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ON THE SOLUTIONS OF ELECTRICAL IMPEDANCE EQUATION: A PSEUDOANALYTIC APPROACH FOR SEPARABLE-VARIABLES CONDUCTIVITY FUNCTION

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Abstract. *Using a quaternionic reformulation of the electrical impedance equation, we consider a two-dimensional separable-variables conductivity function and, posing two different techniques, we obtain a special class of Vekua equation, whose general solution can be approach by virtue of Taylor series in formal powers, for which is possible to introduce an explicit Bers generating sequence.*

1 INTRODUCTION

The study of the electrical impedance equation

$$\mathrm{div}\left(\sigma \mathrm{grad} u\right) = 0, \tag{1}$$

where σ is the conductivity function and u denotes the electric potential, is the base for well understanding the electrical impedance tomography problem, also known as the two-dimensional Calderon's problem [3]. It is remarkable that in many classical works fully dedicated to this topic, the authors could even think that it was *impossible* to express the general solution of (1) in analytic form (see e.g. [20, pag. 99]). But in 2006, K. Astala and L. Päivärinta [1] noticed for the first time that the two-dimensional case of (1) was closely related with a Vekua equation [19], and indeed it was through this relation that they solved the Calderon's problem in the plane. As a consequence, in 2007 V. Kravchenko and H. Oviedo [11] used the theory developed by L. Bers [2] to express the general solution of a two-dimensional electrical impedance equation with a certain class of conductivity functions σ. In this work we review two different techniques that based onto a quaternionic differential equation completely equivalent to (1), allow us to obtain a Vekua equation when considering the two-dimensional case, and using recent discoveries on the field of pseudoanalytic function theory [7], we show how to build the general solution for this equation in terms of Taylor series in formal powers, when the conductivity function σ is at least once differentiable in the plane, and it is separable-variables. This class of conductivity functions constitute an useful and quite common approach for the electrical impedance tomography problem [5].

2 PRELIMINARIES

2.1 Elements of quaternionic analysis

Let us denote the algebra of real quaternions by $\mathbb{H}\left(\mathbb{R}\right)$ [6][10]. The elements belonging to $\mathbb{H}\left(\mathbb{R}\right)$ have the form $q = \sum_{k=0}^{3} q_k e_k$, where q_k, $k = \overline{0,3}$ are real-valued functions, $e_0 = 1$ and e_k, $k = \overline{1,3}$ are the standard quaternionic units possessing the multiplication properties

$$e_1 e_2 e_3 = -1;$$
$$e_k^2 = -1, \ k = \overline{1,3}.$$

We will use the notation $q = q_0 + \overrightarrow{q}$, where q_0 is *the scalar part of* q and $\overrightarrow{q} = \sum_{k=1}^{3} q_k e_k$ is the *vectorial part of* q. Notice every purely-vectorial quaternionic-valued function $q = \overrightarrow{q}$ can be identified with a three-dimensional vectorial-valued function $\overrightarrow{q} \in \mathbb{R}^3$, and that their relation is one-to-one. In virtue of this isomorphism, we can write the multiplication $q = q_0 + \overrightarrow{q}$ and $p = p_0 + \overrightarrow{p}$ as follows

$$q \cdot p = q_0 p_0 + q_0 \overrightarrow{p} + p_0 \overrightarrow{q} - \langle \overrightarrow{q}, \overrightarrow{p} \rangle + \left[\overrightarrow{q} \times \overrightarrow{p} \right].$$

It is evident that, in general, $q \cdot p \neq p \cdot q$, thus we will use the notation

$$M^p q = p \cdot q$$

for indicating the multiplication by the right-hand side of q by p. In the set of at least once-differentiable quaternionic-valued functions, it is defined the Moisil-Theodoresco differential operator D [6]:

$$D = e_1\frac{\partial}{\partial x_1} + e_2\frac{\partial}{\partial x_2} + e_3\frac{\partial}{\partial x_3}.$$

It acts onto a quaternionic-valued function $q = q_0 + \overrightarrow{q}$ as follows

$$Dq = \text{grad}q_0 - \text{div}\overrightarrow{q} + \text{rot}\overrightarrow{q}.$$

2.2 Elements of pseudoanalytic functions

According to [2], let us consider a pair of complex valued functions (F, G) such that

$$\text{Im}\left(\overline{F}G\right) > 0, \tag{2}$$

where $\overline{F}$ is the complex conjugation of F : $\overline{F} = \text{Re}F - i\text{Im}F$, and i is the standard imaginary unit $i^2 = -1$. Therefore, any complex-valued function W can be represented as

$$W = \phi F + \psi G,$$

where ϕ and ψ are real-valued functions. A pair of functions satisfying (2) is called a *Bers generating pair.* The *derivative in the sense of Bers* or (F, G)-*derivative* of W is introduced as

$$\frac{d_{(F,G)}W}{dz} = (\partial_z\phi)\, F + (\partial_z\psi)\, G, \tag{3}$$

where $\partial_z = \frac{\partial}{\partial x} - i\frac{\partial}{\partial y}$. This derivative will exist if and only if

$$(\partial_{\overline{z}}\phi)\, F + (\partial_{\overline{z}}\psi)\, G = 0, \tag{4}$$

where $\partial_{\overline{z}} = \frac{\partial}{\partial x} + i\frac{\partial}{\partial y}$. Let us introduce the the *characteristic coefficients* of the pair (F, G):

$$A_{(F,G)} = -\frac{\overline{F}\partial_z G - \overline{G}\partial_z F}{F\overline{G} - \overline{F}G},\; B_{(F,G)} = \frac{F\partial_z G - G\partial_z F}{F\overline{G} - \overline{F}G}, \tag{5}$$
$$a_{(F,G)} = -\frac{\overline{F}\partial_{\overline{z}} G - \overline{G}\partial_{\overline{z}} F}{F\overline{G} - \overline{F}G},\; b_{(F,G)} = \frac{F\partial_{\overline{z}} G - G\partial_{\overline{z}} F}{F\overline{G} - \overline{F}G}.$$

Using these notations, the (F, G)-*derivative* of W (3) can be written as

$$\frac{d_{(F,G)}W}{dz} = \partial_z W - A_{(F,G)}W - B_{(F,G)}\overline{W}, \tag{6}$$

and (4) becomes

$$\partial_{\overline{z}} W - a_{(F,G)}W - b_{(F,G)}\overline{W} = 0. \tag{7}$$

This equation is called the *Vekua equation* [19]. The functions W fulfilling (7) are known as (F, G)-*pseudoanalytic functions*.

The following statements were proposed in [2].

Remark 1 *The functions F and G of the generating pair (F, G) are (F, G)-pseudoanalytic, and their (F, G)-derivatives fulfill*

$$\frac{d_{(F,G)}F}{dz} = \frac{d_{(F,G)}G}{dz} = 0.$$

Definition 2 *Let (F,G) and (F_1,G_1) be two generating pairs. If their characteristic coefficients satisfy the relations*

$$a_{(F,G)}=a_{(F_1,G_1)},\ B_{(F,G)}=-b_{(F_1,G_1)}. \tag{8}$$

(F_1,G_1) will be called the successor *pair of (F,G), as well (F,G) will be the* predecessor *pair of (F_1,G_1) .*

Theorem 3 *Let W be a (F,G)-pseudoanalytic function, and let (F_1,G_1) be a successor pair of (F,G). Therefore the (F,G)-derivative of W will be a (F_1,G_1)-pseudoanalytic function.*

Definition 4 *Let (F,G) be a generating pair. Its* adjoint pair *(F^*,G^*) is defined by the formulas*

$$F^*=-\frac{2\overline{F}}{F\overline{G}-\overline{F}G},\quad G^*=\frac{2\overline{G}}{F\overline{G}-\overline{F}G}.$$

It is also possible to introduced an (F,G)-*integral* of a complex-valued function W as

$$\int_{z_0}^{z_1}Wd_{(F,G)}z=F\left(z_1\right)\operatorname{Re}\int_{z_0}^{z_1}G^*Wdz+G\left(z_1\right)\operatorname{Re}\int_{z_0}^{z_1}F^*Wdz,$$

such that if $W=\phi F+\psi G$ is (F,G)-pseudoanalytic, we will have that

$$\int_{z_0}^{z}\frac{d_{(F,G)}W}{dz}d_{(F,G)}z=W\left(z\right)-\phi\left(z_0\right)F\left(z\right)-\psi\left(z_0\right)G\left(z\right).$$

The last integral expression represents the *antiderivative in the sense of Bers* of the complex-valued function

$$\frac{d_{(F,G)}W}{dz}$$

since the (F,G)-derivatives of F and G are zero.

The complex-valued function w will be (F,G)-*integrable* iff

$$\operatorname{Re}\oint G^*wdz+i\operatorname{Re}\oint F^*wdz=0.$$

Theorem 5 *The (F,G)-derivative of a (F,G)-pseudoanalytic function W is (F,G)-integrable.*

Theorem 6 *Let the pair (F,G) be a predecessor of (F_1,G_1). A complex-valued function will be (F_1,G_1)-pseudoanalytic iff it is (F,G)-integrable.*

Definition 7 *Let $\{(F_m,G_m)\}$, $m=0,\pm1,\pm2,\pm3,...$ be a sequence of generating pairs, and let (F_{m+1},G_{m+1}) be a successor of (F_m,G_m) . Thus we call $\{(F_m,G_m)\}$ a* generating sequence. *If $(F_0,G_0)=(F,G)$ we will say (F,G) to be* embedded *in $\{(F_m,G_m)\}$.*

Definition 8 *The formal power $Z_m^{(0)}\left(a,z_0;z\right)$ with center at z_0, coefficient a and exponent 0 is defined as the linear combination of the elements of the generating pair F_m and G_m with real constant coefficients λ and μ, satisfying the condition*

$$\lambda F_m\left(z_0\right)+\mu G_m\left(z_0\right)=a.$$

The higher exponents $n=1,2,...$are defined by

$$Z_m^{(n)}\left(a,z_0;z\right)=n\int_{z_0}^{z}Z_{m+1}^{(n-1)}\left(a,z_0;\varsigma\right)d_{(F_m,G_m)}\varsigma.$$

The formal powers possesses the following properties.

1. $Z_m^{(n)}(a, z_0; z)$ is (F_m, G_m)-pseudoanalytic.

2. For a_1 and a_2 being real constants, we have
$$Z_m^{(n)}(a_1 + ia_2, z_0; z) = a_1 Z_m^{(n)}(1, z_0; z) + a_2 Z_m^{(n)}(i, z_0; z).$$

3. The asymptotic formulas hold:
$$\lim_{z \to z_0} Z_m^{(n)}(a, z_0; z) = a(z - z_0)^n.$$

Remark 9 *As shown in [2], any complex-valued function W, satisfying (7), accepts the expansion*
$$W = \sum_{n=0}^{\infty} Z^{(n)}(a_n, z_0; z), \tag{9}$$
where the missing subindex m means that all formal powers belong to the generating pair (F, G). This is: expression *(9)* is an analytic representation of the general solution of *(7)*.

Definition 10 *[14] A function $\Phi = \phi + i\psi$ of a complex variable $z = x + iy$ is called p-analytic if*
$$\frac{\partial}{\partial x}\phi = \frac{1}{p}\frac{\partial}{\partial y}\psi \text{ and } \frac{\partial}{\partial y}\phi = -\frac{1}{p}\frac{\partial}{\partial x}\psi. \tag{10}$$

Theorem 11 *[8] The complex-valued function*
$$W = \phi \cdot \sqrt{p} + \psi \cdot \frac{i}{\sqrt{p}}$$
will be a solution of the Vekua equation
$$\partial_{\overline{z}} W - \frac{\partial_{\overline{z}}\sqrt{p}}{\sqrt{p}}\overline{W} = 0 \tag{11}$$
if and only if the real-valued functions ϕ and ψ satisfy (10).

Theorem 12 *[7] Let (F, G) be a generating pair of the form*
$$\begin{aligned} F &= \sqrt{p} = U(x)V(y), \\ G &= \frac{i}{\sqrt{p}} = \frac{i}{U(x)V(y)}. \end{aligned}$$
Thus this generating pair is embedded in the generating sequence $\{(F_m, G_m)\}$, $m = 0, \pm 1, \pm 2, \pm 3, ...$ defined as
$$\begin{aligned} F_m &= 2^m U(x)V(y), \\ G_m &= i\frac{2^m}{U(x)V(y)}; \end{aligned}$$
for even m, and
$$\begin{aligned} F_m &= \frac{2^m}{U(x_1)}V(x_2), \\ G_m &= i\frac{2^m}{V(x_2)}U(x_1) \end{aligned}$$
for odd m.

3 FROM THE QUATERNIONIC ELECTRICAL IMPEDANCE EQUATION TO THE VEKUA EQUATION

As it was shown in [13],[15],[17] and [18], when we introduce the notations

$$\begin{aligned} \overrightarrow{\mathcal{E}} &= \sqrt{\sigma}\text{grad}u, \\ \overrightarrow{\sigma} &= \frac{\text{grad}\sqrt{\sigma}}{\sqrt{\sigma}}, \end{aligned} \tag{12}$$

the equation (1) turns into

$$\left(D + M^{\overrightarrow{\sigma}}\right)\overrightarrow{\mathcal{E}} = 0. \tag{13}$$

Essentially, we will study two different paths to obtain the Vekua equation from (13). The first one [16] is to consider at once

$$\begin{aligned} \overrightarrow{\mathcal{E}} &= \mathcal{E}_1 e_1 + \mathcal{E}_2 e_2, \\ \sigma &= \sigma(x_1, x_2) \end{aligned}$$

then, introducing the notation

$$\sigma_1 = \frac{1}{\sqrt{\sigma}}\frac{\partial}{\partial x_1}\sqrt{\sigma}, \quad \sigma_2 = \frac{1}{\sqrt{\sigma}}\frac{\partial}{\partial x_2}\sqrt{\sigma}, \tag{14}$$

the function $\overrightarrow{\sigma}$ in (12) will take the form

$$\overrightarrow{\sigma} = \sigma_1 e_1 + \sigma_2 e_2,$$

thus from (13) we will have

$$D(\mathcal{E}_1 e_1 + \mathcal{E}_2 e_2) + (\mathcal{E}_1 e_1 + \mathcal{E}_2 e_2)(\sigma_1 e_1 + \sigma_2 e_2) = 0,$$

from which the following system is obtained

$$\begin{aligned} \frac{\partial}{\partial x_1}\mathcal{E}_1 + \frac{\partial}{\partial x_2}\mathcal{E}_2 &= -\mathcal{E}_1\sigma_1 - \mathcal{E}_2\sigma_2, \\ \frac{\partial}{\partial x_1}\mathcal{E}_2 - \frac{\partial}{\partial x_2}\mathcal{E}_1 &= \mathcal{E}_3\sigma_1 - \mathcal{E}_1\sigma_2, \\ \frac{\partial}{\partial x_3}\mathcal{E}_1 &= \frac{\partial}{\partial x_3}\mathcal{E}_2 = 0. \end{aligned}$$

Multiplying the second equation by $-i$ and adding to the first, it follows

$$\left(\frac{\partial}{\partial x_1} + i\frac{\partial}{\partial x_2}\right)(\mathcal{E}_1 - i\mathcal{E}_2) + (\sigma_1 - i\sigma_2)(\mathcal{E}_1 - i\mathcal{E}_2) = 0,$$

but according to (14)

$$\sigma_1 - i\sigma_2 = \frac{1}{\sqrt{\sigma}}\left(\frac{\partial}{\partial x_1} - i\frac{\partial}{\partial x_2}\right)\sqrt{\sigma}.$$

Considering this, and introducing the notation

$$\begin{aligned} \mathcal{E} &= \mathcal{E}_1 - i\mathcal{E}_2, \qquad (15)\\ \partial_{z_1} &= \frac{\partial}{\partial x_1} - i\frac{\partial}{\partial x_2} \\ \partial_{\bar{z}_1} &= \frac{\partial}{\partial x_1} + i\frac{\partial}{\partial x_2} \end{aligned}$$

we have

$$\partial_{\bar{z}_1}\mathcal{E} + \frac{\partial_{z_1}\sqrt{\sigma}}{\sqrt{\sigma}}\overline{\mathcal{E}} = 0. \qquad (16)$$

This is a Vekua equation for which we are able to build the general solution in terms of Taylor series in formal powers, as we will show further, for the case when σ is a separable-variables function. We must mention that a similar Vekua equation, but considering a bicomplex case, had been already deduced in [4] from a quaternionic Dirac equation corresponding to a massive particle under the influence of a special class of potentials. Let us study now a secund method for obtaining a Vekua equation from (1).

In [12] is given the proof of the following statement.

Theorem 13 *Let the purely-vectorial quaternionic-valued functions* $\overrightarrow{\mathcal{E}}_1$, $\overrightarrow{\mathcal{E}}_2$ *and* $\overrightarrow{\mathcal{E}}_3$ *be linear independent, and be all solutions of (13). Let the real scalar functions* φ_1, φ_2 *and* φ_3 *be all solutions of the equation*

$$\sum_{k=1}^{3} (D\varphi_k)\, \overrightarrow{\mathcal{E}}_k = 0. \qquad (17)$$

Hence, the purely-vectorial quaternionic-valued function

$$\overrightarrow{\mathcal{E}} = \sum_{k=1}^{3} \varphi_k \overrightarrow{\mathcal{E}}_k \qquad (18)$$

is the general solution of (13).

It is easy to check [15] that the following quaternionic-valued functions constitute a set of three linear independent solutions for the equation (13).

$$\begin{aligned} \overrightarrow{\mathcal{E}}_1 &= e_1 K_1 e^{-\int \sigma_1 dx_1 + \int \sigma_2 dx_2}, \\ \overrightarrow{\mathcal{E}}_2 &= e_2 K_2 e^{\int \sigma_1 dx_1 - \int \sigma_2 dx_2}, \\ \overrightarrow{\mathcal{E}}_3 &= e_3 K_3 e^{\int \sigma_1 dx_1 + \int \sigma_2 dx_2}, \end{aligned}$$

where K_1, K_2 and K_3 are real constants, and σ_1, σ_2 have the form (14). Let us consider the case when $\varphi_3 = 0$ in (17), which precisely corresponds, as we will see, to the two-dimensional case of (14). We have then

$$D\varphi_1 \cdot \overrightarrow{\mathcal{E}}_1 + D\varphi_2 \cdot \overrightarrow{\mathcal{E}}_2 = 0.$$

Expanding the expression, we obtain the following system of differential equations

$$\frac{\partial \varphi_1}{\partial x_2} = \frac{1}{p_1}\frac{\partial \varphi_2}{\partial x_1}, \quad \frac{\partial \varphi_1}{\partial x_1} = -\frac{1}{p_1}\frac{\partial \varphi_2}{\partial x_2},$$
$$\frac{\partial \varphi_1}{\partial x_3} = \frac{\partial \varphi_2}{\partial x_3} = 0;$$

where $p_1 = Ke^{-2\int \sigma_1 dx_1 + 2\int \sigma_2 dx_2}$ and K is a real constant. The first pair of equations constitute the differential system of the so called p-analytic functions, introduced in Definition 10, and by Theorem 11, its equivalent Vekua equation will have the form

$$\partial_{\bar{z}_2}\mathcal{W} - \frac{\partial_{\bar{z}_2}\sqrt{p_1}}{\sqrt{p_1}}\overline{\mathcal{W}} = 0, \tag{19}$$

where $\partial_{\bar{z}_2} = \frac{\partial}{\partial x_2} + i\frac{\partial}{\partial x_1}$ and

$$\mathcal{W} = \varphi_1 p_1 + i\frac{\varphi_2}{p_1},$$

that in fact we can also write as

$$\mathcal{W} = \mathcal{W}_1 + i\mathcal{W}_2,$$

where $\mathcal{W}_1$ and $\mathcal{W}_2$ are real-valued functions.

An small but interesting detail that was not pointed out in [15] is that

$$\frac{\partial_{\bar{z}_2}\sqrt{p_1}}{\sqrt{p_1}} = -i\left(\sigma_1 + i\sigma_2\right).$$

Substituting this into (19) we have

$$\partial_{\bar{z}_2}\mathcal{W} + i\left(\sigma_1 + i\sigma_2\right)\overline{\mathcal{W}} = 0,$$

but in fact

$$\partial_{\bar{z}_2} = i\left(\frac{\partial}{\partial x_1} - i\frac{\partial}{\partial x_2}\right),$$

therefore

$$\left(\frac{\partial}{\partial x_1} - i\frac{\partial}{\partial x_2}\right)\mathcal{W} + \left(\sigma_1 + i\sigma_2\right)\overline{\mathcal{W}} = 0.$$

Considering the complex conjugation of this expression we have

$$\left(\frac{\partial}{\partial x_1} + i\frac{\partial}{\partial x_2}\right)\overline{\mathcal{W}} + \left(\sigma_1 - i\sigma_2\right)\mathcal{W} = 0,$$

and taking into account the notations (15), by simply identifying $\mathcal{W}_1 = \mathcal{E}_1'$ and $\mathcal{W}_2 = \mathcal{E}_2'$, the last equation turns into

$$\partial_{\bar{z}_1}\mathcal{E}' + \frac{\partial_{z_1}\sqrt{\sigma}}{\sqrt{\sigma}}\overline{\mathcal{E}'} = 0, \tag{20}$$

where $\mathcal{E}' = \mathcal{E}_1' - i\mathcal{E}_2'$, a completely equivalent equation to (16).

3.1 Building the general solution for the two-dimensional electrical impedance equation when the conductivity function is separable-variables

First of all, we should notice when we select a generating pair [9] [11]

$$F = \sqrt{\sigma},\ G = \frac{i}{\sqrt{\sigma}}; \tag{21}$$

the corresponding characteristic coefficients (6) become

$$A_{(F,G)} = a_{(F,G)} = 0,$$
$$B_{(F,G)} = \frac{\partial_{z_1}\sqrt{\sigma}}{\sqrt{\sigma}},$$
$$b_{(F,G)} = \frac{\partial_{\overline{z}_1}\sqrt{\sigma}}{\sqrt{\sigma}}.$$

Therefore the Vekua equation will have the form

$$\partial_{\overline{z}_1} W - \frac{\partial_{\overline{z}_1}\sqrt{\sigma}}{\sqrt{\sigma}}\overline{W} = 0. \tag{22}$$

According to Definition 2, the successor pair (F_1, G_1) of (21) must have characteristic coefficients

$$a_{(F_1,G_1)} = 0,$$
$$b_{(F_1,G_1)} = -\frac{\partial_{z_1}\sqrt{\sigma}}{\sqrt{\sigma}},$$

and by virtue of Theorem 3, the derivative in the sense of Bers $\frac{d_{(F,G)}W}{dz}$ of a (F, G)-pseudoanalytic function W solution of (22) will be (F_1, G_1)-pseudoanalytic. This is, it will be a solution of the Vekua equation

$$\partial_{\overline{z}_1}\left(\frac{d_{(F,G)}W}{dz}\right) - B_{(F_1,G_1)}\overline{\left(\frac{d_{(F,G)}W}{dz}\right)} = 0,$$

or more precisely

$$\partial_{\overline{z}_1}\left(\frac{d_{(F,G)}W}{dz}\right) + \frac{\partial_{z_1}\sqrt{\sigma}}{\sqrt{\sigma}}\overline{\left(\frac{d_{(F,G)}W}{dz}\right)} = 0.$$

Considering (15), by simply denoting

$$\mathcal{E} = \frac{d_{(F,G)}W}{dz},$$

the last Vekua equation becomes

$$\partial_{\overline{z}_1}\mathcal{E} + \frac{\partial_{z_1}\sqrt{\sigma}}{\sqrt{\sigma}}\overline{\mathcal{E}} = 0, \tag{23}$$

which evidently coincides with the Vekua equations (16) and (20) corresponding to the two-dimensional electrical impedance equation (1).

For the case when $\sigma = U^2(x_1)\,V^2(x_2)$, a very important and general case in the field of electrical impedance tomography, according to Theorem 12, we can introduce an explicit generating sequence $\{(F_m, G_m)\}$, $m = 0, \pm1, \pm2, \pm3, ...$ in which the generating pair

$$F = \sqrt{\sigma} = U(x_1)\,V(x_2),$$
$$G = \frac{i}{\sqrt{\sigma}} = \frac{i}{U(x_1)\,V(x_2)},$$

is embedded. Therefore, by Definitions, Theorems and Remarks 2-9, we are able to express the general solution of (22) in term of Taylor series in formal powers

$$W = \sum_{n=0}^{\infty} Z^{(n)} (a, z_0, z),$$

where $z = x_1 + ix_2$, and by Theorem 3, the (F, G)-derivative of W will be the general solution of (23). It follows, using (12), to obtain the electric potential u, which will be the general solution of (1).

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