COMPUTER-AIDED STATIC ANALYSIS OF COMPLEX PRISMATIC ORTHOTROPIC SHELL STRUCTURES BY THE ANALYTICAL FINITE STRIP METHOD

Dr. Eng. Christo T. Christov, Associate Professor,
M. Sc. Eng. Lillya B. Petrova, Chief Assistant,
Higher Military School of Transrort, Sofia, Bulgaria

1. INTRODUCTION

The finite strip method (FSM) is effective for linear analysis of long, single-span thin-walled structures simply supported at their transverse ends. Basic unknowns in FSM with direct determination of the displacements are the three displacements and the rotation of the linear joints. In longitudinal direction unknown quantities and external loads are presented by single Fourier series [1], [2], [3]. In the semi-analytical FSM in transverse direction of the strip an approximation with power polynomials is applied [1]. In the analytical FSM hyperbolic functions are used, presenting an exact solution of the corresponding differential equations of the straight strip [2], [3]. The analytical FSM for linear static analysis of simply supported prismatic isotropic shell structures under arbitrary external loads is developed in [2] and [3].

The present paper describes a development of the analytical FSM in displacements for linear elastic static analysis of simply supported at their transverse ends orthotropic prismatic shell structures with open or closed deformable profile of the cross-section.

2. STRUCTURAL THEORETICAL MODEL AND INITIAL DIFFERENTIAL EQUATIONS

Thin-walled prismatic structures with an arbitrary cross-section profile and with constant wall thickness are studied. The structure is supported at its transverse ends by diaphragms which are assumed as infinitely rigid in their own plane and ideally flexible out it. The material is homogeneous, orthotropic and linear elastic. General external loads are assumed. A great part of the bridge top structures, some roof structures and others are related to the examined class of structures.

By longitudinal sections (linear joints) the prismatic thin-walled structure is discretized to a limited number of plane straight strips (Fig. 1). As basic unknowns are accepted the three displacements of the points from the linear joints and the rotation towards these lines. Each strip from the basic system is simply supported at its transverse ends and fixed at its longitudinal ends. The global co-ordinate system $OX_1X_2X_3$ of the structure and the local co-ordinate system $x_1\,x_2\,x_3$ of a finite strip s with a start joint i and end joint j are shown in Figs. 1 and 2 respectively. The generalized displacements and external loads of each joint from the strip are described to the global co-ordinate system $OX_1\,X_2\,X_3$ to directions with unit vectors $\left\{e^p\right\}(p=1,2,3,4)$ and the local co-ordinate system $x_1\,x_2\,x_3$ to directions with unit vectors $\left\{e_s^p\right\}\ (p=1,2,3,4)$.

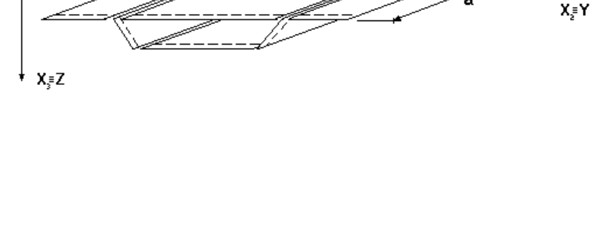


Fig. 1. Discretized structure and a global co-ordinate system

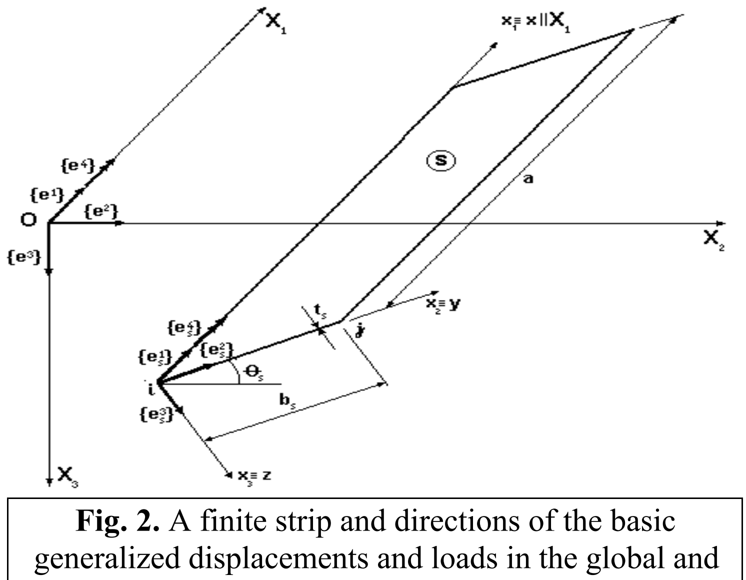


Fig. 2. A finite strip and directions of the basic generalized displacements and loads in the global and local co-ordinate systems

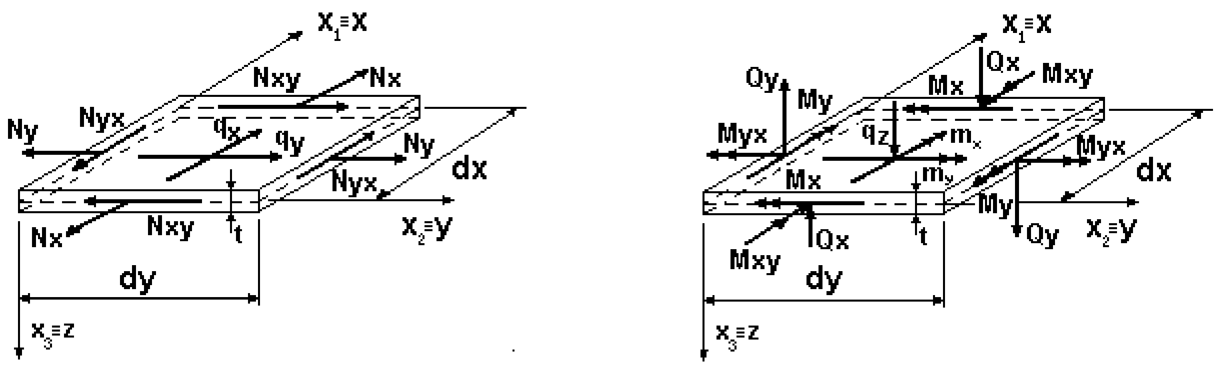
The membrane and bending stressed and

strained states of the strip are described respectively by the following two independent partial differential equations to the local co-ordinate system (Figs. 2,3):

$$\frac{1}{E_y}\frac{\partial^4 F}{\partial x^4}+\left(\frac{1}{G_{xy}}-\frac{\nu_x}{E_x}-\frac{\nu_y}{E_y}\right)\frac{\partial^4 F}{\partial x^2 \partial y^2}+\frac{1}{E_x}\frac{\partial^4 F}{\partial y^4}=0, \quad (1a)$$

$$D_x\frac{\partial^4 w}{\partial x^4}+2D_{xy}\frac{\partial^4 w}{\partial x^2 \partial y^2}+D_y\frac{\partial^4 w}{\partial y^4}=q(x,y), \quad (1b)$$

where $F(x,y)$ is a function of the membrane stresses, $w(x,y)$ is a function of the vertical displacements, $q(x,y)$ is the distributed generalized external normal loads, E_x and E_y are the Young's moduli on directions of axes x and y respectively, $G(x,y)$ is the shear modulus along axes x and y, ν_x and ν_y are the Poisson's ratios on directions of axes x and y respectively and D_x, D_y and D_{xy} are the bending rigidities along axes x and y respectively and the torsion rigidity. The linearly distributed membrane loads q_x and q_y are assumed. The length of the structure is denoted by a. The width and the thickness of the strip are designated by b_s and t_s respectively.



a) Infinitesimal element of shear-wall, membrane forces and loads

b) Infinitesimal element of a plate, bending forces and loads

Fig. 3. External loads, internal forces and a local co-ordinate system of a infinesimal element of the prismatic shell

3. BASIC EQUATIONS AND RELATIONS OF A STRAIGHT ORTHOTROPIC FINITE STRIP

In longitudinal direction the unknown functions of displacements and internal forces as well as the external loads are decomposed into Fourier series as for example:

$$F(x,y)=\sum_{n=1}^{\infty}F_n(y).\sin\alpha_n x, \quad w(x,y)=\sum_{n=1}^{\infty}w_n(y).\sin\alpha_n x, \quad \alpha_n=\frac{n\pi}{a}, \quad (n=1,2,3,\ldots,\infty) \quad (2)$$

The unknown functions $F_n(y)$ and $w_n(y)$ $(n=1,2,3,\ldots,\infty)$ are determined by the condition $F(x,y)$ and $w(x,y)$ to satisfy equations (1a) and (1b) respectively as well as the boundary conditions at the strip longitudinal ends ($y=0$ and $y=b$). For determination of $F_n(y)$ and $w_n(y)$ are obtained two similar fourth-order differential equations with constant coefficients from the following type:

$$Y_n^{IV}(y)-2\gamma_n Y_n^{II}(y)+\delta_n^2 Y_n(y)=f_n(y), \quad (n=1,2,3,\ldots,\infty), \quad (3)$$

where for the membrane state (lower index N) of the strip:

$$2\gamma_{N,n}=E_x\left(\frac{1}{G_{xy}}-\frac{\nu_x}{E_x}-\frac{\nu_y}{E_y}\right)\alpha_n^2, \quad \delta_{N,n}^2=\frac{E_x}{E_y}\alpha_n^4, \quad f_{N,n}(y)=0, \quad (4a)$$

and for the bending state (lower index M) of the strip:

$$2\gamma_{M,n}=\frac{2\alpha_n^2 D_{xy}}{D_y}, \quad \delta_{M,n}^2=\frac{D_x\alpha_n^4}{D_y}, \quad f_{M,n}(y)=\frac{q_n(y)}{D_y}. \quad (4b)$$

Depending on the values of coefficients in equations (3) all cases are studied and three physically possible solutions of the equations are derived.

Finally the analytical solution of equations (1) obtains the following form:

$$F(x,y)=\sum_n(\{C_{N,n}\}^T\{h_n\})\sin\alpha_n x\,,\qquad \alpha_n=\frac{n\pi}{a}, \tag{5a}$$

$$w(x,y)=\sum_n(\{C_{M,n}\}^T\{h_n\}+\{\overline{w_{q,n}}\})\sin\alpha_n x\,,\ (n=1,2,3,\ldots,\infty), \tag{5b}$$

where $\{C_{N,n}\}=[A_{N,n}\,B_{N,n}\,C_{N,n}\,D_{N,n}]^T$ and $\{C_{M,n}\}=[A_{M,n}\,B_{M,n}\,C_{M,n}\,D_{M,n}]^T$ are vectors of the integration constants for the $F_n(y)$ and $w_n(y)$ respectively and $\{h_n\}$ is a vector of the hyperbolic functions which is different for each of the three physically possible cases:

1) When $\gamma_n>0$ and $\delta_n<\gamma_n^2$, $\{h_n\}=[ch\lambda_n y\ \ sh\lambda_n y\ \ ch\nu_n y\ \ sh\nu_n y]^T$, (6a)

where $\lambda_n=\sqrt{\gamma_n+\sqrt{\gamma_n^2-\delta_n^2}}$, $\nu_n=\sqrt{\gamma_n-\sqrt{\gamma_n^2-\delta_n^2}}$;

2) When $\gamma_n>0$ and $\gamma_n^2<\delta_n^2$,

$$\{h_n\}=[\cos\mu_n y\,ch\varepsilon_n y\ \ \sin\mu_n y\,ch\varepsilon_n y\ \ \cos\mu_n y\,sh\varepsilon_n y\ \ \sin\mu_n y\,sh\varepsilon_n y]^T, \tag{6b}$$

where $\varepsilon_n=\frac{\sqrt{2}}{2}\sqrt{\delta_n+\gamma_n}$, $\mu_n=\frac{\sqrt{2}}{2}\sqrt{\delta_n-\gamma_n}$;

3) When $\gamma_n>0$ and $\gamma_n^2=\delta_n^2$, $\{h_n\}=[ch\lambda_n y\ \ sh\lambda_n y\ \ y\,ch\lambda_n y\ \ y\,sh\lambda_n y]^T$, (6c)

where $\lambda_n=\nu_n=\sqrt{\gamma_n}$, $(n=1,2,3,\ldots,\infty)$.

For the assumed linear functions of q_x and q_y, the partial integral $\overline{F_{N,n}}(y)$ of the non-homogeneous differential equation (3) for $F_n(y)$ is equal to zero. The partial integral of the similar equation (3) for $w_n(y)$ is denoted by $\overline{w_n}(y)$.

By the functions F(x,y) and w(x,y) using the relations known in the theory of elasticity, the following expressions for internal forces and displacements in the strip (Fig. 3) are obtained:

membrane state:

$$\{N\}=\begin{Bmatrix}N_x\\N_y\\N_{xy}\end{Bmatrix}=\begin{Bmatrix}\sum_{n=1}^{\infty}\left(\{C_{N,n}\}^T\{h_n^{II}\}\right)\cos\alpha_n x\\ -\sum_{n=1}^{\infty}\left(\{C_{N,n}\}^T\{h_n\}\right)\alpha_n^2\sin\alpha_n x\\ -\sum_{n=1}^{\infty}\left(\{C_{N,n}\}^T\{h_n^{I}\}\right)\alpha_n\cos\alpha_n x\end{Bmatrix}, \tag{7}$$

$$\{\delta\}=\begin{Bmatrix}u\\v\end{Bmatrix}=\begin{Bmatrix}-\sum_{n=1}^{\infty}\left(\{C_{N,n}\}^T\left[\delta_x\frac{1}{\alpha_n}\{h_n^{II}\}+\nu_y\delta_y\alpha_n\{h_n\}\right]\right)\cos\alpha_n x\\ -\sum_{n=1}^{\infty}\left(\{C_{N,n}\}^T\left[\delta_y\alpha_n^2(\int_y\{h_n\}dy+\nu_x\delta_x(\int_y\{h_n^{II}\}dy\right]\right)\sin\alpha_n x\end{Bmatrix}, \tag{8}$$

bending state:

$$\{M\}=\begin{Bmatrix}M_x\\M_y\\M_{xy}\end{Bmatrix}=\begin{Bmatrix}-D_x\sum_{n=1}^{\infty}\left(\{C_{M,n}\}^T\{h_{1n}\}+[\nu_y\overline{W}_{q,n}-\alpha_n^2\overline{W}_{q,n}]\right)\sin\alpha_n x\\ -D_y\sum_{n=1}^{\infty}\left(\{C_{M,n}\}^T\{h_{2n}\}+[-\nu_x\alpha_n^2\overline{W}_{q,n}+\overline{W}_{q,n}^{III}]\right)\sin\alpha_n x\\ -D_{xy}(1-\nu_x\nu_y)\sum_{n=1}^{\infty}\left(\{C_{M,n}\}^T\{h_n^{I}\}+\overline{W}_{q,n}\right)\cos\alpha_n x\end{Bmatrix}, \tag{9}$$

$$
\{Q\}=\begin{Bmatrix} Q_x \\ Q_y \\ Q_x^* \\ Q_y^* \end{Bmatrix}=\begin{Bmatrix}
-D_x\sum_{n=1}^{\infty}\left(\{C_{M,n}\}^T\{h_{3n}\}+\left[-\alpha_n^3\overline{w}_{q,n}+\left(\nu_y+2\frac{G_{xy}}{E_x}(1-\nu_x\nu_y)\right)\alpha_n\overline{w}_{q,n}^{III}\right]\right)\cos\alpha_n x \\
-D_y\sum_{n=1}^{\infty}\left(\{C_{M,n}\}^T\{h_{4n}\}+\left[-\left(\nu_x+2\frac{G_{xy}}{E_y}(1-\nu_x\nu_y)\right)\alpha_n^2\overline{w}_{q,n}^{I}+\overline{w}_{q,n}^{III}\right]\right)\sin\alpha_n x \\
-D_x\sum_{n=1}^{\infty}\left(\{C_{M,n}\}^T\{h_{3n}\}+\left[-\alpha_n^3\overline{w}_{q,n}+\left(\nu_y+2\frac{G_{xy}}{E_x}(1-\nu_x\nu_y)\right)\alpha_n\overline{w}_{q,n}^{II}\right]\right)\cos\alpha_n x-2D_{xy}(1-\nu_x\nu_y)\sum_{n=1}^{\infty}\alpha_n\left(\{C_{M,n}\}^T\{h_n^{II}\}+\overline{w}_{q,n}^{II}\right)\cos\alpha_n x \\
-D_y\sum_{n=1}^{\infty}\left(\{C_{M,n}\}^T\{h_{4n}\}+\left[\overline{w}_{q,n}^{III}-\alpha_n^2\left(\nu_x+2\frac{G_{xy}}{E_y}(1-\nu_x\nu_y)\right)\overline{w}_{q,n}^{I}\right]\right)\sin\alpha_n x+2D_{xy}(1-\nu_x\nu_y)\sum_{n=1}^{\infty}\alpha_n^2\left(\{C_{M,n}\}^T\{h_n^{I}\}+\overline{w}_{q,n}^{I}\right)\sin\alpha_n x
\end{Bmatrix}, \quad (10)
$$

$$
\{\varphi\}=\begin{Bmatrix}\varphi_x \\ \varphi_y\end{Bmatrix}=\begin{Bmatrix}\sum_{n=1}^{\infty}\left(\alpha_n\{C_{M,n}\}^T\{h_n\}+\overline{w}_{q,n}\right)\cos\alpha_n x \\ \sum_{n=1}^{\infty}\left(\{C_{M,n}\}\{h_n^I\}+\overline{w}_{q,n}\right)\sin\alpha_n x\end{Bmatrix}, \quad (11)
$$

where $(...)'=\frac{d(...)}{dx}$, $\delta_x=\frac{1}{hE_x}$, $\delta_y=\frac{1}{hE_y}$, $\{h_{1n}\}=-\nu_y\{h_n^{II}\}-\alpha_n^2\{h_n\}$, $\{h_{2n}\}=-\nu_x\alpha_n^2\{h_n\}+\{h_n^{II}\}$,

$\{h_{3n}\}=-\alpha_n^3\{h_n\}+\left(\nu_y+\frac{2G_{xy}}{E_x}(1-\nu_x\nu_y)\right)\alpha_n\{h_n^{II}\}$, $\{h_{4n}\}=-\left(\nu_x+\frac{2G_{xy}}{E_y}(1-\nu_x\nu_y)\right)\alpha_n^2\{h_n^I\}+\{h_n^{III}\}$.

As degrees of freedom of the finite strip are taken the generalized displacements of its joints in the global co-ordinate system (Fig. 2):

$$
\{Z_s\}=[\{Z_i\}\{Z_j\}]^T=\left[Z_i^1\,Z_i^2\,Z_i^3\,Z_i^4\,Z_j^1\,Z_j^2\,Z_j^3\,Z_j^4\right]^T. \quad (12)
$$

Their relevant generalized joint reactive forces are $\{R_s\}$.

The respective to $\{Z_s\}$ and $\{R_s\}$ vectors in the local co-ordinate system are $\{z_s\}$ and $\{r_s\}$.

The transformation of joint displacements and their corresponding reactions from the global into the local co-ordinate system is done by the expressions:

$$
\{z_s\}=[T_s]\{Z_s\}, \quad (13a) \qquad \{r_s\}=[T_s]\{R_s\}, \quad (13b)
$$

where the transformation matrix for transition has the following form:

$$
[T_s]=\begin{bmatrix}[t_s] & \\ & [t_s]\end{bmatrix}, \quad (14a) \qquad [t_s]=\begin{bmatrix}1 & 0 & 0 & 0 \\ 0 & \cos\theta_s & -\sin\theta_s & 0 \\ 0 & \sin\theta_s & \cos\theta_s & 0 \\ 0 & 0 & 0 & 1\end{bmatrix}, \quad (14b)
$$

The following correspondence exists between the components of vectors $\{r\}$ and $\{z\}$, and displacements and internal forces of the finite strip (Figs. 2,3) :

$$
z^1=u, \qquad z^2=v, \qquad z^3=w, \qquad z^4=\varphi_x, \quad (15)
$$

$$
r_i^1=-N_{xy},\ r_i^2=-N_y,\ r_i^3=-Q_y^*,\ r_i^4=M_y, \quad r_j^1=N_{xy},\ r_j^2=N_y,\ r_j^3=Q_y^*,\ r_j^4=-M_y.
$$

The components of vectors $\{Z_S\}$, $\{z_s\}$ and $\{R_S\}$, $\{r_s\}$ are expanded into Fourier series:

$$
\{Z_s\}^T=\sum_n\{Z_{s,n}\}^T[d_n(x)], \qquad \{z_s\}^T=\sum_n\{z_{s,n}\}^T[d_n(x)], \quad (16)
$$

$$
\{R_s\}^T=\sum_n\{R_{s,n}\}^T[d_n(x)], \qquad \{r_s\}^T=\sum_n\{r_{s,n}\}^T[d_n(x)], \quad (17)
$$

where $\{Z_{s,n}\}$, $\{z_{s,n}\}$, $\{R_{s,n}\}$ and $\{r_{s,n}\}$ are vectors the elements of which are Fourier coefficients and

$$
[d_n(x)]=\mathrm{diag}\left[\cos\alpha_n x\ \ \sin\alpha_n x\ \ \sin\alpha_n x\ \ \sin\alpha_n x\right]. \quad (18)
$$

4. LOCAL RIGIDITY MATRICES OF STRAIGHT ORTHOTROPIC FINITE STRIP

Equalizing the Fourier coefficients of generalized joint reactive forces and the corresponding joint displacements of the strip, the following basic relationship is obtained:

$$
\{r_n\}=[A_n]\{z_n\}, \qquad (n=1,2,3,\ldots,\infty), \quad (19)
$$

where $\{r_n\}$ is a vector of reactions in the strip linear joints, $[A_n]$ is a matrix of the strip rigidity, $\{z_n\}$ is a vector of displacements of the strip joints and

$$\{r_n\}=\left[\{r_{i,n}\}\{r_{j,n}\}\right]^T=\left[r^1_{i,n}\,r^2_{i,n}\,r^3_{i,n}\,r^4_{i,n}\,r^1_{j,n}\,r^2_{j,n}\,r^3_{j,n}\,r^4_{j,n}\right]^T,\ \{z_n\}=\left[\{z_{i,n}\}\{z_{j,n}\}\right]^T=\left[z^1_{i,n}\,z^2_{i,n}\,z^3_{i,n}\,z^4_{i,n}\,z^1_{j,n}\,z^2_{j,n}\,z^3_{j,n}\,z^4_{j,n}\right]^T$$

The rigidity matrix $[A_n]$ has a symmetric block structure:

$$[A_n]=\begin{bmatrix}[A_{ii,n}] & [A_{ij,n}]\\ [A_{ji,n}] & [A_{jj,n}]\end{bmatrix}_{(8x8)},\qquad (n=1,2,3,\ldots,\infty). \tag{20}$$

The separate sub-matrices of $[A_n]$ have the following form:

$$[A_{rr}]=\begin{bmatrix}[A_{N,rr}] & 0\\ 0 & [A_{M,rr}]\end{bmatrix}_{(4x4)},\qquad (r=i,j), \tag{21}$$

.where $$[A_{N,rr}]=\begin{bmatrix}a^{11}_{rr} & a^{12}_{rr}\\ a^{21}_{rr} & a^{22}_{rr}\end{bmatrix},\qquad [A_{M,rr}]=\begin{bmatrix}a^{33}_{rr} & a^{34}_{rr}\\ a^{43}_{rr} & a^{44}_{rr}\end{bmatrix}.$$

The rigidity matrix of the orthotropic finite strip is obtained by application of the principle of single displacements in the way revealed in [2] and [3] for a isotropic strip.

4.1. Membrane state of the strip

By analogy with equation (19) is obtained:

$$\{r_{N,n}\}=[A_{N,n}]\{z_{N,n}\},\qquad (n=1,2,3,\ldots,\infty), \tag{22}$$

where $\{r_{N,n}\}=\left[r^1_{i,n}\,r^2_{i,n}\,r^1_{j,n}\,r^2_{j,n}\right]^T$, $\{z_{N,n}\}=\left[z^1_{i,n}\,z^2_{i,n}\,z^1_{j,n}\,z^2_{j,n}\right]^T$, $[A_{N,n}]=\begin{bmatrix}[A_{N,ii}] & [A_{N,ij}]\\ [A_{N,ji}] & [A_{N,jj}]\end{bmatrix}$.

According to the rigidity matrix definition and from expressions (7) and (15) follows:

$$[A_{N,n}]=[H_{N,n}][\widetilde{C}_{N,n}],\qquad (n=1,2,3,\ldots,\infty), \tag{23}$$

where $$[H_{N,n}]=\begin{Bmatrix}-N_{xy,n}(0)\\ -N_{y,n}(0)\\ +N_{xy,n}(b)\\ +N_{y,n}(b)\end{Bmatrix}=\begin{bmatrix}\alpha_n\{h^I_n(0)\}^T\\ \alpha^2_n\{h_n(0)\}^T\\ -\alpha_n\{h^I_n(b)\}^T\\ -\alpha^2_n\{h_n(b)\}^T\end{bmatrix}, \tag{24}$$

$[\widetilde{C}_{N,n}]$ is a matrix of unknown integration constants generated from unit values of n-order Fourier coefficients for displacements.

By the strip boundary conditions, formulas (8) and (15), replacing consecutively the unit values of displacements $[\Delta_{N,n}]$, for determining the constants $[\widetilde{C}_{N,n}]$ the following matrix equation is derived

$$[H_{\Delta,n}][\widetilde{C}_{N,n}]-[E]=0, \tag{25}$$

where $$[H_{\Delta,n}]=\left[\Delta_{i,n}(0)\ \Delta_{j,n}(b)\right]^T=\left[u_n(x,0)\quad v_n(x,0)\quad u_n(x,b)\quad v_n(x,b)\right]^T, \tag{26}$$

is a matrix of the strip displacements , caused by unit displacements of its joints and $[E]$ is the unit matrix.

Multiplying both sides of (25) from the left and from the right by $[H_{\Delta,n}]^{-1}$ obtains:

$$[\widetilde{C}_{N,n}]=[H_{\Delta,n}]^{-1},\qquad (n=1,2,3,\ldots,\infty). \tag{27}$$

Replacing (27) in (23) follows:

$$[A_{N,n}]=[H_{N,n}][H_{\Delta,n}]^{-1},\qquad (n=1,2,3,\ldots,\infty). \tag{28}$$

4.2. Bending state of the strip

By analogy with equation (19) is derived:

$$\{r_{M,n}\} = [A_{M,n}]\{z_{M,n}\}, \qquad (n = 1,2,3,\ldots,\infty), \tag{29}$$

where $\{r_{M,n}\} = \left[r^3_{i,n}\ r^4_{i,n}\ r^3_{j,n}\ r^4_{j,n}\right]^T$, $\{z_{M,n}\} = \left[z^3_{i,n}\ z^4_{i,n}\ z^3_{j,n}\ z^4_{j,n}\right]^T$, $[A_{M,n}] = \begin{bmatrix} [A_{M,ii}] & [A_{M,ij}] \\ [A_{M,ji}] & [A_{M,jj}] \end{bmatrix}$.

In analogical way to the membrane state for the bending rigidity matrix is get:

$$[A_{M,n}] = [H_{M,n}][H_{w,n}]^{-1}, \qquad (n = 1,2,3,\ldots,\infty). \tag{30}$$

where $[H_{M,n}] = \begin{Bmatrix} M_{y,n}(x,0) \\ Q^*_{y,n}(x,0) \\ M_{y,n}(x,b) \\ Q^*_{y,n}(x,b) \end{Bmatrix} = \begin{bmatrix} \{h_{2n}(0)\}^T \\ \{h_{4n}(0)\}^T \\ \{h_{2n}(b)\}^T \\ \{h_{4n}(b)\}^T \end{bmatrix}$, $[H_{w,n}] = \begin{Bmatrix} w_n(x,0) \\ \varphi_x(x,0) \\ w_n(x,b) \\ \varphi_x(x,b) \end{Bmatrix} = \begin{bmatrix} \{h_n(0)\}^T \\ \{h_n(0)\}^T \\ \{h_n(b)\}^T \\ \{h_n(b)\}^T \end{bmatrix}$. (31)

The blocks of the global rigidity matrix $[B_n]$ of the orthotropic finite strip are determined as:

$$[B_{rr,n}] = [t_s][A_{rr,n}][t_s]^T, \qquad (r = i,j), \qquad (n = 1,2,3,\ldots,\infty). \tag{32}$$

5. SYSTEMS OF LINEAR ALGEBRAIC EQUATIONS AND FINAL FORCES AND DISPLACEMENTS

The system of linear algebraic equations has the known form:

$$[K_n][Z_n] + [R_{fn}] = 0, \qquad (n = 1,2,3,\ldots,\infty). \tag{33}$$

The rigidity matrix of the structure $[K_n]$ and the vector of external loads $\{R_{fn}\}$ are generated by the known way [1], [2], [3]. The matrix $[K_n]$ has a symmetric block structure.

The vector of joint loads is given in the global co-ordinate system by its four components (Fig. 2). It is also can be presented by Fourier series with known coefficients, as for example for a joint k:

$$\{B_{fk}(x)\} = \sum_{n.}\left[B^1_{fk,n}(x)\quad B^2_{fk,n}(x)\quad B^3_{fk,n}(x)\quad B^4_{fk,n}(x)\right]^T.[d_n(x)]. \tag{34}$$

Surface loads on the finite strip are given to the local co-ordinate system by their four components (Fig. 2). Their statically equivalent nodal loads to the same system are the following:

$$\{A_{fs,n}\} = \left[A^1_{fi,n}\quad A^2_{fi,n}\quad A^3_{fi,n}\quad A^4_{fi,n}\quad A^1_{fj,n}\quad A^2_{fj,n}\quad A^3_{fj,n}\quad A^4_{fj,n}\right]^T, \qquad (n = 1,2,3,\ldots,\infty), \tag{35}$$

and with respect to the global co-ordinate system is obtained:

$$\{B_{fn}\} = \left[\{B_{fin}\}\{B_{fjn}\}\right]^T, \quad \{B_{fin}\} = [t_s]^T\{A_{fin}\}, \quad \{B_{fjn}\} = [t_s]^T\{A_{fjn}\}, \quad (n = 1,2,3,\ldots,\infty) \tag{36}$$

The vector of external load $\{R_{fn}\}$ has a block structure. Its elements are gained by the vectors $\{B_{fn}\}$.

The obtained nodal displacements are in the global and transformed in the local co-ordinate system.

For each finite strip s the following can be written:

$$S_s = \sum_n \left(S^0_{s,n} + \{S_{si,n}\}\{z_{in}\} + \{S_{sj,n}\}\{z_{jn}\}\right)\begin{matrix}\sin\alpha_n x \\ \cos\alpha_n x\end{matrix}, \qquad (n = 1,2,3,\ldots,\infty), \tag{37}$$

where $S^0_{s,n}$, $S_{si,n}$ and $S_{sj,n}$ are n-order coefficients of the internal force (displacement) S, caused respectively by the given external load, by generalized displacement $z_{in} = 1$ and by displacement $z_{jn} = 1$.

6. CONCLUSIONS

For long structures FSM is more efficient for linear elastic static analysis than the classic FEM since the problem dimension is reduced by one and the number of unknowns is significantly decreased. The analytical FSM leads to a practically precise solution in comparison with the semi-analytical FSM, especially for wider strips, and provides compatibility of displacements and of forces along the longitudinal joint lines between the adjacent finite strips.

REFFERENCES

1. Cheung, Y.K., Finite Strip Method in Structural Analysis, Pergamon Press, Oxford-New York-Toronto-Sydney-Paris-Frankfurt, 1976.
2. Kolchakow M., Dragolow A., About the Finite Strip Method, Sofia, Construction journal, 1980, vol.9 (in Bulgarian).

3. Dragolow A., Analytical Finite Strip Method for Examination of Prismatic Shell, Sofia, Construction journal, 1996, vol.4 (in Bulgarian).