A SIMPLE FEM BEAM ELEMENT WITH AN ARBITRARY NUMBER OF CRACKS

Matjaz Skrinar
University of Maribor, Faculty of Civil Engineering
Maribor, Slovenia

INTRODUCTION

To fulfil safety requirements the changes in the static and/or dynamic behaviour of the structure must be analysed with great care. These changes are often caused by local reduction of the stiffness of the structure caused by the irregularities in the structure, as for example cracks. The presence of the crack does not necessary mean that the structure will collapse, but it is a signal that the behaviour of the structure should be considered more carefully. An accurate prediction of the initiation of cracks and the subsequent monitoring of their behaviour during the structure response play a significant role in the prediction of the general safety of the structure. In simple structures such analysis can be performed directly, by solving equations of motion, but for more complex structures a different approach, usually numerical, must be applied. The problem of crack implementation into the structure behaviour has been studied by many authors who have usually used either plane 2D finite elements or differential governing equation to analyse the behaviour of the cracked structure.
Analytical expressions between the change in the eigenfrequency and the location and severeness of the damage for structures with rectangular cross-sections can be found in Okamura et al. (1969), Liang et al. (1992) and Krawczuk and Ostachowicz (1993) and expression for circular cross section in Rajab et al. (1991). They modelled the crack as a massless rotational spring of suitable stiffness placed at the beam at the location where the crack occurs.
The changes of vibrational characteristics of beam-like structures with only one transverse part-through surface crack were studied by El-Dannah and Farghaly (1994), and Liang et al. (1992) for various boundary conditions. They have studied differential equation for the transverse vibration of the shaft for obtaining the eigenfrequencies of structure with crack. Their solution consists of finding four boundary conditions that have to be satisfied. Four of the boundary conditions are associated with two ends of the element and other four come from continuity conditions as continuity of deflection, continuity of moments, continuity of shear force and discontinuity of slope across the crack interface between the left and the right part. Using the eight boundary conditions results in eight homogeneous equations, the solution of which exists if the determinant vanishes. Setting the determinant of the system of equations to zero results in the characteristic equations, which is a function of crack depth, crack location on the shaft, shaft natural frequencies, shaft material, shaft length and shaft cross-section geometry.
Although their method can be theoretically straightforwardly extended to arbitrary number of cracks and other boundary conditions for more complex examples it is obvious that such approach is effective only with simple structures. Even replacing symbolic manipulations with numerical calculation does not lead to a solution for general structures.
The aim of this work is to present a generalisation of a simple mathematical FEM based model for transverse motion of a beam with cracks. The numerical procedure for the computation of a beam element with a single transverse crack was first introduced by Gounaris and Dimarogonas (1988). The element stiffness matrix written in fully symbolic form was presented by Skrinar and Umek (1996). Battelino and Skrinar (1995) presented a study where they demonstrated the implementation of such element. In the detailed study a comparison of the results obtained by using 200 2D finite elements with those obtained with a single cracked beam element is presented, confirming the usefulness of such element.
The advantage of this element is that such an element can be easily incorporated into existing software for FEM structural analysis. The element requires two additional types of data: locations of the cracks on the element and depths of cracks.

THEORETICAL MODEL

Figure 1 shows the discrete model for an analysed cracked beam. The local flexibilities introduced by cracks located at the distances $\overline{x}_i$ from the left joint is represented by massless rotational spring, with a constant K_i. Such model, which is relatively simple is widely used in the literature with the governing equation solutions.

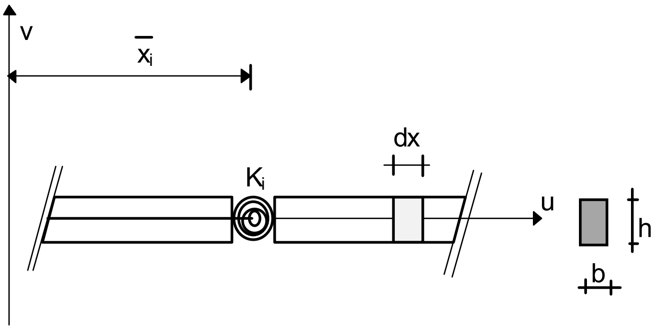

Figure 1: Beam finite element with transverse cracks

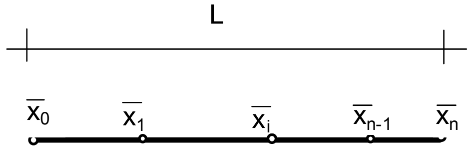

Figure 2: Points and sections

Let n represent the number of uncracked sections of beam between n+1 points (n-1 cracks and 2 nodal points). If we denote the starting point of the beam with $\bar{x}_o$ and the end point of the beam with $\bar{x}_n$ we thus obtain $n+1$ points and n sections between them (Fig. 2).
The transverse displacements can be written as the sum of the displacements over distinct particular sections between two neighbouring cracks. Each part has thus specific displacement function v_i.

DEFINITION OF INTERPOLATION FUNCTIONS FOR TRANSVERSE DISPLACEMENTS

The transverse displacements over the element can be written as:

$$v = \sum_{i=1}^{n} v_i \tag{1}$$

where each displacement function (i=1,2,3,..., n) is defined as vector product of vector of the interpolation functions $f_j^{(i)}$ with the vector of unknown nodal displacements $\mathbf{q}$:

$$\begin{aligned} &v_i(x) = f_1^{(i)} \cdot Y_1 + f_2^{(i)} \cdot \Phi_1 + f_3^{(i)} \cdot Y_2 + f_4^{(i)} \cdot \Phi_2 = \left\{f_1^{(i)}, f_2^{(i)}, f_3^{(i)}, f_4^{(i)},\right\} \cdot \left\{Y_1, \cdot\Phi_1, Y_2, \Phi_2\right\}^T = \mathbf{f}^{(i)} \cdot \mathbf{q}^T \qquad \bar{x}_{i-1} \le x \le \bar{x}_i \\ &v_i(x) = 0 \qquad x < \bar{x}_{i-1} \quad \& \quad \bar{x}_i < x \end{aligned} \tag{2}$$

where the subscripts denote the corresponding section and not the derivative.
Each interpolation function $f_j^{(i)}(x)$ (j=1,2,3,4) in equation (2) is a polynomial of the third degree:

$$f_j^{(i)} = a_j^{(i)} + b_j^{(i)} \cdot x + c_j^{(i)} \cdot x^2 + d_j^{(i)} \cdot x^3 \tag{3}$$

To obtain the interpolation functions completely 16·n unknown coefficients have to be defined. Interpolation functions must satisfy boundary conditions at both ends of the element and continuity conditions at each crack as well, and thus coefficients $a_j^{(i)}$, $b_j^{(i)}$, $c_j^{(i)}$ and $d_j^{(i)}$ (j=1,2,3,4; i=1,...,n) can be obtained.
The following 4 boundary conditions at both ends of the element give us 16 constants:

$$v_1(0) = Y_1 \qquad \varphi_1(0) = \Phi_1 \qquad v_n(L) = Y_2 \qquad \varphi_n(0) = \Phi_2 \tag{4}$$

where $\varphi_i = \dfrac{dv_i}{dx}$.
Remaining (n-1)·4 boundary conditions are defined at n-1 crack locations as continuity of deflection, continuity of moments, continuity of shear force and discontinuity of slope across the crack interface between the left and right part.
These continuity conditions are written as:

$$v_i(\bar{x}_i) = v_{i+1}(\bar{x}_i) \quad \text{the equivalence of displacements} \tag{5a}$$

$$v_i'(\bar{x}_i) + \left(\frac{EI}{K_i}\right) \cdot v_i''(\bar{x}_i) = v_{i+1}'(\bar{x}_i) \quad \text{the difference of slopes} \tag{5b}$$

$$v_i''(\bar{x}_i) = v_{i+1}''(\bar{x}_i) \quad \text{the equivalence of moments} \tag{5c}$$

$$v_i'''(\bar{x}_i) = v_{i+1}'''(\bar{x}_i) \quad \text{the equivalence of shear forces} \tag{5d}$$

where ψ_i is the ratio between the flexural rigidity of the uncracked cross section EI and the compliance or rotational stiffness K_i of the i-th spring (crack) $\dfrac{EI}{K_i} = \psi_i$.
Introduction of equation (2) into continuity conditions (5a-5d) simplifies the determination of some coefficients.
For the equivalence of shear forces (5d) the following expression is obtained:

$$Y_1 \cdot \left(f_1^{(i)}(\bar{x}_i)\right)''' + \Phi_1 \cdot \left(f_2^{(i)}(\bar{x}_i)\right)''' + Y_2 \cdot \left(f_3^{(i)}(\bar{x}_i)\right)''' + \Phi_2 \cdot \left(f_4^{(i)}(\bar{x}_i)\right)''' =$$
$$Y_1 \cdot \left(f_1^{(i+1)}(\bar{x}_i)\right)''' + \Phi_1 \cdot \left(f_2^{(i+1)}(\bar{x}_i)\right)''' + Y_2 \cdot \left(f_3^{(i+1)}(\bar{x}_i)\right)''' + \Phi_2 \cdot \left(f_4^{(i+1)}(\bar{x}_i)\right)''' \tag{6}$$

Equalling the terms that belong to the same degree of freedom we obtain in general notation

$$\left(f_j^{(i)}(\bar{x}_i)\right)''' = \left(f_j^{(i+1)}(\bar{x}_i)\right)''' \tag{7}$$

wherefrom it follows

$$\left(f_j^{(i)}(\bar{x}_i)\right)''' = 6 \cdot d_j^{(i)} = \left(f_j^{(i+1)}(\bar{x}_i)\right)''' = 6 \cdot d_j^{(i+1)} \tag{8}$$

and

$$d_j^{(i)} = d_j^{(i+1)}$$

what proves that coefficients d_j are equal for all segments thus proving the following notation

$$d_j^{(i)} = d_j \,. \tag{9}$$

The request for the equivalence of moments similarly gives:

$$Y_1 \cdot \left(f_1^{(i)}(\bar{x}_i)\right)'' + \Phi_1 \cdot \left(f_2^{(i)}(\bar{x}_i)\right)'' + Y_2 \cdot \left(f_3^{(i)}(\bar{x}_i)\right)'' + \Phi_2 \cdot \left(f_4^{(i)}(\bar{x}_i)\right)'' =$$
$$Y_1 \cdot \left(f_1^{(i+1)}(\bar{x}_i)\right)'' + \Phi_1 \cdot \left(f_2^{(i+1)}(\bar{x}_i)\right)'' + Y_2 \cdot \left(f_3^{(i+1)}(\bar{x}_i)\right)'' + \Phi_2 \cdot \left(f_4^{(i+1)}(\bar{x}_i)\right)'' \tag{10}$$

and equalling again the coefficients that belong to the same degree of freedom we obtain:

$$\left(f_j^{(i)}(\bar{x}_i)\right)'' = \left(f_j^{(i+1)}(\bar{x}_i)\right)'' = 2 \cdot c_j^{(i)} + 6 \cdot d_j \cdot \bar{x}_i = 2 \cdot c_j^{(i+1)} + 6 \cdot d_j \cdot \bar{x}_i \tag{11}$$

$$c_j^{(i)} = c_j^{(i+1)} \rightarrow c_j^{(i)} = c_j \tag{12}$$

The remaining two boundary conditions do not give any further simplification.

The boundary conditions at both ends give following equations:

$$v_1(0) = Y_1 \qquad f_j^{(1)}(0) = 0 \rightarrow a_j^{(1)} = \delta_{j1} \tag{13}$$

$$v_1'(0) = \Phi_1 \qquad \left.\left(f_j^{(1)}(x)\right)'\right|_{x=0} = \delta_{j2} \rightarrow b_1^{(1)} = \delta_{j2} \tag{14}$$

$$v_n(L) = Y_2 \qquad f_j^{(n)}(L) = \delta_{j3} \rightarrow a_j^{(n)} + b_j^{(n)} \cdot L + c_j \cdot L^2 + d_j \cdot L^3 = \delta_{j3} \tag{15}$$

$$v_n'(L) = \Phi_2 \qquad \left.\left(f_j^{(n)}(x)\right)'\right|_{x=L} = \delta_{j4} \rightarrow b_j^{(n)} + 2 \cdot c_j \cdot L + 3 \cdot d_j \cdot L^2 = \delta_{j4} \tag{16}$$

The equations for the unknown coefficient can be thus summarised in a compact form as:

$$\begin{array}{ll}
a_j^{(1)} = \delta_{1j} & \\
b_j^{(1)} = \delta_{2j} & \\
a_j^{(n)} + b_j^{(n)} \cdot L + c_j \cdot L^2 + d_j \cdot L^3 = \delta_{3j} & \qquad j=1,2,3,4 \\
b_j^{(n)} + 2 \cdot c_j \cdot L + 3 \cdot d_j \cdot L^2 = \delta_{4j} & \\
\left.\begin{array}{r} a_j^{(i)} + b_j^{(i)} \cdot \bar{x}_i = a_j^{(i+1)} + b_j^{(i+1)} \cdot \bar{x}_i \\ b_j^{(i)} + \psi_i \cdot \left(2 \cdot c_j + 6 \cdot d_j \cdot \bar{x}_i\right) = b_j^{(i+1)} \end{array}\right\} i = 1,2,..,n-1 &
\end{array} \tag{17}$$

which represents four systems of equations, each of rank (2·n+2) by (2·n+2) with (2·n+2) unknowns. By solving systems of equations (17) unknown coefficients $a_j^{(1)}, a_j^{(2)}, \ldots, a_j^{(n)}, b_j^{(1)}, b_j^{(2)}, \ldots, b_j^{(n)}, c_j, d_j$ for interpolation functions (3) are obtained.

It is instructive to realise that with either n=1 or $\psi_i \equiv 0$ (no cracks) these functions represent well known interpolation functions for an uncracked beam element.

Although equations (17) represent four systems it should be noticed that all four systems differ only in the vectors of unknowns and the right hand vectors. Thus the generalised procedure for the determination of 16·n coefficients transforms into the inversion of (n+1)·2 by (n+1)·2 matrix. The unknown coefficients are found in first 4 columns in the following order: $a_j^{(1)}, b_j^{(1)}, a_j^{(2)}, b_j^{(2)}, \ldots, a_j^{(n)}, b_j^{(n)}, c_j, d_j$ where *j* represents the column number.

STIFFNESS MATRIX

Once the interpolation functions for the displacements are known the deformation energy can be expressed in terms of unknown nodal displacements as :

$$U = \frac{1}{2}\int_{x=0}^{x_1} E\cdot I_z \cdot \left(\frac{\partial^2 v_1}{\partial x^2}\right)^2 dx + \frac{1}{2}\int_{x=x_1}^{x_2} E\cdot I_z \cdot \left(\frac{\partial^2 v_2}{\partial x^2}\right)^2 dx + \ldots + \frac{1}{2}\int_{x=x_{1n-}}^{x_n} E\cdot I_z \cdot \left(\frac{\partial^2 v_n}{\partial x^2}\right)^2 dx + +\frac{1}{2}\sum_{i=1}^{n-1} K_i \cdot \left(\left.\frac{\partial v_i}{\partial x}\right|_{x_i} - \left.\frac{\partial v_{i+1}}{\partial x}\right|_{x_i}\right)^2 \quad (18)$$

where the last term represents the strain energy in the cracks. In general form Equation (18) can be thus rewritten as:

$$U = \frac{1}{2}\sum_{i=1}^{n}\int_{x=x_{i-1}}^{x_i} E\cdot I_z \cdot \left(\frac{\partial^2 v_i}{\partial x^2}\right)^2 \cdot dx + \frac{1}{2}\sum_{i=1}^{n-1} K_i \cdot \left(\left.\frac{\partial v_i}{\partial x}\right|_{x_i} - \left.\frac{\partial v_{i+1}}{\partial x}\right|_{x_i}\right)^2 \quad (19)$$

Introducing equations (3) and (4) into equation (18) the latest becomes:

$$U = \frac{1}{2}\sum_{i=1}^{n}\int_{x=x_{i-1}}^{x_i} E\cdot I_z \, \mathbf{q}^T \cdot \left(\mathbf{N}_i^{''}\right)^T \cdot \left(\mathbf{N}_i^{''}\right) \cdot \mathbf{q} \cdot dx + \frac{1}{2}\sum_{i=1}^{n-1} K_i \cdot \mathbf{q}^T \cdot \left(\mathbf{N}_i^{'} - \mathbf{N}_{i+1}^{'}\right)^T \cdot \left(\mathbf{N}_i^{'} - \mathbf{N}_{i+1}^{'}\right) \cdot \mathbf{q} \quad (20)$$

Where:

$$\mathbf{N}_i = \left\{f_1^{(i)}, f_2^{(i)}, f_3^{(i)}, f_4^{(i)}\right\} \rightarrow \left(\mathbf{N}_i\right)^{'} = \left\{\left(f_1^{(i)}\right)^{'}, \left(f_2^{(i)}\right)^{'}, \left(f_3^{(i)}\right)^{'}, \left(f_4^{(i)}\right)^{'}\right\}$$
$$\left(\mathbf{N}_i\right)^{''} = \left\{\left(f_1^{(i)}\right)^{''}, \left(f_2^{(i)}\right)^{''}, \left(f_3^{(i)}\right)^{''}, \left(f_4^{(i)}\right)^{''}\right\} \quad (21)$$

Considering equation (3) the inner terms become

$$f_j^{(i)} = a_j^{(i)} + b_j^{(i)} \cdot x + c_j \cdot x^2 + d_j \cdot x^3 \rightarrow \left(f_j^{(i)}\right)^{'} = b_j^{(i)} + 2\cdot c_j \cdot x + 3\cdot d_j \cdot x^2$$
$$\left(f_j^{(i)}\right)^{''} = 2\cdot c_j + 6\cdot d_j \cdot x \rightarrow \left(\mathbf{N}_i\right)^{''} = \mathbf{N}^{''} \quad (22)$$

and equation (20) thus becomes:

$$U = \frac{1}{2} E\cdot I_z \cdot \mathbf{q}^T \cdot \int_{x=0}^{L} \left(\mathbf{N}^{''}\right)^T \cdot \left(\mathbf{N}^{''}\right) \cdot dx \cdot \mathbf{q} + \frac{1}{2}\sum_{i=1}^{n-1} K_i \cdot \mathbf{q}^T \cdot \left(\mathbf{N}_i^{'} - \mathbf{N}_{i+1}^{'}\right)^T \cdot \left(\mathbf{N}_i^{'} - \mathbf{N}_{i+1}^{'}\right) \cdot \mathbf{q} \quad (23)$$

Introducing equation (3) into the last term of equation (23) and denoting

$$\Delta\mathbf{B}_i = \left\{b_1^{(i)} - b_1^{(i+1)}, b_2^{(i)} - b_2^{(i+1)}, b_3^{(i)} - b_3^{(i+1)}, b_4^{(i)} - b_4^{(i+1)}\right\}^T \quad (24)$$

equation (23) becomes:

$$U = \frac{1}{2} E\cdot I_z \cdot \mathbf{q}^T \cdot \int_{x=0}^{L} \left(\mathbf{N}^{''}\right)^T \cdot \left(\mathbf{N}^{''}\right) \cdot dx \cdot \mathbf{q} + \frac{1}{2}\sum_{i=1}^{n-1} K_i \cdot \mathbf{q}^T \cdot \Delta\mathbf{B}_i^T \cdot \Delta\mathbf{B}_i \cdot \mathbf{q} \quad (25)$$

and finally:

$$U = \frac{1}{2} \cdot \mathbf{q}^T \cdot \left(E\cdot I_z \cdot \int_{x=0}^{L} \left(\mathbf{N}^{''}\right)^T \cdot \left(\mathbf{N}^{''}\right) \cdot dx + \sum_{i=1}^{n-1} K_i \cdot \Delta\mathbf{B}_i^T \cdot \Delta\mathbf{B}_i \right) \cdot \mathbf{q} \quad (26)$$

The stiffness matrix is thus:

$$\mathbf{K} = E\cdot I_z \cdot \int_{x=0}^{L} \left(\mathbf{N}^{''}\right)^T \cdot \left(\mathbf{N}^{''}\right) \cdot dx + \sum_{i=1}^{n-1} K_i \cdot \Delta\mathbf{B}_i^T \cdot \Delta\mathbf{B}_i \quad (27)$$

Introducing

$$\frac{d^2 f_j^{(i)}}{dx^2} = \frac{d^2}{dx^2}\left(a_j^{(i)} + b_j^{(i)} \cdot x + c_j \cdot x^2 + d_j \cdot x^3\right) = \frac{d}{dx}\left(b_j^{(i)} + 2\cdot c_j \cdot x + 3\cdot d_j \cdot x^2\right) = 2\cdot c_j + 6\cdot d_j \cdot x \quad (28)$$

and rewriting the stiffness matrix as the sum of two matrixes

$$\mathbf{K} = \mathbf{k}_1 + \mathbf{k}_2 \quad (29)$$

The i-th row and j-th column term of first matrix can be computed as:

$$\mathbf{k}_{1(i,j)} = E\cdot I_z\cdot\int_{x=0}^{L}(2\cdot c_i+6\cdot d_i\cdot x)\cdot(2\cdot c_j+6\cdot d_j\cdot x)\cdot dx =$$

$$= E\cdot I_z\cdot\int_{x=0}^{L}\left(4\cdot c_i\cdot c_j+12\cdot c_j\cdot d_i\cdot x+12\cdot c_j\cdot d_i\cdot x+36\cdot d_i\cdot d_j\cdot x^2\right)\cdot dx = \tag{30}$$

$$= E\cdot I_z\cdot\left.\left(4\cdot c_i\cdot c_j\cdot x+12\cdot(c_j\cdot d_i+c_j\cdot d_i)\cdot\frac{x^2}{2}+36\cdot d_i\cdot d_j\cdot\frac{x^3}{3}\right)\right|_0^L =$$

$$= E\cdot I_z\cdot\left(4\cdot c_i\cdot c_j\cdot L+6\cdot(c_j\cdot d_i+c_j\cdot d_i)\cdot L^2+12\cdot d_i\cdot d_j\cdot L^3\right)$$

and the equivalent term for the second matrix is computed as:

$$\mathbf{k}_{2(i,j)} = \sum_{m=1}^{n-1}K_m\cdot\left(b_i^m-b_i^{m+1}\right)\cdot\left(b_j^m-b_j^{m+1}\right) \tag{31}$$

Once again it can be noticed that for the uncracked case ($\mathbf{k}_2=\mathbf{0}$ as $b_i^m=b_i^{m+1}$) the stiffness matrix is reduced into the stiffness matrix for the uncracked beam.

Although the crack locations are not directly presented in Equation (31) they influence the computation of coefficients of the interpolation function and thus indirectly also the stiffness matrix.

LOAD VECTOR

The load vector is computed as

$$\mathbf{q} = \int_0^L p(x)\cdot v(x)\cdot dx = \int_0^L p(x)\cdot\sum_{i=1}^{n}v_i\cdot dx \tag{32}$$

For a uniform load (p=const.) over the whole length of the element we obtain:

$$\mathbf{q} = p\cdot\sum_{i=1}^{n}\int_0^L v_i\cdot dx = p\cdot\sum_{i=1}^{n}\int_{x=x_{i-1}}^{x_i}\left(\left\{f_1^{(i)},f_2^{(i)},f_3^{(i)},f_4^{(i)}\right\}\cdot\begin{Bmatrix}Y_1\\ \Phi_1\\ Y_2\\ \Phi_2\end{Bmatrix}\cdot dx\right) = p\cdot\sum_{i=1}^{n}\int_{x=x_{i-1}}^{x_i}\left\{f_1^{(i)},f_2^{(i)},f_3^{(i)},f_4^{(i)}\right\}\cdot dx\cdot\begin{Bmatrix}Y_1\\ \Phi_1\\ Y_2\\ \Phi_2\end{Bmatrix} \tag{33}$$

where

$$\sum_{i=1}^{n}\int_{x=\bar{x}_{i-1}}^{\bar{x}_i}f_j^{(i)}\cdot dx = \sum_{i=1}^{n}a_j^{(i)}\cdot(\bar{x}_i-\bar{x}_{i-1})+\sum_{i=1}^{n}b_j^{(i)}\cdot\left(\frac{\bar{x}_i^{\,2}}{2}-\frac{\bar{x}_{i-1}^{\,2}}{2}\right)+c_j\cdot\frac{L^3}{3}+d_j\cdot\frac{L^4}{4} \tag{34}$$

COMPUTATION

The numerical procedure can be summarised as follows: from the inversion of equation (17) the coefficients are determined and consequently from equations (30) and (31) the stiffness matrix is obtained. Considering the straightforwardness of the described procedure at one hand and relatively complex symbolic description of the stiffness matrix with a single crack at the other, it is evident that in a general case the stiffness matrix must be obtained numerically. The model used in the derivation of the stiffness matrix and load vector has also more general importance. Namely as with the increase of the crack depth the stiffness of the rotational string is decreasing, the element will in reality fall into two parts in the case when the whole depth of the element is cracked; but the model presented here will describe an element with a perfect hinge placed where the crack is. Therefore is it possible with this procedure to describe an element with a hinge, placed at arbitrary points on the element and not only in the end nodes.

NUMERICAL EXAMPLE

Let us consider the static behaviour of a cantilever beam with following dimension:

length L=2 m, b/h = 0.1/0.2 m, $E=2.1\cdot10^7$ N/m^2, and $\rho=2500$ kg/m^3 with three cracks (crack depth=0.05 m). The cracks are located at distances 0.7 m, 0.9 m and 1.3 m from the fixed end.

The example was computed using 10, 8, 4, 2 and 1 element. Table 1 shows the comparison of vertical displacements and rotations for the point at the free end for various degrees of discretisation.

Table 1: The comparison of vertical displacements and rotations for the point at the free end

point/discret	10 elements	8 elements	4 elements	2 elements	1 element
displacement	-0.784826577	-0.784826579	-0.784826597	-0.784826587	-0.784827283
rotation	-0.537966122	-0.537966118	-0.537966140	-0.537966135	-0.537966801

From the Table 1 it can be noticed the results exhibit very good agreement. Small discrepancy of the results is a clear consequence of numerical error.

CONCLUSIONS

In the paper the expansion of the single cracked finite element to multicracked finite element is briefly presented. Although it is evident that a structure with an arbitrary number of cracks can be adequately described with single cracked elements, yielding also identical results, the usage of this new element is more convenient. Its main advantage against the old element lies in the fact that the initiation of new cracks within one element does not demand new elements, thus preserving the rank of the problem constant. As both elements offer the same results, all conclusions, valid for the single cracked element are substantial also for multicracked finite element.

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