certain subregions, i.e. in those subregions where the quality is bad and the demanding algorithm is interested in. In areas where the available data set can fully represent the calculation model no additional solver run is needed. Whenever a calculation is made its data is pushed back in a database and so it can be used as support point for the following steps. Through this the quality in the region of interest grows. The entire functionality of the approach can be seen in fig.13.

Especially in RDO-problems this strategy leads to massive reduction of the number of required solver runs. The main reason is that the outer looped optimization converges in a small area of the total space. In this region the meta model is refined until the needed quality is achieved. Subsequently, calculations are only necessary if the optimizer finds new subregions of interest. Starting with low quality demands and increasing them with every iteration of the outer loop shows additional potential to reduce the number of solver runs.

5 EXAMPLE

As example a beam under dynamic load is chosen. The example was first introduced in [Bucher(2005)]. The objective is to minimize the mass with a constraint on the probability of exceeding a maximum displacement. Since it was first introduced various methods were tested with it and a lot of improvement was achieved. Width d and height h are control-

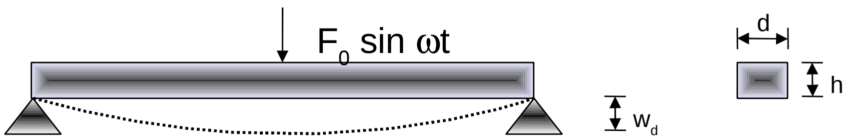

Figure 14: beam design principle

lable (design) parameters. The parameters of the harmonic load $F_0 \quad (meanvalue = 20000)$, $\omega \quad (meanvalue = 60)$ are random variables with a coefficient of variation of 0.1. Constant values are young's modulus $E = 3E10$, poisson ratio $\nu = 0.2$, mass density $\rho = 2500$ and the length of the beam $L = 10$. The problem is defined as:

$$min(d \cdot h) \tag{9}$$

subject to

$$\begin{aligned} 0 \leq d \leq 1 \\ 0 \leq h \leq 1 \\ P[w_d < 0.005] \leq 0.01 \end{aligned} \tag{10}$$

For the reason of a fair comparison with respect to [Roos et al.(2006)] the problem is solved with ARSM for outer loop and ARSM for inner loop [Roos and Bucher(2003)]. Starting at the deterministic optimum $d = 0.06$ / $h = 1.00$ it takes 1060 solver runs with the classical approach. The robust optimum lies at $d = 0.08/h = 1.00$. The estimated probability of failure is 0.01. Compared with the analytical solution this is the real optimal solution.

The sample recycling is used for the same algorithm settings with ARSM on ARSM in 2 steps of quality criteria. Starting with a lower quality criteria the RDO needed 61 real calculations. This means that approximately 1000 designs could be calculated on the response surface. Using the calculated design with a higher quality level, it was necessary to perform 18 additional calculations to find the same optimal point as the classical approach did. The effort was reduced by more than 90% without any loss of accuracy. The differences of effort between classical and suggested approach can be seen in fig.15 and fig.16.

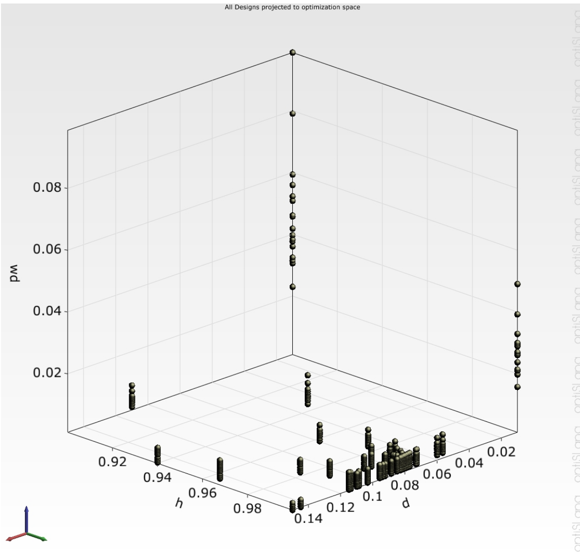

Figure 15: classical approach(1060samples)

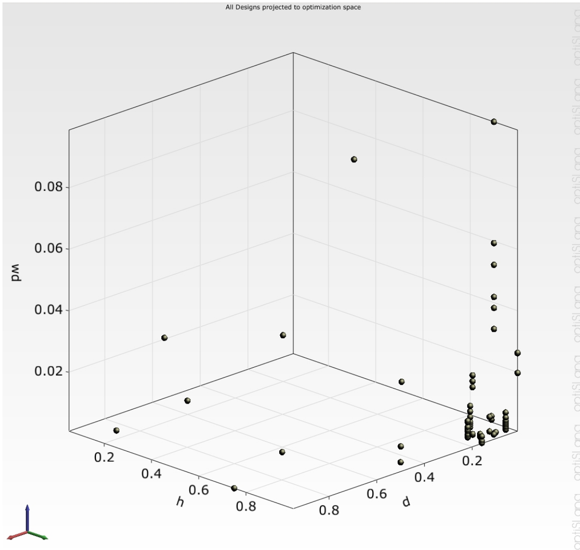

Figure 16: sample recycling(79samples)

To proof the usability for small probabilities the same example as above is used but now with a much smaller demanded probability of failure.

$$min(d \cdot h) \tag{11}$$

with respect to

$$\begin{aligned} 0 \leq d \leq 1 \\ 0 \leq h \leq 1 \\ P[wd < 0.005] \leq 3.4E - 6 \end{aligned} \tag{12}$$

Therefore the same two step evaluation as mentioned before was used. For the first step 63 solver runs were required. The calculated optimum was found at $(0.20/1.0)$. In the second step 18 additional solver runs were needed. The resulting optimum was calculated at $(0.185/1.0)$ with the demanded probability of failure. Which is the same point as the classical approach found with 4094 solver runs. This example shows that there is no loss of accuracy. Also, the number of necessary solver runs is increased only by a small amount compared to the lower probability. Hence, the suggested approach is applicable for small probabilities of failure, too.

6 CONCLUSION

Providing the required quality is seen by far as the most essential advantage in international business competition. Because of the widely known cost of change curve "built-in" quality should be aimed at. One approach for doing that is using RDO. However, in classical approaches of RDO the effort of stochastic analysis multiplies with the complexity of the optimization algorithm. The suggested approach for sample recycling shows that it is possible to reduce the expenditure enormously by re-using previously obtained results. It has been proofen in an example that this is possible without loss of accuracy.

If the outer algorithm allows parallelization it is also possible for the proposed method. Sample recycling can be used independent from underlying metamodel as well as from demanding superior algorithm. The database grows with every calculated design. Subsequently, calculations are only necessary if the optimizer finds new subregions of interest. As far as it has enough information, no more simulations are needed.

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