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SIMULATION MODEL OF TRAM ROUTE OPERATION

M. Bauer*, **M. Richter and H. Weiß**

** Cracow University of Technology, Chair of Transport Systems, Ul. Warszawska 24, 31-155 Cracow, Poland*
E-mail: mbauer@transys.wil.pk.edu.pl

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Abstract. *From passenger's perspective, punctuality is one of the most important features of tram route operation. We present a stochastic simulation model with special focus on determining important factors of influence. The statistical analysis bases on large samples (sample size is nearly 2000) accumulated from comprehensive measurements on eight tram routes in Krakow. For the simulation, we are not only interested in average values but also in stochastic characteristics like the variance and other properties of the distribution.*

A realization of trams operations is assumed to be a sequence of running times between successive stops and times spent by tram at the stops divided in passengers alighting and boarding times and times waiting for possibility of departure .

The running time depends on the kind of track separation including the priorities in traffic lights, the length of the section and the number of intersections. For every type of section, a linear mixed regression model describes the average running time and its variance as functions of the length of the section and the number of intersections. The regression coefficients are estimated by the iterative re-weighted least square method. Alighting and boarding time mainly depends on type of vehicle, number of passengers alighting and boarding and occupancy of vehicle. For the distribution of the time waiting for possibility of departure suitable distributions like Gamma distribution and Lognormal distribution are fitted.

1 INTRODUCTION

Each tramway line operation is strongly connected with basic functions of public transport – the carriage of passengers between stops and the passenger's exchange on stops. High quality of this operation makes the line attractive for passengers and causes big passengers streams on line. Passengers pay attention to various public transport features, but in many surveys led in European cities, first of all they prefer high punctuality of operation [7]. Similar situation is observed in Krakow, for majority of passengers – the most important feature of tram operation is punctuality, frequency of operation and travel time (Figure 1).

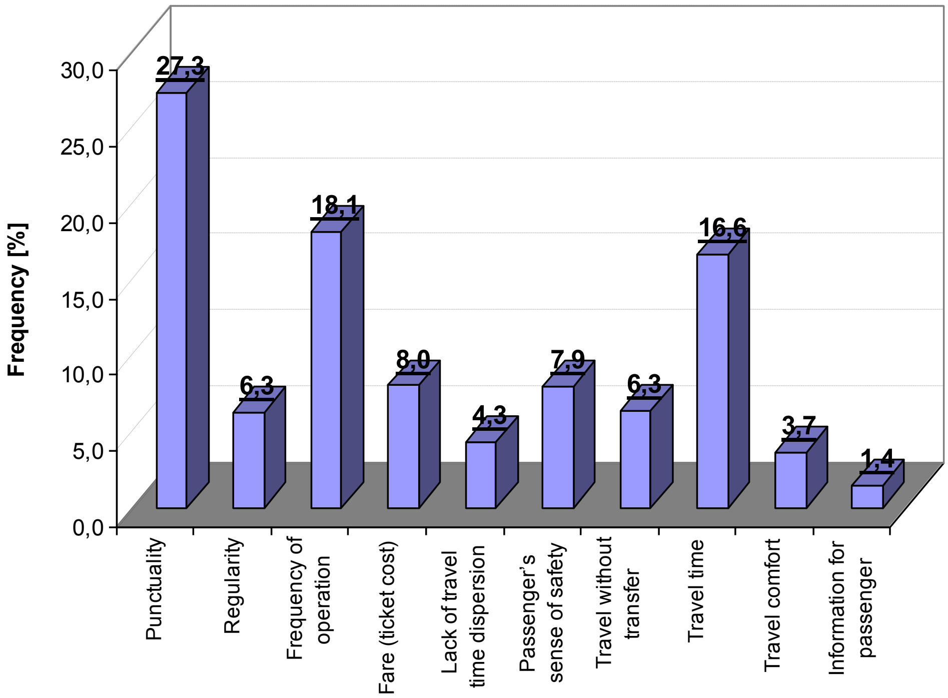


Figure 1. Preferences of tram passengers in Krakow

Nevertheless, urban traffic conditions have very significant influence onto tram lines operation. Big traffic volumes on street sections and at the intersections areas (especially during peak hours) result in longer and much variable running times and difficulties with punctuality and regularity assurance [4, 5]. Each tram line is susceptible to effect of many factors, connected with street conditions (e.g. insufficient capacity of sections and stops, lack of separated tracks, priorities for trams in traffic lights on intersections), traffic conditions (traffic volumes of trams and other vehicles, passenger streams at stops, occupancies), human factors (driver and passengers behaviours), public transport organization (realistic schedules, types of vehicles, frequency), environmental and local conditions.

It is necessary, already on the stage of modelling, to take into consideration the most important features of tramway line operation and main disturbing factors [1]. In this paper a single tramway line model in mezzo scale will be presented, which could help to close the gap between existing micro and macro simulation models. This model will make easier the comprehensive analysis of any tramway line operating in urban area. It could be used in:

- Scheduling procedures, especially in case of new tram lines, as a first approach, before starting and during first phase of operation (designed schedules have to be verified by measurements during tram line operation in real conditions).
- Network analysis of public transport, in estimating input data for macro-simulation models of tram networks (e.g. VISUM software). Using of model gives the possibility of better network calibration.
- Planning and designing tram routes in urban conditions.
- Feasibility studies of new tram tracks building, when a more detailed approach is not required.

2 GENERAL ASSUMPTIONS OF THE MODEL

The structure of tram line operation model is based on graphs and events theories. Graph theory gives a possibility of selection the most important elements of any tram line: stops and sections between them. Any tram line – in mathematical description – has a simple digraph structure, where the set of vertexes consists of all stops on the line (in both directions), and set of edges consists of sections between following stops. Majority of tram lines in cities operates in two directions. Last stop in first direction is situated on the same loop that first stop of second direction. The section between those stops (technical section) must be also taken into consideration. If operational time on the loop is too short then big delays on first direction influence onto moment of departure during operation in opposite direction. Digraph structure for two ways line is presented on Figure 2.

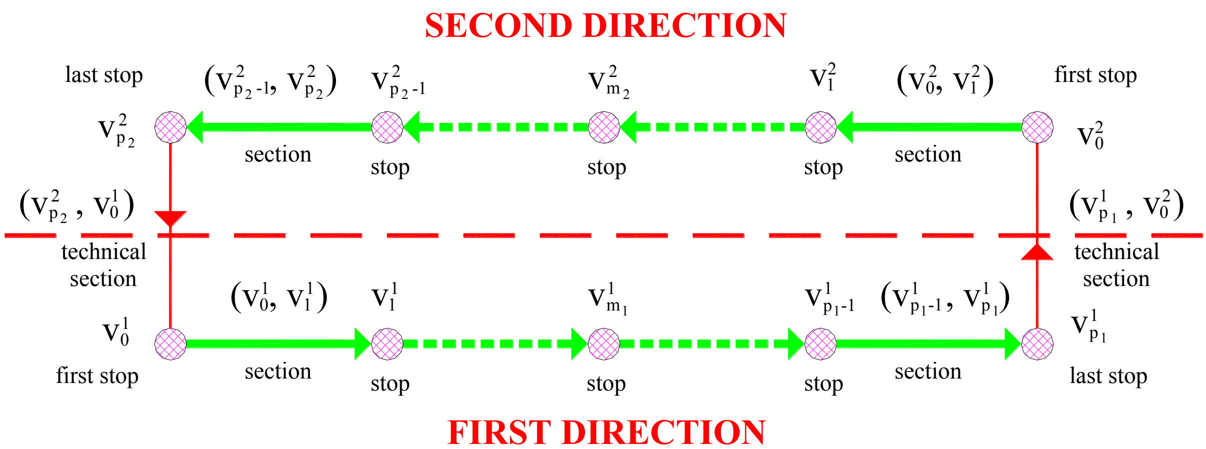


Figure 2. Graph structure of tram line operation model

A large group of possible events influence onto line operation at every stop and on every section. They are connected with any possible stopping and starting on the line operation (at the stops, at the intersections, at clear sections, because of queue) and with processes of alighting and boarding passengers at the stops (e.g. starting alighting, finishing boarding, waiting for possibility of departure, change of stopping place). They have fundamental influence on state of the tram line, but their number is individual for any course on the line. Only few of them are obligatory, the rest occurs or not.

In the presented model, it is assumed that only three of possible events have strong influence onto state of line: moment of starting alighting and boarding passengers, moment of finishing alighting and boarding passengers, and moment of departure from stop (Figure 3). At corresponding time points three processes start: alighting and boarding passengers, waiting for possibility of departure from stop after alighting and boarding passengers, but before departure (because of red signal or other vehicles traffic) and running of section (between current and following stop).

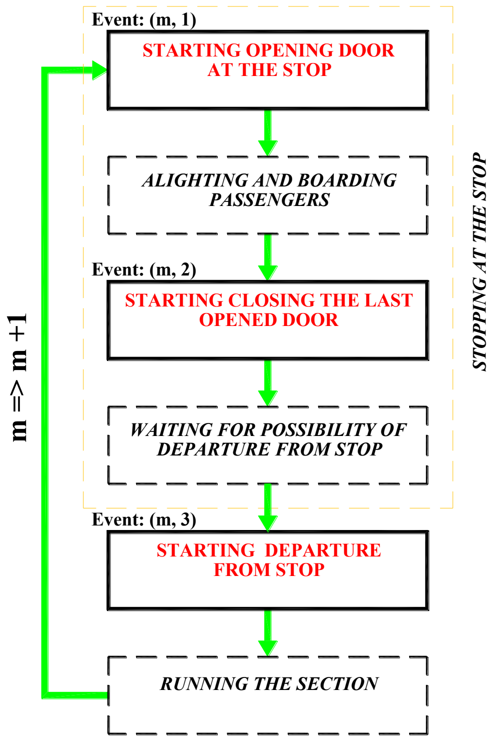


Figure 3. The most important events on tram line

All remaining events are aggregated to those three processes on the line. The biggest simplification concerns section running time, where the number of possible influences is the largest. Gathering complete and credible data appears to be problematic – events occur randomly, their registration is very difficult. Because of that and with the aim of making a useful model this simplification was assumed.

Three main processes will be defined. The running time is described by the time between moment of departure from first stop on the section and the moment of arrival on the following stop [9]. The stopping time is defined as the time from the moment of vehicle stopping on stop to the moment of the start of vehicle's movement. According to Figure 3, the stopping time consists of alighting and boarding time and time lost on stop before departure, because of impossibility of start its movement. Alighting and boarding time is the time from the moment of starting opening doors to the moment of starting closing the last opened doors. Therefore, time of waiting for possibility of departure is described by the time from the moment of starting closing last opened doors to the moment of beginning running from stop.

There is an adequate way to describe lines as a group of module sequences: section between following stops – stop at the end of the section, with three events happen each time. The first stop on every line should be taken into consideration separately. Alighting and boarding time at this kind of stop is not so important, in majority of cases tram arrives much earlier than moment of scheduled departure. The real moment of departure depends mainly from the driver's decision.

3 AVAILABLE DATA

Estimation of parameters for the specified components (running time, alighting and boarding time, time of waiting for possibility of departure) was done on basis of measurements leading by observers inside vehicles, in Krakow, in years 2005 – 2008 during typical workdays. There were two ways of gathering the results, both with the aim of assuring enough accuracy. The first part of measurements was done with manual registration, with radio-controlled clocks using (eight tram urban lines). During the second part, GPS receivers (model Garmin 60CSx) were being used (four lines). Using of GPS receivers gave additional possibility of automatic stops' location on the map (Figure 4), and decreased number of mistakes. Registration of interesting events was done only by setting waypoints – three times at every stop: in moments of starting opening door, starting closing the last opened door and starting departure from stop.

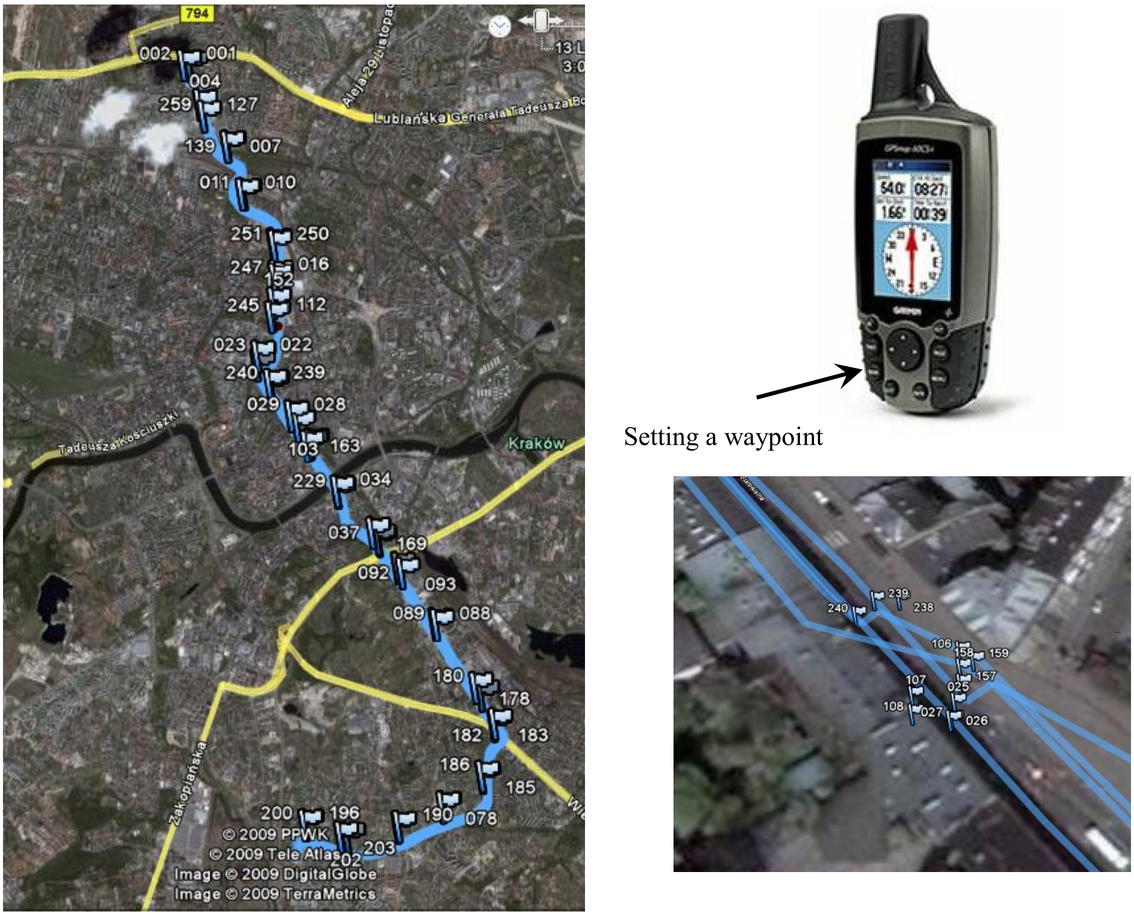


Figure 4. Exampled results from measurements made with GPS receiver using Google™ Earth

In both cases (with and without GPS), accuracy of obtained results is very high – events were registered with one second precision. Less precision of stops' location with GPS using (average +/- 5 [m]) has not consequences – average locations of stops correspond with real locations. Additionally, on nine of those lines, the numbers of alighting and boarding passengers were also registered.

Overall, there were recorded more than 2500 running times (sections with lengths from 0.15 to 0.93 [km]) and more than 2400 tram visits at the stops (Table 1). The sample sizes for the elements of the model are different. In some cases, there occur request stops, on which the tram passed the stop without stopping. In these cases, the corresponding sections were excluded from the statistical analyses.

Element of the model	Sample size	Sample Minimum	Sample Maximum	Mean	Standard deviation
Running time [min]	2539	0.32	11.30	1.51	0.83
Alighting and boarding time [sec]	2433	2	85	19	12
Time lost before departure [sec]	2433	1	104	13	17
Number of alighting passengers [pas]	1710	0	53	7	8
Number of boarding passengers [pas]	1710	0	60	7	8
Velocity [km/h]	2539	3.2	81.7	23.7	10.6

Table 1. Sample for model's parameters estimation

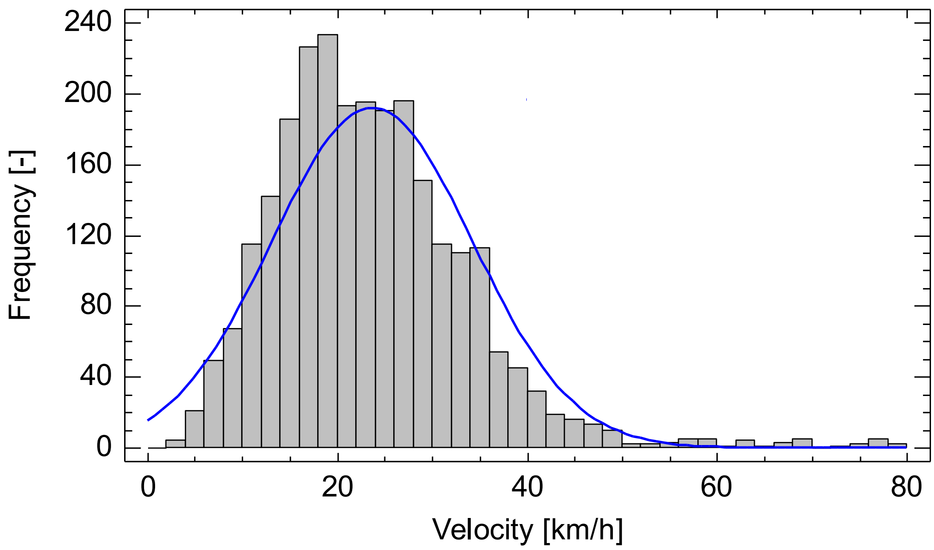


Figure 5. Distribution of velocity

Presented results have mainly statistical character. Velocity on sections can be considered on general level, its distribution for all gathered results together, with trace of fitted Normal distribution is presented on Figure 5. The other components have individual character depending of section length, type of vehicle, kind of stop, etc.

4 RUNNING TIME BETWEEN FOLLOWING STOPS

The section running time is defined as the time from moment of starting departure from stop at the beginning of the section and moment of starting opening door at the following stop (at the end of the section). Due to very various conditions on tram lines in practice, there were considered different types of tram tracks. There were taken into consideration tram tracks and common tram-bus lanes, separated from other traffic (by green, by curb, by painted line) or tracks not separated, located on street. For all mentioned cases, there was carried out the comparison of average tram velocities. On a basis of analysis of variance results and the significance of the Kruskal-Wallis test, four types of sections are defined (Figure 6):

- **Type A** – Tram track or shared tram-bus lane, separated by green or by curb and the priorities in traffic lights for trams (and buses) are assured. There is no possibility of track using by other vehicles (except privileged cars: police, ambulance on tram-bus lanes).
- **Type B** – Tram track or shared tram-bus lane, separated by green, by curb or by painted line (only in case of tram-bus lanes). Priorities for public transport in traffic lights on intersections are not assured. There is no possibility of legal tram track using by other vehicles, except privileged cars.

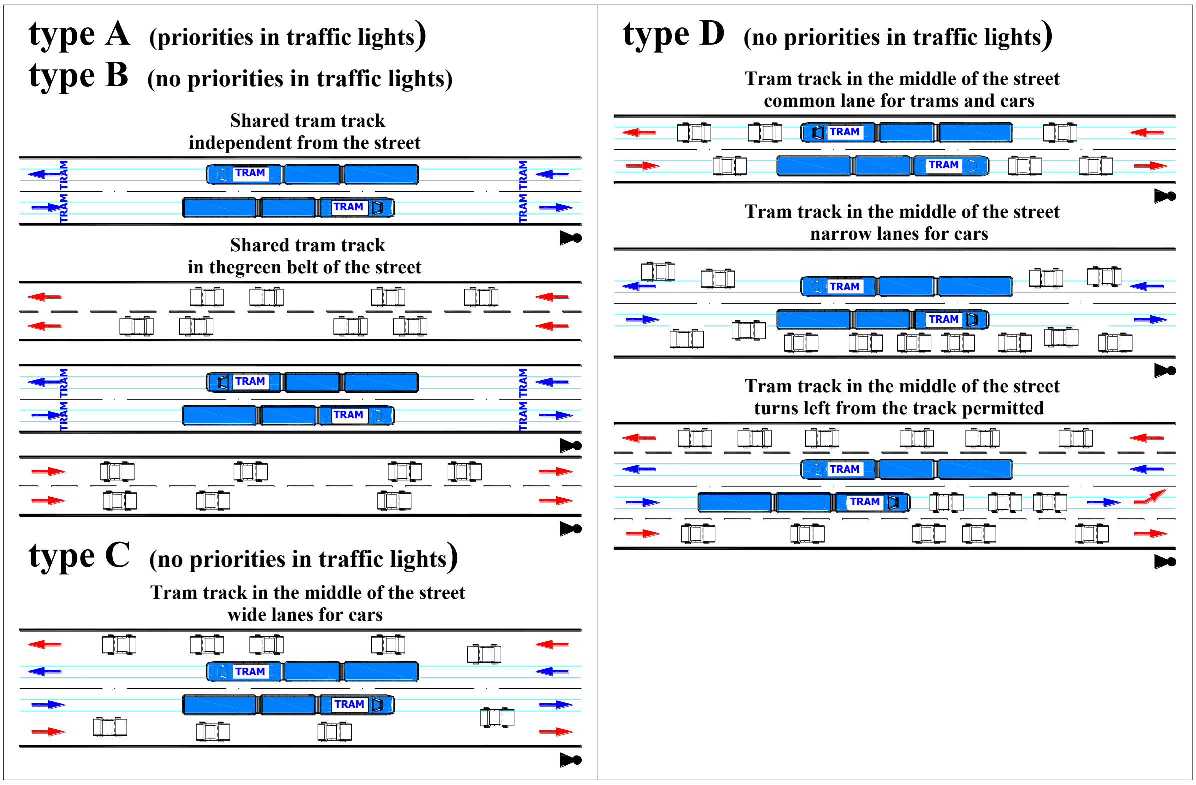


Figure 6. Types of sections

- **Type C** – Tram track not separated. It is located in the middle part of street with wide lanes for other vehicles. Curb or sidewalk parking does not significant matter for trams movement. Cars could use the tram track area sporadically.
- **Type D** – Tram track not separated. It is located in the middle part of street, with too narrow lanes for other vehicles or car; turns from tram track area are permitted. Curb or sidewalk parking could be permitted as well. It appears high probability that cars using even part of tram track area are blocking trams.

In practice, also mixed sections are used, where track separating is applied only at a part of the section.

The aim of the presented model is to predict the running time in dependence of the length of the section and the number of signalized intersections at the section separately for every type of section. In Table 2, some descriptive statistics are shown.

Type of section	Short description	Sample size	Range	
			Lengths [km]	Number of intersections [-]
Type A	Separated tram track or tram-bus lane, priorities in traffic lights on intersections	294	0.40 – 0.93	0 – 2
Type B	Separated tram track or tram-bus lane, lack of priorities in traffic lights	1128	0.15 – 0.90	0 – 3
Type C	Tram track located in the middle part of the street, track sporadically used by other vehicles	748	0.25 – 0.85	0 – 2
Type D	Tram track located in the middle part of the street, track often used by other vehicles	369	0.15 – 0.70	0 – 2

Table 2. Descriptive statistics of chosen factors and explanatory variables

The ordinary linear regression model

$$t_{r,i} = \beta_S S_i + \beta_L L_i + \tilde{\varepsilon}_i$$

with the zero-mean random terms $\tilde{\varepsilon}_i$ is useful to predict the mean value of the running time $t_{r,i}$ in dependence of the length of the section L_i and the number of intersections S_i of the i-th section. Thereby β_S and β_L are the average values of the waiting time per intersection and the pure running time per unit length. In practice, the variance of the running time grows with increasing length and increasing number of intersections. The linear mixed regression model

$$t_{r,i} = \beta_S S_i + \beta_L L_i + \alpha_{L,i}\sqrt{L_i} + \alpha_{S,i}\sqrt{S_i} + \varepsilon_i$$

takes into account this heteroskedasticity. The random variables $\alpha_{L,i}$ and $\alpha_{S,i}$ specify the random effects connected with the randomness of the pure running time and the waiting time per intersection of the i-th section.

For the statistical analysis it is assumed that all variables $\alpha_{L,i}$ are identically independent distributed with zero mean and variance $\sigma_L^2 = Var(\alpha_{L,i})$, all variables $\alpha_{S,i}$ are identically independent distributed with zero mean and variance $\sigma_S^2 = Var(\alpha_{S,i})$ and all variables ε_i are identically independent distributed with zero mean and variance $\sigma_\varepsilon^2 = Var(\varepsilon_i)$.

Under the assumption of the independence of $\alpha_{L,i}$, $\alpha_{S,i}$ and ε_i for all $i = 1,\ldots,n$ the variance of the running time is calculated by

$$\sigma_i^2 = Var(t_{r,i}) = Var(\tilde{\varepsilon}_i) = \sigma_S^2 S_i + \sigma_L^2 L_i + \sigma_\varepsilon^2 .$$

In case of heteroskedasticity with known variances $\sigma_1^2,\ldots,\sigma_n^2$ the traditional method of estimating the coefficients β_S and β_L is weighted least squares. For the linear model

$$t_r = X\beta + \tilde{\varepsilon}$$

where $t_r = \begin{pmatrix} t_{r,1} \\ \vdots \\ t_{r,n} \end{pmatrix}$, $X = \begin{pmatrix} S_1 & L_1 \\ \vdots & \vdots \\ S_n & L_n \end{pmatrix}$, $\beta = \begin{pmatrix} \beta_S \\ \beta_L \end{pmatrix}$ and $\tilde{\varepsilon} = \begin{pmatrix} \tilde{\varepsilon}_1 \\ \vdots \\ \tilde{\varepsilon}_n \end{pmatrix}$

the weighted least square estimator of β is given by

$$\hat{\beta} = (XWX)^{-1} X^T W^T t_r \quad \text{with the weighting matrix } W = diag(1/\sigma_1^2,\ldots,1/\sigma_n^2).$$

This is known as best linear unbiased estimator.

The ordinary least square estimator of $\sigma^2 = (\sigma_S^2, \sigma_L^2, \sigma_\varepsilon^2)^T$ in case of known predictor $x = X\beta$ of the linear model describing the variances

$$y = Z\sigma + \hat{\varepsilon} \quad \text{with } y = \begin{pmatrix} (t_{r,1} - x_1)^2 \\ \vdots \\ (t_{r,n} - x_n)^2 \end{pmatrix} \text{ and } Z = \begin{pmatrix} S_1 & L_1 & 1 \\ \vdots & \vdots & \vdots \\ S_n & L_n & 1 \end{pmatrix}$$

is

$$\hat{\sigma}^2 = (Z^T Z)^{-1} Z^T y .$$

In lack of Gaussian distribution of $\tilde{\varepsilon}_i$ it is not possible to use maximum likelihood method or Henderson method 3.

In our case both β and σ^2 are unknown. We define the following iteration:

i. Start the iteration with the estimation of $\hat{\beta}$ with the ordinary least square estimator ($W = diag(1,\ldots,1)$).

ii. Evaluate $\hat{\sigma}^2$ with the value of $\hat{\beta}$ from the iteration step before. If one component of $\hat{\sigma}^2$ is too small, delete the according factor from the model.

iii. Evaluate $\hat{\beta}$ with the value of $\hat{\sigma}^2$ from the iteration step before.

It should be mentioned, that in case of Gaussian distribution iterative re-weighted least square algorithm converges to a solution of the maximum likelihood equation [6, 8].

The following coefficients have to be estimated:

β_S … Expected value of waiting time per intersection,

β_L … Expected value of pure running time per unit length,

σ_S^2 … Variance of waiting time per intersection,

σ_L^2 … Variance of pure running time per unit length and

σ_ε^2 … Variance of ε.

The iteration converges for all types of section very fast. In Table 3, the results after 10 iterations are shown.

Type of section	β_S	β_L	σ_S^2	σ_L^2	σ_ε^2
Type A	0.1507	1.5043	0	0.2244	0
Type B	0.2153	2.3475	0	0.8777	0
Type C	0.2825	2.9861	0.4020	1.0965	0
Type D	0.3943	3.2343	0.2814	1.0961	0

Table 3. Estimations of the regression coefficients

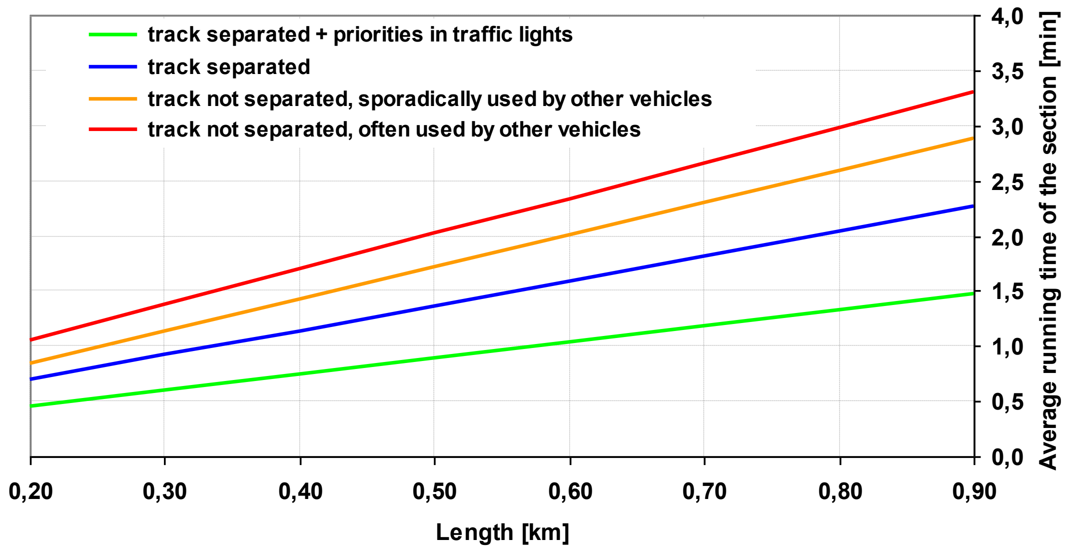


Figure 7. Average running time in dependence on length of section with two signalized intersections

The smallest influence of section's length onto running time appears for sections with separated track and priorities in traffic lights on intersections, the highest for sections with non-separated tracks. The same results are observed in case of dependence from number of intersections. Analysis of sensitivity of obtained regression formulas, for sections with two signalized intersections is presented at Figure 7.

In Figure 8 the measurements as well as the regression lines for the running time in dependence on the length of the section is shown for type of section B (track separated). Here several values of the number of intersections are compared directly.

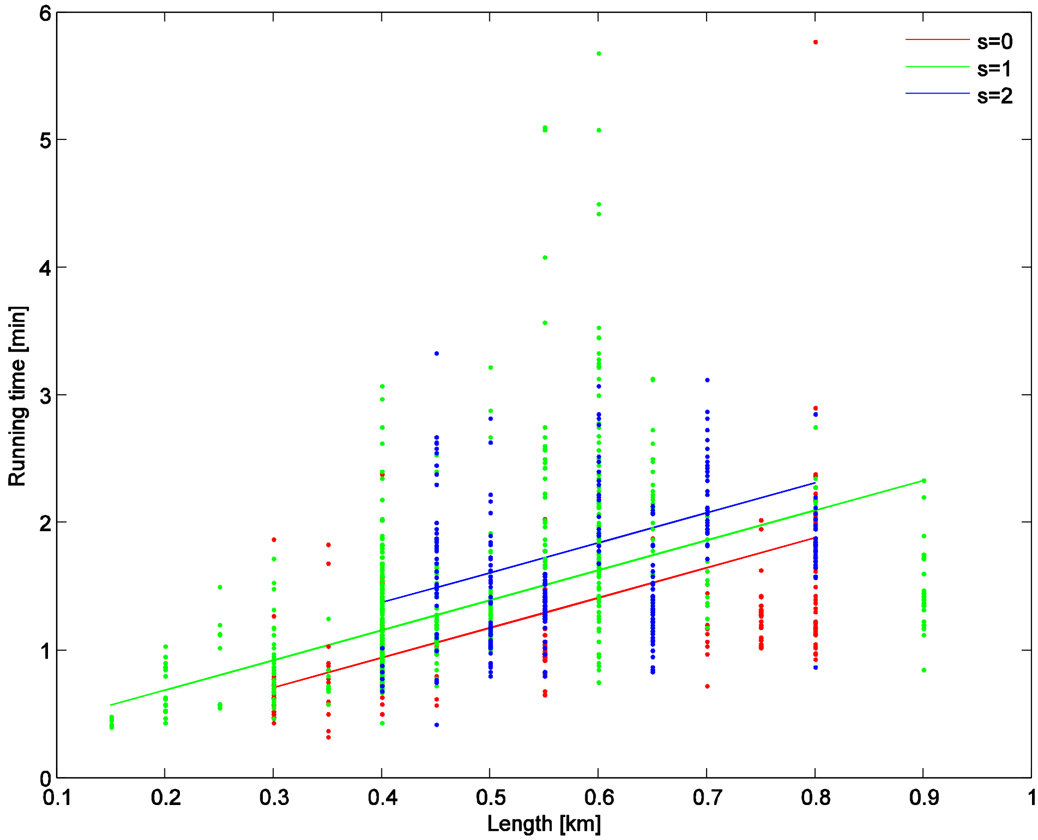


Figure 8. Running time in dependence on length of section

5 PASSENGERS' ALIGHTING AND BOARDING TIME

Alighting and boarding time states the main part of the stopping time on every tram stop. It depends mainly on the number of alighting and boarding passengers but also on the number of passengers inside vehicle reaching the stop. It depends also on the type of vehicle. Different types of trams were taken into consideration – old, high-floor trams GT6, 105N (single, double or triple composition of cars) and modern, low-floor trams NGT6 (Figure 9). Those types of trams are classified to three groups (Table 4).

A) B) C) D)

Figure 9. Tram fleet operating during measurements led in Krakow.

A) Modern NGT6 with low floor, B) GT6, C) N8S, D) 105N

Type of tram	Models	Nominal capacity [pas]	Floor level [mm]
Normal with high floor (NH)	2x105N, N8S, GT6	140 - 210	880 / 910
Normal with low floor (NL)	NGT6	185	290 / 560
Long with high floor (LH)	3x105N	315	910

Table 4. Defined tram types

For those three types of vehicles, the multiple regression formulas for alighting and boarding time were fitted. In all cases, alighting and boarding time was modelled by a linear function of the number of alighting passengers a, boarding passengers b and occupancy of vehicle incoming to the stop P (Table 5). It is the simplest way to describe this process – nonlinear models give similar results.

Type of tram	Range			Multiple regression formula for mean value of alighting and boarding time [min]
	Alighting [pas]	Boarding [pas]	Inside vehicle [pas]	
Normal with high floor (NH)	0 – 46	0 – 57	0 – 110	$\bar{t}^{NH} = 0.48 \cdot a + 0.88 \cdot b + 0.17 \cdot P$

Normal with low floor (NL)	0 – 52	0 – 60	0 – 148	$\bar{t}^{NL} = 0.52 \cdot a + 0.69 \cdot b + 0.11 \cdot P$
Long with high floor (LH)	0 – 53	0 – 57	0 – 173	$\bar{t}^{LH} = 0.49 \cdot a + 0.49 \cdot b + 0.10 \cdot P$

Table 5. Estimation of alighting and boarding time

While the number of alighting passengers has nearly the same influence on the alighting and boarding time for all types of vehicles, the number of boarding passengers and the occupancy of vehicle causes longer alighting and boarding time for normal tramways as for long tramways. Moreover, as an example the comparison of different values of alighting and boarding time for 15 passengers alighting and 15 passengers boarding to the vehicle in dependence on the occupancy of the vehicle is shown on the Figure 10. Regression formulas are characterized by rather big values of determination coefficient, close to 85 [%]. For the random behaviour of the alighting and boarding time Normal distribution is assumed.

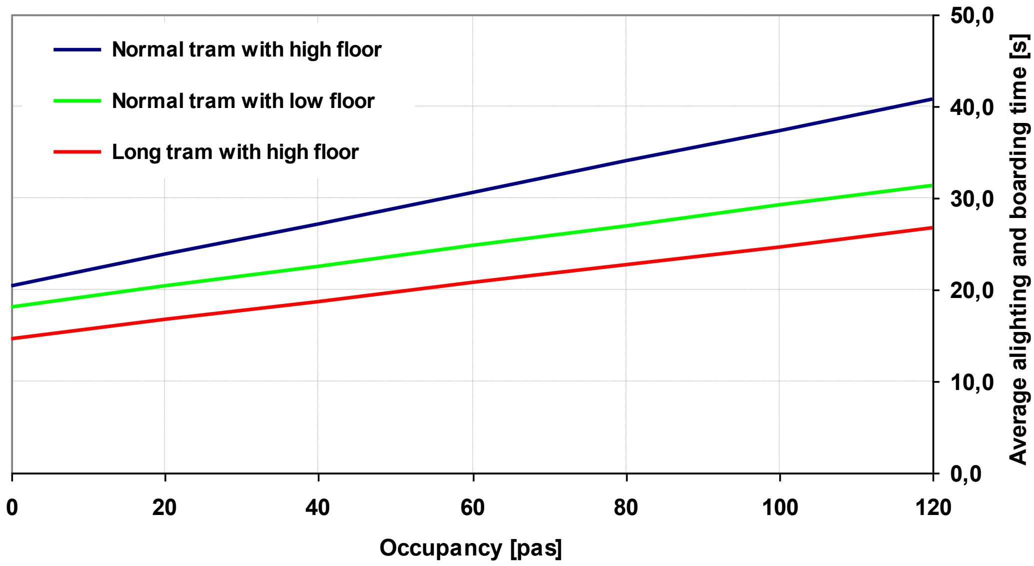


Figure 10. Average alighting and boarding time in dependence on occupancy of incoming vehicle (15 alighting and 15 boarding passengers)

6 TIME WAITING FOR POSSIBILITY OF DEPARTURE

Time waiting for possibility of departure from stop after finishing alighting and boarding passengers depends on stop location and its character. There are two basic types of tram stops in Polish cities: stops, where every tram has to stop and request stops, where passengers request tram stopping. Request tram stops are very seldom, and they have not been taken into consideration in current analysis. There are stops shared by trams and buses especially in cases, when trams and buses use a common lane. Most of analyzed stops are intended only for trams. A large part of tram stops in Krakow is located on separated tracks, and because of that, independent from other vehicle's traffic. The other stops are located in the middle of the street, which causes the possibility of blocking trams by other vehicles at stop's areas.

In addition to the considerations in Section 5 tram stops can be classified with respect to the distance to the nearest intersection. They can be located in the close neighbourhood of the nearest intersection (when stop is situated within range of intersection) or mid block (when the distance from stop to nearest intersection is significant). Kind of neighbouring intersection is important as well – whether it is signalized or not. In case of stops, which are located near side signalized intersections it is also worth to take into consideration whether priorities for trams in traffic lights are assured. Another relevant classification of tram stops is based on their location in relation to various parts of the city. The traffic conditions are different in close city area, where partial restrictions for private cars are assured, and remaining areas of the city. Finally, four types of stops were defined (Figure 11):

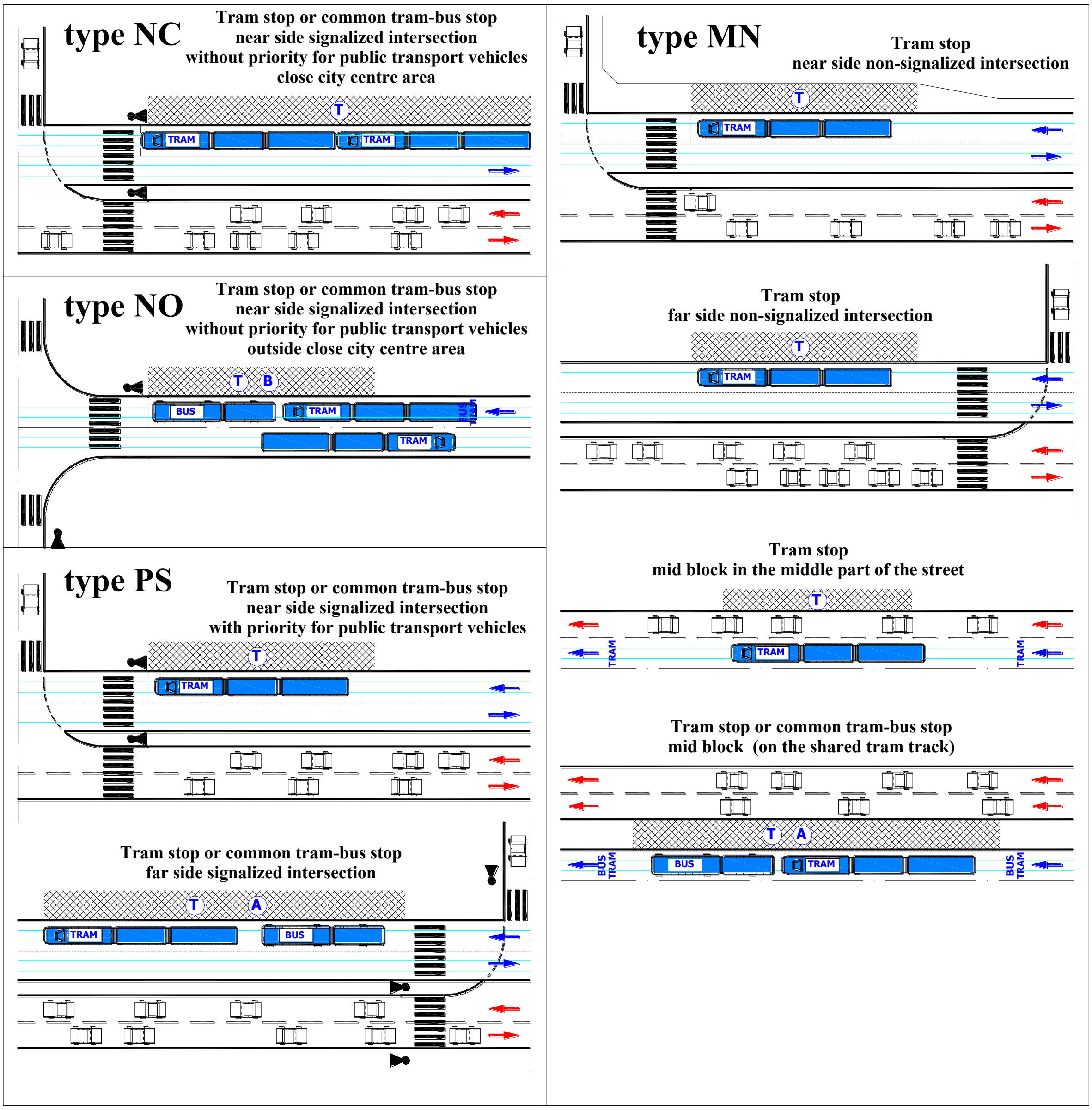


Figure 11. Types of tram stops

- **Type NC** – Tram stop or common tram-bus stop located in close city centre, near side signalized intersection, there are no priorities for trams in traffic lights.

- **Type NO** – Tram stop or common tram-bus stop located outside close city centre, near side signalized intersection, there are no priorities in traffic lights.
- **Type PS** – Tram stop or common tram-bus stop located near side signalized intersection (with priorities in traffic lights – green signal is given to vehicle, which has finished alighting and boarding passengers) or far side signalized intersection.
- **Type MN** – Tram stop located near side or far side non-signalized intersection, or mid block.

For considered types of stops there were estimated the means and standard deviations. The results are shown in Table 6 and in Figure 12.

Type of stop	Sample size	Range	Mean	Standard deviation
	[-]	[s]	[s]	[s]
Tram stop (tram-bus stop) located in close city centre, near side signalized intersection, no priorities in traffic lights (NC)	788	1 – 104	21.1	21.3
Tram stop or tram-bus stop, located outside close city centre, near side signalized intersection, no priorities in traffic lights (NO)	597	1 – 80	14.1	17.2
Tram stop (tram-bus stop) located near side signalized intersection (with priorities in traffic lights) or far side signalized intersection (PS)	406	1 – 61	7.0	7.3
Tram stop, near or far side non-signalized intersection, or mid block (MN)	620	1 – 55	5.4	5.3

Table 6. Characteristics of waiting time for possibility of departure from stop

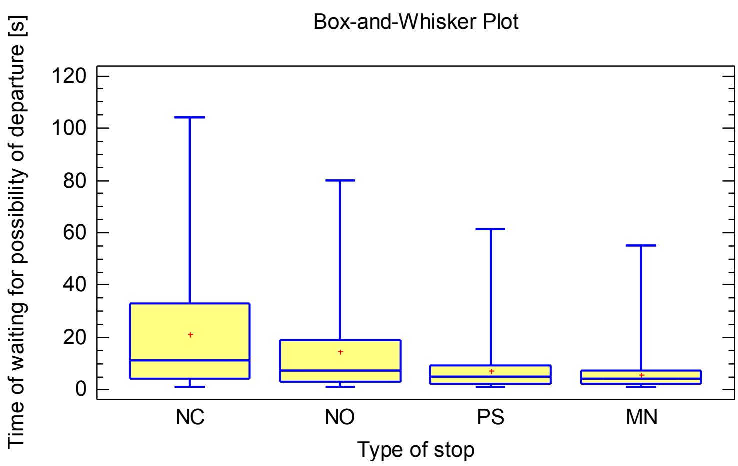
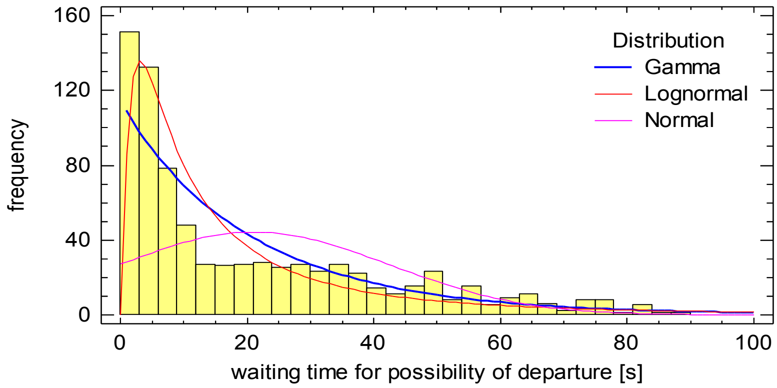


Figure 12. Box and Whisker plot of waiting time for possibility of departure in dependence on type of stop

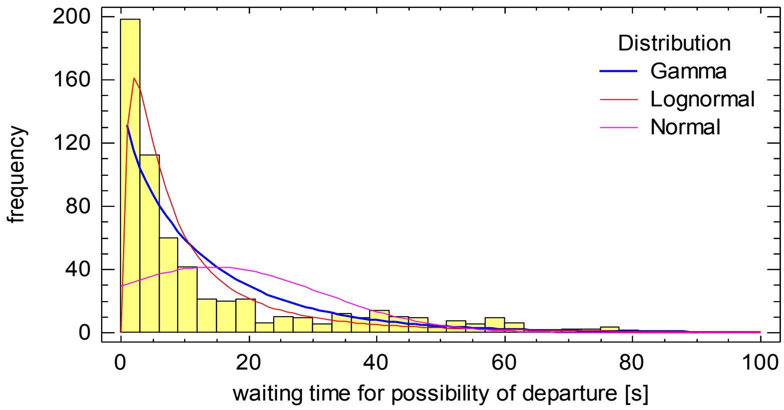
The results of multiple comparison tests done for this classification, show that there is a significant difference between the distributions of time lost for all considered cases. Especially, as a result of a Kruskal-Wallis test there is a significant difference amongst the medians.

For the four described cases of time lost, distributions were fitted (Figure 13). From the large group of possible distributions, finally there were taken into consideration Normal, Gamma and Logarithmic Normal distribution. However, in most of the cases a χ^2-test of goodness of fit rejects the corresponding hypothesis. Only in case PS both Gamma and Logarithmic Normal distribution can be fitted, the better results are obtained for Logarithmic Normal distribution. For further results reference is made to [2].

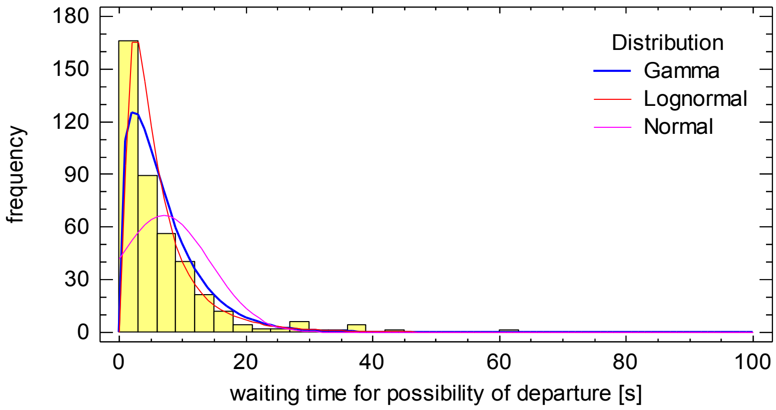
Type NC



Type NO



Type PS



Type MN

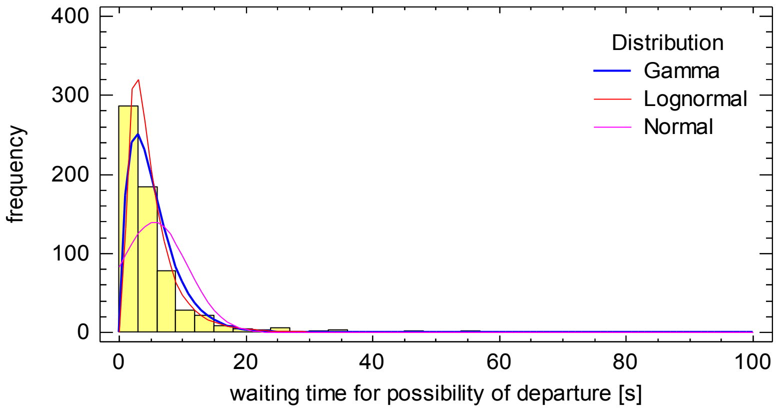


Figure 13. Fitted distributions of time waiting for possibility departure from stop

7 SIMULATION MODEL

To demonstrate the developed simulation model in Figure 14 the results of seven realizations the simulation of tram route no. 7 in Krakow are shown. Additionally the prediction values are printed. In Figure 15 the time-table and the predicted times are compared. It should be mentioned, that the corresponding variables are assumed to be independent. Moreover, in contrast to the considerations in Section 5 the alighting and boarding time is simulated directly by the help of the corresponding empirical distribution. The development of a suitable model for the description of the processes for number of alighting passengers a, boarding passengers b and occupancy of vehicle incoming to the stop P is still in progress and counts to the tasks of further research.

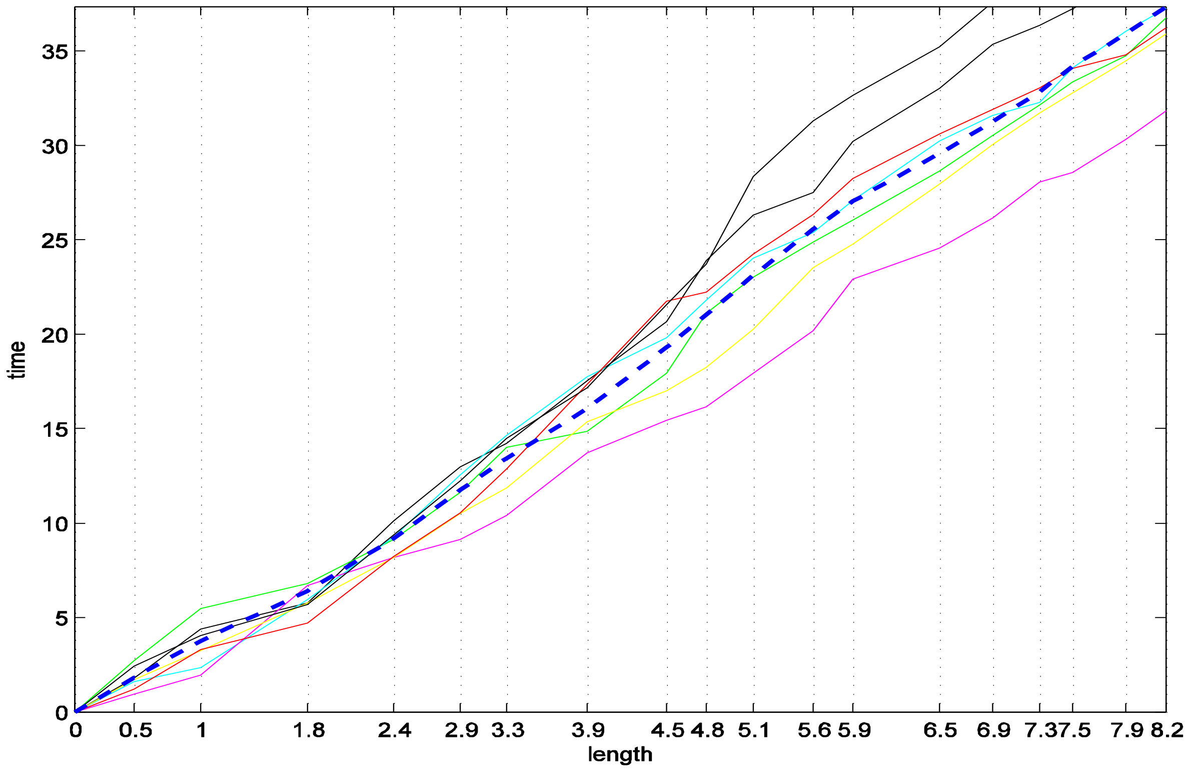


Figure 14. Mean time and seven realizations (lane number 7 in Krakow)

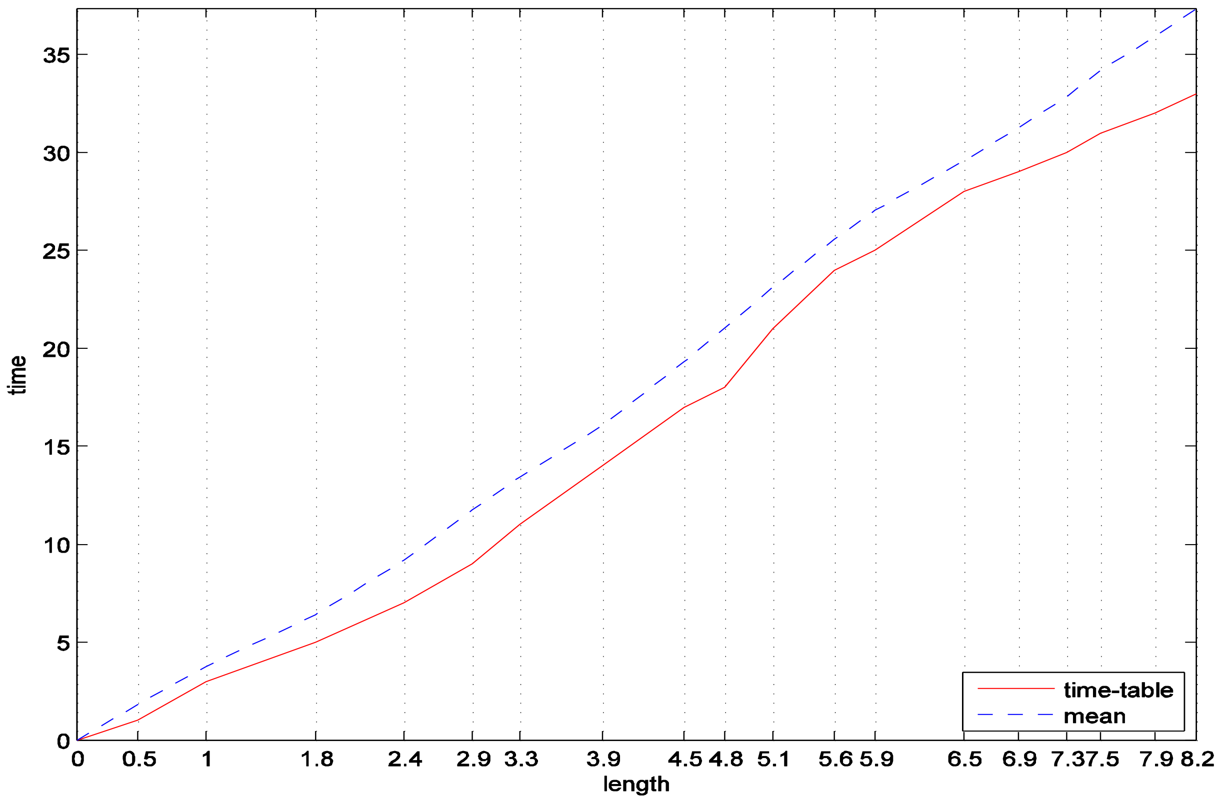


Figure 15. Comparison of mean time and the time-table (lane number 7 in Krakow)

8 MAIN CONCLUSIONS

1. Punctuality is one of the most important features of tram lines operation. It should be especially taken into consideration during modelling lines and whole public transport systems.
2. The Graph Theory and Events Theory are very useful in tram line modelling, as a structure of modules: section – stop, with repeating events: moment of starting alighting and boarding passengers, moment of finishing alighting and boarding passengers, and moment of departure from stop.
3. Classifications of sections and stops, proposed in model could be useful in other models, even for micro-simulation.
4. Stochastic simulation model contains deterministic and probabilistic components, which describe processes appearing in real tram line operation.
5. Running time could be explained by advanced regression formulas including length of section and total number of intersections on this section with taking into consideration the way of tram track separating.
6. The presented model can be helpful for time-table‘s better designing, in scheduling procedures, in planning and designing tram routes, in feasibility studies of new tram tracks, and in network analysis of public transport, in estimating input data for macro-simulation models of tram networks (e.g. VISUM software).

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