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NUMERICAL AND LABORATORY ANALYSIS OF STRESS DISTRIBUTION IN THE REINFORCED CONCRETE SUPPORT BEAM BRACKET EXPOSED TO DAMAGE

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Abstract. *The presented case refers to the ready built structure, where some cracks on the cross-section of the support beam bracket was observed during construction activities. Designers and contractors applied to the Building Institute of Wrocław University of Technology for consultation and explanation of the reasons of such slant cracks occurrence.*

The paper describes the model of support beam bracket as a spatial structure by means of three dimensional finite elements method. The results of numerical analysis and laboratory tests are given in abridged form. Estimation of the applied method is included as well as some conclusions.

1 INTRODUCTION

The article presents analysis of stress distribution in the reinforced concrete support beam bracket which is a component of prefabricated reinforced concrete building.

The building structure is spatial frame where dilatations were applied. The proper stiffness of its structure is provided by frames with stiff joints, monolithic lift shifts and staircases.

The prefabricated slab floors are supported by beam shelves which are shaped as inverted letter 'T'. Beams are supported by the column brackets. In order to lower the storey height and fulfill the architectural demands at the same time, the designer lowered the height of beam at the support zone (Fig.1 and 2).

The analyzed case refers to the bracket zone where the slant crack (Fig.3). on the support beam bracket was observed. It could appear as a result of overcrossing of allowable tension stresses in reinforced concrete, in the bracket zone.

It should be noted that the construction solution applied, i.e. concurrent support of the "undercut" beam on the column bracket causes local concentration of stresses in the undercut zone where the strongest transverse forces and tangent stresses occur concurrently. Some additional rectangular stresses being a result of placing the slab floors on the lower part of beam shelves sum up with those described above.

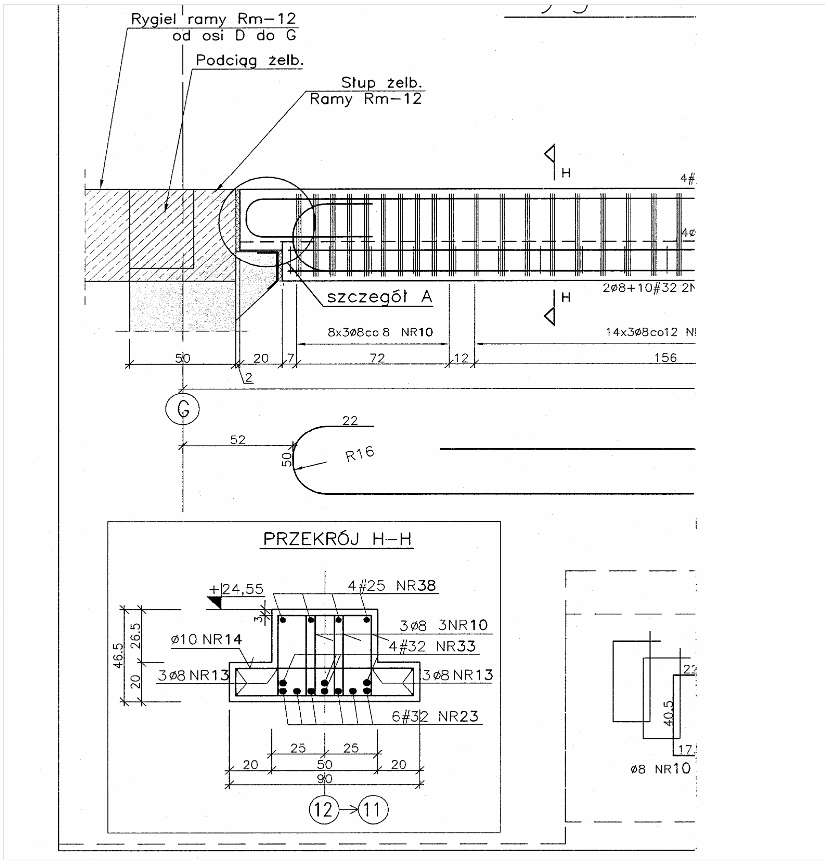


Fig.1. Reinforcement detail of beam bracket and the way it is placed on the column bracket

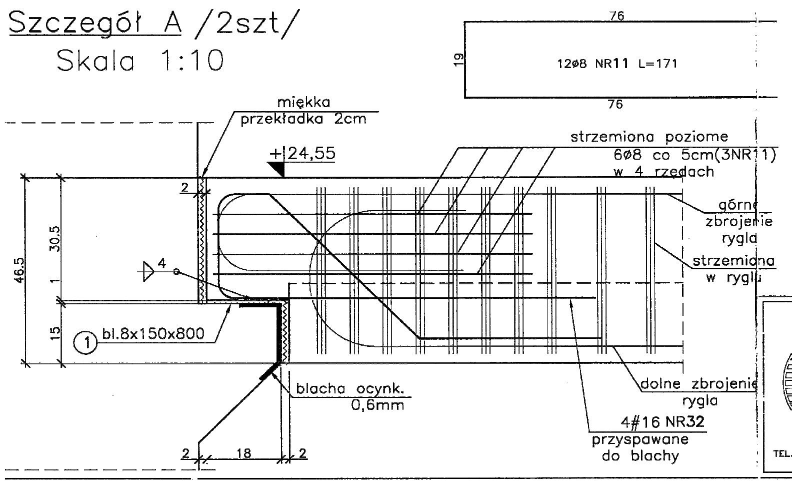


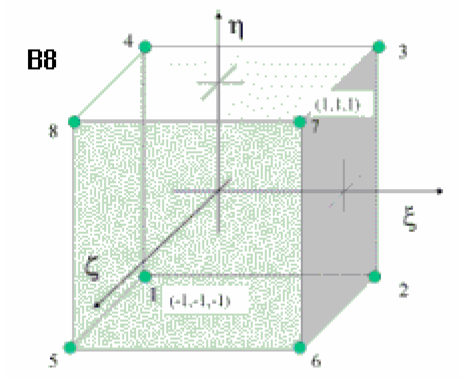
Fig.2. Beam with brackets under slab floors and beam bracket

Fig.3. Cracked beam bracket

2 NUMERICAL ANALYSIS OF STRESS STATE

Numerical analysis of stress state using Finite Element Method was carried out. The support beam bracket was modeled as three dimensional structure using isoperimetric, volumetric finite elements.

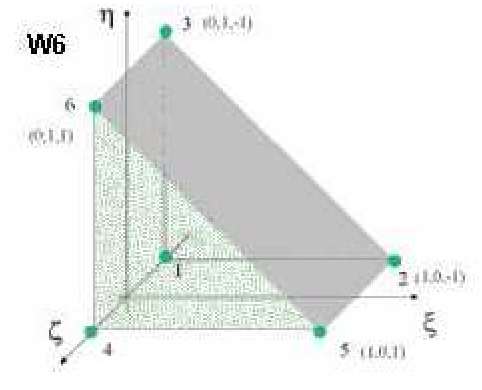
Three types of elements , such as: cuboid (eight joints), wedge (six joints and tetrahedron (four joints) were applied to create numerical model. Shape functions and numeration of joints described on 3D standard elements is presented below.



$$N_i = \frac{1}{8}(1+\xi_i\xi)(1+\eta_i\eta)(1+\zeta_i\zeta)$$

i=1,n (n – number joint)

Fig.4. Cuboid standard element and its shape function

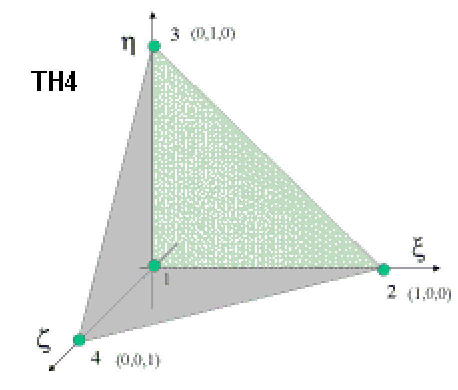


$$N_i = \frac{1}{2}(1+\zeta_i\zeta)\cdot N^{T6}_{k(i)}(\xi,\eta)$$

$$k(i) = ((i-1)(\mathrm{mod}\ 3))+1$$

$$N_1^{T6} = 1-\xi-\eta$$
$$N_2^{T6} = \xi$$
$$N_3^{T6} = \eta$$

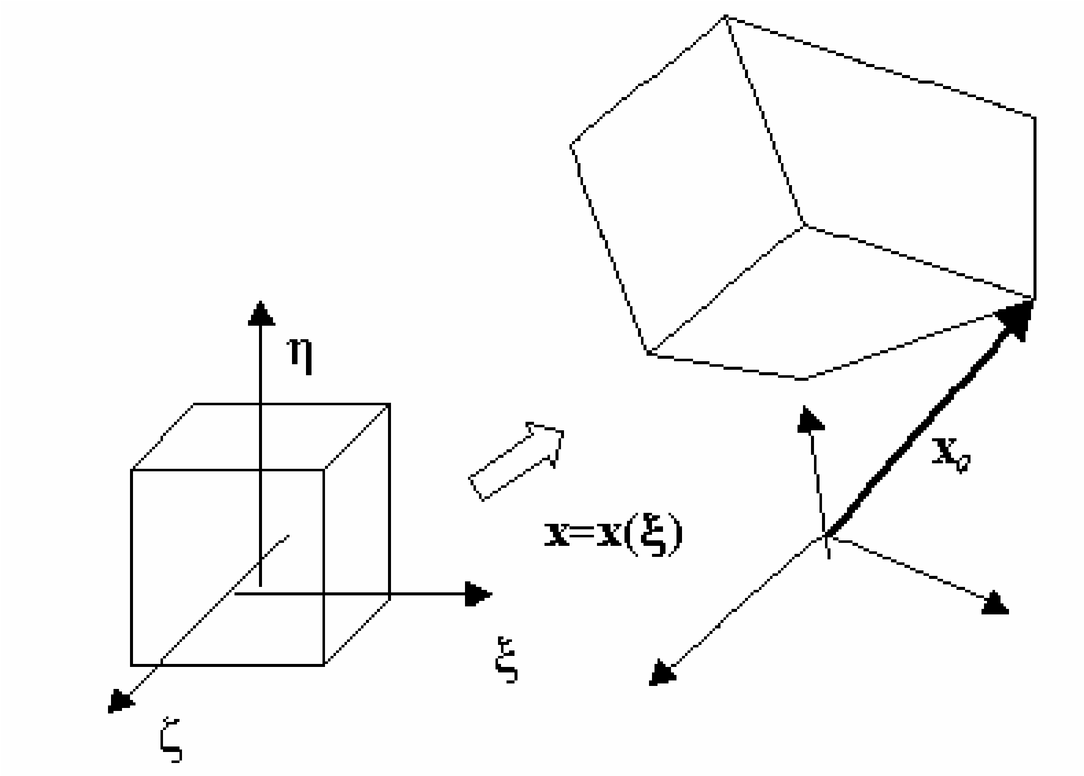
Fig.5. Wedge standard element and its shape function



$$N_1 = 1-\xi-\eta-\zeta$$
$$N_2 = \xi$$
$$N_3 = \eta$$
$$N_4 = \zeta$$

Fig.6. Tetrahedron standard element and its shape function

Geometry of element is specified by isoparametric representation of the standard element on an optional spatial element (Fig.7.).



$$x(\xi)=\sum_{a=1,n} x_a N_a(\xi)$$

Fig.7. Standard element representation on an optional element

Displacement field inside the element is represented by the following equation:

$$u=[u,v,w]^T$$

$$u(\xi)=\sum_{a=1,n} u_a N_a(\xi)$$

Deformations are described by the equation given below:

$$\varepsilon=[\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}, \gamma_{xx}, \gamma_{yy}, \gamma_{zz}]^T$$

$$\varepsilon(\xi)=\mathbf{B}(\xi)u=\sum_{a=1,n} B_a(\xi) u_a$$

where matrices B are determined as:

$$\mathbf{B} = [\mathbf{B}_a], \quad \mathbf{B}_a = \begin{bmatrix} \frac{\partial N_a}{\partial x} & 0 & 0 \\ 0 & \frac{\partial N_a}{\partial y} & 0 \\ 0 & 0 & \frac{\partial N_a}{\partial z} \\ \frac{\partial N_a}{\partial y} & \frac{\partial N_a}{\partial x} & 0 \\ \frac{\partial N_a}{\partial z} & 0 & \frac{\partial N_a}{\partial x} \\ 0 & \frac{\partial N_a}{\partial z} & \frac{\partial N_a}{\partial x} \end{bmatrix}, \quad a = 1, n$$

The derivatives of shape function which occur in elements of matrix B are described as:

$$\frac{\partial N_a}{\partial \mathbf{x}} = (\mathbf{J}^{-1})^T \frac{\partial N_a}{\partial \xi}, \qquad \mathbf{J} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial x}{\partial \eta} & \frac{\partial x}{\partial \zeta} \\ \frac{\partial y}{\partial \xi} & \frac{\partial y}{\partial \eta} & \frac{\partial y}{\partial \zeta} \\ \frac{\partial z}{\partial \xi} & \frac{\partial z}{\partial \eta} & \frac{\partial z}{\partial \zeta} \end{bmatrix}$$

Stresses in linear – elastic range are determined as:

$$\boldsymbol{\sigma} = [\sigma_{xx}, \quad \sigma_{yy}, \quad \sigma_{zz}, \quad \sigma_{xy}, \quad \sigma_{xz}, \quad \sigma_{yz}]^T,$$

$$\boldsymbol{\sigma} = \mathbf{D}(\mathbf{Bu} - \varepsilon^0),$$

where $\boldsymbol{\varepsilon}^0$ are the imposed deformations caused by contraction or temperature and **D** is constitutive matrix of isotropic linear – elastic material presented below:

$$\mathbf{D} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 0 & 0 & 0 \\ & 1 & \frac{\nu}{1-\nu} & 0 & 0 & 0 \\ & & 1 & 0 & 0 & 0 \\ & & & \frac{1-2\nu}{2(1-\nu)} & 0 & 0 \\ & sym. & & & \frac{1-2\nu}{2(1-\nu)} & 0 \\ & & & & & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix}$$

The modeled loading of element corresponds with the loading values of the real structure. The articulated joint support located on beam brackets was used. In order to check the influence of support on the stresses distribution in the support beam zone, the first point of support was located on the one end of the beam support edge and the other one in a distance of 5 cm from the undercut beam support edge.

Below (Fig.8 – 10) the chosen stresses map is presented for the "wings" loading of 100 kN/m^2

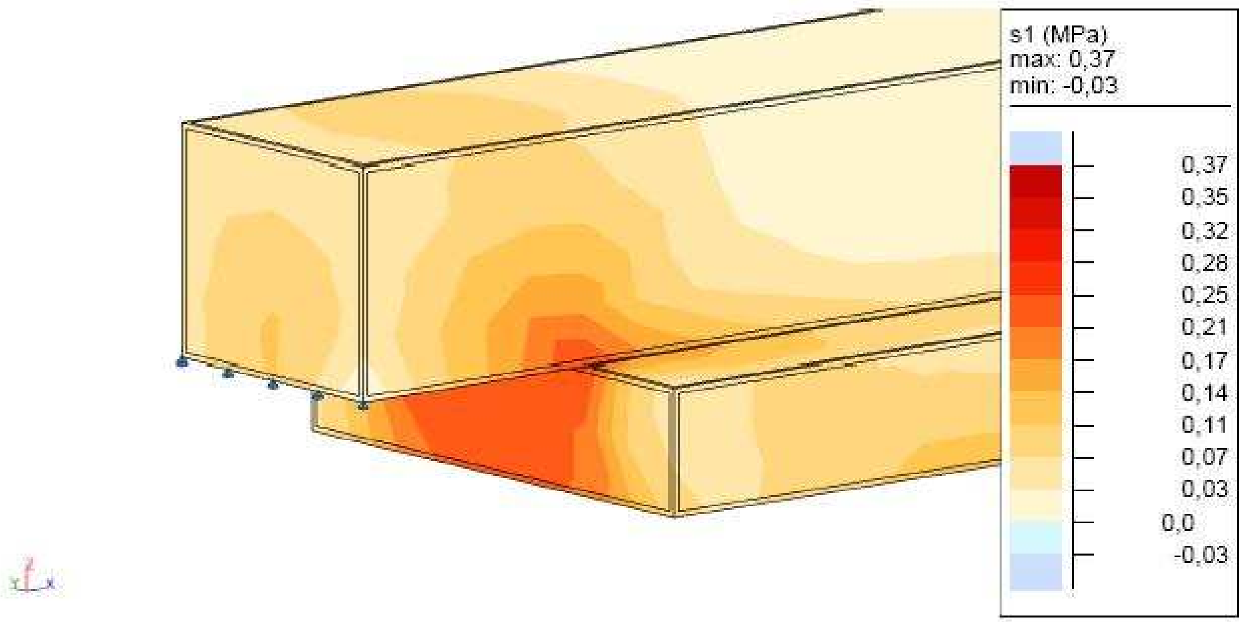


Fig.8. Main stress trajectory map (support located on the edge)

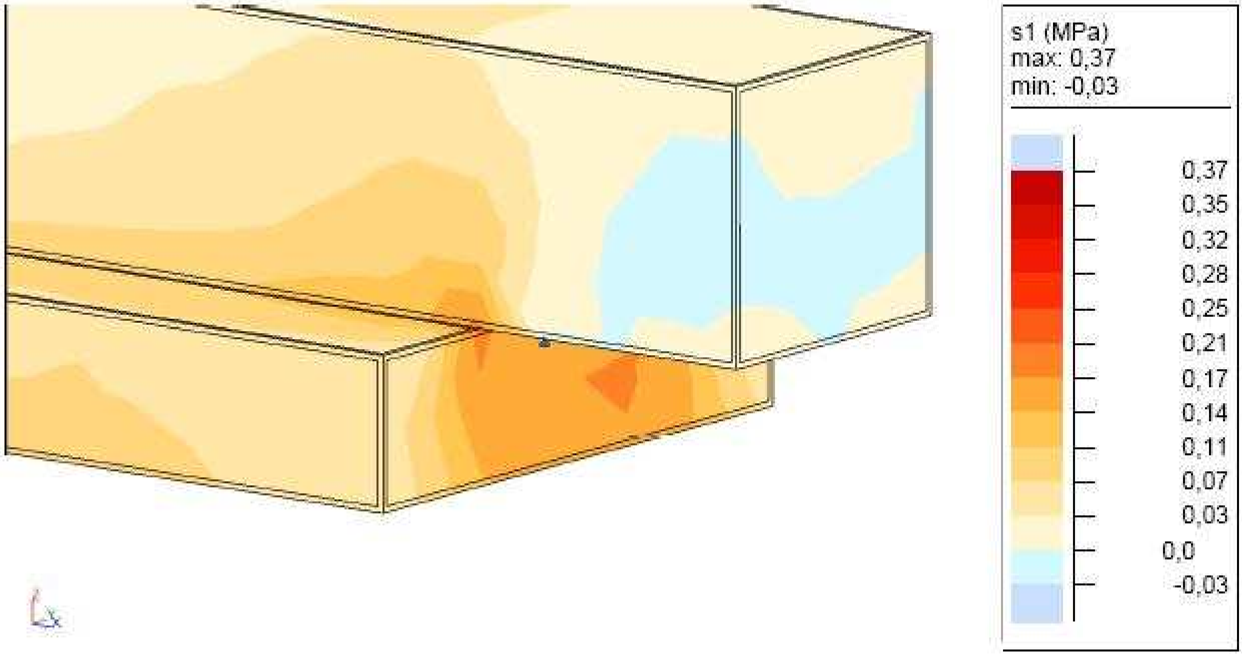


Fig.9. Main stress trajectory map (support located 5 cm from the undercut)

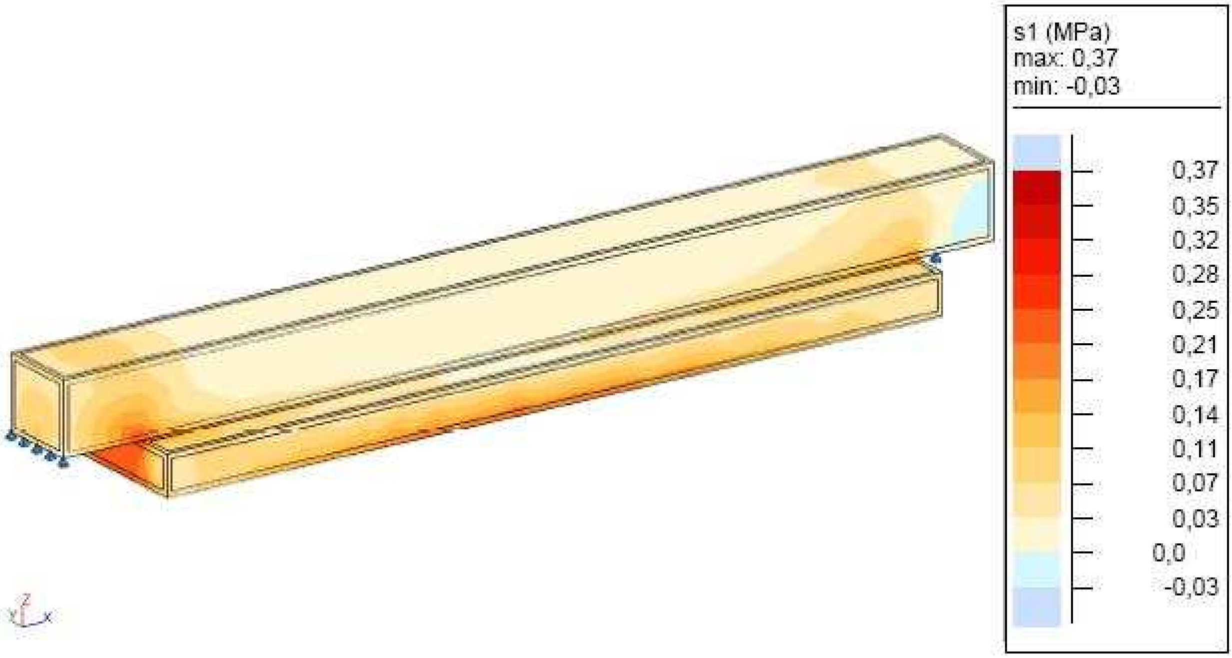


Fig.10. Main stress trajectory map (general view of the whole element)

The results of static analysis are presented as colourful maps of main trajectories. The scale on the right side allows to estimate the value of stresses in elements.

3 LABORATORY TESTS

In order to verify the assumed numerical model some laboratory tests of elements in natural scale were carried out. General scheme of the test stand is given in Fig. 11.

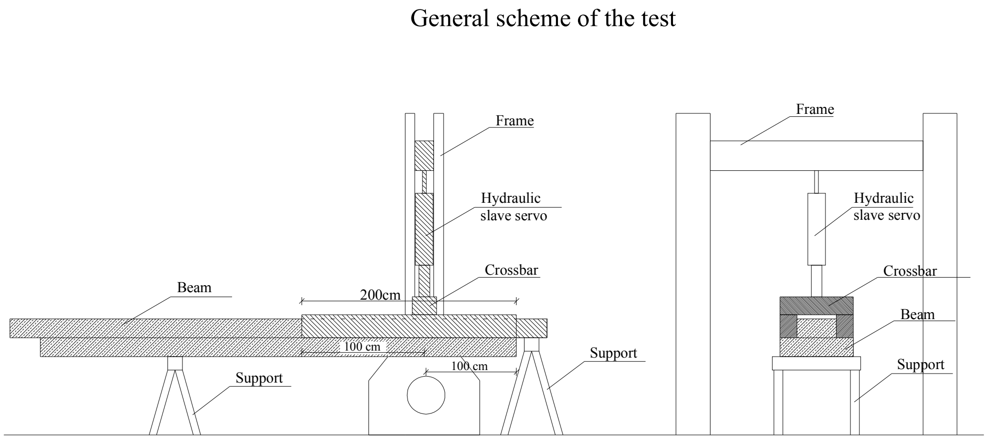


Fig.11. General scheme of the test stand used to provide test on supports of the reinforced concrete beams

The element was cyclically loaded up to the determined force value and the unloading. Continuous measurement of the element deformations was performed by placing electroresistant tensometers on the reinforcement steel and the reinforced concrete surface (Fig.12 and 13).

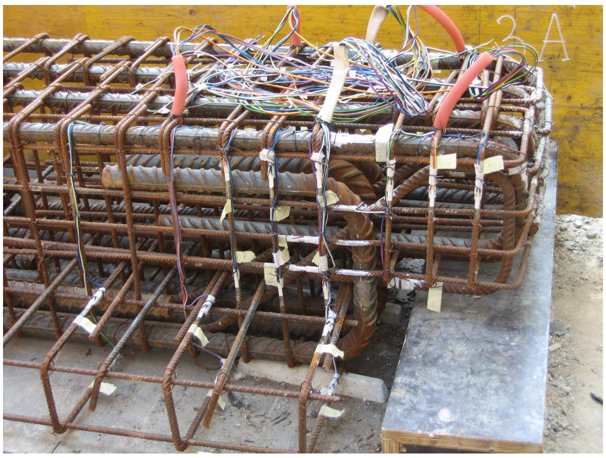


Fig.12. Arrangement of electroresistant tensometers on the reinforcement bars

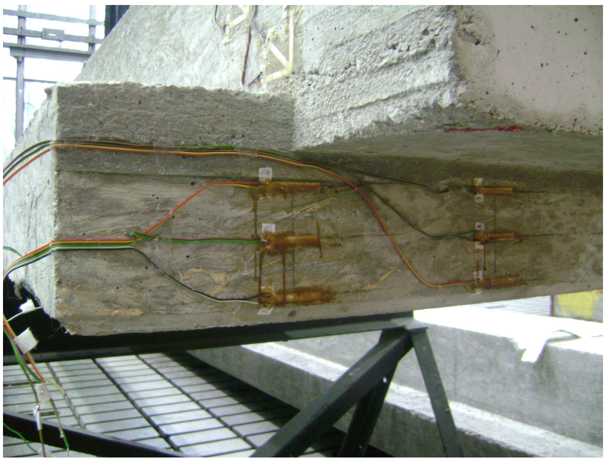


Fig.13. Arrangement of electroresistant tensometers on the element surface

In each loading cycle values of the applied force was increased. Finally, in the last loading cycle the beam support was destroyed (Fig.14 and 15.).

Fig.14 i 15. View of the destroyed element (side and underneath view)

4 CONCLUSIONS

In order to maximize the usable area ratio in relation to the cubature of objects some solutions including beam supports have been increasingly developed. On the other hand, this kind of the economic pressures makes designing of beam supports more and more difficult.

One of the most important problems arising is a difficulty to properly estimate the values and directions of the main stresses. In addition it is much more complicated in case of beams with compound variable-section.

In such situation, assessment of the correct stress trajectory by using traditional methods may cause miscalculation in measurements of elements causing serious damage to the object.

The similarity of records obtained in laboratory tests and those from numerical analysis proves that analytical method applied allows to reconstruct the element performance with much accuracy. Using the obtained values and stress trajectory maps a designer is able to distribute reinforcement quite precisely and in the amount demanded for the failure-free performance of element.

These types of constructions require using big amount of reinforcement that causes difficulties in its distribution and proper concrete vibration, so this may have a considerable impact on the quality of elements and safety of the structures.

Further experimental tests supported by numerical analysis are planned considering spatial character of this type of structures and including numerical model of cracks and damage.

REFERENCES

[1] PN – 84/B – 03264: „Konstrukcje betonowe, żelbetowe i sprężone. Obliczenia statyczne i projektowanie"

[2] PN – B – 03264/2002: „Konstrukcje betonowe, żelbetowe i sprężone. Obliczenia statyczne i projektowanie"

[3] W. Starostolski „Konstrukcje żelbetowe", PWN 2005

[4] K. Godycka – Ćwirko.: „Wsporniki żelbetowe. Badania, teoria, projektowanie." Wydawnictwo Politechniki Gdańskiej, Gdańsk 2001

[5] E. Rusiński, J. Czmochowski, T. Smolnicki „Zaawansowana metoda elementów skończonych w konstrukcjach nośnych", Oficyna Wyd. Pol. Wroc., Wrocław, 2000