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NUMERICAL ANALYSIS OF THE CRACKED REINFORCED CONCRETE BEAMS

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Abstract. *The article presents the way of calculation of displacement in the bent reinforced concrete bar elements where rearrangement of internal forces and plastic hinge occurred. The described solution is based on prof. Borcz's mathematical model. It directly takes into consideration the effects connected with the occurrence of plastic hinge, such as for example a crack, by means of a differential equation of axis of the bent reinforced concrete beam. In order to present the assumed method an example of numerical solution of the cracked reinforced concrete beam was described where plastic hinge occurs in the predetermined zone.*

1 INTRODUCTION

Standards designed to calculate reinforced concrete structures assume that bent beams can be used in ultimate and serviceability limit states, i.e. cracks are allowable. Traditional methods of calculation take into account the effect of crack by means of an average decreasing of stiffness. Actually the effect of crack occurs only locally and depends on the kind of internal force in the cross-section (moment, shearing force). Moreover, in case of statically indeterminable structures, there is a possibility of internal forces rearrangement after cracking occurrence. It is connected with the formation of the so called plastic hinge which is either omitted from static calculations or referred to as discreet angular dislocation with the limited rotation stiffness.

The presented model allows to simulate the calculation under the assumption that plastic hinge occurs in the predetermined zone. This kind of approach to numerical solution makes it possible to calculate more precisely internal forces and displacements at the time of the hinge formation as well as to achieve the expected effects while simulating the rearrangement of internal forces.

The paper describes the application of mathematical model to calculate the bent bar structure elements which allows numerical modelling of the effects connected with plastic hinge formation.

2 CALCULATION MODEL FOR NONCRACKED AND CRACKED REINFORCED CONCRETE BAR STRUCTURES

The model is based upon differential equation of the bent axis beam made of elastic material which is as follows:

$$EJ\frac{d^4 v(x)}{dx^4} = p(x) \tag{1}$$

where:

EJ – beam stiffness,

x – beam coordinate,

v(x) – deflection rectangular to the beam axis,

p(x) – external load.

After cracking, the stiffness of the reinforced concrete beam EJ is being changed with the load increase. Making such an assumption, it can be accepted that the beam stiffness before the cracking is approximately constant and following the cracking it decreases.

In order to calculate the deflection value of the cracked beam standards usually assume the average stiffness of the cracked cross-section which depends on the load level. Most well known theories differentiate in a discreet way (local) or continual (continuous) change of stiffness at the beam length, but does not directly include the effects of plastic hinge in numerical calculations.

It is worth mentioning that one of the possible approaches can be Borcz's model [2, 3] which makes it possible to include the effects characteristic for concrete structures, such as: cracking, violation of steel and concrete collaboration or material plasticization. It assumes that the left side of differential equation (1) describes ideally elastic material while on the right side, operator $\mathfrak{D}$ (2) describing the material qualities, such as cracking, rheological effects and the resulting secondary stiffness changes can be introduced.

$$EJ\,\frac{d^4 v(x)}{dx^4} = p(x) + \mathfrak{D} \tag{2}$$

In this kind of equation general integral (2) is independent from stiffness changes resulting from the load character.

Part $\mathfrak{D}$ for the rectangular cracks is as follows:

$$\mathfrak{D}_1 = \sum_i \frac{d^2\delta(x - x_i)}{dx^2} r_i \tag{3}$$

where:

$$r_i^M = -r_{0i} + r_{1i} M(x_i) \tag{4}$$

Parameter r can be interpreted as a descreet pitch of rotation angle dv/dx located at coordinate x_i described by Dirac's delta, while the components consist of respectively - r_{0i} – residual part and $r_{1i}M(x_i)$ – elastic part. The rotation angle increment on the ordinate where the crack occurs describes the effects in a crack and includes all the effects between the cracks, for example creeping.

After the equation (2) was four times integrated the following deflection has been obtained:

$$v(x) = \frac{p(x)l^4}{24EJ} + \frac{1}{EJ}\left[l^3 r_i \sum_i (x - x_i) h(x - x_i) + \frac{1}{6}Ax^3 + \frac{1}{2}Bx^2 + Cx + D \right] \tag{5}$$

The equation (5) becomes a sum of deflections obtained from the ideally elastic beam and deflections from imperfections (residual and elastic crack opening).

$$v(x) = v_1(x) + v_2(x)\,(x - x_i)h(x - x_i) \tag{6}$$

$$h(x - x_i) = \begin{cases} 0 & x \le x_i \\ 1 & x > x_i \end{cases}$$

3 THE CHOSEN RESULTS OF INVESTIGATIONS (TAKEN FROM THE LITERATURE) ON THE REINFORCED CONCRETE BEAMS WITH PLASTIC HINGE

The investigations were carried out on beam elements having geometry as shown in Fig. 1.

The elements in series differed from each other in amount of reinforcement ρ and grain diameter of aggregate $d_{a\,max}$.

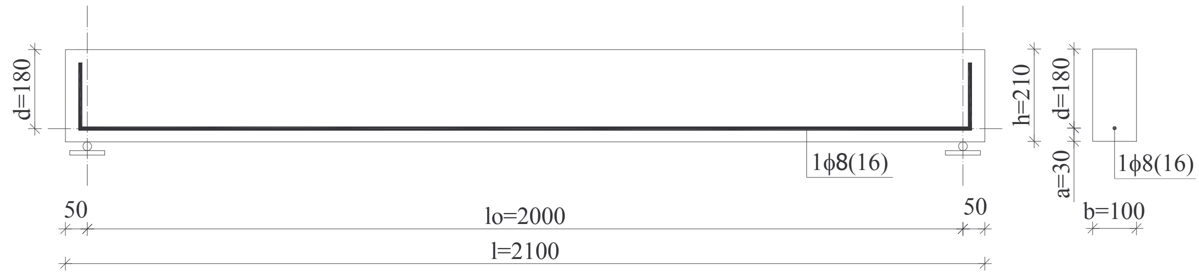

Fig. 1. Geometry of elements B1 ÷ B4

Table 1. Geometry of tested beams.

Geometrical data of tested beams									
Beam signature	h [mm]	d [mm]	b [mm]	Reinforce ment	A_s [mm^2]	ρ_s [%]	l_o [mm]	l [mm]	$d_{a\,max}$ [mm]
B1	210	180	100	1ϕ8 mm	50	0,28	2100	2000	16
B2	210	180	100	1ϕ8 mm	50	0,28	2100	2000	4
B3	210	180	100	1ϕ16 mm	201	1,12	2100	2000	16
B4	210	180	100	1ϕ16 mm	201	1,12	2100	2000	4

A_s – reinforcement cross-section area,

$d_{a\,max}$– maximum diameter of aggregate.

The reinforced beams simply supported were loaded by the force concentrated in the middle of the span. $P \geq 0{,}9P_u$ is the value of loading force. The sensors measuring distortions were located on both the reinforcement concrete and concrete surfaces with particular attention given to the zone where plastic hinge is formed, i.e. in the middle of the tested element (Fig. 2).

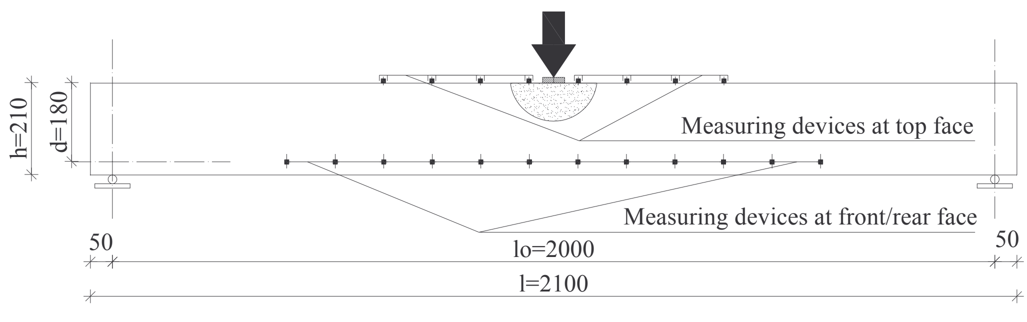

Fig. 2 Tensometers arrangement on elements B1 ÷ B4

4 THE RESULTS OF NUMERICAL ANALYSIS OF THE REINFORCED CONCRETE BEAMS WITH PLASTIC HINGE

Numerical analysis of the cracked reinforced concrete beams based upon Borcz's model where differential equation of the beam deflection axis is represented by the following formula (2) was carried out.

The deflection was calculated as a sum of displacement of ideally elastic beam (8) and additional displacements caused by the cracks appearance (9).

It is described by the following equation for the simple supported beam loaded by the concentrated load:

$$v = v_1 + v_2 \tag{7}$$

where:

$$v_1 = \frac{Pl^3}{48EJ} \tag{8}$$

v_1 – deflection of a beam made of ideally elastic material,

P – concentrated force,

$$v_2 = l\sum_{i=1}^{n_r}\left(r_{0i} + r_{1i}v,_{xx}\right) \tag{9}$$

v_2 – additional deflection caused by the appearance of cracks in a beam.

The deflections of the reinforced concrete beams with geometry as shown in Fig. 1 were calculated and according to the material data taken from the Bigaj investigations [1] for B1 ÷ B4 beam series.

The deflection value depends on the length of the crack zone l_t and the crack spacing s_{rm}.

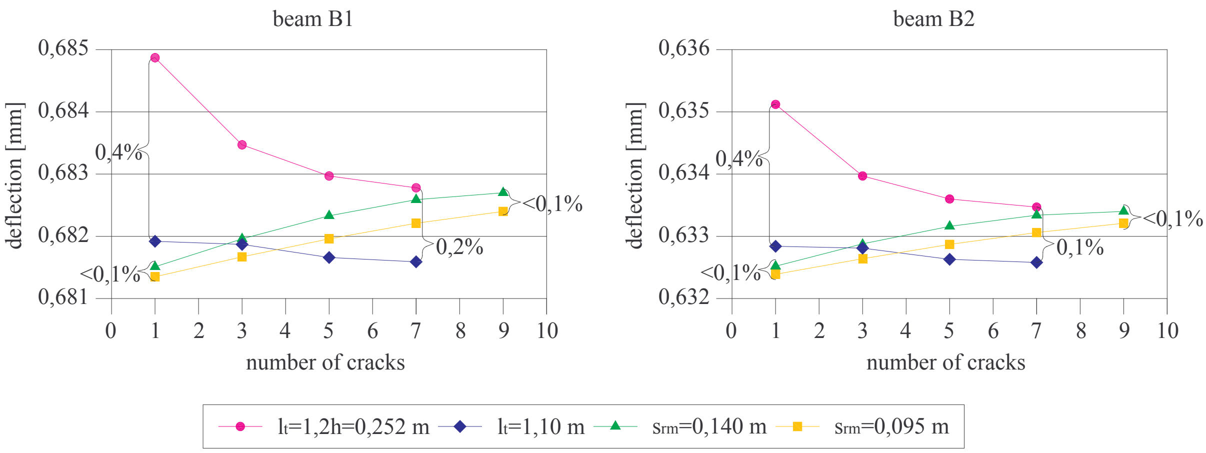

Fig. 3. Deflection of beams with ρ = 0,28 % - B1 and B2

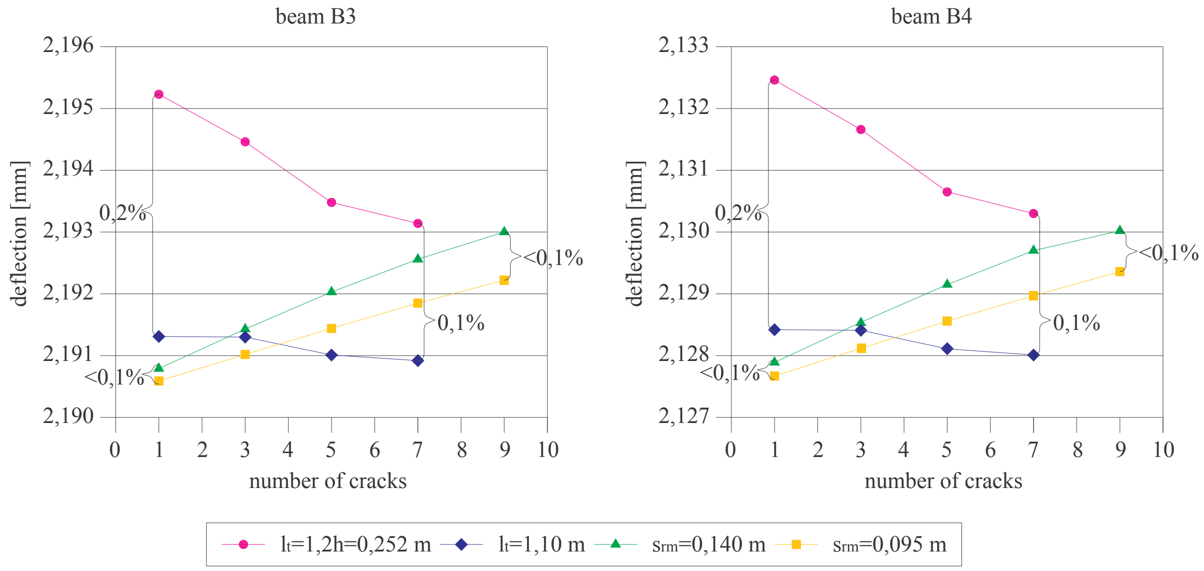

Fig. 4. Deflection of beams with ρ = 1,12% - B3 and B4

The deflections obtained from the calculations are shown in Fig. 3 and 4. The presented results prove that they are similar for the beams having the same geometry regardless of the reinforcement amount and maximum diameter of aggregate.

It is important to describe sensitivity of the applied parameters while modelling. In case of the above described model the following factors were taken into consideration: the width of the plastic hinge formation zone, the average crack spacing in this zone and the amount of the cross-section reinforcement. As the above diagrams show the most important parameter which determines the deflection value is just the reinforcement amount of the cross-section ρ. In case of the reinforcement amount included within the following interval where ρ = <0,3% - 1,1%> deflections increase even three times. When the plastic hinge formation zone is assumed according to the Eurocode 2 (l_t = 1,2h ≈ 0,25 m) as well as observations made upon testing the final deflection result has changed by 0,1 – 0,4 % where (l_t ≈ 1,0 m). It can thus be concluded that the average crack spacing s_{rm} has much less influence on on the deflections. For the values within interval s_{rm} = <0,09 – 0,14 m> the deflection change is less than 0,1 %. Increasing or decreasing character of these diagrams has no practical value for the calculation of the structure deflections.

However, attention must be given to the quality of aggregate. When the same amount of reinforcement and maximum diameter of aggregate grain $d_{a\,max}$ = 4 mm and $d_{a\,max}$ = 16 mm are considered, the deflection values decrease by 3 to 8 % (Fig. 5).

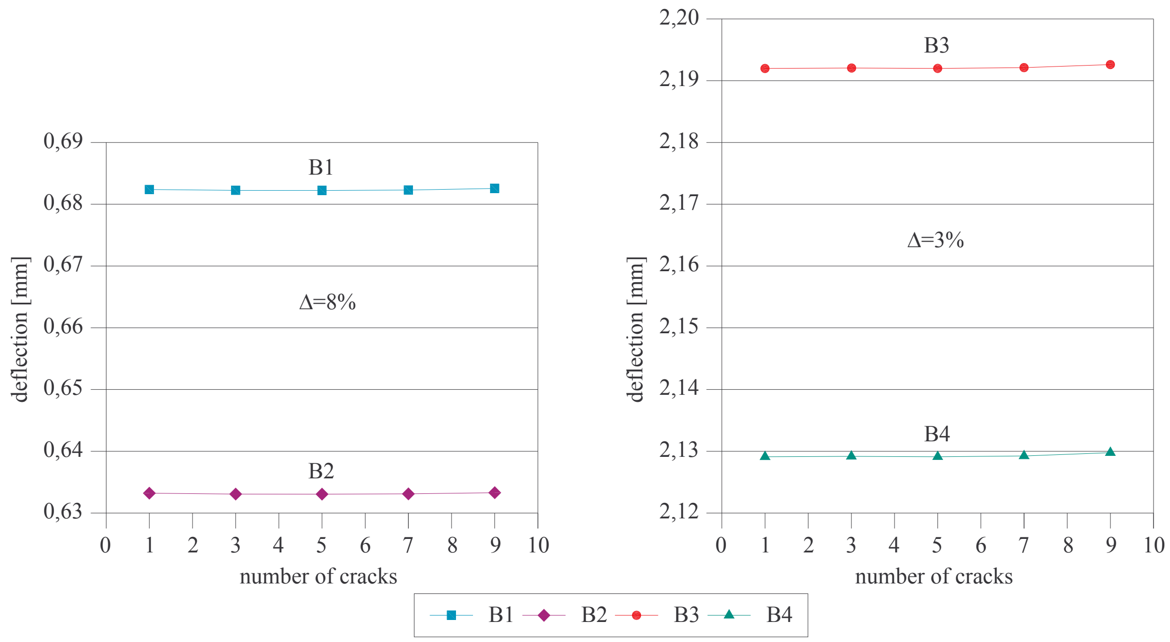

Fig. 5. Average deflection of beams B1 ÷ B4

5 SUMMARY

The EN Eurocode 2 makes it possible to consider the influence of plastic hinge on the values of the reinforced concrete structures. This influence can also be assumed using other analytical methods, such for example the above presented Borcz's method. However, the results obtained by the application of Eurocode 2 are higher from those received in testing.

Just comparably big error level occurs when calculations are made by means of Borcz's method, but in the latter case, the results depend on the assumptions made beforehand. This method makes it possible to apply the experimental results using parameters r_1 i r_0. When the experimental results are taken into account, one could observe the compatibility between the calculations and actual deflections of the structure.

Though the method presented in this paper refers to the bent reinforced concrete bar structures, the authors have been currently developing the subject and trying to apply the method for slab structures as well.

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