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MODEL DESCRIBING STATIC AND DYNAMIC DISPLACEMENT OF SILO WALLS DURING DISCHARGE OF GRANULAR SOLID

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Abstract.*Correct evaluation of wall displacements is a key matter when designing silos. This issue is important from both the standpoint of design engineer (load-bearing capacity of structures) and end-consumer (durability of structures). Commonplace methods of silo design mainly focus on satisfying limit states of load-bearing capacity. Current standards fail to specify methods of dynamic displacements analysis.*
Measurements of stressacting on silo walls prove that the actual stress is sum of static and dynamic stresses. Janssen came up with differential equation describing state of static equilibrium in cross-section of a silo. By solving the equation static stress of granular solid on silo walls can be determined. Equations of motion were determined from equilibrium equations of feature objects. General solution, describing dynamic stresses was presented as parametric model.
This paper presents particular integrals of differential equation, which enable analysing displacements and vibrations for different rigidities of silo walls, types of granular solid and its flow rate.

1 INTRODUCTION - TECHNICAL PROBLEM STATEMENT

Silos for storing grain or other granular solids, under normal operating conditions exhibit wall vibration caused by flow of material generating friction against silo wall. That vibration normally has quasi-harmonic waveform alternately increasing and decreasing. Its strength depends on various factors, however, observations made thus far prove that crucial parameters are wall flexibility and its decrement rate.

Stresspeaks upon silo emptying are widely known phenomena. Since those greater stresses used to cause silo failures, it is currently a common practice to factor them in and compensate their impact through experimentally determined by increasing coefficients. Those coefficients usually enable satisfying limit states of structural capacity, however, they could not be used to: compute maximum amplitudes in walls, determine material fatigue hazards or potential resonance threats.

The Authors set out to develop a computational model for describing fluctuation of vibration in silo walls during granular solid and grain flows. The model was devised based on analysis of continuous stress on silo walls.

2. ASSUMPTIONS TAKEN FOR MODELLING

The following assumptions were taken to formulate the model:

- Actual stress is superposition of static and dynamic stress (fig. 1):
- Vibration has harmonic waveform alternately increasing and decreasing.
- Conditions for static equilibrium defined by Janssen

$$\frac{\mathrm{dp_v}}{dx} + \mathrm{p_v}\ \mathrm{k}\ \mathrm{tg}\delta \frac{\mathrm{U}}{\mathrm{F}} = \gamma \qquad (1.1)$$

  p_v – vertical stress; F – silo cross-sectional area; U – silo circumference; δ - angle of wall friction; γ – bulk solid weight by volume;

  Solution to differential equation (1) in accordance with Janssen:

$$p_h^{stat} = \frac{\gamma F}{k\, tg\delta\ U}\left(1 - e^{-\frac{k\ tg\delta\ U z}{F}}\right)$$

- The displacement equation should be a variable dependent on silo height *z* and time *t*, and other derivatives will enable solving for longitude of the nodal line φ(z,t), bending moment *M*(z,t) and transverse force *V*(z,t);
- The model factors in wall give and decrement rate of the structure;

Special assumption taken to describe self-excited vibration is proportionality of exciting force - stressp_h(x,t), with derivatives of perpendicular translations relative to silo wall – w(x,t), $\frac{dw(x,t)}{dt}$, $\frac{d^2w(x,t)}{dt^2}$.

Pomiar naporu na ściany silosu z ciągłą rejestracją naporu na taśmie magnetycznej (badania dr Jerzego Kmity) [7]

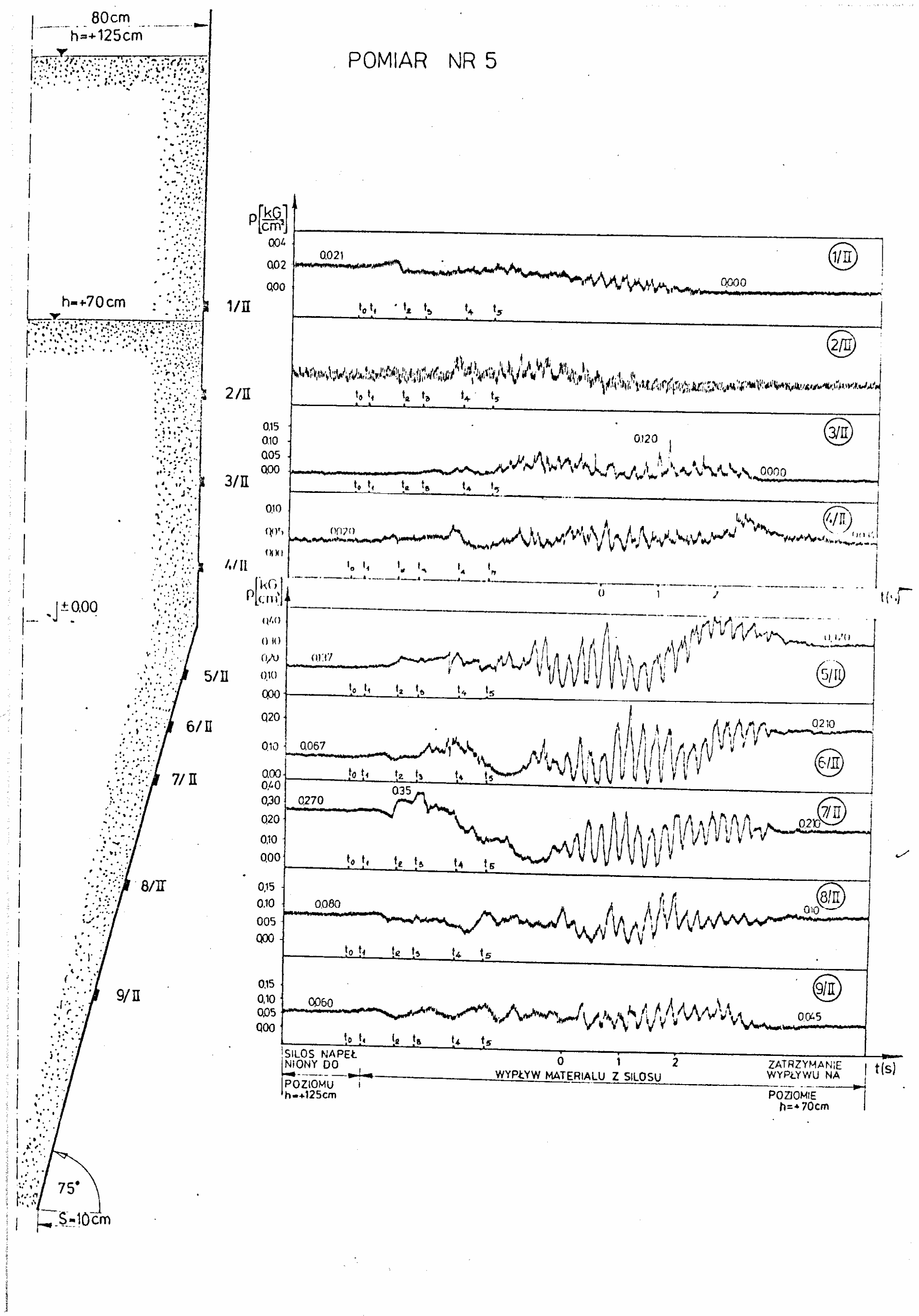


Fig. 2.1. Stress σ_n acting on cylindrical silo wall [Kmita, Ubysz].

Continuous recording silo wall stress measurement

3. FORMULATION OF THE MODEL

According to Boroch [2] and Langer [7] physical objects are continuous systems with indefinite degrees of freedom. Mathematics normally tends to discretise tasks. An assumption can be made that desired effect is obtained through force acting as point mass (granular solid, physical discretisation). The mass, however, can be divided into regular partitions, and the state of displacements can be given by limited number of parameters (mathematical discretisation). The latter method was employed to develop model describing silo wall displacement under load of granular solid.

The square one is equations of motion for uni-dimensional continuous arrangements and underlying dynamics assumptions. This model is justified by silo geometry. In many case this task could be approximately reduced to one dimension.

An important part of every analysis is defining the right-hand side of the equation. In order to correctly describe effect of self-excited vibration, the exciting force has to be proportional to derivatives of perpendicular translations relative to silo wall. In order to more comprehensively analyse experimentally a given phenomenon and more accurately describe vibrations, adequate parameters have to be introduced into RHS of equation. In many cases, numerous parameters would prevent from obtaining a closed solution and that is the objective authors did set for themselves.

Another issue important from viewpoint of modelling is rigidity of the silo wall. As far as dynamic parameters are concerned - frequency and amplitude - the wall is approximated by a beam of adequate flexural rigidity, or by membrane (string) whose flexural rigidity is ignored.

3.1. Linear membrane model of self-excited system

Base model was assumed as unit width membrane, fixed at both ends. The task was reduced down to unit width. In authors' view this is the most fundamental model taken from classic mechanics, which gives a good representation of self-excited vibration generated by dry friction. Figure 3.1 illustrates cut-out of wall under horizontal stress $p_{h1}(x)$ and quasi-harmonic stress $p_{h2}(x,t)$. N(x,t), N(x+Δx,t) expresses local longitudinal force, α(x,t), α(x+Δx,t) – vertical tilt. The load applied to membrane $p_{h2}(x,t)$ is a transverse force per unit of length. From condition for equilibrium Σx (vertical axis) factoring in forces generated by motion and forces of inertia (D'Alembert's principle) we get:

$$N(x+\Delta x,t) \sin \alpha (x+\Delta x,t) - N(x,t) \sin \alpha (x,t) = \Delta x \left[m \frac{d^2 w(x,t)}{dt^2} - p_h (x,t)\right] \quad (3.1)$$

m - mass per unit of length, $\frac{d^2 w(x,t)}{dt^2}$ - acceleration

Formulation *m* is an inertial force inferred from II Newton's laws of motion.

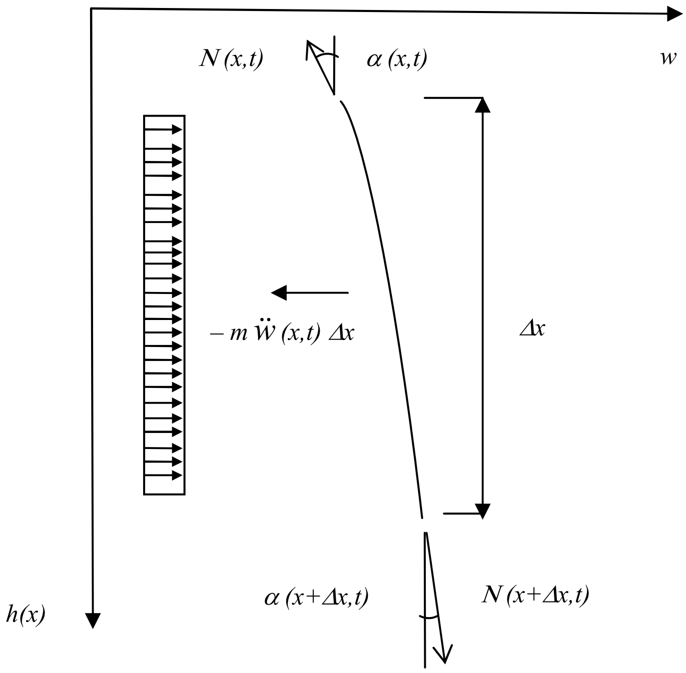


Fig. 3.1. Cut-out of membrane and distribution of forces.

By dividing both sides of equation 3.1 by Δx we get difference quotient on LHS and given $\Delta x \rightarrow 0$ we get derivative at dx (from definition of derivative). For constant x over segment Δx at small angles $\sin\alpha \rightarrow dw/dx$ we get longitudinal force N:

$$N - m\frac{d^2 w(x,t)}{dt^2} = -p_h(w,t) \qquad (3.2)$$

This equation, however, has more a general nature. For the exciting force to generate self-excited vibration it has to depend on displacement and its derivatives. Here, on wall displacement, its speed and acceleration. Some authors take static friction f_s as an auxiliary parameter. Forces occurring in the self-excited system, however, have to be explicitly separated from static and quasi-static stress generated through storage, filling and discharging the silo. Hence the general form of non-homogeneous equation can be written as:

$$N\frac{d^2 w(x,t)}{dx^2} - m\frac{d^2 w(x,t)}{dt^2} = -p_{h1}(w,t) - p_{h2}\left(w, \frac{dw(x,t)}{dt}, \frac{d^2 w(x,t)}{dt^2}, f_s, t\right) \qquad (3.3)$$

– $p_{h1}(w,t)$ – static component of stress; $p_{h2}\left(w, \frac{dw(x,t)}{dt}, \frac{d^2 w(x,t)}{dt^2}, f_s, t\right)$ – dynamic component of stress.

In similar fashion both the linear flexural model of self-excited system and the model of vibration along generatrix of silo's plating can be described. This will be discussed in dissertation currently in progress. Their final shape is presented in the following subsection.

3.2. Linear flexural membrane model of self-excited system

In case of silo wall with little give, its rigidity has to be factored in as well. Hence - similarly to flexible wall - the entire side surface is assumed to have the same rigidity. From general relationships from strength of materials, geometric relationships and by factoring in physical conditions, we get for small displacements:

$$M(x,t) = -EI(x)\ \frac{d^2 w(x,t)}{dx^2} \tag{3.4}$$

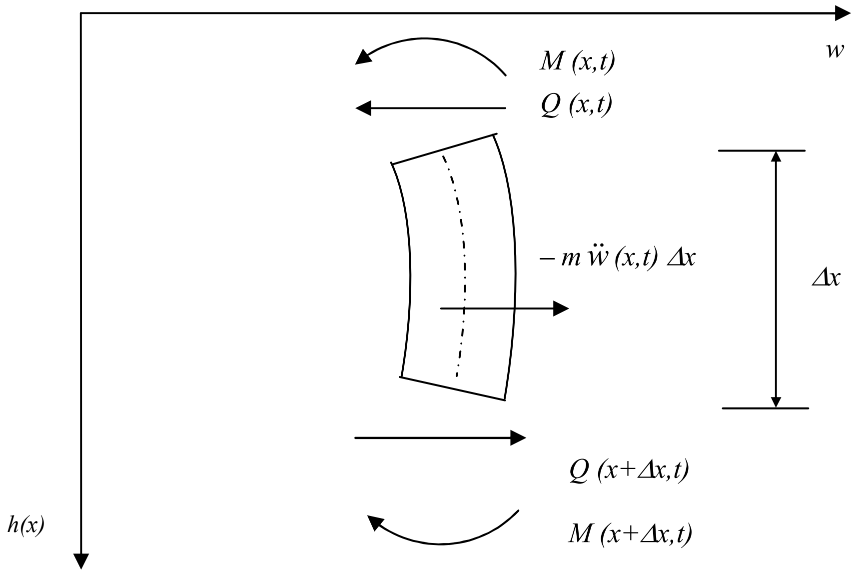


Fig. 3.2. Cut-out of silo wall and distribution of forces.

From conditions for equilibrium (fig. 3.2) we get:

$$-m\,\ddot{w}(x,t)\,\Delta x + Q(x+\Delta x,t) - Q(x,t) + p_h(x,t)\,\Delta x = 0 \tag{3.5}$$

Similarly to equation 3.1, the equation 3.5 can be divided by Δx, and given Δx→0 we get:

$$Q'(x,t) = m\frac{d^2 w(x,t)}{dx^2} - p_h(x,t) \tag{3.6}$$

From equation 3.4 we get:

$$Q(x,t) = \frac{d}{dx}M(x,t) = \frac{d}{dx}\left[-EI(x)\ \frac{d^2 w(x,t)}{dx^2}\right] \tag{3.7}$$

After substituting (3.7) to (3.6) we get:

$$\frac{d^2}{dx^2}M(x,t) = \frac{d^2}{dx^2}\left[-EI(x)\ \frac{d^2 w(x,t)}{dx^2}\right] = m\frac{d^2 w(x,t)}{dx^2} - p_h(x,t) \tag{3.8}$$

In majority of cases, silos have high longitudinal rigidity constant across segments, hence EI(x) = const. and 3.8 is reduced to:

$$EI(x)\left[\frac{d^4 w(x,t)}{dx^4}\right] + m\,\ddot{w}(x,t) = p_h(x,t) \tag{3.9}$$

Similarly to previous case, the component of excitation generated by vibration in self-excited system can by isolated:

$$EI(x)\left[\frac{d^4 w(x,t)}{dx^4}\right] + m\ddot{w}(x,t) = p_{h1}(x,t) + p_{h2}\left(x, \frac{dx}{dt}, \frac{d^2x}{dt^2}, f_s, t\right) \quad (8.9)$$

3.3. Model of vibration along generatrix of silo's plating

In high silos elastic strain occurs along generatrix of the silo. Here, vibrations are excited longitudinally. System of forces along the wall is illustrated in figure 3.3 and equation 3.10 shows equilibrium:

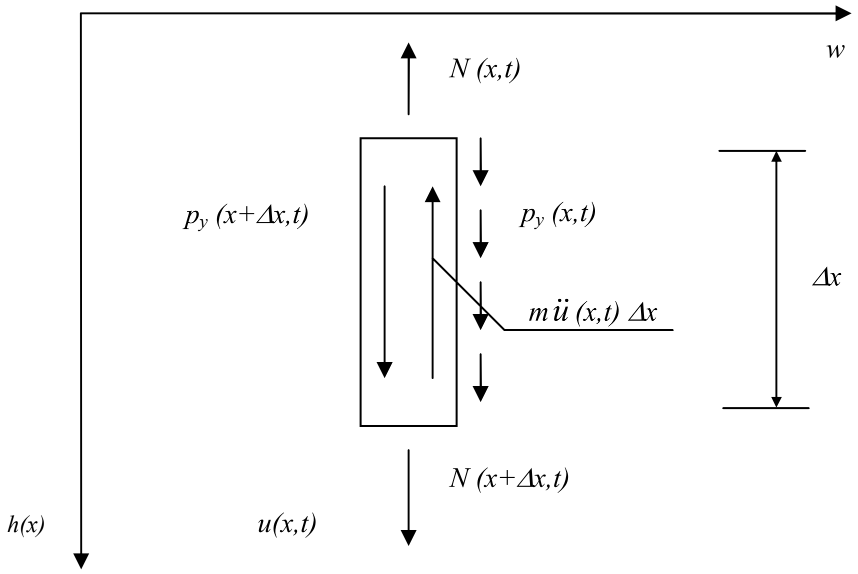


Fig. 3.3. Cut-out of silo wall and distribution of forces.

$$N(x+\Delta x,t) - N(x,t) = m\ddot{u}(x,t)\,\Delta x - p_v(x,t)\,\Delta x \quad (3.10)$$

Similarly dividing by Δx in line with definition of differential quotient, given $\Delta x \rightarrow 0$, the differential equation is:

$$\frac{dN}{dx} = m\ddot{u}(x,t) - p_v(x,t) \quad (3.11)$$

Then in accordance with physical relationship (Hooke's law) we get:

$$\sigma = E\,\varepsilon \quad (3.12)$$

$$\frac{N}{A} = E\frac{du}{dx} \quad (3.13)$$

By substituting (3.13) to (3.11):

$$\frac{d}{dx}\left(EA\frac{du}{dx}\right) = m\ddot{u}(x,t) - p_v(x,t) \quad (3.14)$$

and for quantities constant over segments EA = const.:

$$EA\frac{d^2u}{dx^2} = m\ddot{u}(x,t) - p_v(x,t) \quad (3.15)$$

After rearranging equation 3.15 and singling out factor generating self-excited vibration it becomes:

$$m\ddot{u}(x,t) - E A\frac{d^2u}{dx^2} = p_{v1}(x,t) + p_{v2}(x, \frac{dx}{dt}, \frac{d^2x}{dt^2}, f_s, t) \tag{3.16}$$

However, terms p_v are significantly lower than p_h, are recorded longitudinal vibrations have amplitudes significantly lower than vibrations perpendicular to silo walls. Thus they are considerably less important for silo wall vibrations.

4. SOLUTIONS TO SELECTED MODELS DESCRIBING SILO WALL VIBRATION

Models of self-excited systems were based on equations of motion. Solutions were sought after for complete structure of mass and using approximated description of state of displacements with limited number of parameters expressing characteristics of silo wall stresses.

4.1. Laminar flow stress

Stress generated by granular solid acting on silo walls - provided there is no pulsation (laminar) - can be described by differential equation, which - since stress is assumed proportional to horizontal translation of silo wall - will take form of general equation of motion. If there is no pulsation upon bulk solid being discharged from the silo (laminar flow), the phenomenon can be expressed by the following differential equation:

$$\frac{\partial^2 u(x,t)}{\partial t^2} = -t^n \frac{\partial u(x,t)}{\partial t} - u(x,t); \qquad n \in <½; 2> \tag{4.1}$$

For initial conditions:

$$u(x,t) = 0; \; u'(x,t) = 1 \tag{4.2}$$

we get relationship describing silo wall displacements over time. This is the case where there are no extra loads applied of oscillatory nature. By following up on the assumption wall displacements are proportional to wall stress, we get relationship expressing that stress over discharge time. Depending on assumed *n*, usually between <½; 2>, the stress will fade over various time. This corresponds to different readings of measurements taken at different heights. Due to sheer complexity of factors influencing actual stresses, the following models use theoretical normal stress $p_h^* = 1$, which for given silo has to be multiplied by static stress at given height determined through e.g. Janssen equations. The following figures show example stress over time charts. To better illustrate the function, waveforms of stress fluctuations were shown for three levels: upper, middle and lower part of the silo. According to the function, the highest stress occurs once the discharge process starts and is most intensive towards the hopper.

Due to numerous factors influencing stress acting on different levels of silo, a parametric function is envisaged to describe the phenomenon.

$$p_h(x,t) = m t^L e^{\frac{-ht}{n\pi}} + c_1 t + c_2 \tag{4.3}$$

This function is general integral of equation (4.2) expressed in real numbers. Initial and boundary conditions for determining parameters and constants of integration are determined based on results of experimental study. Parameters of "low vulnerability" to function fluctuation were taken for further deliberations as constants:

m = coefficient describing initial discharge <0,7÷1,00> – assumed 0.8;

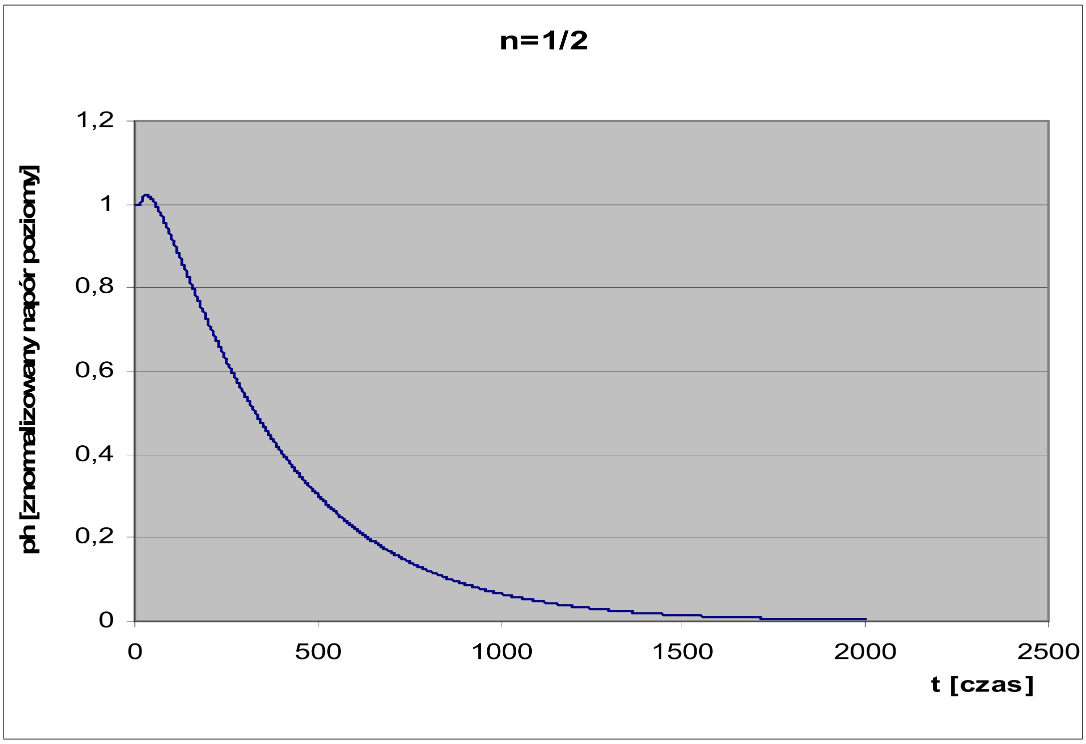
L = coefficient describing variability of the function fluctuation - assumed 0.1

h = coefficient describing variability of the function fluctuation (convexity, concavity, inflection point) - assumed 0.005

The solids discharge starts at t_0. Hereunder the function is:

$$p_h(x,t) = \begin{cases} 1 & t \le t_0 \\ m t^{0,1} e^{\frac{-0,005\,t}{n\pi}} & t > t_0 \end{cases} \qquad (4.4)$$

Both the function fluctuation and parameters were taken based on experimental data. Arriving at the solution enables adjusting (correcting) those parameters for flow of different substances



$$p_h(x,t) = 0,8\,t^{0,1}\,e^{\frac{-0,005\,t}{0,5\pi}} \qquad (4.5)$$

Fig. 4.1. Relationship between horizontal stress and time - upper silo section (n = ½).

4.2. Stress at flow with self-excited vibration

Solids discharge from silo is often distorted by dynamic factors occurring over the course of that process. The most important from the viewpoint of day-to-day operation are seemingly:

- tumble of overburden material due to arching or doming;
- vibration generated by turbulent flow;
- vibration excited by "dry friction" as called in literature self-excited vibration.

The nature on impact stress exerts depends on multiple factors and can be vary both qualitatively and quantitatively. Some of the factors are:

- caking (humidity, granulation, re-solidification),
- angle of internal friction of bulk solid,
- wall friction coefficient,
- silo wall give,
- flow rate of bulk solid,
- boundary conditions (wall joints, silo supports, foundation),
- mass flow problems (relief equipment, segregation);
- other.

Presented next is model built based on available to authors results of experimental study, where parameters of bulk solid (sand) can be assumed homogeneous, the silo is regular shape and discharge flow is consistent. Silo walls show some give, and the wall where wall stress measurements were taken had natural frequency $\nu = 33$ Hz.

For purposes of formulating the model, only tape sections where vibrations were the strongest were used. Based on continuous readings it was concluded that an important oscillatory factor is self-excited vibration.

The equation describing silo wall strain over time was formulated using forecasting method. Presented was superposition function of carrier wave and function describing oscillation. The function describing carrier wave was described in detail in subsection 4.5.1. The function describing the oscillatory wave was introduced with - apart from conditions enabling modulation of frequency and amplitude - conditions enabling to describe excitation and extinction of vibration.

The function describing horizontal displacement of silo wall in any given section is also given by:

$$u(x,t) = u_1(x,t) + u_2(x,t), \tag{4.6}$$

where: $$u_1(x,t) = Ate^{-Bt}(\sin Ct - \sin Dt), \tag{4.7}$$

$$u_2(x,t) = M\,t^L\,e^{-Ht}. \tag{4.8}$$

Thus the full equation is:

$$u(x,t) = Ate^{-Bt}(\sin Ct - \sin Dt) + M\,t^L\,e^{-Ht} \tag{4.9}$$

or

$$u(x,t) = Ate^{-Bt}\sin Ct - Ate^{-Bt}\sin Dt + M\,t^L\,e^{-Ht} \tag{4.10}$$

Next in this subsection presented is rearrangement of the function, leading to final differential equation describing silo wall vibration. To keep working outs concise, derivative symbol used for single-variable function $(\ )^{!}$ was substituted into the RHS of equation.

$$\frac{\partial u(x,t)}{\partial t} = \left(Ate^{-Bt}\right)^{!}(\sin Ct - \sin Dt) + Ate^{-Bt}((\sin Ct - \sin Dt))^{!} + \left(M\,t^L\right)^{!} e^{-Ht} + M\,t^L\left(e^{-Ht}\right)^{!}$$

(4.11)

$$\left(Ate^{-Bt}\right)^{!} = Ae^{-Bt} + A(-B)te^{-Bt}$$

$$\left((\sin Ct - \sin Dt)\right)^{!} = \left(C\cos Ct - D\cos Dt\right)$$

$$\left(M\,t^{L}\right)^{!} e^{-Ht} + M\,t^{L}\left(e^{-Ht}\right)^{!} = MLt^{(L-1)}e^{-Ht} + M(-H)t^{L}\,e^{-Ht}$$

$$\frac{\partial u(x,t)}{\partial t} = \left(Ae^{-Bt} - AB\,te^{-Bt}\right)(\sin Ct - \sin Dt) + Ate^{-Bt}\left(C\cos Ct - D\cos Dt\right) + ML\,t^{(L-1)}\,e^{-Ht} - MH\,t^{L}\,e^{-Ht}$$

$$\frac{\partial u(x,t)}{\partial t} = \left(Ae^{-Bt}\sin Ct - AB\,te^{-Bt}\sin Ct - Ae^{-Bt}\sin Dt + AB\,te^{-Bt}\sin Dt\right) +$$

$$+\left(Ate^{-Bt}C\cos Ct - Ate^{-Bt}D\cos Dt\right) + ML\,t^{(L-1)}\,e^{-Ht} - MH\,t^{L}\,e^{-Ht}$$

$$\frac{\partial^{2} u(x,t)}{\partial t^{2}} = \left(\left(Ae^{-Bt} - AB\,te^{-Bt}\right)(\sin Ct - \sin Dt) + Ate^{-Bt}\left(C\cos Ct - D\cos Dt\right) + ML\,t^{(L-1)}\,e^{-Ht} - MH\,t^{L}\,e^{-Ht}\right)^{!}$$

$$\frac{\partial^{2} u(x,t)}{\partial t^{2}} = \begin{pmatrix} \left(Ae^{-Bt}\sin Ct - AB\,te^{-Bt}\sin Ct - Ae^{-Bt}\sin Dt + AB\,te^{-Bt}\sin Dt\right) + \\ +\left(Ate^{-Bt}C\cos Ct - Ate^{-Bt}D\cos Dt\right) + ML\,t^{(L-1)}\,e^{-Ht} - MH\,t^{L}\,e^{-Ht} \end{pmatrix}^{!}$$

$$\left(Ae^{-Bt}\sin Ct\right)^{!} = -ABe^{-Bt}\sin Ct + ACe^{-Bt}\cos Ct$$

$$\left(ABte^{-Bt}\sin Ct\right)^{!} = \left(ABe^{-Bt} + AB(-B)te^{-Bt}\right)\sin Ct + ABCte^{-Bt}\cos Ct$$

$$\left(ABte^{-Bt}\sin Ct\right)^{!} = ABe^{-Bt}\sin Ct - AB^{2}te^{-Bt}\sin Ct + ABCte^{-Bt}\cos Ct$$

$$\left(Ae^{-Bt}\sin Dt\right)^{!} = -ABe^{-Bt}\sin Dt + ACe^{-Bt}\cos Dt$$

$$\left(ABte^{-Bt}\sin Dt\right)^{!} = \left(ABe^{-Bt} + AB(-B)te^{-Bt}\right)\sin Dt + ABDte^{-Bt}\cos Dt$$

$$\left(ABte^{-Bt}\sin Dt\right)^{!} = ABe^{-Bt}\sin Dt - AB^{2}te^{-Bt}\sin Dt + ABDte^{-Bt}\cos Dt$$

$$\left(ACte^{-Bt}\cos Ct\right)^{!} = \left(ACe^{-Bt} + AC(-B)te^{-Bt}\right)\cos Ct - AC^{2}te^{-Bt}\sin Ct$$

$$\left(ACte^{-Bt}\cos Ct\right)^{!} = ACe^{-Bt}\cos Ct - ABCte^{-Bt}\cos Ct - AC^{2}te^{-Bt}\sin Ct$$

$$\left(ADte^{-Bt}\cos Dt\right)^{!} = \left(ADe^{-Bt} + AD(-B)te^{-Bt}\right)\cos Dt - AD^2te^{-Bt}\sin Dt$$

$$\left(ADte^{-Bt}\cos Dt\right)^{!} = ADe^{-Bt}\cos Dt - ABDte^{-Bt}\cos Dt - AD^2te^{-Bt}\sin Dt$$

$$\left(ML\,t^{(L-1)}e^{-Ht}\right)^{!} = ML(L-1)t^{(L-2)}e^{-Ht} + ML(-H)t^{(L-1)}\,e^{-Ht}$$

$$\left(ML\,t^{(L-1)}e^{-Ht}\right)^{!} = ML(L-1)t^{(L-2)}e^{-Ht} - MLH\,t^{(L-1)}\,e^{-Ht}$$

$$\left(MH\,t^L\,e^{-Ht}\right)^{!} = MHLt^{(L-1)}e^{-Ht} + MH(-H)t^L\,e^{-Ht}$$

$$\left(MH\,t^L\,e^{-Ht}\right)^{!} = MHLt^{(L-1)}e^{-Ht} - MH^2\,t^L\,e^{-Ht}$$

$$u(x,t) = u_1(x,t) + u_2(x,t) = Ate^{-Bt}\sin Ct - Ate^{-Bt}\sin Dt + M\,t^L\,e^{-Ht}$$

$$\frac{\partial u(x,t)}{\partial t} = \left(Ae^{-Bt}\sin Ct - AB\,te^{-Bt}\sin Ct - Ae^{-Bt}\sin Dt + AB\,te^{-Bt}\sin Dt\right) +$$

$$+\left(Ate^{-Bt}C\cos Ct - Ate^{-Bt}D\cos Dt\right) + ML\,t^{(L-1)}\,e^{-Ht} - MH\,t^L\,e^{-Ht}$$

$$\left(Ae^{-Bt}\sin Ct\right)^{!} = -ABe^{-Bt}\sin Ct + ACe^{-Bt}\cos Ct$$

$$-\left(ABte^{-Bt}\sin Ct\right)^{!} = -ABe^{-Bt}\sin Ct + AB^2te^{-Bt}\sin Ct - ABCte^{-Bt}\cos Ct$$

$$-\left(Ae^{-Bt}\sin Dt\right)^{!} = ABe^{-Bt}\sin Dt - ADe^{-Bt}\cos Dt$$

$$\left(ABte^{-Bt}\sin Dt\right)^{!} = ABe^{-Bt}\sin Dt - AB^2te^{-Bt}\sin Dt + ABDte^{-Bt}\cos Dt$$

$$\left(ACte^{-Bt}\cos Ct\right)^{!} = ACe^{-Bt}\cos Ct - ABCte^{-Bt}\cos Ct - AC^2te^{-Bt}\sin Ct$$

$$-\left(ADte^{-Bt}\cos Dt\right)^{!} = -ADe^{-Bt}\cos Dt + ABDte^{-Bt}\cos Dt + AD^2te^{-Bt}\sin Dt$$

$$\left(ML\;t^{(L-1)}e^{-Ht}\right)^{!} = ML\,(L-1)t^{(L-2)}e^{-Ht} - MLH\;\;t^{(L-1)}\,e^{-Ht}$$

$$-\left(MH\,t^L\,e^{-Ht}\right)^{!} = -MHLt^{(L-1)}e^{-Ht} + MH^2\,t^L\,e^{-Ht}$$

After substitution:

$$\frac{\partial^2 u_1(x,t)}{\partial t^2} = \left(B^2 - C^2\right)u_1(x,t) - 2B\frac{\partial u_1(x,t)}{\partial t} + AB\,te^{-Bt}(\sin Ct - \sin Dt) + 2Ae^{-Bt}(C\cos Ct - D\cos Dt)$$

and

$$\frac{\partial^2 u_2(x,t)}{\partial t^2} = ML(L-1)t^{(L-2)}e^{-Ht} - 2MLH\,t^{(L-1)}\,e^{-Ht} + MH^2\,t^L\,e^{-Ht} + (2MH^2\,t^L\,e^{-Ht} - 2MH^2\,t^L\,e^{-Ht})$$

and after rearranging

$$\frac{\partial^2 u_1(x,t)}{\partial t^2} = \left(B^2 - 2B - C^2\right)u_1(x,t) - 2B\frac{\partial u_1(x,t)}{\partial t} + \frac{2}{t}\frac{\partial u_1(x,t)}{\partial t} - \left(\frac{2}{t^2} + \frac{B}{t}\right)u_1$$

$$\frac{\partial^2 u_2(x,t)}{\partial t^2} = -2M\frac{\partial u_2(x,t)}{\partial t} - u_2(x,t) + ML(L-1)t^{(L-2)}e^{-Ht}$$

When considering complete solids discharge, the parameter of time might reach high values and last two equation components will tend to naught. Then the final equations of motion become:

$$\frac{\partial^2 u_1(x,t)}{\partial t^2} - \left(B^2 - 2B - C^2\right)u_1(x,t) + 2B\frac{\partial u_1(x,t)}{\partial t} = \frac{2}{t}\frac{\partial u_1(x,t)}{\partial t} - \left(\frac{2}{t^2} + \frac{B}{t}\right)u_1 \qquad (4.12)$$

$$\frac{\partial^2 u_2(x,t)}{\partial t^2} + 2M\frac{\partial u_2(x,t)}{\partial t} + u_2(x,t) - ML(L-1)t^{(L-2)}e^{-Ht} = 0 \qquad (4.13)$$

In equation (4.12) the RHS is the function of displacement and first derivative of displacement - this is consistent with initial assumptions made for self-excited vibrations. The equation (4.13) can be interpreted as "main wave" −describing characteristics of stress on silo wall during solids discharge.

Similarly to laminar flow, stress was assumed proportional to horizontal translations of silo walls, however, as opposed to that case, apart from regular stress upon solids discharge, also other loads occur with oscillatory waveform. Based on that assumption, we get the relationship between stress and solids discharge time. To make possible comparing graphs of functions, the parameter was assumed to fluctuate within n = <½; 2>. Hence, those stresses will be described for different levels of silo wall. These models also use the theoretical normal stress $p_h^* = 1$, which for given silo has to be multiplied by actual stress at given height. The following figures show example stress over time charts. To better illustrate the function, waveforms of stress fluctuations were shown for three levels: upper, middle and lower part of the silo.

The function describing stress acting on silo wall is given by superposition of main wave and oscillatory stresses. Introduced parameters enable experimentally describing the function both in terms of its graph and expected values.

$$p_h(x,t) = Ate^{-Bt}(\sin Ct - \sin Dt) + M\,t^L\,e^{\frac{-ht}{n\pi}} + c_1 t + c_2 \qquad (4.14)$$

Similarly to solution for laminar flow, the function is general integral of equation () expressed in real numbers. Initial and boundary conditions for determining parameters and constants of integration are determined based on results of experimental study.

Parameters found in equations have the following physical representation:

- A - vibration amplitude; when describing stress acting on silo wall where amplitude of oscillatory component to fixed component is 5 ÷ 20 %, the parameter fluctuates within <0,0005 ÷ 0,002>;

- $B = \frac{b}{n\pi}$, b – range of damping; n ∈<½; 2>; when self-excited vibrations extinct at 0.7 p_h acting on silo wall, b = 0.03, provided self-excited vibrations extinct at 0.2 p_h, b = 0.07;
- C, D - parameters describing frequency of resonant excitation; quantities closest to observed excitations are obtained for C/D=<0,85 ÷ 0,95>;

and previously assumed for laminar flow

- m = coefficient describing initial discharge <0,7÷1,00> – assumed 0.8;
- L = coefficient describing variability of the function fluctuation - assumed 0.1;
- $H = \frac{h}{n\pi}$ h = coefficient describing variability of the function fluctuation (convexity, concavity, inflection point) - assumed 0.005

The solids discharge starts at t_0. Finally the formula became:

$$p_h(x,t) = \begin{cases} 1 & t \le t_0 \\ Ate^{-Bt}(\sin 0{,}9t - \sin t) + M\, t^{0{,}1}\, e^{\frac{-0{,}005t}{n\pi}} & t > t_0 \end{cases} \qquad (4.15)$$

In case of rigid silos with high decrement rate vibration has nature of short-term excitation, which causes temporary overload of the silo wall. For that eventuality silo overload was modelled for stresses beyond 1.2 p_h. Because overload has short-term nature, silo overload has characteristics similar to random structural overload during solids discharge, where expected stresses are a band of expected loads as opposed to non-ambiguous values.

In case of dynamically flexible walls with high decrement rate, higher amplitude vibrations might occur, but they who quickly extinct (fig. 4.2).

Another example is dynamically susceptible wall with low decrement rate, where periodically excited are cyclical high-amplitude vibrations. Description of these vibrations is rather characteristic with explicit self-excited vibration (fig.4.3).

Selected examples are seemingly the most representative based on available experimental studies as far as analysis of vibrations and stresses acting on silo walls are concerned.

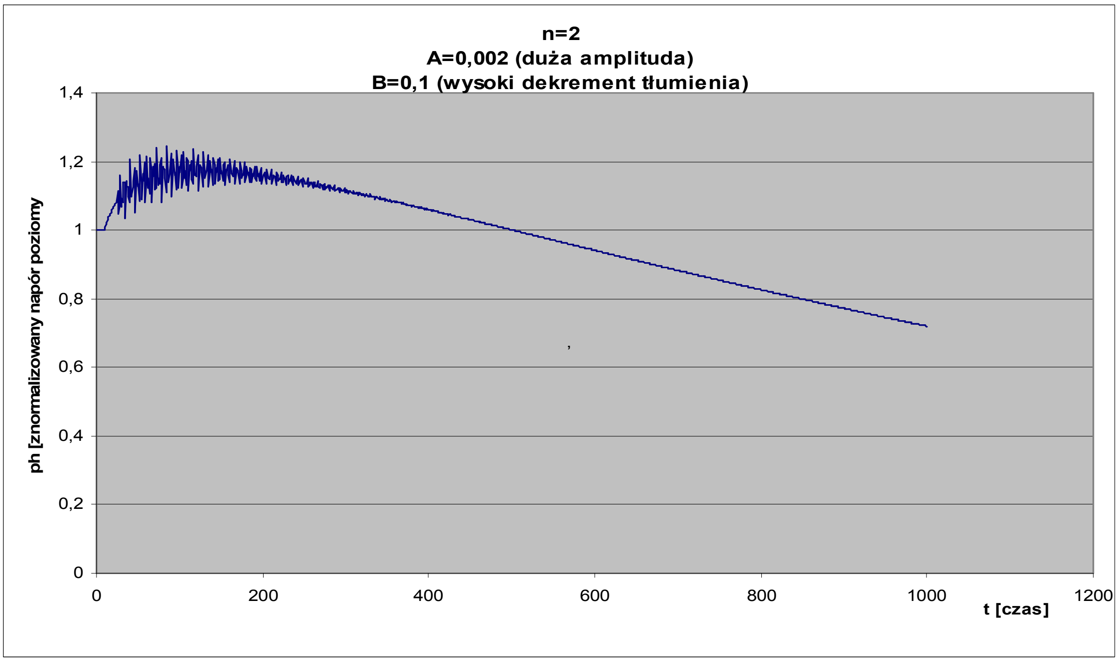


Fig. 4.2.Horizontal stress acting on silo flexible wall to vibration with high decrement rate.

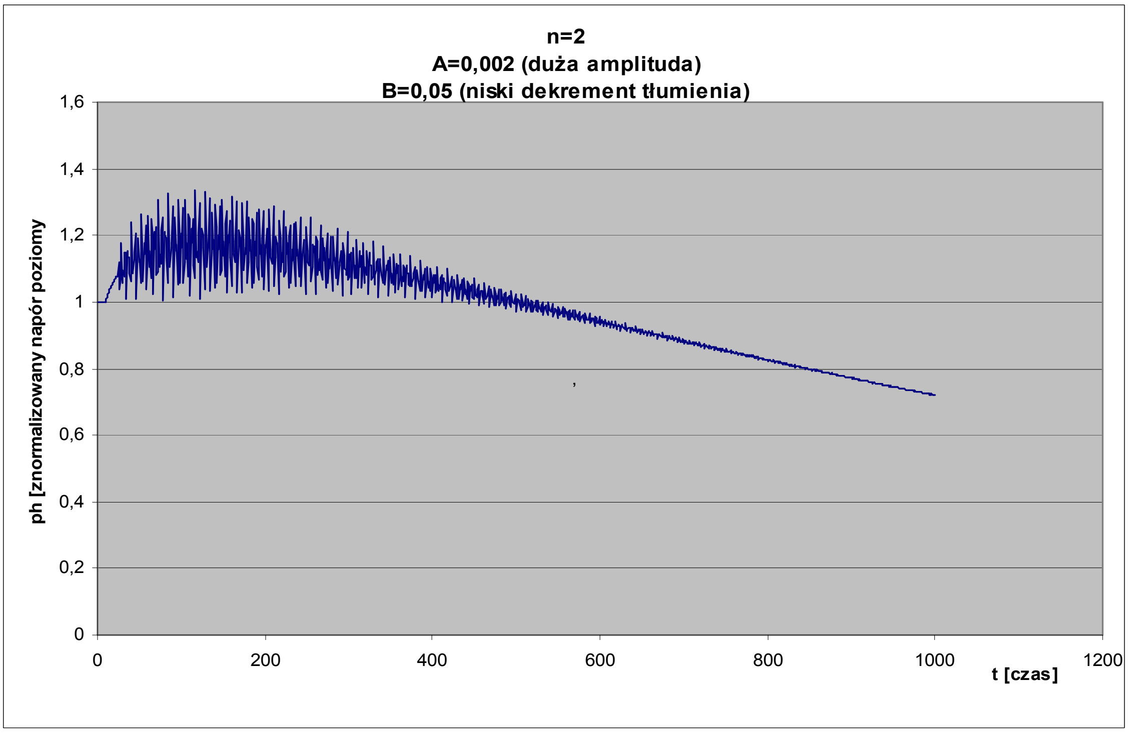


Fig. 4.3. Deformable wall with low decrement rate.

5. SUMMARY, RESEARCH PROBLEMS AND SUPPLEMENTARY RESEARCH PLAN

From designer's point of view, issues of stress acting on silo wall are synonymously related to wall displacements, strength of materials problems and fatigue problems. The proposed model is a tool intended for forecasting loads and some processes related to day-to-day silo operation.

This proposition of stress&strain model was devised to give original contribution to current state of arts in behaviour of silo wall during storage, filling and discharging. This particularly applies to subject area most extensively covered in specialist publications which seems crucial from standpoint of structural safety i.e. real stresses generated by granular solid acting on silo wall. The proposed model aims to make computational models more true to real structural behaviour.

In this paper, authors attempted to compromise between universality of model and its practical applicability, hence it was limited to analyse variability of only selected elements. Taken assumptions are the square one in developing more complex models which would describe with greater accuracy existing cases or enable analysing new practical cases, which were not included in this paper.

Practical applications for the model are:

– Approximated description of vibration cycles (amplitude and frequency of vibrations), key for determining fatigue strength of material,

– Estimation of expected stress acting on silo wall during granular solid flow.

This model - based on research data - requires further verification against greater number of experimental data from natural-scale structures. Beyond doubt, however, it can serve well for purposes of experimental studies.

REFERENCES

[1]. Borcz A., Kmita J., Ubysz A., Drgania samowzbudne podczas wypływu materiału z silosu o ścianach odkształcalnych. Pr. Nauk. Inst, Bud. Polit. Wrocł. Nr 33

[2]. Boroch H. Monographic lecture, TU Wrocław, Wrocław 1976

[3]. Eibl J., Landachl H., Haussler U., Gladen W. Zur Frage des Silodrückes. Beton und Stahlbetonbau 4, 1982 s. 104-110

[4]. Jenike A., Johanson J.R.: On the theory of bin loads, Trans. ASME. J. Eng. Ind. Vol. 91. ser. B2/1969

[5]. Kamiński M. Untersuchungen des Getreidedrücks in Silozellen Die Bautechnik 58, 1981, H. 1, s. 19-23

[6]. Kmita J., Silo as a system of self-induced vibration. Journal of Structural Engineering, vol. 111, nr 1985, s. 190–204

[7]. Langer J.: Dynamika budowli. Wyd. Polit. Wrocł. Wrocław 1980.

[8]. Pieper K., Wenzel F.: Druckverhältnisse in Silozellen, Berlin-München, Verlag von Wilhelm Ernst und Sohn. Berlin 1964

[9]. Schwedes J. , Wilms H. Spannungs Verteilung in divergenten Bunkern. Aufbereitungs-technik H.11, 1979 s. 603-608

[10]. Tejchman J., Niedostatkiewicz M., Experimental and theoretical studies on resonance dynamic effects during silo flow. Powder Handling & Processing, 15, 1 (2003), pp. 36–42.

[11]. Wenzel K, Probleme des Silodruckes. Dissertation TU Braunschweig 1971.