# Additional bending moment for shear-lag phenomenon in tube structures

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## Summary

Framed-tube system with multiple internal tubes is analysed using an orthotropic box beam analogy approach in which each tube is individually modelled by a box beam that accounts for the flexural and shear deformations, as well as the shear-lag effects. A simple numerical modeling technique is proposed for estimating the shear-lag phenomenon in tube structures with multiple internal tubes. The proposed method idealizes the framed-tube structures with multiple internal tubes as equivalent multiple tubes, each composed of four equivalent orthotropic plate panels. The numerical analysis is based on the minimum potential energy principle in conjunction with the variational approach. The shear-lag phenomenon of such structures is studied taking into account the additional bending moments in the tubes. A detailed work is carried out through the numerical analysis of the additional bending moment. The moment factor is further introduced to identify the shear lag phenomenon along with the additional moment.

## 1 Introduction

Modern highrise buildings of the framed-tube system exhibit a considerable degree of shear-lag with consequential loss of cantilever efficiency. Despite this drawback, the framed-tube structures are accepted as an economical system capable of maximising the structural efficiency for highrise buildings over a wide range of building heights. In particular, the framed-tube structures with multiple internal tubes, which consist of a smaller size of internal tubes, are widely used due to their high stiffness in resisting lateral loads. In addition, this type of structures shows a reduced shear-lag due to the existence of the internal tubes.

It has been noted that existing analysis models not only ignore the contribution of the internal tubes to the overall lateral stiffness but also neglect the negative shear-lag effects in the tubes. Thus, these models can cater only for the structural analysis of the external tube but fail to consider the shear-lag phenomenon of the internal tubes. For the analysis, the existing methods are not adequate in capturing the true behaviour of such structures.

Note that the existence of the tube-tube interaction coupled with the negative shear-lag in the tubes further complicates the estimation of the structural performance and the accurate analysis of the structures. The additional bending moments due to the tube-tube interaction are considered to be a way of explaining the shear-lag phenomenon of the tube(s)-in-tube structures. However, existing simple analytical methods and existing commercial 3-D frame analysis programs cannot handle the additional bending moments and hence, they can not interpret the cause of the shear-lag phenomenon existing in the tubes. In view of this, the proposed method is made to analyse the additional bending moments.

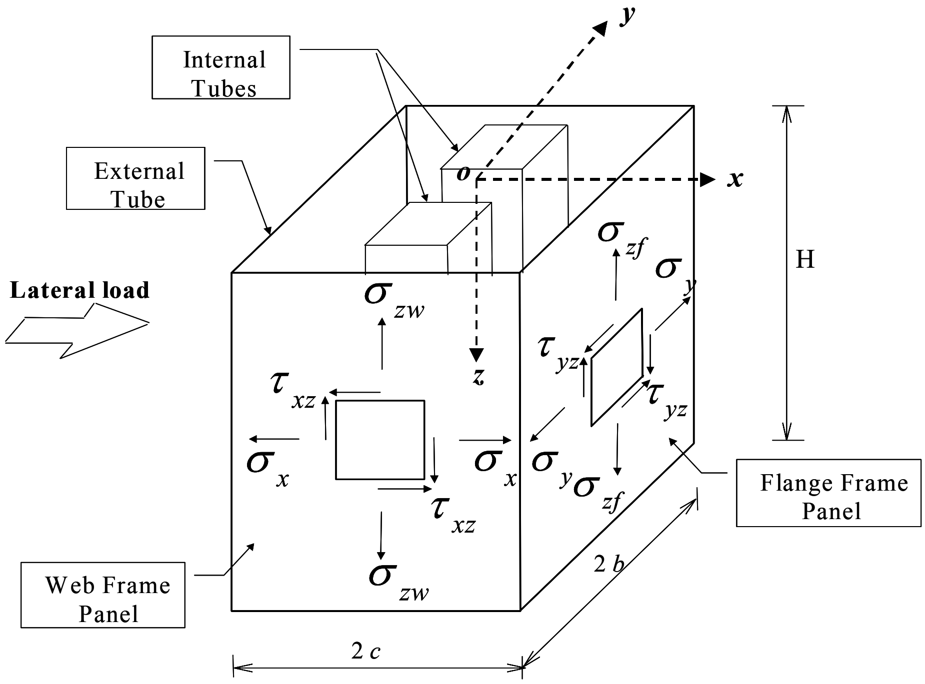
The proposed method takes into account the net shear-lag effects for the additional bending moments in the tubes. As a result, the method is adequate in capturing the true shear behaviour as well as the true bending behaviour of such structures. The numerical analysis so developed is based on the minimum potential energy principal in conjunction with the variational approach.

Three 40-storey framed-tube structures with single, two and three internal tubes are investigated in this study to estimate the shear-lag phenomenon of such structures. To estimate the shear-lag behaviour and its effect on the tube(s)-in-tube structures, the additional bending moments and the shear-lag reversal are identified. The additional bending moment distribution, affected by the tube-tube interaction, is considered to be a structural parameter capable of revealing the shear lag phenomenon in tube(s) in tube structures. The moment factor is further introduced to identify the shear lag phenomenon along with the additional moment.

# 2 Analysis method

## 2.1 Structural modelling

A discrete framed-tube structure with multiple internal tubes (2 in this case) is shown below. The structure is modelled using equivalent multiple tubes, each composed of four equivalent orthotropic plate panels of uniform thickness. Consequently, a framed-tube structure may be analysed as a continuum. The floor slabs in the structure are also considered to be rigid diaphragms within their own plane. Thus, the high in-plane stiffness of the slabs restricts the relative lateral displacements between the multiple tubes at each level.




Equivalent tube structure with multiple (two) internal tubes

The analytical method of the discrete tube structure with multiple internal tubes was proposed previously (Lee, *et al* 2001). The simplicity and accuracy of the proposed method was verified through the comparison of deflection and stress distributions. The stress of each member in the structure is expressed in terms of a family of linear functions of its second moment of area, member property and geometry of the structure.

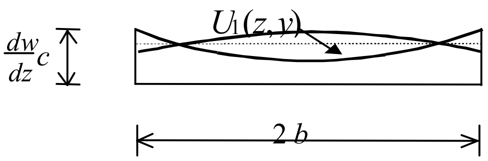
The shape functions are assumed to describe the variation of displacements in flange and web frame panels of each tube. The shape functions can be varied with change of the number of bays and storeys.

## 2.2 Vertical displacement distributions

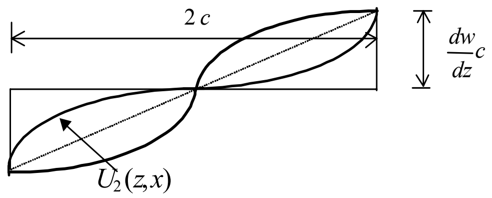
The figures shown below are, respectively, the variations of displacement distributions in the flange and web frame panels. The structure behaves differently from that predicted by the primary bending theory, in that the distribution of stresses in the flange frame panels is not uniform, and that in the web frame panels is nonlinear. This phenomenon is referred to as shear-lag. From the left figure, which shows the distribution of axial stresses across the flange frame, it can be seen that when the degree of the shear-lag varies along the height, the distribution of the axial stresses in the flange frame changes concave or convex. Similarly, from the right figure, which shows the distribution of axial stress across the web frame, it can be also seen that when the degree of the shear-lag varies, the axial stresses near the centre of the web significantly lag down or up in the linear distribution.

For the displacement distributions in the flange frames of tube structures, the shape functions are :

$$U_1(z,y) = c\left[\frac{dw}{dz} + \left(1 - \left(\frac{y}{b}\right)^3\right)u_1(z)\right] \tag{1}$$




. Distribution of vertical displacement in the flange frame panel




Distribution of vertical displacement in the web frame panel

The axial displacement distributions in the web frame panels are :

$$U_2(z,x) = \left[\frac{dw}{dz}x + \left(\frac{x}{c} - \left(\frac{x}{c}\right)^3\right)u_2(z)\right] \tag{2}$$

In equations (1) and (2), $u_1$(z) and $u_2$(z) are shear-lag coefficients of the flange and web frames, respectively, due to the shear deformation, and the expressions can be found elsewhere (Lee, *et al* 2001, 2003).

By the simplified assumptions regarding the patterns of the displacement distributions in external and internal tubes, the complex structural behaviour is reduced to the solution of a single second order linear differential equation. The numerical analysis is based on the minimum potential energy principle in conjunction with the variational approach. This total potential energy must be minimised which can be achieved by using the governing differential equation and the required set of boundary conditions based on the variational approach. The governing differential equations can describe the global behaviour of the framed-tube structures with multiple internal tubes, and those are :

$$\begin{aligned} &\frac{d}{dz}\left[u_1'(z)\alpha_2(z)+w''(z)\ \alpha_1(z)\right]-u_1(z)\alpha_3(z)=0\,;-\frac{d}{dz}\left[EI_e w''(z)-u'(z)_1\alpha_1(z)\right]=P_e(z);\\ &\frac{d}{dz}\left[u_{i1}'(z)\beta_2(z)+w''(z)\beta_1(z)\right]-u_{i1}(z)\beta_3(z)=0;\ -\frac{d}{dz}\left[EI_i w''(z)-u_{i1}'(z)\beta_1(z)\right]=P_i(z)\end{aligned} \tag{3}$$

where $u_1(z)$ and $u_{i1}(z)$ are the undetermined functions including shear-lag coefficients of the external and internal tubes respectively; $P_e$ and $P_i$ are the shear forces in the external and internal tubes respectively; $\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2$ and $\beta_3$ are the constants to be determined.; and $I_e$, $I_i$ and $I$ are second moments of area of the external tube, internal tubes and whole tube(s)-in-tube system respectively.

A pilot study of the proposed displacement functions (Lee, *et al* 2001) indicates that they are adequate to cover the important characteristics of the shear-lag phenomenon in assessing the global behaviour of the tube structures with multiple internal tubes. The anticipated distribution of additional bending moment as affected by the shear-lag in the web and flange panels is presented in the following sections.

## 2.3 Additional bending moments

The structural analysis of the framed-tube structures can be derived from the governing differential equations. Note that the shear-lag phenomenon is due to the distributions of the additional bending moments. The expressions for the additional bending moments are derived from those for the axial bending stresses.

The shear-lag phenomenon of the tube structures can be identified through the additional bending moments due to the tube-tube interaction, which are considered to be a way of explaining the shear-lag phenomenon of the structures. Note again that the existing simple analytical methods and the existing commercial 3-D frame analysis programs cannot handle the additional bending moments and hence, they cannot interpret the cause of the shear-lag phenomenon in the tubes. The additional bending moment distributions are also expressed in terms of a series of linear functions by its second moment of area and the corresponding geometric and material properties. The bending stresses can be written in the following form:

For a tube(s) in tube structure,

$$\sigma_{zf} = -\frac{c}{I}\left[M(z) + M_{fs}(z)\right] \tag{4a}$$

for the external flange frame, where $M_{fs}(z) = -\left(\left(1-\left(\frac{y}{b}\right)^3 + \frac{I_N}{I}\right)IEu'_1(z) + EI_{iN}u'_{i1}(z)\right)$; and

$$\sigma_{zw} = -\frac{x}{I}\left[M(z) + M_{ws}(z)\right] \tag{4b}$$

for the external web frame, where $M_{ws}(z) = -\left(\left(\frac{I_N}{I} - 7.5\frac{b}{c} + 7.5b\frac{x^2}{c^3}\right)IEu'_1(z) + EI_{iN}u'_{i1}(z)\right)$.

The bending stresses for the internal flange frames are

$$\sigma_{zif} = -\frac{c_i}{I_i}\left[M_i(z) + M_{fis}\right] \tag{4c}$$

for a structure with single internal tube, where $M_{fis} = -\left(1-\left(\frac{y}{b_i}\right)^3 + \frac{I_{iN}}{I_i}\right)EI_i u'_{i1}(z)$ ;

$$\sigma_{zif} = -\frac{c_i}{I_i}\left[M_i(z) + M_{fis}\right] \tag{4d}$$

for a structure with an even number of internal tubes, where

$M_{fis} = -\left(1 \pm 8\left(\frac{y-n_i}{2b_i} - \frac{1}{2}\right)^3 + \frac{I_{iN}}{I_i}\right)EI_i u'_{i1}(z)$, in which $n_1 = \frac{(n-1)}{2}a + 2\left(\frac{n}{2}-1\right)b_i$ ,

$n = 2\left(\frac{N}{2}\right)!$ and $N = 2, 4, 6$, etc; and

$$\sigma_{zif} = -\frac{c_i}{I_i}\left[M_i(z) + M_{fis}\right] \quad (4e)$$

for a structure with an odd number of internal tubes.

In equation (4e), $M_{fis} = -\left(1 \pm 8\left(\frac{y-n_i}{2b_i} - \frac{1}{2}\right)^3 + \frac{I_{iN}}{I_i}\right) EI_i u'_{i1}(z)$ where

$n_1 = \frac{(n-1)}{2}a + 2\left(\frac{n}{2} - 1\right)b_i$ , $n = 2\left(\frac{N-1}{2}\right)! + 1$ and $N$ = 3, 5, 7, etc.

Finally for the internal web frame, the bending stress is :

$$\sigma_{ziw} = -\frac{x}{I_i}\left[M_i(z) + M_{wis}\right] \quad (4f)$$

where $M_{wis} = 7.5\frac{b_i}{c_i} - 7.5b_i\frac{x^2}{c^3} - \frac{I_{iN}}{I}$ .

In equations (4), $M(z)$ and $M_i(z)$ are respectively the external and internal tube bending moments computed by the elementary bending theory; $M_{fs}(z)$ and $M_{fis}(z)$ are the additional bending moments in the external and internal flange frames, respectively; and $M_{ws}(z)$ and $M_{wis}(z)$ are the additional bending moments in the external and internal web frames, respectively.

At the intersection between the flange and web frames, the axial stresses are :

$$\sigma_e = \sigma_{zf(y=b)} = -\frac{c}{I}\left(M(z) + M_{fs}(z)_{(y=b)}\right) \quad (5)$$

for the external frame, where $M_{fs} = -E\left(I_N u'_1(z) + I_{iN} u'_{i1}(z)\right)$; and

$$\sigma_i = \sigma_{zif(y=b_i \text{ or } a/2)} = -\frac{c_i}{I_i}\left(M_i(z) + M_{fis}(z)_{(y=b \text{ or } a/2)}\right) \quad (6)$$

for the internal frames, where $M_{fis} = -EI_{iN} u'_{i1}(z)$ .

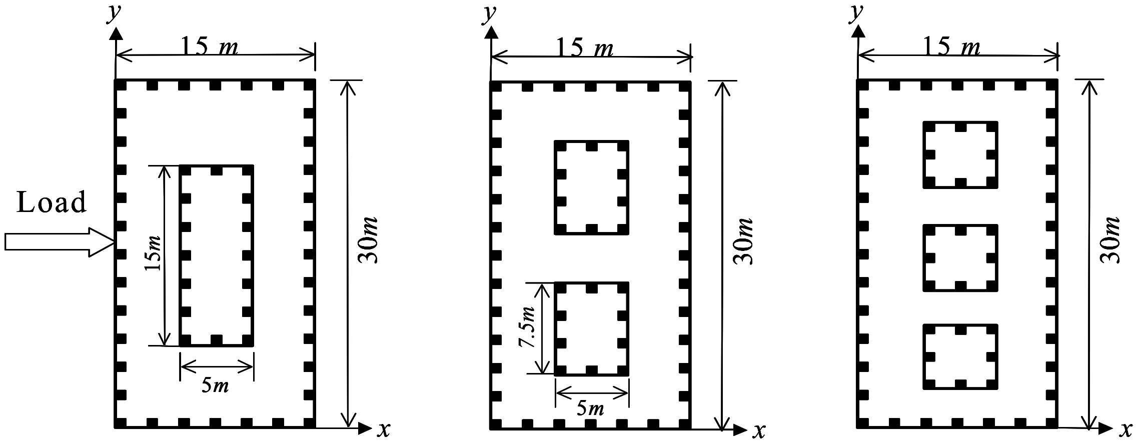
In equations (5) and (6), $M_{fs(y=b)}$ and $M_{fis(y=b_i \text{ or } a/2)}$ are respectively the additional bending moments in the corner columns of the external and internal flange frames due to the shear-lag effect. If the additional bending moment ($M_{fs}$) has the same sign as the external tube bending moment, then $\sigma_e$ given by equation (5) is larger than that obtained by the elementary bending theory. This is due to the effect of the positive shear-lag. In a reverse case, however, it is very difficult to predict whether $\sigma_e$ is larger or smaller than that computed by the elementary bending theory. This is due to the effect of the negative shear-lag. The magnitude of this effect depends upon the ratio of $M_e$ (total bending moment) to $M$ (the elementary bending moment). Thus the additional bending moment, $M_{fs}$, plays an important role in representing the effect of the positive and negative shear-lag. A similar procedure can also be applied to evaluate the additional bending moments in the internal flange frames.

## 3 Comparison of analysis results

A series of tube structure with multiple internal tubes subjected to lateral loading is analysed to verify the simplicity and accuracy of the proposed method in the additional bending moment distributions. Three 40-story tube(s) in tube structures consisting of horizontal beams and vertical columns are analysed using the proposed method.

Each building has a 3.0m story height, 2.5m centre-to-centre column spacing and a uniformly distributed lateral load along the entire height of the structure. The cross-sectional area of all the columns and beams in the external tube of the example structures is taken to be 0.64 $m^2$, and

Young's modulus E and shear modulus G are equal to $2.06\times10^{10}\ N/m^2$ and $0.824\times10^{10}\ N/m^2$, respectively. The second moment of area of the internal tube of each the example structures is taken to be 90 $m^4$. In order to consider the critical case of the structures, a uniformly distributed lateral load of 88.24*KN/m* is assumed to be applied to long side frame panel(flange frame panel) parallel to y-axis as below.

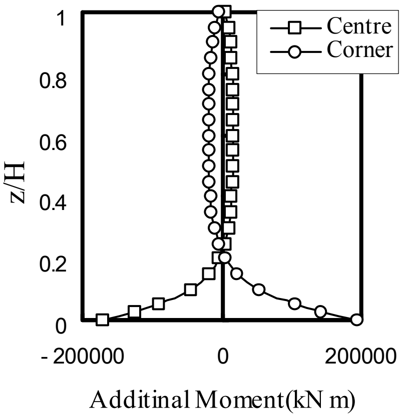



(a) Tube in tube structure (b) 2 tubes in-tube structure (c) 3 tubes in tube structure

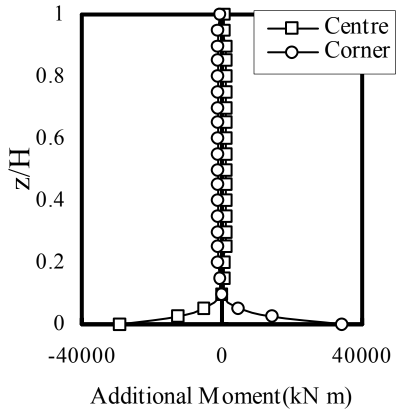
Plans of the three tube(s) in tube structures

Shown below are the distributions of the additional bending moments in the center and corner columns of the flange frames for the three tube structures. To investigate the shear-lag reversal points and the shear-lag distribution due to the tube-tube interaction, the additional bending moments in the center and corner columns of the flange frames are plotted against *z*/*H*. It is found that as the second moments of area of the internal tubes are identical, an increasing number of internal tubes results in a gradually decreasing increment in the additional bending moments from center to corner columns in the internal tubes. Hence, a reduction occurs in the additional bending moments between center and corner columns. As a result, the shear-lag is also reduced. However, the internal tubes with the same second moment of area have little effect on the additional bending moments in the external tube. It is further observed, in the external tubes, that the effect of the positive shear-lag is greater at the bottom of the structures, whereas the negative shear-lag occurs at around 1/4 of the building height. The points of shear-lag reversal for the internal tubes exist at a lower level than those for the external tubes.

The shear-lag phenomenon varies along the height of the tube structure. The degree of shear-lag depends upon the actual difference in the bending moments with shear-lag and without shear-lag. Such effects are quantified in terms of a dimensionless moment factor λ, which is equal to *Me/M*. Note that *Me* and *M* denote the total bending moment by the proposed method and the bending moment by the elementary bending theory respectively, and the story level where λ=1 denotes the shear-lag reversal point. The variations of λ in the corner and center columns of the flange frames along the height for the three tube(s) in tube structures are shown as below. It is observed that the variation of λ in the flange frame of the external tube changes from positive to negative shear-lag along the height of the structures. On the other hand, in the flange frame of the internal tube, λ changes from positive to negative and to positive shear-lag again. This means that shear-lag reversals in the internal flange frames take place at two level points (around levels 4 and 12), whereas that in the external flange frames takes place at one level point only (around level 10). It is further observed that near the top of the structures, the effect of negative shear-lag is so predominant that the corner column develops the moments opposite to that in the center column.

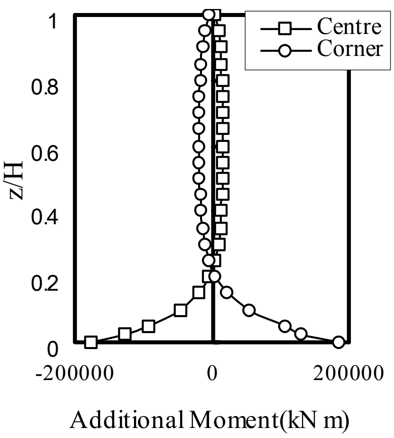



External frame

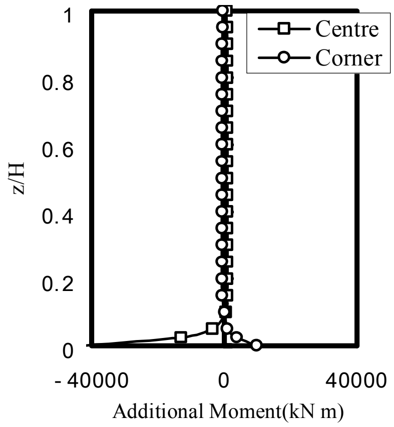



Internal frame

(a) Tube structure

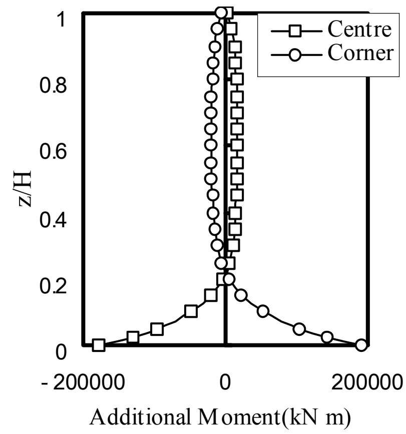



External frame

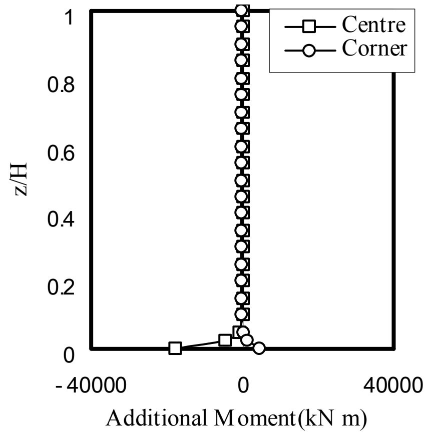



Internal frame

(b) Tube in tube structure



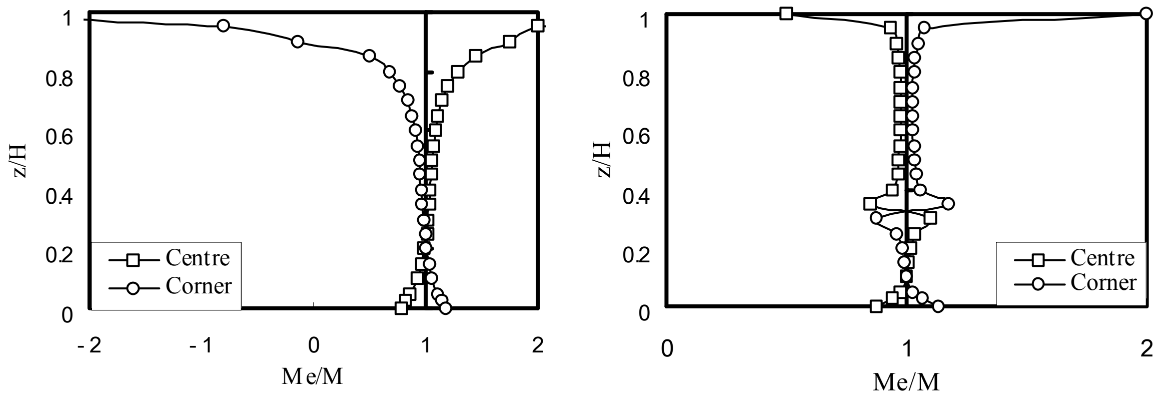

External frame




Internal frame

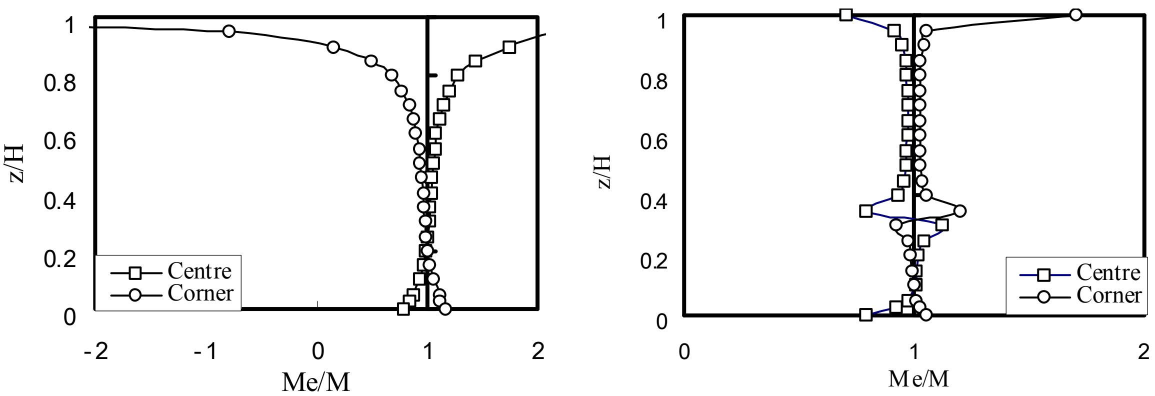
(c) 2 tubes in tube structure

Additional bending moment distributions in external and internal flange frame columns of the three tube(s) in tube structures:



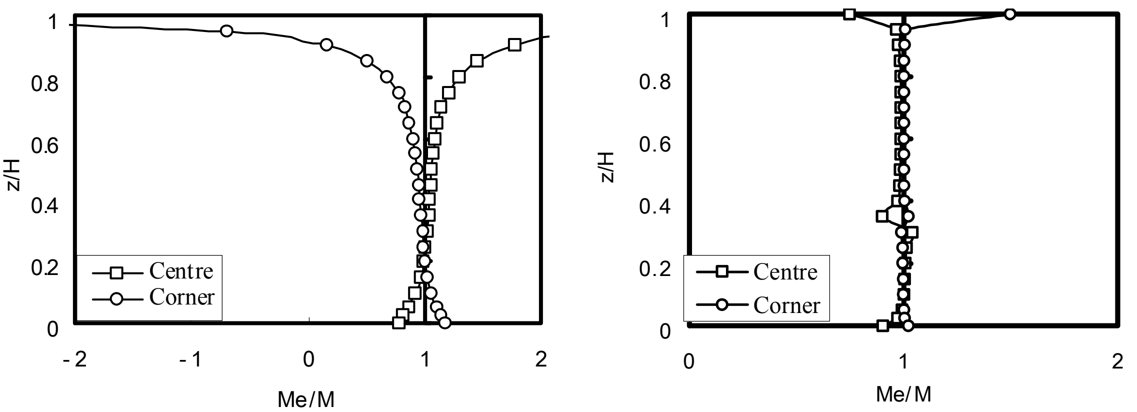

External frame    Internal frame

(a) Tube structure




External frame    Internal frame

(b) Tube in tube structure




External frame    Internal frame

(c) 2tubes in tube structure

Variation of Me/M in flange frame columns of the three tube(s) in tube structures

## 4 Conclusion

A simple mathematical model is proposed for the approximate additional bending moment analysis of the tube structure with multiple internal tubes. The numerical analysis is based on the minimum potential energy principle in conjunction with the variational approach. The net shear-lag effects and the lateral stiffness of the internal tube are taken into consideration to estimate the additional bending moments in the tubes in tube structures.

The additional bending moments are analysed for investigating the shear-lag phenomenon in the framed-tube structures with multiple internal tubes. The shear-lag phenomenon is explained by quantifying the additional bending moments and the moment factors. Along with the moment factor, the additional bending moment is also considered to be a structural parameter capable of enhancing the understanding of the structural behaviour of tube(s) in tube structures as well as the shear-lag phenomenon.

## 5 References

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