The use of genetic algorithms in finite element model identification

S. Bernstein*, M. Richter**

*Bauhaus-Universität Weimar, Mathematische Optimierung,
Coudraystr. 13B, 99421 Weimar, Germany,
**Technische Universität Chemnitz, Fakultät für Mathematik,
09107 Chemnitz, Germany

Abstract

We present a procedure to identify parameters of structures from given vibration data. A model of the structure is given by finite element method and the parameters are identified by an optimization procedure based on genetic algorithms. Frequency response functions are used for the stopping criterion. The feasibility of the present method is verified through some numerically simulated damage identification tests for a beam and a plate structure.

1 Introduction

A realistic and reliable model is an important precondition for the estimation of the system properties of existing buildings as well as for the simulation of revitalization tasks. Thereby, the main focus lies on the parameter identification, the optimization strategies and the preparation of experiments.

Usually complex structures are modelled using the finite element method. This as well as other techniques are based on idealizations and empiric material properties. Within one theory the parameters of the model should be approximated by gradually performed experiments and their analysis. This approximation method is performed by solving an optimization problem. Thereby non-convex and non-differentiable objective functions in spaces with high dimensions occur [1].

The basic scheme of the identification procedure is shown in Figure 1.

The identification problem involves estimating the physical properties that are used in a finite element model such that some measure of the errors between experimental data and those produced by the mathematical model are minimized. Damage within a structure causes changes in dynamical characteristics such as natural frequencies, mode shapes and modal damping. The close relation between these modal quantities and the frequency response functions enables the possibility of using the frequency response in the objective function of the optimization problem. Thereby, difficulties concerned with a direct identification of modal data are avoided. In contrast to the modal data frequency response functions are directly measured from the structure. Moreover, a large frequency range is considered, which can provide much more information of a damage than modal data (which are extracted only from a small frequency range around resonance).

Iterative procedures for solving this problem fall into two general classes, calculus-based approaches and naive processes. The use of calculus-based approaches depends upon being able to estimate the derivative of the objective function. These procedures also perform hill-climbing. Which is best for unimodal functions. But our problem typically possesses several solutions or near-by solutions that create several (local) minima of the optimization problem.

Naive procedures fall into the following two classes, enumerative and guided random. It is clear that in our case enumerative techniques cannot be used because they need to much processing time. Guided random techniques are generally started with purely random elements. But, during the
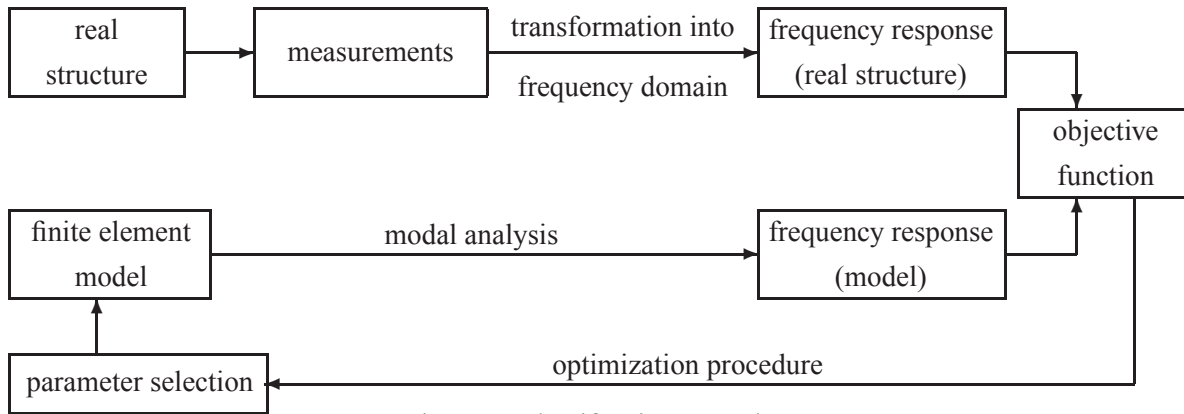
Figure 1: Identification procedure.

process they focus more and more the optimal solutions whilst still employing the use of stochastic elements.

We use genetic algorithms which are a guided random procedure. The striking feature of genetic algorithms is a probabilistic search that has its roots in the principles of genetics.

2 Genetic Algorithms

Genetic algorithms are optimization methods that mimic evolution in nature. That means, natural evolution does not occur by looking ahead and attempting to determine which features will improve the fitness of a species, but rather tries out different features and those which prove beneficial are preferentially selected. This preferential selection is through an increased likelihood of mating, leading to a higher probability that a fit individual's genes will spread throughout the species over subsequent generations.

The beginning of genetic algorithms is credited to John Holland [2], [3], who developed the basic ideas. Figure 2 shows the simplified flow chart of a genetic algorithm (cf. [4]) to illustrate the basic ideas. It should be mentioned that we use greycode which allows a better performance of the genetic algorithm.

The realized modification of the genetic algorithm is described more detailed. In a first step, the necessary parameters of the algorithm (like the mutation and crossover rates, the size of the population as well as the domain and the number of bits for each gene) are defined. The optimization parameters, the so-called genes of each individual are coded sequentially in a chromosome like a designvector.

The initialization of all genes in the population is done randomly. The bit-streams are translated into floats and the corresponding fitness values of the chromosomes are calculated. The rank of every chromosome is determined according to its fitness. The stopping criterion is checked (in the first run of the procedure usually it is not satisfied) and the algorithm turns to the step of selection. Thereby, a fixed percentage of the chromosomes from the population according to their rank is selected to be a part of the set of parents of the next generation. Another part of the chromosomes (those with a medium-grade fitness) has also the opportunity to become parents, while a fixed percentage of the chromosomes is killed.

After the selection the evolution procedure is performed, which consists in a crossover and a mutation step. In order to generate offsprings, at first singlepoint crossover methods are used. Two parents generate two offsprings by passing on parts of their chromosomes. The method exchanges the bits of the chromosome's bitstream beginning at a single random point and store them in the offspring's chromosomes. Instead the singlepoint crossover it is also possible to use n-point or shuffle crossover methods.

$k := 0$
Form $\mathbf{P}(0)$

Evaluate $\mathbf{P}(k)$

Stopping criterion satisfied? — Yes → STOP

No

Fitness

Selection

Evolution
Crossover ← p_c
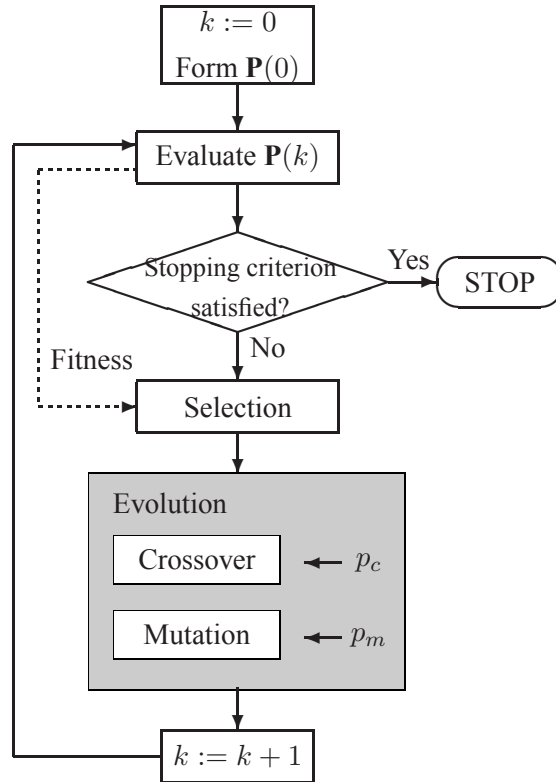Mutation ← p_m

$k := k + 1$

Figure 2: Flow chart of a genetic algorithm.

Mutation is performed simply by switching a fixed (low) percentage of all bits in the population. As mentioned, thereby greycode is used. The greycode transformation copies the first bit of the actual gene, the consecutive bits indicate whether the corresponding bit has changed (1) or not (0) according to the bit before. To give an example, if the normal code of the actual gene looks like 0000011111, the corresponding greycode is 0000010000. The result of a single mutation of bit 5 is 0000110000 in greycode or 0000100000 in normal bit code. The aim of the greycode procedure is in this example obviously, the gene 0000100000 is clearly "near" the gene 0000011111, in contrast to the case of the primitive mutation, which would provide the gene 0000111111.

After evolution, the fitness of all altered individuals is evaluated and the next selection step starts by checking the stopping criterion. Moreover, it should be mentioned, that the algorithm was improved in that way, that it is possible to privilege several elitist genes by preventing them from deletion and mutation.

The selection of the parameters of the genetic algorithm has an important influence on the performance. The main work is done by mutation but if p_m is to big the performance is very slow. Crossover works more or less like hill-climbing. If there is high crossover rate p_c the procedure may end in a local optimum. Therefore it is important to choose an appropriate influence of mutation and crossover.

3 Identification of physical parameters

Normally modal data gives a much more attractive search space for non-linear identification techniques, whereas frequency response data typically results in a search space which leaves traditional non-linear identification procedure very dependent upon good initial estimates (cf. [5]). But, as mentioned in Section 1, modal data has already been subjected to an identification procedure resulting in reduced data which is likely to be biased by the modal identification procedure (cf. [6]). Therefore we will use frequency response data.

The equation of motion for a damped dynamic system using finite element method can be formulated as

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{g}(t).$$

Here $\mathbf{M}, \mathbf{C}, \mathbf{K} \in \mathbb{R}^{n \times n}$ denote the mass, damping and stiffness matrix, whereas $\mathbf{g}(t)$ denotes the load vector. The modal matrix $\mathbf{\Phi} = \{\phi_k\}_{k=1}^n$ consists of the eigenvectors ϕ_k of $\mathbf{K} = \lambda\mathbf{M}$ with respect to the eigenvalues ω_k^2. The transformation $\Phi^\tau \cdot \Phi$ transforms the mass and the stiffness matrix into diagonal matrices $\mathbf{m} = diag\{m_1, \ldots, m_n\}$ and $\mathbf{k} = diag\{m_1\omega_1^2, \ldots, m_n\omega_n^2\}$ respectively. The usual assumption of modal analysis is that the damping matrix $\mathbf{C}$ is also transformed into an diagonal matrix. Let $\mathbf{f}(t) = \Phi^\tau\mathbf{g}(t)$ and $\mathbf{x}(t) = \Phi\mathbf{q}(t)$ we obtain a system of n decoupled equations:

$$\ddot{q}_k(t) + 2\xi_k\omega_k\dot{q}_k(t) + \omega_k^2 q_k(t) = \frac{f_k(t)}{m_k}, \quad k = 1, \ldots, n.$$

Transforming these equations into the frequency domain leads to:

$$\hat{q}_k(i\omega) = h_k(i\omega) \cdot \hat{f}_k(i\omega),$$

where $h_k(i\omega)$ is given by

$$h_k(i\omega) = \frac{1}{m_k} \frac{1}{\omega_k^2 - \omega^2 + 2i\xi_k\omega\omega_k}.$$

Finally, assembling the diagonal matrix

$$\mathbf{h} = diag\{h_1(i\omega), \ldots, h_n(i\omega)\}$$

and forming the matrix $\mathbf{H} = \mathbf{\Phi}\mathbf{h}\mathbf{\Phi}^\tau$ which will be called matrix of frequency response functions we get:

$$\hat{\mathbf{x}}(i\omega) = \mathbf{H}(i\omega)\hat{\mathbf{g}}(i\omega),$$

i.e. the matrix of frequency response functions connects the displacement and load vector in the frequency domain.

It is obvious that by experiments we are only able to determine selected components $H_{kl}(i\omega)$ of the matrix $\mathbf{H}(i\omega)$. For the identification procedure we need such a set of frequency response functions $\{H_{kl}(i\omega), (k, l) \in \mathcal{O} \subset \{1, \ldots, n\} \times \{1, \ldots, n\}\}$, which contains sufficient information. We know (cf. [1]) that passive systems are completely determined by a row or a column of the matrix of frequency response functions. I.e. it is enough to place the load in one node and measure the vibrating response of the system at all other nodes. But on the one hand this is only an upper bound for a successful identification procedure and on the other hand for real life problems it will be simply impossible to measure at all nodes. Simulation results show, that it is possible to use a proper subset for the identification procedure. Therefore the objective function F of the optimization procedure is given by

$$
\begin{aligned}
F(\mathbf{p}) &= \sum_{(k,l)\in\mathcal{O}} \left|\left|H_{kl,meas}(i\omega) - H_{kl,calc}(i\omega, \mathbf{p})\right|\right|^2 \\
&= \sum_{(k,l)\in\mathcal{O}} \left(\left(\Re H_{kl,meas}(i\omega) - \Re H_{kl,calc}(i\omega, \mathbf{p})\right)^2 + \left(\Im H_{kl,meas}(i\omega) - \Im H_{kl,calc}(i\omega, \mathbf{p})\right)^2\right),
\end{aligned}
$$

where $\mathcal{O}$ denotes a proper subset of $\{1, \ldots, n\} \times \{1, \ldots, n\}$ and $\mathbf{p}$ denotes the vector of physical parameters which should be identified.

It is well-known that structural damage leads to a drop of the stiffness of the structure. We would like to model this stiffness reduction as a reduction in Young's modulus of the damaged region. The quantity of the reduction may characterize the state of damage. Therefore, in what follows Young's moduli play an important role in the parameter vector $\mathbf{p}$ which is in the interest of identification. Besides this, in general modal damping coefficients cannot assumed to be known and

therefore might be added to the vector **p**. Questions of minimizing identification errors (caused by experimental measurement errors, errors of the structural model and other influences) have been an important issue in damage identification research (cf. [7]). A further point of interest is the question, how large the range of measured frequencies has to be. Obviously, the most important mode shapes are associated to "lower" frequencies. We should take into consideration all frequencies that are associated to mode shapes we use in our model. Furthermore there is a "natural" upper bound given by the measurement equipment. Using experiences from in-situ measurements (cf. [8]) we choose the range from 0 to 100 Hz. The problem of choosing a sufficient number of frequency response functions in case of a simply supported beam is discussed in [9] and [10]. There is also shown how additional conditions for the objective function may help to overcome the lack of information if there are not enough frequency response functions available. In [9] the procedure is also applied to a real bridge.

4 Numerical Simulation

Our first example is a beam structure shown in Figure 3. The load is placed at node F. As mentioned above, for a successful identification it would be necessary to simulate measurements in all other nodes, which is an impractical situation. We changed the procedure and simulated measurements of the response of the beam structure only in the nodes M1 and M2. Because this is not enough for the identification in general, we modified the objective function and added a condition. This additional term reflects the fact that we expect only a few drops of Young's modulus.
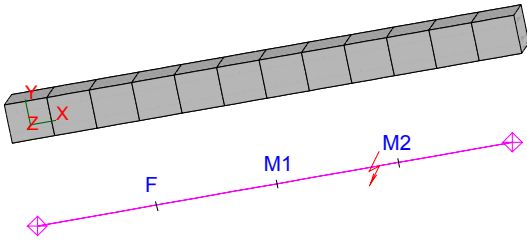

length $[m]$	4
width $[m]$	0.3
height $[m]$	0.4
mass density $\left[kg/m^3\right]$	2500
Poisson's ratio	0
damping ratio	0.02
Young's modulus $\left[N/m^2\right]$	
(excepting 9th element)	$3\cdot10^{10}$
(9th element)	$2\cdot10^{10}$

Figure 3: Beam structure.   Table 1: Physical parameters for the beam.

The additional condition is as follows. Let us denote by E_j Young's modulus of the jth element of our beam structure. By E^m we denote the mean value of the 12 Young's moduli: $E^m := \frac{1}{12}\sum_{j=1}^{12} E_j$, then we minimize the additional term

$$\sum_{j=1}^{12} |E^m - E_j|^2 \to \min. \qquad (*)$$

Thus the fitness of a chromosome increases if the term $(*)$ decreases, such that solutions with only a small deviation from the mean value of Young's modulus are preferred by our procedure. This means in fact that we choose solutions with only a few drops. There is no problem in using the condition $(*)$ if it is well handled. We have to ensure that the main condition for our procedure is given by the frequency response functions. We realize this by multiplying our condition $(*)$ with a suitable small number. From the mathematical point of view we regularize the problem.

5

To design the beam structure the software package PreSlang (cf. [11]) is used. The beam is modelled by RECT-elements which are geometrically linear with 2 nodes on the generalized coordinate finite element formulation. These RECT-elements use linear elastic and bilinear material models. The local element displacements in vertical direction are interpolated with a third-order polynomial function

$$v(x_1, x_2) = \alpha_1 + \alpha_2 x_1 + \alpha_3 x_2 + \alpha_4 x_1^2 + \alpha_5 x_1 x_2 + \alpha_6 x_2^2 + \alpha_7 x_1^3 + \alpha_8 x_1^2 x_2 + \alpha_9 x_1 x_2^2 + \alpha_{10} x_2^3.$$

There are 4 integration points in every cross section and 4 cross sections. Besides this, there are 16 integration points per element. Each node has 3 translation degrees of freedom and 3 rotation degrees of freedom. But for our beam only translations in vertical direction are allowed, all other degrees of freedom are barred. For the reference model, Young's modulus is chosen as the same constant in all elements with exception of the 9th element, where the Young's modulus has been reduced. To state more precisely, the material constants of the reference model are listed in Table 1.

As our second example, we consider the plate structure of Figure 4. The load is placed at node M13 and measurements of the response of the system are simulated at the nodes M1, M2 and M3. The plate is designed once again with aid of the software package PreSLang [11]. Especially Shell6N-elements which are geometrically linear triangular plate and plane stress elements with 6 nodes were employed. Kirchhoff plate theory and linear elastic isotropic and elastoplastic material models are used. 3 translation degrees of freedom and 2 rotation degrees of freedom per node are considered and 26 integration points are situated in 2 layers. The local element displacements are interpolated with a complete fifth-order polynomial containing 21 terms given by

$$u(x_1, x_2) = \alpha_1 + \alpha_2 x_1 + \alpha_3 x_2 + \alpha_4 x_1^2 + \ldots + \alpha_{21} x_2^5.$$

For more details it is referred to the element description in Slang [12].

As mentioned above, simulated data which are obtained from the calculated vibrating response of a reference finite element model of the plate were used. For the reference model, Young's modulus is chosen as the same constant in all elements with the exception of the 4th element, where it has been reduced. The physical parameters and material constants of the reference model are listed in Table 2.
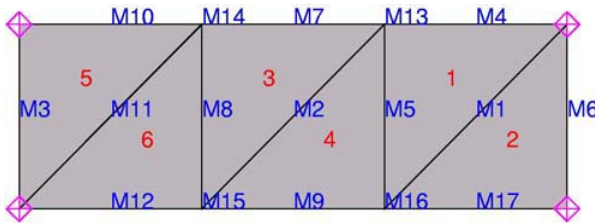

Figure 4: Plate structure.

length $[m]$	9.0
width $[m]$	3.0
thickness $[m]$	0.2
mass density $[kg/m^3]$	2500
Poisson's ratio	0.2
Yield stress	1
damping ratio	0.02
Young's modulus $[N/m^2]$ (excepting 4th element)	$3 \cdot 10^{10}$
(4th element)	$2 \cdot 10^{10}$

Table 2: Physical parameters for the plate.

5 Results

As it can be seen in Figure 5 we are successful in identifying Young's modulus for the beam structure which allows us to conclude in which element a drop of stiffness occurs. A drop of stiffness itself represents a damage of the structure. In Figure 5 the results of 10 runs of the identification procedure are shown and compared with the reference parameters.
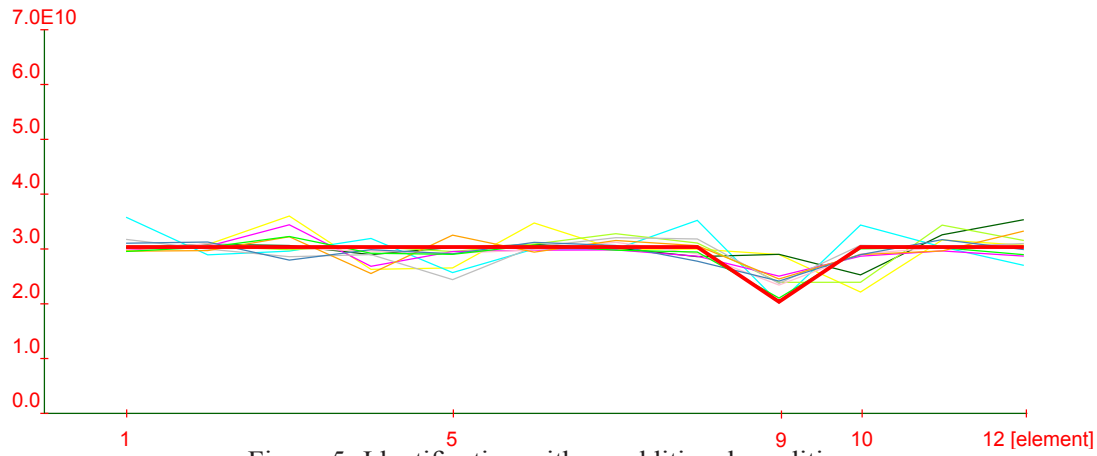
Figure 5: Identification with an additional condition.

But it is also easily seen that there is a "symmetry problem". The procedure is likely to identify the drop in the 4th element which is symmetric to the 9th element with respect to the center of the beam. This has its background in the symmetry of the mode shapes.
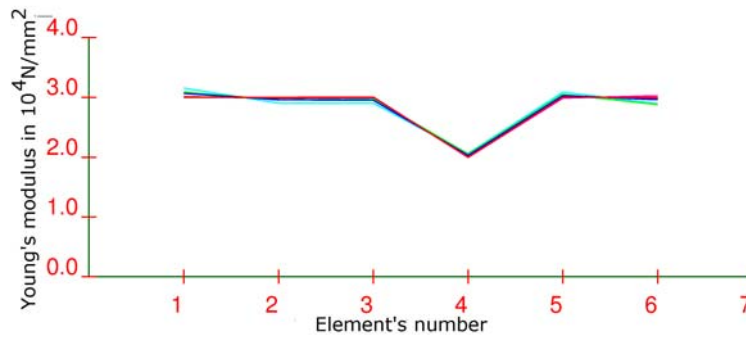

Figure 6: Young's modulus for the elements of the plate.

The identification of Young's modulus was also successful in case of the plate structure. In contrast to the beam an additional condition like (∗) was not necessary. Moreover, for the plate structure there was not a similar phenomenon to the "symmetry problem" for the beam structure. The results for the plate structure are illustrated in Figure 6 and Figure 7. In a first attempt only Young's moduli were identified. In Figure 6 the results of 5 different runs of the identification procedure are shown.

A major disadvantage is that due to the assumption of modal analysis in general unknown damping coefficients are incorporated. We tried to overcome this problem by considering these damping coefficients as unknowns which have to be identified by the procedure too. The result of 5 different runs of the identification procedure to identify 6 Young's moduli and 9 damping coefficients is shown in Figure 7.
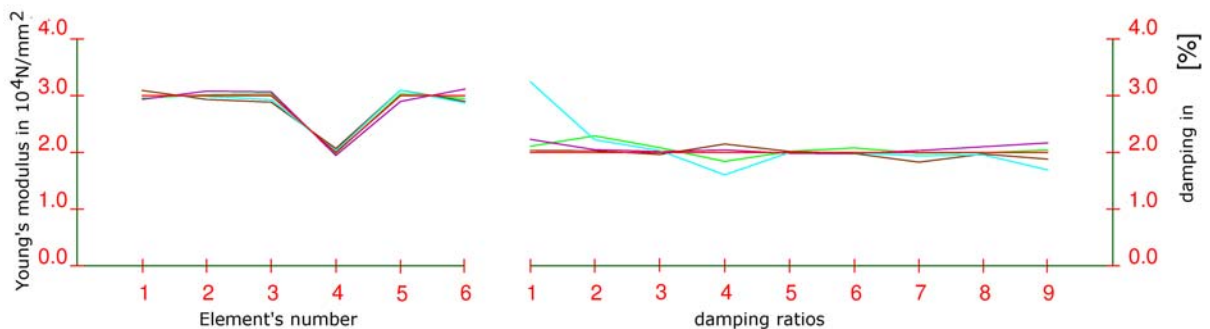

Figure 7: Young's modulus for the elements of the plate and damping ratios associated to 9 mode shapes.

The identification is not that as good as it is in case of Figure 6 for Young's moduli only but very reasonable. The identification of the damping coefficients is less good than of Young's moduli which reflects the higher influence of Young's moduli in the objective function.

6   Conclusions

The paper describes a procedure to locate damage by identifying Young's modulus. In fact a drop in Young's modulus represents a drop in stiffness which means a damage of the structure. The special type of damage cannot be explained by our procedure and must be investigated by other methods. The advantage is that these additional methods need not to investigate the whole structure but only the localized damaged region.

The optimization procedure is based on genetic algorithms which are easy to handle and robust but need also a lot of resources. The objective function of the optimization procedure relies on frequency response function data instead of modal data. By doing so we avoid to identify Young's moduli from data which are not given explicitly and therefore uncertain. To improve the procedure damping coefficients were included into the procedure.

A great deal of further work is still necessary to prove the efficacy of the technique in problems of greater complexity.

7   Acknowledgement

References

[1]   Natke, H.G.: *Einführung in Theorie und Praxis der Zeitreihen- und Modalanalyse.* Vieweg, Wiesbaden, 1992.

[2]   Holland, J.H.: *Genetic algorithms.* Scientific American, July 1992; 9:, 44-50.

[3]   Holland, J.H.: *Adaption in Natural and Artificial Systems: An Introductory Analysis with Applications to Biology, Control and Artificial Intelligence.* Cambridge, MA: The MIT Press, 1992.

[4]   Chong, E.K.P. Chong and Żak, S.H.: *An Introduction to Optimization.* John Wiley & Sons Inc., New York, 1996.

[5]   Dunn, S.A.: *Issues concerning the updating of finite element models from experimental data.* NASA TM 109116, June 1996.

[6]   Dunn, S.A.: *The use of genetic algorithms and stochastic hill-climbing in dynamic finite element model identification.* Computers and Structures, 1998; 66(4): 489-497.

[7]   Lee, U., Shin, J.: *A frequency response function-based structural damage identification method,* Computers and Structures, 2002; 80: 117-132.

[8]   Huth, O., Riedel, J., Bucher, C.: *Finite Element Modell Optimierung einer Stahlbetonbrücke auf der Grundlage von in-situ Experimenten.* Tagungsbericht D-A-C-H Tagung, Berlin, 1999.

[9]   Bernstein, S., Hempel, L., Richter, M. and Riedel, J.: *Genetic Algorithms for the Identification of Structural Parameters.* ECCM-2001 (European Conference on Computational Mechanics) Cracow, Poland, 2001.

[10]   Bernstein, S. and Riedel, J.: *Parameteridentification by genetic algorithms.* Proc. Annual GAMM meeting 2001, Zürich, www.interscience.wiley.com, Proc. Appl. Math. Mech. 1, 2001.

[11]   *PreSLang.* Institute of Structural Mechanics, Bauhaus-University Weimar, Weimar, 2000.

[12]   *Slang, the Structural Language.* Institute of Structural Mechanics, Bauhaus-University Weimar, Weimar, 2000.