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T-SPLINE BASED XIGA FOR ADAPTIVE MODELING OF CRACKED BODIES

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Abstract. *Safety operation of important civil structures such as bridges can be estimated by using fracture analysis. Since the analytical methods are not capable of solving many complicated engineering problems, numerical methods have been increasingly adopted. In this paper, a part of isotropic material which contains a crack is considered as a partial model and the proposed model quality is evaluated. EXtended IsoGeometric Analysis (XIGA) is a new developed numerical approach [1, 2] which benefits from advantages of its origins: eXtended Finite Element Method (XFEM) and IsoGeometric Analysis (IGA). It is capable of simulating crack propagation problems with no remeshing necessity and capturing singular field at the crack tip by using the crack tip enrichment functions. Also, exact representation of geometry is possible using only few elements. XIGA has also been successfully applied for fracture analysis of cracked orthotropic bodies [3] and for simulation of curved cracks [4]. XIGA applies NURBS functions for both geometry description and solution field approximation. The drawback of NURBS functions is that local refinement cannot be defined regarding that it is based on tensor-product constructs unless multiple patches are used which has also some limitations. In this contribution, the XIGA is further developed to make the local refinement feasible by using T-spline basis functions. Adopting a recovery based error estimator in the proposed approach for evaluation of the model quality and performing the adaptive processes is in progress. Finally, some numerical examples with available analytical solutions are investigated by the developed scheme.*

1 INTRODUCTION

Increasingly development of computers made the possibility to apply numerical methods for simulation of civil structures as an alternative to analytical methods which are not feasible in resolving complex problems. This has attracted many researchers' interests for developing more accurate and efficient computational approaches in the last decades.

Fracture analysis of structures is of great importance for estimation of their safety operation. A reliable and efficient numerical method is required for analysis of cracked part of a structure.

The remeshing necessity and existence of a singular field around a crack tip in simulation of crack propagation problems led to the development of a new generation of computational approaches such as meshfree methods [5-14] and the extended FEM (XFEM) [15-20] which belongs to the class of Partition of Unity Methods (PUM). Moving discontinuous problems such as crack propagation can be analyzed by these methods without the requrirement of remeshing or rearranging of the nodal points. In the XFEM, a priori knowledge of the solution is locally added to the approximation space. This enrichment allows for capturing particular features such as discontinuities and singularities which are present in the solution exactly.

More recently, a numerical approach called extended isogeometric analysis (XIGA) [1, 2] has been developed for simulation of stationary and propagating cracks by incorporating the concepts of the XFEM into the isogeometric analysis [21, 22]. Some superiorities of the isogeometric analysis in comparison with the conventional FEM are: simple and systematic refinement strategies, an exact representation of common and complex engineering shapes, robustness and higher accuracy. XIGA has also been successfully applied for fracture analysis of cracked orthotropic bodies [3] and for simulation of curved cracks [4].

XIGA applies NURBS functions for both geometry description and solution field approximation. The drawback of NURBS functions is that local refinement cannot be defined regarding that it is based on tensor-product constructs unless multiple patches are used which has also some limitations. In this contribution, T-spline basis functions are applied in the XIGA to make local refinement feasible.

Finally, for quality evaluation of the proposed model, some numerical simulations with available analytical solutions are studied.

2 BASIS FUNCTIONS

2.1 NURBS

Non-uniform rational B-splines (NURBS) are a generalization of piecewise polynomial B-spline curves. The B-spline basis functions are defined in a parametric space on a knot vector Ξ. A knot vector in one dimension is a non-decreasing sequence of real numbers:

$$\Xi = \{\xi_1, \xi_2, \ldots, \xi_{n+p+1}\} \tag{1}$$

where ξ_i is the i^{th} knot, i is the knot index, $i = 1, 2, \ldots, n + p + 1$, p is the order of the B-spline, and n is the number of basis functions. The half open interval $[\xi_i, \xi_{i+1})$ is called the i^{th} knot span and it can have zero length since knots may be repeated more than once, and the interval $[\xi_1, \xi_{n+p+1}]$ is called a patch. In the isogeometric analysis, always open knot vectors are employed. A knot vector is called open if it contains $p+1$ repeated knots at the two ends.

With a certain knot span, the B-spline basis functions are defined recursively as,

$$N_i^0(\xi) = \begin{cases} 1 & \text{if } \xi_i \le \xi \le \xi_{i+1} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

and

$$N_i^p(\xi) = \tfrac{\xi-\xi_i}{\xi_{i+p}-\xi_i} N_i^{p-1}(\xi) + \tfrac{\xi_{i+p+1}-\xi}{\xi_{i+p+1}-\xi_{i+1}} N_{i+1}^{p-1}(\xi) \quad p = 1, 2, 3, \ldots \tag{3}$$

where $i = 1, 2, \ldots, n$.

A B-spline curve of order p is defined by:

$$\mathbf{C}(\xi) = \sum_{i=1}^{n} N_i^p(\xi)\,\mathbf{P}_i \tag{4}$$

where $N_i^p(\xi)$ is the i^{th} B-spline basis function of order p and $\{\mathbf{P}_i\}$ are control points, given in d-dimensional space $\mathbf{R}^d$.

The non-uniform rational B-spline (NURBS) curve of order p is defined as:

$$\mathbf{C}(\xi) = \sum_{i=1}^{n} R_i^p(\xi)\,\mathbf{P}_i \tag{5}$$

$$R_i^p(\xi) = \frac{N_i^p(\xi)\, w_i}{\sum_{j=1}^{n} N_j^p(\xi)\, w_j} \tag{6}$$

where $\{R_i^p\}$ are the NURBS basis functions, $\{\mathbf{P}_i\}$ are the control points and w_i is the i^{th} weight that must be non-negative. In the two dimensional parametric space $[0,1]^2$, NURBS surfaces are constructed by tensor product through knot vectors $\mathbf{\Xi}^1 = \left\{\xi_1^1, \xi_2^1, \ldots, \xi_{n+p+1}^1\right\}$ and $\mathbf{\Xi}^2 = \left\{\xi_1^2, \xi_2^2, \ldots, \xi_{n+p+1}^2\right\}$. It yields to:

$$R_{i,j}^{p,q}\left(\xi^1, \xi^2\right) = \frac{N_i^p(\xi^1)\, M_j^q(\xi^2)\, w_{i,j}}{\sum_{k=1}^{n}\sum_{l=1}^{m} N_k^p(\xi^1)\, M_l^q(\xi^2)\, w_{k,l}} \tag{7}$$

For more details on NURBS, refer to [23].

2.2 T-splines

T-splines is a generalization of NURBS enabling local refinement [26, 27]. For defining the T-spline basis functions, an index space called T-mesh is defined. It is similar to the index space representation of a NURBS, with the difference that T-junctions, which are vertices connecting three edges, are allowed. An example of T-mesh is illustrated in Fig. 1. It is noted that each line

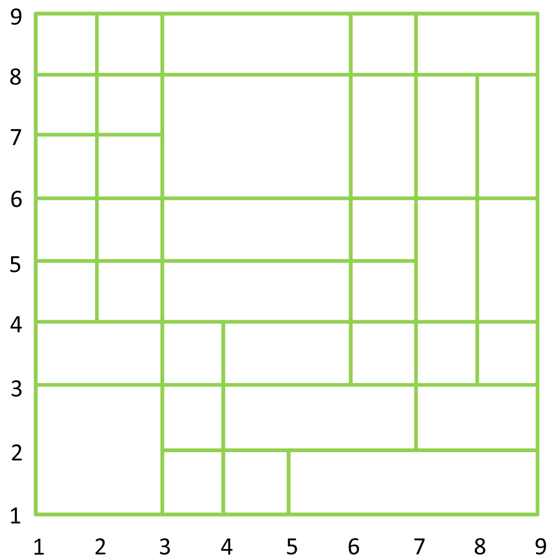


Figure 1: A sample of T-mesh.

in the mesh corresponds to a knot value. Then, anchors are defined on the T-mesh to identify the location of each basis function. They are located at the intersections of knot lines if the polynomial order is odd, otherwise their location are in the center of the cells. Regardless of degree, an anchor location is at the center of the support of a function in the index space.

For definition of T-splines, local knot vectors are defined instead of using the global knot vectors since each basis function has the compact support of $(p+1) \times (q+1)$ knots. As illustrated in Fig. 2, local knot vectors in each direction are defined by horizontally or vertically marching from the anchors backward and forward [27]. Afterwards, each basis function can be defined using the Eqs. 2, 3 and 7 and its corresponding local knot vectors.

In order to refine the mesh, knot insertion process is performed. It consists of adding new knots to the present mesh/T-mesh and correspondingly, modifying and adding some control points. For more information about T-spline and local refinement, readers are referred to [26, 27].

3 EXTENDED ISOGEOMETRIC ANALYSIS

Extended isogeometric analysis (XIGA) is a newly developed computational approach which uses the superiorities of the extended finite element method (XFEM) within the isogeometric analysis. It is capable of crack propagation simulation without the remeshing necessity since element edges are defined independent of the crack location.

Solution field approximation is extrinsically enriched by the Heaviside and branch functions for crack face and singular field (around the crack tip) modeling, respectively.

$$\begin{aligned} \mathbf{u}^h\left(\xi^1,\xi^2\right) = & \textstyle\sum_{i=1}^{n_{en}} R_i^{p,q}\left(\xi^1,\xi^2\right)\mathbf{u}_i + \sum_{j=1}^{n_H} R_j^{p,q}\left(\xi^1,\xi^2\right) H\mathbf{a}_j \\ & + \textstyle\sum_{k=1}^{n_Q} R_k^{p,q}\left(\xi^1,\xi^2\right)\sum_{\alpha=1}^{4} Q_\alpha \mathbf{b}_k^\alpha \end{aligned} \tag{8}$$

The first term in the right hand side is standard IGA approximation. $\{R_i^{p,q}\left(\xi^1,\xi^2\right)\}$ are the T-Spline basis functions of orders p and q in ξ^1 and ξ^2 directions, respectively, at the point (ξ^1,ξ^2)

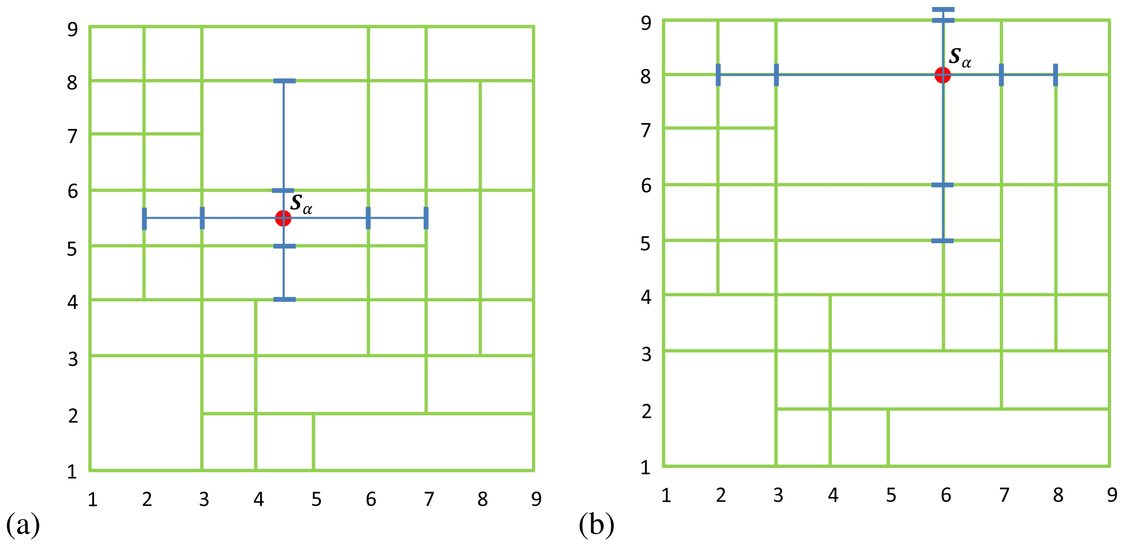


Figure 2: Schematic view of defining local knot vectors for the anchor $\mathbf{S}_\alpha$: (a) quadratic polynomial order: $\Xi_\alpha^1 = \{\xi_2^1, \xi_3^1, \xi_6^1, \xi_7^1\}$ and $\Xi_\alpha^2 = \{\xi_4^2, \xi_5^2, \xi_6^2, \xi_8^2\}$; (b) cubic polynomial order: $\Xi_\alpha^1 = \{\xi_2^1, \xi_3^1, \xi_6^1, \xi_7^1, \xi_8^1\}$ and $\Xi_\alpha^2 = \{\xi_5^2, \xi_6^2, \xi_8^2, \xi_9^2, \xi_9^2\}$.

in the parametric space $[0,1] \times [0,1]$. $\{\mathbf{a}_j\}$ are the vectors of additional degrees of freedom which are related to the modeling of crack faces, $\{\mathbf{b}_k^\alpha\}$ are the vectors of additional degrees of freedom for modeling the crack tip, n_{en} is the number of nonzero basis functions for a given knot span, n_Q is the number of n_{en} basis functions which have been selected as branch enriched basis functions. They can be selected using the topological enrichment strategy or geometrical enrichment one. In topological enrichment scheme, the basis functions which contain the crack tip in their influence domains are selected as the branch enriched basis functions while in geometrical enrichment method, branch enriched basis functions consist of the basis functions chosen from the previous strategy and the ones which are selected according to considering a constant domain around the crack tip. In this contribution, geometrical enrichment method is adopted and a circular domain with a predefined radius at the center crack tip is considered and basis functions whose influence domains contain the crack tip and whose anchors located in the circle are selected as branch enriched basis functions. n_H is the number of n_{en} basis functions that have crack face in their support domains and have not been selected as branch enriched basis functions. H is the generalized Heaviside function [24],

$$H(\mathbf{X}) = \begin{cases} +1 & \text{if } (\mathbf{X} - \mathbf{X}^*) . \mathbf{e}_n > 0 \\ -1 & \text{otherwise} \end{cases} \tag{9}$$

where $\mathbf{e}_n$ is the unit normal vector of crack alignment in point $\mathbf{X}^*$on the crack surface which is the nearest point to $\mathbf{X}(\xi^{\mathbf{1}}, \xi^{\mathbf{2}})$.

In Eq. 8, $Q_\alpha \{\alpha = 1, 2, 3, 4\}$ are the crack tip enrichment functions whose roles are reproducing the singular field around crack tips,

$$\{Q_\alpha\}_{\alpha=1}^4 = \left\{ \sqrt{r} \sin\frac{\theta}{2}, \sqrt{r}\cos\frac{\theta}{2}, \sqrt{r}\sin\theta\sin\frac{\theta}{2}, \sqrt{r}\sin\theta\cos\frac{\theta}{2} \right\} \tag{10}$$

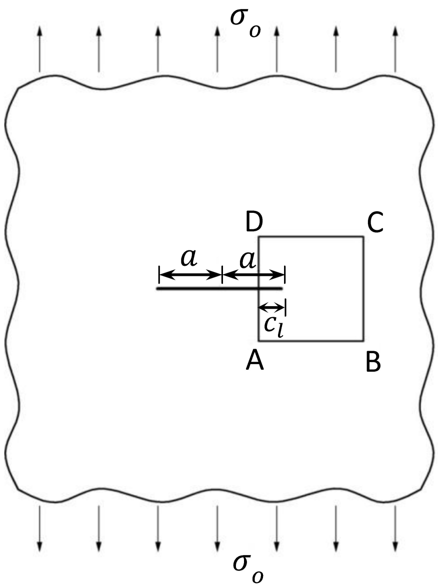


Figure 3: A mode I crack model in an infinite plate.

where (r, θ) are the local crack tip polar coordinates with respect to the tangent to the crack tip in the physical space.

Readers are referred to [2] for more information about XIGA formulation and implementation.

4 NUMERICAL EXAMPLES

In this section, two numerical examples are investigated by the proposed approach. The first one contains a mode I crack while the other includes a mixed mode crack. New knots added for refinement satisfy the conditions of analysis-suitable T-splines [28]. Basis functions of cubic order are considered. Gauss quadrature rule with 4x4 Gauss points for normal elements is utilized. For integration over split and tip elements, sub-triangles and almost polar techniques with 13 and 7x7 Gauss points for each sub-triangles are adopted.

4.1 Mode I crack model in the infinite plate

An infinite plate including a straight crack under pure fracture mode I is considered, as depicted in Figure 3. The plate is in plane strain state. Then, a local finite square domain ABCD which includes the crack tip in the center is defined. The domain ABCD, which includes the $c_l = 5$ mm part of the crack, is smaller than the crack length $2a = 200$ mm in the infinite plate. The size of this analytical domain ABCD is 10×10 mm. Other parameters are: Young's modulus $E = 10^7$ N/mm^2, Poisson's ratio $\nu = 0.3$ and prescribed uniaxial stress $\sigma_o = 10^4$ N/mm^2.

The analytical solution for the displacement and stress fields in terms of local polar coordinates in a reference frame (r, θ) centered at the crack tip are:

$$\begin{aligned} u_x\left(r,\theta\right) &= \tfrac{2(1+\nu)}{\sqrt{2\pi}}\tfrac{K_I}{E}\sqrt{r}\cos\tfrac{\theta}{2}\left(2-2\nu-\cos^2\tfrac{\theta}{2}\right) \\ u_y\left(r,\theta\right) &= \tfrac{2(1+\nu)}{\sqrt{2\pi}}\tfrac{K_I}{E}\sqrt{r}\sin\tfrac{\theta}{2}\left(2-2\nu-\cos^2\tfrac{\theta}{2}\right) \end{aligned} \qquad (11)$$

Table 1: Error norms (in percent) of the three models before and after local refinement using NURBS and T-splines.

model	local refined	basis functions	control points	elements	DOFs	error norm (%)	
						L_2	energy
I	no	NURBS	64	25	272	0.1341	2.4945
	yes	NURBS	140	77	500	0.0698	2.1441
	yes	T-spline	112	77	444	0.0706	2.1595
II	no	NURBS	324	225	1044	0.0516	1.6823
	yes	NURBS	680	527	2488	0.0101	0.9486
	yes	T-spline	442	391	1820	0.0101	0.9252
III	no	NURBS	784	625	2592	0.0230	1.2012
	yes	NURBS	1620	1377	5864	0.0039	0.6358
	yes	T-spline	972	891	3848	0.0040	0.6238

$$\begin{aligned} \sigma_{xx}(r,\theta) &= \tfrac{K_I}{\sqrt{2\pi r}}\cos\tfrac{\theta}{2}\left(1-\sin\tfrac{\theta}{2}\sin\tfrac{3\theta}{2}\right) \\ \sigma_{yy}(r,\theta) &= \tfrac{K_I}{\sqrt{2\pi r}}\cos\tfrac{\theta}{2}\left(1+\sin\tfrac{\theta}{2}\sin\tfrac{3\theta}{2}\right) \\ \sigma_{xy}(r,\theta) &= \tfrac{K_I}{\sqrt{2\pi r}}\sin\tfrac{\theta}{2}\cos\tfrac{\theta}{2}\cos\tfrac{3\theta}{2} \end{aligned} \tag{12}$$

where $K_I = \sigma_o\sqrt{\pi a}$ is the mode I stress intensity factor. Analytical displacement field (Eq. 11) is prescribed on the boundaries except for the crack boundary. Unlike homogeneous essential boundary conditions, inhomogeneous boundaries can not be imposed in a straightforward approach in isogeometric analysis; because the non-interpolating natures of NURBS and T-splines do not allow for satisfaction of the kronecker delta property. For imposition of essential boundary conditions, the least-squares minimization method [1] is applied.

Three models with uniformly distributed elements are considered: model I with 5×5 elements, model II with 15×15 elements, and model III with 25×25 elements. For this purpose, the h-refinement (knot insertion) process is utilized. In order to locally refine the mesh around the crack, the elements intersected with the crack are chosen for uniform refinement in a 3×3 mesh. Both NURBS and T-spline basis functions are applied for each model. Mesh and elements for the model III before and after local refinement are displayed in Fig. 4.

The exact L_2 (of displacement) and energy error norms (in percent) of all models are given in Table 1. It is observed that in some cases the models which are locally refined by using T-splines result in even more accurate results than those obtained by using NURBS, although much less number of control points and degrees of freedom are applied.

4.2 Inclined center crack in a square plate under uniaxial tension

Mixed mode stress intensity factors for a square plate with a center inclined crack under remote uniaxial tensile stress (Fig. 5) are investigated. The plate is in plane stress state, with $L = D = 10$ and $2a = 0.7$. Since the plate dimensions are large in comparison to the crack length, the numerical results can be reasonably compared with the analytical solution of infinite plate. For the predefined loading σ_o, the exact mixed mode stress intensity factors are:

$$K_I = \sigma_o\sqrt{\pi a}\cos^2\phi, \quad K_{II} = \sigma_o\sqrt{\pi a}\sin\phi\cos\phi \tag{13}$$

which ϕ is the crack inclination angle with respect to the horizontal line.

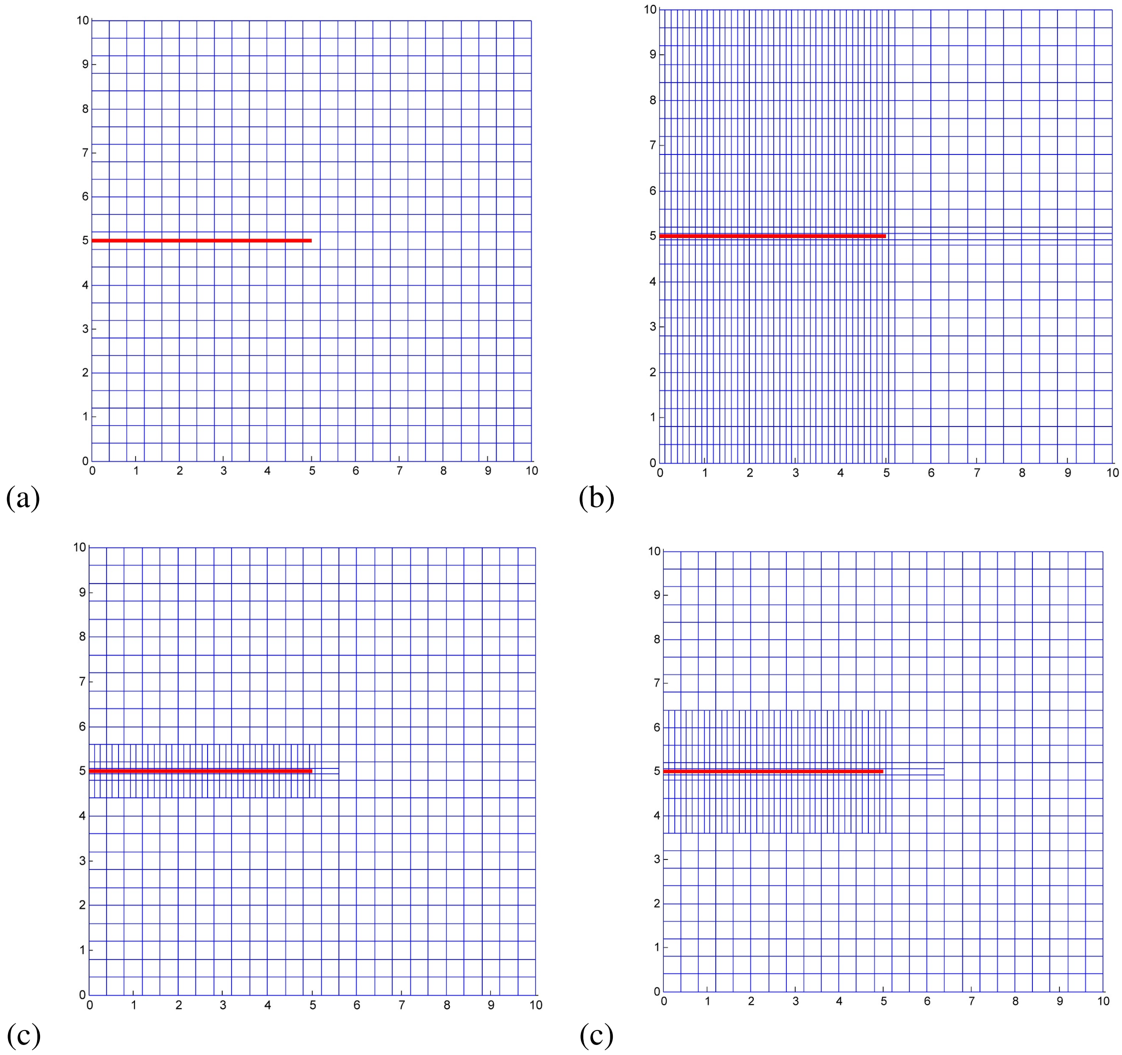


Figure 4: Mesh and elements of the model III before and after local refinement: (a) mesh/elements before local refinement; (b) mesh/elements after local refinement using NURBS; (c) mesh after local refinement using T-splines; (c) elements (for integration) after local refinement using T-splines.

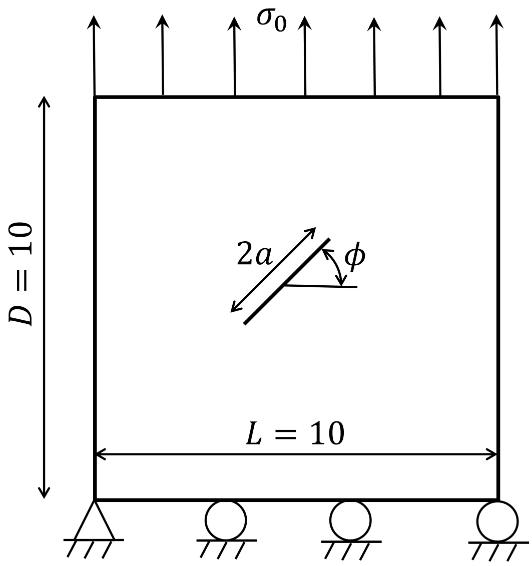


Figure 5: Geometry and loading of a square plate with a center inclined crack.

Table 2: Errors (%) of computed mixed-mode SIFs for different crack inclination angles, ϕ (degree).

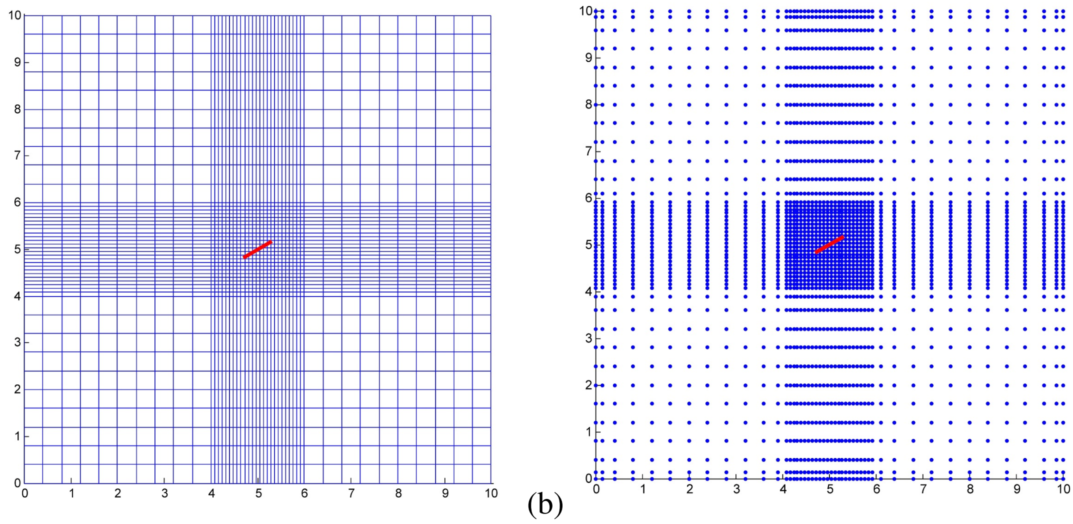
	K_I		K_{II}	
ϕ	NURBS	T-spline	NURBS	T-spline
0	0.4339	0.4339	-	-
15	0.3786	0.3786	0.2912	0.2913
30	0.477	0.477	1.0711	1.07
45	0.4759	0.4759	1.0388	1.0387
60	0.6498	0.6526	1.3027	1.3039
75	0.4746	0.4746	1.1389	1.1389

Since Dirichlet boundary condition is homogeneous in this example, no specific technique is utilized for imposition of essential boundary conditions. For discretizing the model, firstly 25×25 uniformly distributed elements are constructed using the h-refinement, then the elements located in $[4, 6] \times [4, 6]$ are selected for uniform refinement in a 5×5 mesh. Both NURBS and T-spline basis functions are applied for analysis. Model discretizations for models which use NURBS and T-spline basis functions are illustrated in Figs. 6 and 7, respectively.

Different inclination angles have been modeled using the two aforementioned discretizations. It is interesting to note that the both resulted in very similar mixed mode SIFs while 2304 control points and 2025 elements are modeled for the first discretization and 1504 control points and 1465 elements are modeled for the second one. Errors (%) of the computed SIFs are given in Table 2 and the exact and computed normalized mixed mode SIFs are illustrated in Fig. 8. The computed SIFs are close to the exact SIFs.

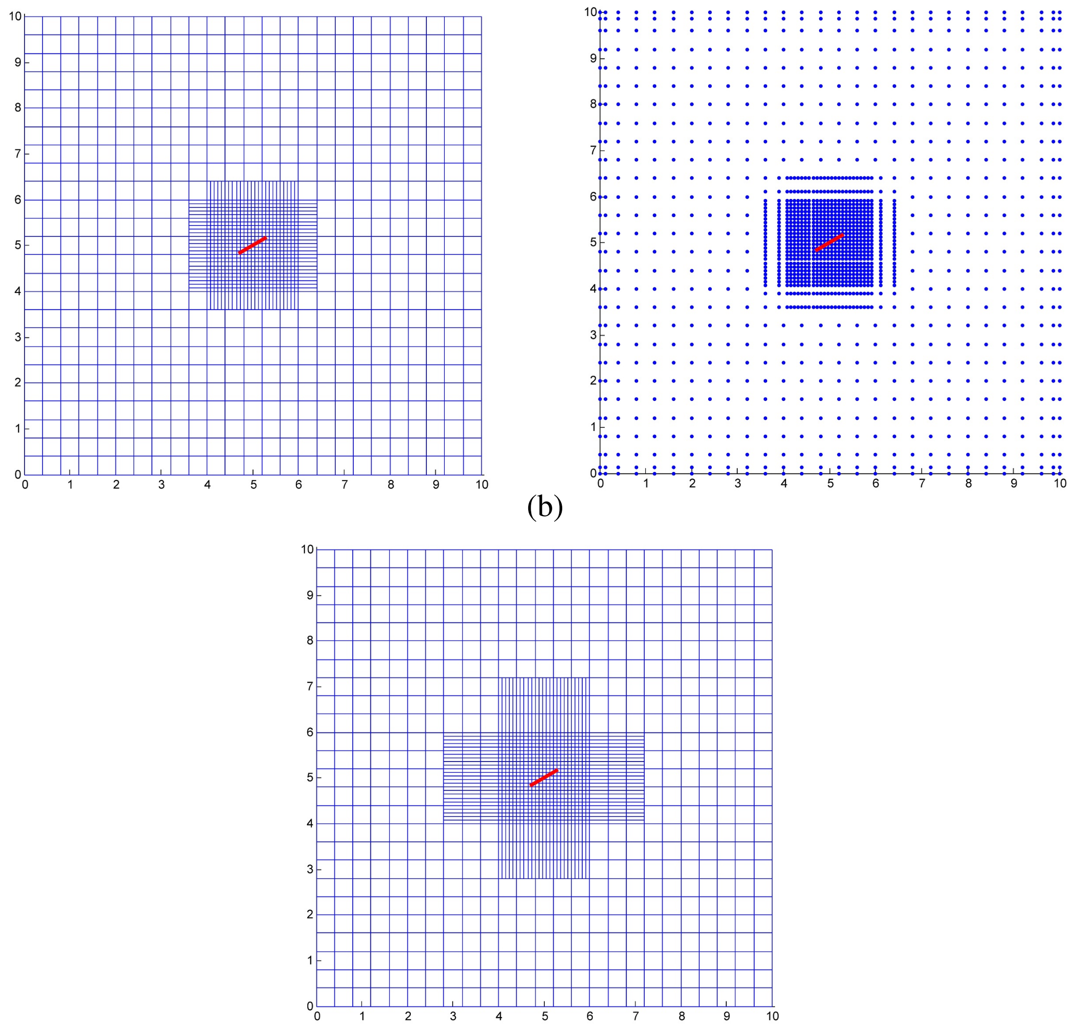
5 CONCLUSION

In this contribution, the XIGA method has been further developed by using the T-spline basis functions. This method is capable of local refinement which is necessary for adaptive procedure.



(a) (b)

Figure 6: Discretization of a square plate with a center inclined crack using NURBS: (a) mesh; (b) control points.



(a) (b) (c)

Figure 7: Discretization of a square plate with a center inclined crack using T-splines: (a) mesh; (b) control points; (c) elements for integration.

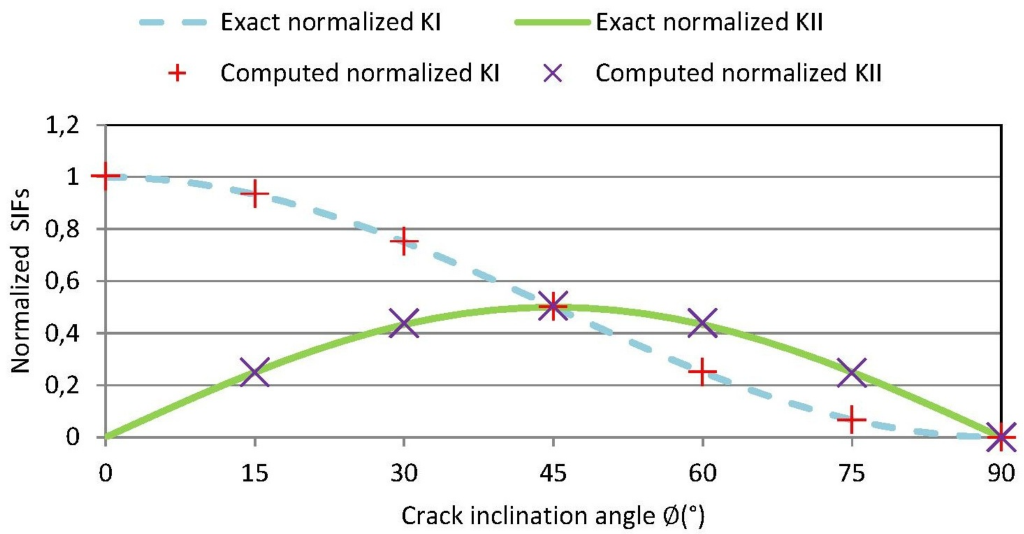


Figure 8: Analytical and computed normalized mixed mode SIFs for several crack inclination angles

Adopting a recovery based error estimator in the proposed approach (which is in progress by the authors), can make XIGA a robust and practical method for fracture analysis of structures.

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