Bauhaus Isometry and Fields

Abstract: this paper is resulting from an experimental talk. The author and some colleagues saw the drawback of our fixations on our own narrow topics and a gradual inhibition of our joy in discussion groups. Therefore, if it could become a little clearer that even in rather technical matters there are connections that we do not immediately see and which are more likely marginalized because of our specializations. We want to awaken creativity and what we may call the spirit of hobbyists and handicraft enthusiasts. The Bauhaus is a school of art, design and architecture where teaching concentrated on functional craftsmanship. It began with our insight, as Klaus Gürlebeck had pointed out, that veil basic craft belongs to art. Today such compartments would be quickly abolished. We encounter the same situation in arts and science. Therefore let us go together into some fundamental questions of creation both human and cosmic. I shall give us a frame for the inquiry. After the session this paper shall be completed by recording our several contributions in the discussion.

The deep unconscious act in creation

Between 1914 and 1930 Carl Gustav Jung wrote the legendary Red Book. It is a large, leather bound volume which contained the 'archetypical nucleus' of his later work. The Red Book, or Liber Novus (New Book) is an important part in his opus. It has remained unpublished till 2009. Jung drew mandalas in vivid colors. He also collected drawings from sick children some of whom would soon die.

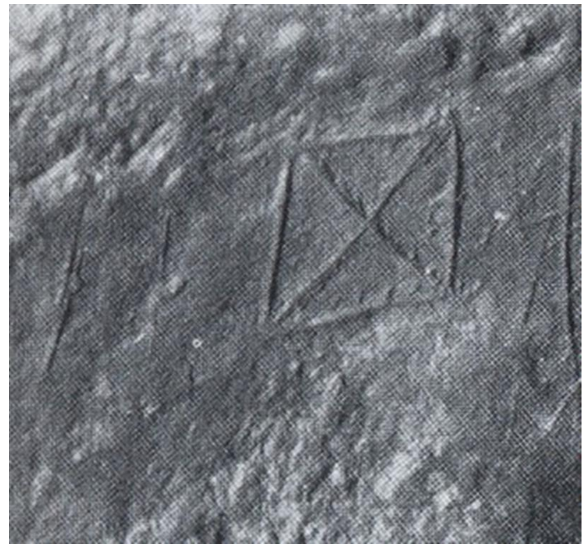
mandala done by a female patient
of C. G. Jung

The Wheel of Life
Hildegard von Bingen

We go into the meaning of these symmetric circle drawings. If the deep subconscious has to do with very old information, it may be connected with the oldest forms of human art, the Paleolithic petroglyphs. Here I show us just a few of these very old pieces.

Cupule and meander petroglyph on a boulder
at the Auditorium Cave,
Bhimbetka, Madhya Pradesh, India

290000 – 700000 BCE, the oldest art



"Ginsterhöhle" am Haute Pierre,
Dép. Seine-et-Oise

Rock engraves from Foppe, Nadro, Val Camonica,
Lombardy Italy, central Alps

Blombos Cave Engravings, incised on
Ochre Stone, about 70000 BCE. East of
Capedown, South Africa

The 'mandala of plane reorientation' as I use to call it, has a dihedral symmetry of the non-abelian group D_4, the space congruence of a square or automorphism of a 'Zweibein'. For me, it represents a neuronal interface between 'extension' and 'cognition', between our images of geometry and the logic of thought. The Gustav Jung Society of the UK has actually posed the question "Is this an Imago Dei"? Well, if we consider some ideas of Spinoza, god shoud unite our two properties of being material and mental, being extended and emotional on the one hand, and being cognitive and logical on the other.

Dihedral group

$$D_{2d}$$

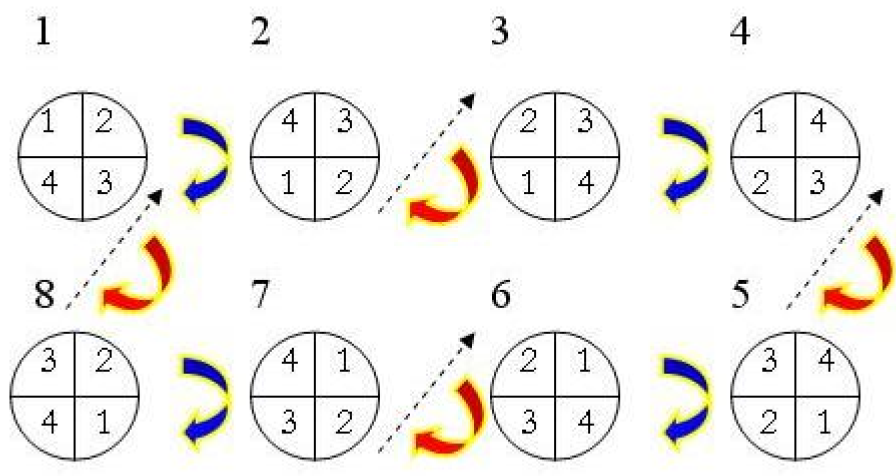
with 2 diagonals

$$d = 2$$

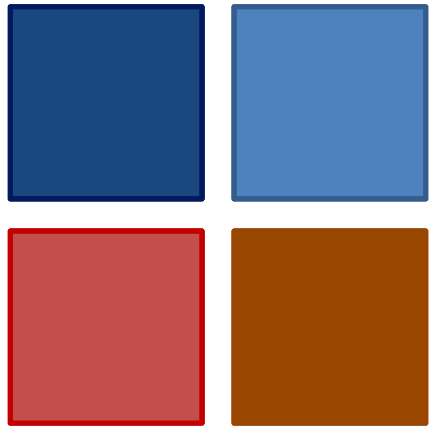
$$\boxplus \quad \times \quad \otimes \quad + \quad \boxdot$$

$$D_4$$

abelian group with 8 elements



We demonstrate the non-commutativity of the group algebra by the following teaching aid:



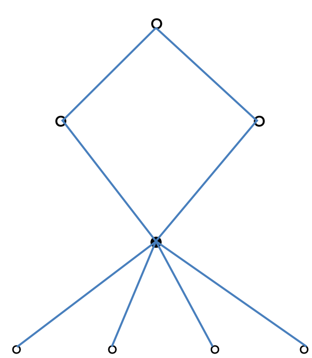
Teaching aid
Dihedral disc
colored on both sides

Representation in $Cl_{1,1} \simeq Cl_{2,0}$

$$Id,\ e_1,\ e_2,\ e_{12},\ -Id\ ,\ -e_1,\ -e_2,\ -e_{12}$$

the ‘Dirac-group’ of $Cl_{1,1}$

is generated by base units e_1, e_2 by Clifford multiplication.



The cycle-graph of the dihedral group

Table 1: constructed by Erich Brendel

'*Dihedral symmetry breaking of multiplication table*'

Significance of D_4 in the Minkowski algebra

Surprisingly, the dihedral group D_4 plays a central role even for the Clifford algebra of the Minkowski space-time. Following the relaxed notation of Pertti Lounesto, we can generate higher order Clifford algebra by the formula

$$Cl_{p,q} \otimes Cl_{1,1} \simeq Cl_{p+1,q+1}$$

This signifies a form of geometric periodicity that had been studied by Johannes G. Maks in his thesis. In physics this recursive relation brings several merits

$$Cl_{3,1} \simeq Cl_{2,2}$$

and likewise

$$Cl_{3,1} \simeq Mat(2, Cl_{1,1})$$

That is, the Clifford algebra of the Minkowski space-time can be represented as a 2×2- matrix algebra with entries in the planar Clifford algebra $Cl_{1,1}$. The most convenient form of $Cl_{3,1}$ is thus $Mat(4, \mathbb{R})$.

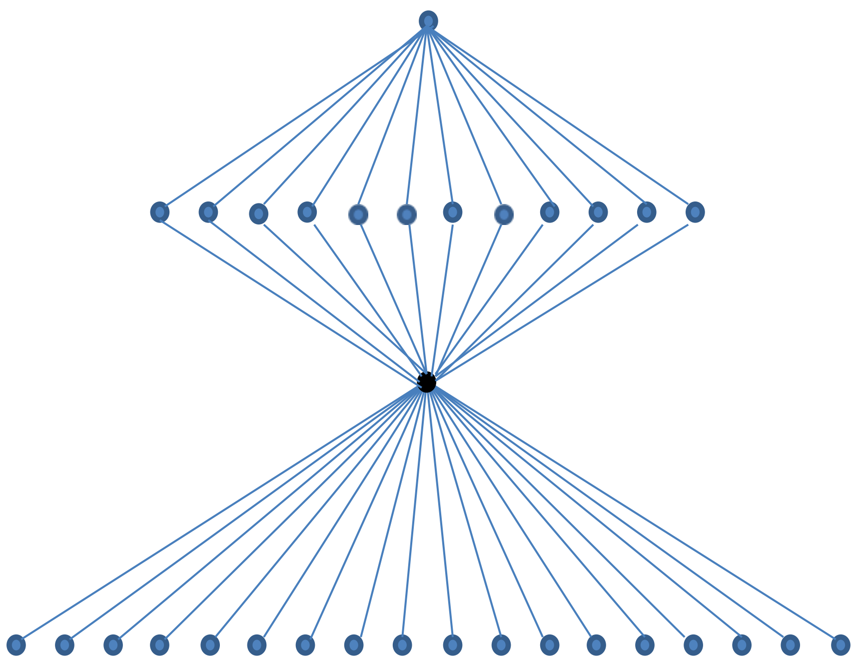
The Dirac group is the finite group generated by the base unit monomials of the Clifford algebra in the form $\{\pm Id,\ \pm e_\alpha\}$. Considering the Pauli algebra as an example, its 'Dirac group' is given by 16 base units

$$G_{3,0} = \{\pm Id, \pm e_1, \pm e_2, \pm e_3, \pm e_{12}, \pm e_{23}, \pm e_{13}, \pm e_{123}\}$$

where the unit vetors are given by the Pauli matrices, that is, $e_\alpha = \sigma_\alpha$. The Dirac group of the Minkowski algebra can be represented by a direct Clifford product of two forms of the dihedral group

$$G_{3,1} = D_4 \times D_4$$

where the two forms have only the identity Id in common.



The cycle-graph of the Dirac group of Minkowski algebra

The story does not halt here. But we have to see the essential difference between reflections in the Pauli algebra and reflections in the Minkowski algebra. We have to investigate the 'reorientation groups' of those spaces.

In some recent, yet unpublished work I explained reflexivity, the meaning of the concept of Eigenform as has been worked out by Heinz von Foerster and Louis Kauffman, and how action and acted on become one in a magma. It is possible to derive the full motion group of leptons and fermions in the Clifford algebra of the Minkowski space-time. This algebra is married to a rank-3 Lie group $\mathfrak{L}_3$. The full motion group is nothing else than the standard model of HEPhy. It is essentially given by an algebra $su(3) \times su(2) \times u(1)$. We show from which primary statements the $su(3)$-component can be derived. The $su(2)$ is constituted by the bivectors (even subalgebra), as usual, but there is also a rank-2 Clifform $\mathfrak{L}_2 \subset \mathfrak{L}_3$ represented by either $sl(3,\mathbb{R})$, or $sl(3,\mathbb{C})$ or $su(3,\mathbb{C})$. A third component that goes far beyond the $u(1)$ has been called volume-time by our friend Hitzer (2007).

Originally the generators of the *Clifform* of that graded Lie group have been found on a long path of trial and error. Today (2012) it is possible to derive it from comparatively few and simple assumptions by a little rigor. These assumptions read:

1. There is a Group $\mathfrak{L}_2 \subset Cl_{3,1}$ with a fixed point f_1
2. Every group element $g \in \mathfrak{L}_2$ as an element of $Cl_{3,1}$ must have the general form $g = x_2 e_1 + x_3 e_2 + \ . \ . \ . + x_{15} e_{234} + x_{16} e_{1234}$
3. Every group element $g \in \mathfrak{L}_2$ transforms elements $\psi \in Cl_{3,1}$ by conjugation just like in a spin group: $\psi \longmapsto g\psi g^{-1}$
4. Generators of the Lie algebra of the group $\mathfrak{L}_2 \subset Cl_{3,1}$ should be unitary.
5. Group elements $g \in G$ are calculated from the algebra by an exponential map.
6. Generators of the group algebra should satisfy commutation relations of some form of $SU(3)$.

This (Cliff)form can have different real forms. If it were a *normal real form*, it should be represented by a matrix algebra $SL(3,\mathbb{R})$ within $Cl_{3,1}$, where the latter is known to be isomorphic with the matrix algebra $Mat\ (4,\mathbb{R})$. If it were a compact form, it should be represented by $SU(3,\mathbb{C})$, but now as matrices $Mat\ (4,\mathbb{C})$ in $\mathbb{C}\otimes Cl_{3,1}$. The Lie algebra $\mathfrak{L}_2$ can be given (2012) by the elements

$$T_1 = \tfrac{1}{2}(-e_{34} + e_{134});\quad T_2 = \tfrac{i}{2}(-e_{23} + e_{123});\quad T_3 = \tfrac{1}{2}(-e_{24} + e_{124});\quad T_4 = -\tfrac{1}{2}(e_3 + e_{234})$$
$$T_5 = -\tfrac{i}{2}(e_{13} + J);\quad T_6 = \tfrac{1}{2}(e_2 + e_{14});\quad T_7 = \tfrac{i}{2}(e_4 + e_{12});\quad T_8 = \tfrac{1}{2\sqrt{3}}(-2e_1 + e_{24} + e_{124})$$

I recall, some of us found beautiful solutions for a red up quark

$$\begin{matrix} SU(2) & \times & U(1) \end{matrix}$$
$$\begin{bmatrix} a & b & -ic & id \\ -b & a & -id & -ic \\ ic & id & a & -b \\ -id & ic & b & a \end{bmatrix} \times \begin{bmatrix} j & 0 & -k & 0 \\ 0 & j & 0 & k \\ k & 0 & j & 0 \\ 0 & -k & 0 & j \end{bmatrix}$$
$$(aId - be_{23} + c\ ie_{13} + d\ ie_{12}) \times (Id\cos\phi - e_{1235}\sin\phi) =$$
$$= (aId - be_{23} + c\ ie_{13} + d\ ie_{12})\cos\phi - (ae_{1235} + be_{15} + c\ ie_{25} - d\ ie_{35})\sin\phi$$
$$(a^2 + b^2) - (c^2 + d^2) = 1 \qquad j^2 + k^2 = 1$$

Look, how easily this can be satisfied. We can just write down the waves without even knowing the equation of motion. Namely, consider the following constants

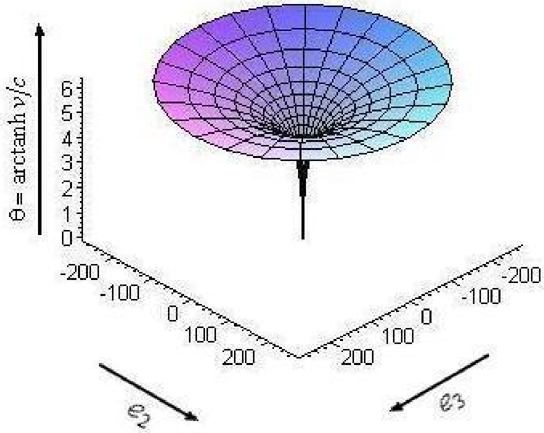
with hyperbolic conditions of unitary

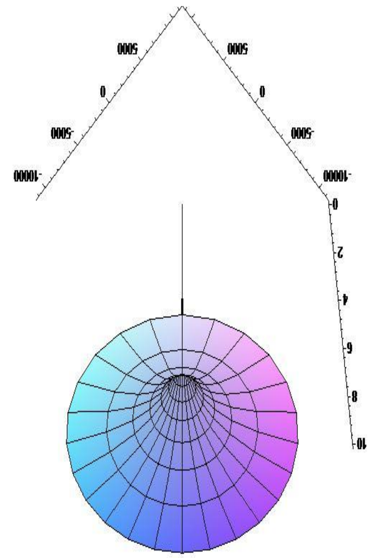
$$a = \cosh\theta\ \cos\frac{1}{2}\psi,\ \ b = \cosh\theta\sin\frac{1}{2}\psi,\ \ c = \sinh\theta\ \cos\frac{1}{2}\psi,\ \ d = \sinh\theta\ \sin\frac{1}{2}\psi$$

$j = \cos\phi,\ \ k = \sin\phi,$ with $\theta = \operatorname{arctanh}\left(\frac{v}{c}\right)$, c ... light velocity, v ... velocity

I came upon a singularity at rest, the deadlock singularity (Stillstand). Space-time seems flat for light and has a singularity at rest.

Figure 46: Isospin deconvoluted soliton
Maximal immersed surface from Lorentz transformation
$(\cos\alpha\cosh\theta)e_2 + (\sin\alpha\cosh\theta)e_3,\ z=\alpha,\ u=\theta=\operatorname{arctanh} v/c$
$z = 0\ldots 2\pi,\ u = 0\ \ldots\ 2\pi$





This wonderful picture must ultimately lead to a celly bag cap like caps of Piff and Paff

Piff and Paff are two boys from a remote planet who land with their spaceship on our earth and experience all sorts of adventures. They amaze the terrestrials with technical trix, but sometimes don’t figure out the meaning of certain terrene utensils. They are landing then on some clothesline or in the sackful of flour. Sometimes they make themselves invisible and flee with their car.

When I was a boy, I read “Wunderwelt”. »Wunderwelt« is the most known children’s magazine in postwar Austria. It appeared from march 1948 as the third and last of three big children’s newspapers following the concept of its predecessors. A costly illustrated fairytale at the centre spread of the paper soon became its recognition mark. It was substantially shaped by the drawer and writer Teja Aicher, who among others created the “dwarf Bumsti”, a figure that served Wunderwelt as flagship until 1976. Aicher was an academic painter, 1909-1979, passed the Academy of Fine Arts Vienna, worked from 1948 to 1976 as writer and drawer for the children’s magazine.

May be, I am not aware that I could not let go the celly bag cap. The personal roots of our interest may be undiscovered by us, or unknown or both. Not knowing we accommodate to a frame of images and emotions. Dreams of early childhood continue in later activities. The feelings, the compassion reverberate … There are pre-cognitive frames we have to follow, and there is much beauty in this, if we can follow them freely.

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