

MODEL COMBINATIONS FOR ASSESSING THE FLUTTER STABILITY OF SUSPENSION BRIDGES

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Abstract. *Long-span cable supported bridges are prone to aerodynamic instabilities caused by wind and this phenomenon is usually a major design criterion. If the wind speed exceeds the critical flutter speed of the bridge, this constitutes an Ultimate Limit State. The prediction of the flutter boundary therefore requires accurate and robust models. This paper aims at studying various combinations of models to predict the flutter phenomenon.*

Since flutter is a coupling of aerodynamic forcing with a structural dynamics problem, different types and classes of models can be combined to study the interaction. Here, both numerical approaches and analytical models are utilised and coupled in different ways to assess the prediction quality of the hybrid model. Models for aerodynamic forces employed are the analytical Theodorsen expressions for the motion-induced aerodynamic forces of a flat plate and Scanlan derivatives as a Meta model. Further, Computational Fluid Dynamics (CFD) simulations using the Vortex Particle Method (VPM) were used to cover numerical models.

The structural representations were dimensionally reduced to two degree of freedom section models calibrated from global models as well as a fully three-dimensional Finite Element (FE) model. A two degree of freedom system was analysed analytically as well as numerically. Generally, all models were able to predict the flutter phenomenon and relatively close agreement was found for the particular bridge. In conclusion, the model choice for a given practical analysis scenario will be discussed in the context of the analysis findings.

1 INTRODUCTION

Long-span bridges are highly flexible, light weight and have low structural damping. They can be subjected to large dynamic motion due to wind actions. The assessment of aerodynamic behaviour, therefore, plays very important role in the design of long-span bridges. For this reason, flutter is also viewed as an essential aeroelastic phenomenon to be studied for these structures. The aeroelastic stability of long-span bridges against flutter is checked by calculating a wind speed at which flutter occurs which is known as the flutter limit. The required aeroelastic properties of the bridge deck section are usually determined in wind tunnel tests.

Scanlan introduced mathematical treatment of flutter in 1960s whereas for the last couple of decades, numerical methods are becoming more popular due to the increasing use of computers in the field of Structural Engineering. In this paper, the study is made on the aerodynamic phenomena, the methods available for flutter analysis for bridges, and to apply the analytical and numerical based analysis on the Lillebælt suspension bridge to calculate ultimately its flutter limit.

2 METHODS OF AERODYNAMIC ANALYSIS

Analytical approaches play very important role in Wind Engineering whereas numerical methods are gaining more importance. All methods apply simplifications to a certain extent such as assuming two-dimensionality of the flow or the shedding process like the wake oscillator model for the case of vortex induced vibration [?]. There are three main types of analysis to deal with the aerodynamic problems:

- Experimental methods
- Analytical methods
- Numerical methods

The last two methods have been used in this study. Analytical solution based on potential flow theory for the motion induced forces on a flat plate exerting sinusoidal heave and pitch motion was given by Theodorsen [?]. In most cases, the empirical models are available which are based on the results of experimental studies. These models are mostly for 2D situations but in reality the 3D effects are present. Also in analytical models, the basic physical causes are attended but the Fluid-structure Interaction is not addressed. The experimental methods are considered relatively accurate compared to the other methods. Wind tunnel testing and full scale models are the examples of experimental methods.

The wind tunnel testing is very expensive for parametric studies but the numerical approach makes it relatively cheaper. With the advancement in the computer modelling and the processing power and by using the principles of CFD, it is now possible to study wind effects on structures in relatively less time. These methods are also efficient, repeatable and economical. Numerical simulations can be used in place of wind tunnel investigation for the fundamental studies. The accuracy of results from these methods not only depends on the quality of the solver but also on the modelling itself. Therefore, the numerical method must be reliable and robust to be used in place of wind tunnel tests.

3 REFERENCE OBJECT

The Lillebælt suspension bridge, Denmark, has been used as a reference object to employ the various model combinations. The data about the bridge is available in [?] and the structural parameters used for this reference object are given in Table ??. For simplicity in the calculation, the railing and other attachments on the deck are not considered in this study.

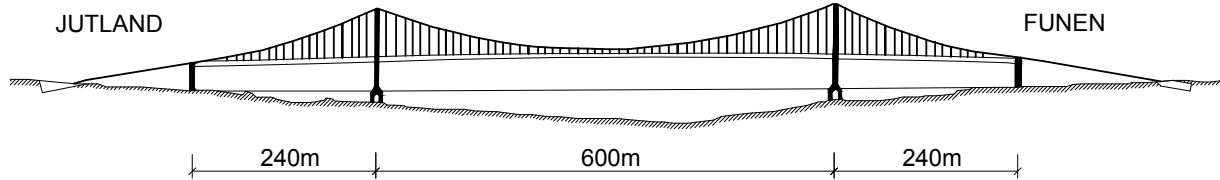


Figure 1: Elevation of the Lillebælt suspension bridge, Denmark used as a reference object in this study.

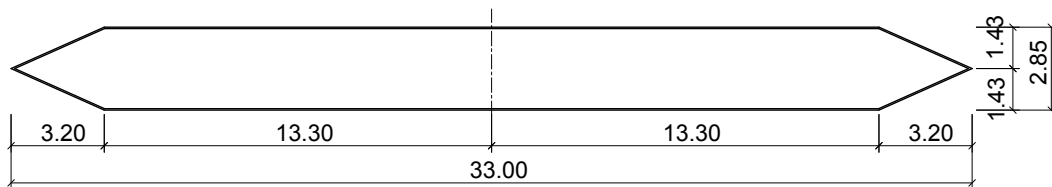


Figure 2: Simplified Lillebælt suspension bridge deck section geometry used in this study (Dimensions: [m]).



Figure 3: Physical model of the Lillebælt suspension bridge.

Table 1: Basic data and structural properties of the Lillebælt suspension bridge section used in this study.

Section width	Mass	Inertial mass	Bending Frequency	Torsional Frequency	Damping ratio
B [m]	m [kg/m]	I [kgm ² /m]	f_h [Hz]	f_α [Hz]	ξ [-]
33	11667	1017778	0.156	0.500	0.01

4 COUPLING OF MODELS

Flutter is a coupling of aerodynamic forcing with a structural dynamics problem. Therefore, different types and classes of models can be combined to study the interaction. In this study, both numerical approaches and analytical models are utilised and coupled in different ways to assess the prediction quality of the hybrid model.

The structural representations were dimensionally reduced to two degree of freedom section models calibrated from global models as well as a fully three-dimensional Finite Element (FE) model. A two degree of freedom system was analysed analytically as well as numerically. The following models were thus derived and analysed: Fully analytical, CFD Derivatives-Analytical and Numerical 2D Structural, CFD Derivatives-Numerical 3D Structural, Fully coupled CFD Numerical 2D Structural. This has allowed to investigate a very broad range of model combinations and to study their merits and drawbacks.

Table 2: Model coupling for the flutter analysis used in this study.

		Aerodynamic		Analytical		Numerical	
				Flat Plate		Flat Plate	Lillebælt Section
Structural							
Analytical		2D	○	●	●	●	
Numerical	VPM (CFD)	2D	-		○	○	
	FE Software	2D	-		●	●	
		3D	-		●	●	

○ Simple/ Regular model

● Meta model

4.1 Analytical Approach (Theodorsen Theory)

Theodorsen investigated the flutter phenomenon for aircraft wings and gave a very popular approach for the flutter analysis. This approach is independent of the shape of the body but on the other hand it neglects the effect originating from the simplification to the flat plate. From the basic principle of potential flow theory, Theodorsen showed that for thin airfoils in incompressible flow, the expressions for lift F_L and moment F_M are linear in displacement h and rotation α and their first and second derivatives [?].

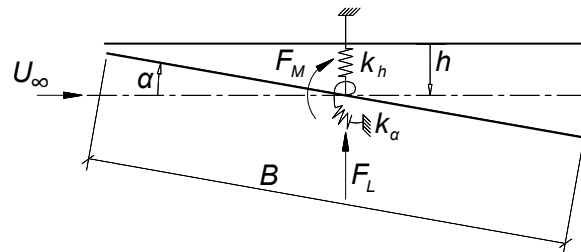


Figure 4: Definition of degrees of freedom (heave h and pitch α) for flutter analysis.

where F_L and F_M are the lift and moment, k_h and k_α are the vertical and rotational spring stiffness and h and α are the vertical displacement and rotation respectively. U_∞ is the oncoming

wind speed. The equations of motion can be written as

$$F_L = m\ddot{h} + 2m\xi_h\omega_h\dot{h} + m\omega_h^2 h \quad (1)$$

$$F_M = I\ddot{\alpha} + 2I\xi_\alpha\omega_\alpha\dot{\alpha} + I\omega_\alpha^2\alpha \quad (2)$$

where ω_h and ω_α are the natural circular frequencies in heave and pitch degree of freedom respectively. The theoretical expressions on a flat plate airfoil for sinusoidal oscillating lift F_L and moment F_M are

$$F_L = -\rho b^2 U_\infty \pi \dot{\alpha} - \rho b^2 \pi \ddot{h} - 2\pi \rho C U_\infty^2 b \alpha - 2\pi \rho C U_\infty b \dot{h} - 2\pi \rho C U_\infty b^2 \frac{1}{2} \dot{\alpha} \quad (3)$$

$$F_M = -\rho b^2 \pi \frac{1}{2} U_\infty b \dot{\alpha} - \rho b^4 \pi \frac{1}{8} \ddot{\alpha} + 2\rho U_\infty b^2 \pi \frac{1}{2} C U_\infty \alpha + 2\rho U_\infty b^2 \pi C \dot{h} + 2\rho \frac{1}{2} U_\infty b^3 \pi C \dot{\alpha} \quad (4)$$

where ρ is the air density, $C(k)$ is the Theodorsens circulation function and $b = B/2$. The system of differential equations (1), (2), (3) and (4) can be written as

$$\begin{bmatrix} \dot{h} \\ \ddot{h} \\ \dot{\alpha} \\ \ddot{\alpha} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} \\ 0 & 0 & 0 & 1 \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} h \\ \dot{h} \\ \alpha \\ \dot{\alpha} \end{bmatrix} \quad (5)$$

This is of the form

$$\dot{X} = AX \quad (6)$$

and assuming the response X is of the form

$$X = \mathbf{R}e^{\lambda t} \quad (7)$$

where \mathbf{R} is real. This simplifies to Eigenvalue problem as follows:

$$[\mathbf{A} - \lambda \mathbf{I}] \mathbf{R}e^{\lambda t} = 0 \quad (8)$$

The solution for $h(t)$ and $\alpha(t)$ is of an exponential form. The Eigenvalues of λ_i of the matrix \mathbf{A} characterize the response of the system as follows:

- Positive real part: Increasing response
- Negative real part: Decaying response
- Imaginary part: Oscillating response

The system will become unstable when an Eigenvalue has a positive real part. When the imaginary part goes towards zero, the oscillatory part vanishes and the phenomenon of static divergence is observed. In this situation, there will be pure heave or pitch motion which can be interpreted as loss of vertical stiffness. The system is solved successively for increasing U_∞ until at least one Eigenvalue becomes positive real. The code for solving the Theodorsens equations was written in Matlab. Using the structural parameters for the Lillebælt bridge (see Table ??), the flutter limit was found 93.8m/s.

4.2 Meta Model (Scanlan Approach)

Scanlan proposed a set of expressions for the aerodynamic forces on a bridge cross section. It assumes that the self-excited lift F_L and moment F_M for a bluff body may be treated as linear in displacement h and rotation α and their first derivatives [?]. Below is commonly used linearised form.

$$F_L = \frac{1}{2}\rho U_\infty^2 B \left[KH_1^* \frac{\dot{h}}{U_\infty} + KH_2^* \frac{B\dot{\alpha}}{U_\infty} + K^2 H_3^* \alpha + K^2 H_4^* \frac{h}{B} \right] \quad (9)$$

$$F_M = \frac{1}{2}\rho U_\infty^2 B^2 \left[KA_1^* \frac{\dot{h}}{U_\infty} + KA_2^* \frac{B\dot{\alpha}}{U_\infty} + K^2 A_3^* \alpha + K^2 A_4^* \frac{h}{B} \right] \quad (10)$$

$$K = \frac{B\omega}{U_\infty} \quad (11)$$

where the non-dimensional coefficients H_i^* and A_i^* are known as aerodynamic or flutter derivatives. The frequency of the bridge oscillation under aerodynamic forcing is known as reduced frequency. As the aerodynamic derivatives are the function of this frequency therefore they can only be measured when the bridge is in the oscillatory state. Normally these are measured in special wind tunnel tests.

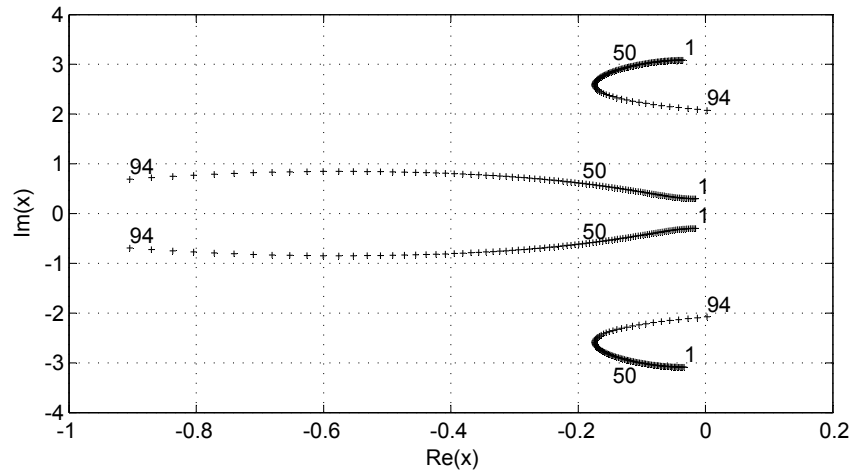


Figure 5: Eigenvalue paths for increasing wind speed U_∞ .

The formulation to the Eigenvalue problem in this case is similar as described in Section ???. The flutter limit is determined when the real part of at least one Eigenvalue becomes positive as shown in Figure ??. The flutter limit was determined as 93.8m/s using the structural parameters of the Lillebælt bridge given in Table ??.

4.3 Forced Vibration Simulation

Forced vibration simulations are used to determine motion-induced forces. The resulting lift and moment time histories are used to compute the aerodynamic derivatives. A computer code, VXFlow, based on VPM has been used here to compute these derivatives. Forced vibration simulations were performed on the Lillebælt section and the flat plate (aspect ratio 1:100) in

sinusoidal heave and pitch motion over a range of reduced frequencies. The reduced frequency is controlled by changing the period of heave and pitch forcing motion.

$$v_r = \frac{2\pi U_\infty}{b\omega} \quad (12)$$

where v_r is reduced frequency. These simulations are performed in heave and pitch motion separately and the aerodynamic derivatives are computed from the resulting force time histories. The simulation gives time histories for $\bar{\mathbf{F}}_L$ and $\bar{\mathbf{F}}_M$ corresponding to known displacement traces \mathbf{h} . Equations (9) and (10) (with $\alpha=0$) thus constitutes a system of equations as

$$\bar{\mathbf{F}}_L = \mathbf{C}_h^L \mathbf{H}_h^* \quad (13)$$

$$\bar{\mathbf{F}}_M = \mathbf{C}_h^M \mathbf{A}_h^* \quad (14)$$

where

$$\mathbf{C}_h^L = \frac{1}{2} \rho U_\infty^2 B K \begin{bmatrix} \frac{\dot{h}}{U_\infty} & K \frac{h}{B} \end{bmatrix} \quad (15)$$

and

$$\mathbf{C}_h^M = B \mathbf{C}_h^L \quad (16)$$

$$\mathbf{H}_h^* = \begin{bmatrix} H_1^* \\ H_4^* \end{bmatrix}, \mathbf{A}_h^* = \begin{bmatrix} A_1^* \\ A_4^* \end{bmatrix} \quad (17)$$

System (13) and (14) can be solved in the least-squares sense by left-multiplying with the \mathbf{C} matrix:

$$\mathbf{C}_h^{LT} \bar{\mathbf{F}}_L = \mathbf{C}_h^{LT} \mathbf{C}_h^L \mathbf{H}_h^* \quad (18)$$

$$\mathbf{C}_h^{MT} \bar{\mathbf{F}}_M = \mathbf{C}_h^{MT} \mathbf{C}_h^M \mathbf{A}_h^* \quad (19)$$

This gives two sets of derivatives in least-square sense. The procedure to calculate these aerodynamic derivatives can be found in [?] and is summarized as follows:

- perform forced vibration tests in either heave or pitch motion
- calculate a best-fit harmonic of the same forcing frequency to obtain lift coefficient and phase shift
- calculate derivatives

The resulting aerodynamic derivatives can be used to calculate flutter limit of the bridge. The forced vibration simulation was performed for the Lillebælt section shown in Figure ?? and the flat plate of the same width (with aspect ratio of 100). For both these cases, structural parameters given in Table ?? were used. The flutter limit for the Lillebælt section was calculated as 94.2m/s and for flat plate as 88.7m/s.

4.4 Fluid-structure Interaction Simulation

VXFlow has been used here for the coupled analysis of the vertical motion and rotation of a two-degree of freedom spring supported section model. The coupling of fluid dynamics solution and the structural dynamics is done at every time step. The pressure on the surface of the body is integrated to get the resultant force in terms of lift and twisting moment. These are associated with the two degrees of freedom of the structural system. The equations of motion for the system are solved by time marching structural dynamics solution. A stiffness matrix is

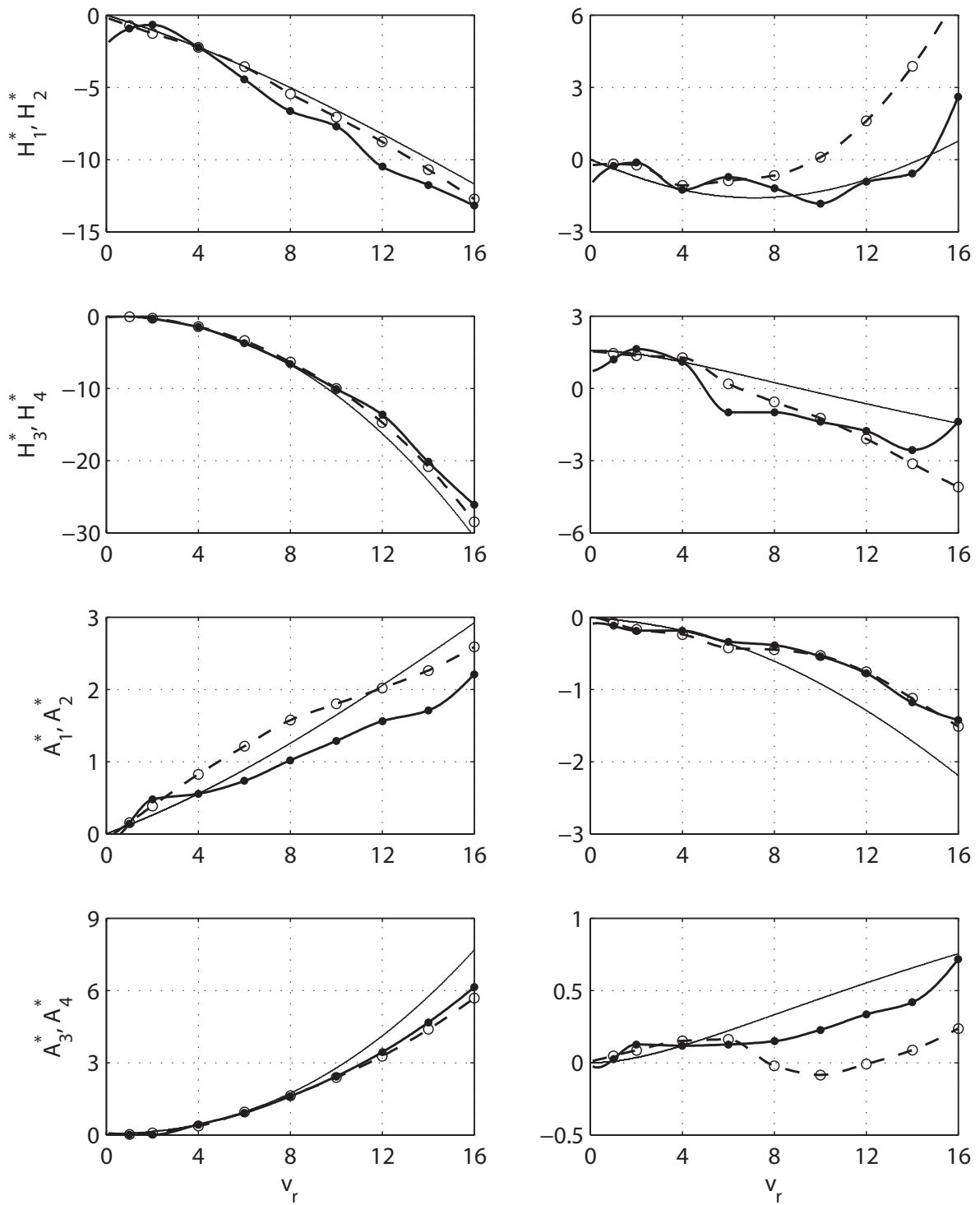


Figure 6: Aerodynamic derivatives (H_i^* and A_i^* where $i=1, 2, 3, 4$) w.r.t. the reduced speed (v_r): flat plate by Theodorsen theory (—), interpolated values from forced vibration analysis on the Liljebælt section (—●—) and the flat plate (- -○- -).

then created and solution is performed. Rayleigh damping is used to model structural damping, for which the damping matrix is proportional to the combination of mass and stiffness matrices.

Structural parameters, given in Table ?? for the Lillebælt suspension bridge, were used for the bridge section (see Figure ??) and the flat plate of aspect ratio 100. The simulations were performed at various wind speed to identify the flutter instability. It was observed that just before the flutter limit, the section goes into the loss of vertical stiffness stage having extreme heave condition. At 95m/s the bridge section becomes unstable after a few hundred time steps. The flutter limit for the plate section was found as 98m/s.

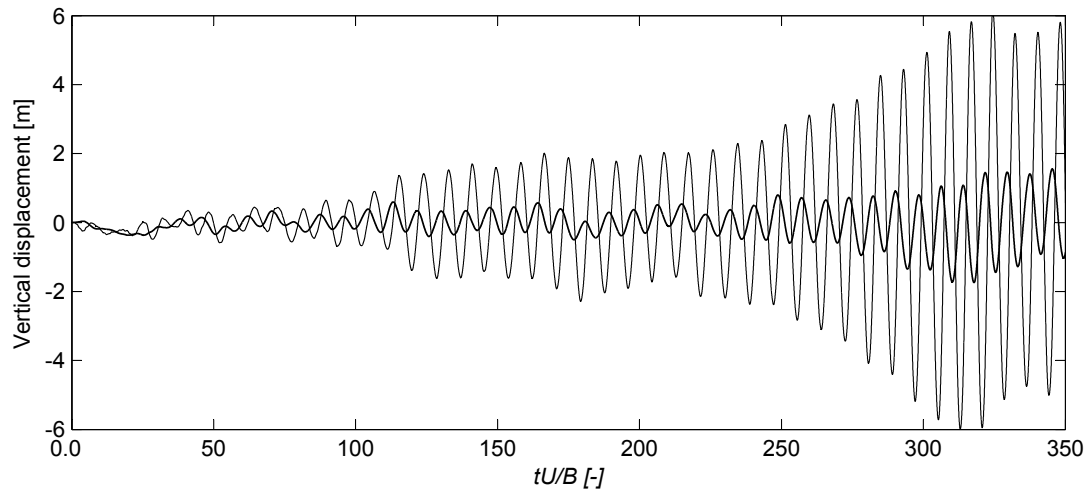


Figure 7: Lillebælt section in Fluid-structure Interaction, displacement time histories at $U_\infty=95\text{m/s}$: leading edge (—), trailing edge (---).

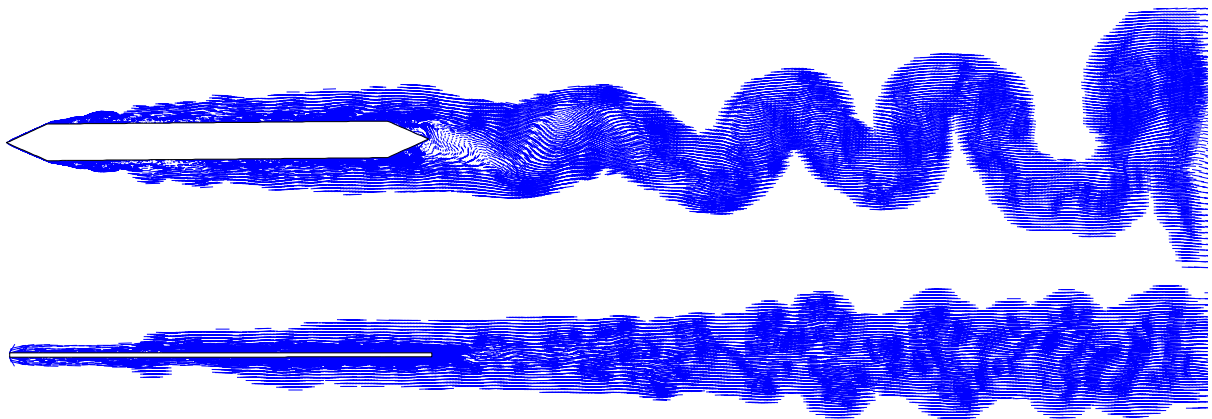


Figure 8: Instantaneous vortex pattern with streak lines for the Lillebælt section (Top) and the flat plate (Bottom).

4.5 Finite Element Model

The section of the Lillebælt suspension bridge deck was modelled as a two degree of freedom beam element of unit length in a Finite Element software. The system was supported on springs with vertical and rotational degrees of freedom. The reduced model was calibrated to represent

the first bending and first torsional mode of the full bridge model. The dimensional reduction to a two-degree of freedom system is a simplification but on the other hand it neglects the effects coming from the higher modes. The aerodynamic derivatives obtained through forced vibration analysis (see Section ??), for the Lillebælt section and the flat plate, were used to calculate the aerodynamic forces. Dynamic wind history analysis was performed on the system. The flutter limit both for the Lillebælt section and the flat plate was found to be 58.5m/s.

A 3D Finite Element model of the Lillebælt suspension bridge was made and calibrated to represent the structural properties of the bridge given in Table ??. In the calibration process some discrepancies were found and the target frequencies were not achieved exactly. The mode shapes of first bending and first torsional mode with their achieved frequencies are shown in Figure ??. The approach from the two degree of freedom model was implemented to the full 3D model of the bridge and the resulting flutter limits were calculated. The effect of higher modes was observed in the deformed model of the bridge at flutter limit. The flutter limit calculated was 49.0m/s both for the Lillebælt section and the flat plate.

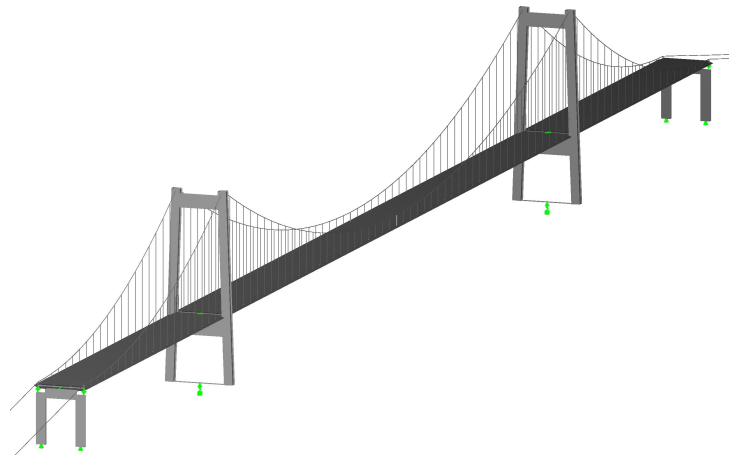


Figure 9: View of full 3D Finite Element bridge model.

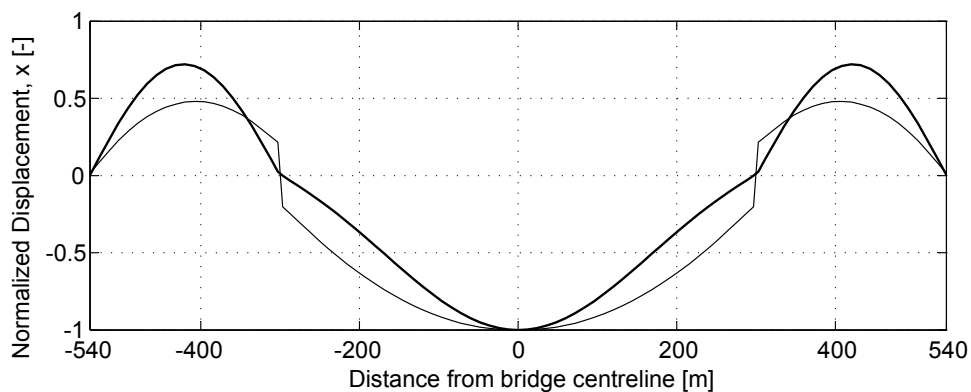


Figure 10: First bending mode (—) (achieved frequency = 0.151 Hz) and first torsional mode (---) (achieved frequency = 0.502 Hz) of full 3D Finite Element bridge model.

5 CONCLUSIONS

The flutter phenomenon was studied employing various combinations of analytical and numerical prediction models. Lillebælt suspension bridge was used as a study object. Generally, all models were able to predict the flutter phenomenon and relatively close agreement was found for the particular bridge. Fully coupled CFD analyses have the advantage that no prior knowledge as to the phenomenon needs to be inserted into the model. Three-dimensional structural representations are superior over dimensionally reduced models in that no prior knowledge as to the modes participating in the flutter coupling is required. Fully analytical models are more direct models and allow a better insight into the force coupling. Simplified aerodynamic models need to be critically assessed with respect to their ability to predict the aerodynamic behaviour of the real cross section. The summary of the results is presented in Table ??.

Table 3: Summary of flutter limits from different model coupling (see Table ??).

		Aerodynamic		Analytical		Numerical	
		Structural		Flat Plate		Flat Plate	Lillebælt Section
Analytical		2D	93.8	93.8	88.7	94.2	
Numerical	VPM (CFD)	2D	-		98.0	95.0	
	FE Software	2D	-		58.5	58.5	
		3D	-		49.0	49.0	

ACKNOWLEDGMENT

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