Risk-Sensitive Markov Decision Process for Underground Construction Planning and Estimating

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Summary

This paper presents an application of dynamic decision making under uncertainty in planning and estimating underground construction. The application of the proposed methodology is illustrated by its application to an actual tunneling project—The Hanging Lake Tunnel Project in Colorado, USA. To encompass the typical risks in underground construction, tunneling decisions are structured as a risk-sensitive Markov decision process that reflects the decision process faced by a contractor in each tunneling round. This decision process consists of five basic components: (1) decision stages (locations), (2) system states (ground classes and tunneling methods), (3) alternatives (tunneling methods), (4) ground class transition probabilities, and (5) tunneling cost structure. The paper also presents concepts related to risk preference that are necessary to model the contractor's risk attitude, including the lottery concept, utility theory, and the delta property. The optimality equation is formulated, the model components are defined, and the model is solved by stochastic dynamic programming. The main results are the optimal construction plans and risk-adjusted project costs, both of which reflect the dynamics of subsurface construction, the uncertainty about geologic variability as a function of available information, and the contractor's risk preference.

1 Introduction

Planning and estimating underground projects during the bidding phase are especially important and challenging tasks for all contractors because of geologic uncertainty. Comprehensive and realistic construction plans strive for optimal decisions that minimize time and cost while addressing all important project risks. This paper presents the application of a risk-sensitive Markov decision process to underground construction planning and estimating. It illustrates the modeling power to quantify and incorporate risk and its effectiveness for choosing optimal plans as functions of the contractor's degree of risk sensitivity.

2 Construction planning and estimating

The primary tasks for contractors during the bidding phase are developing appropriate construction plans and estimating their costs. These are extremely important and challenging tasks because they directly influence the contractor's competitiveness and profit. To establish an appropriate construction plan, contractors need to make several decisions based on available information and personal experience. These decisions include major construction methods and equipment, sequence of construction operations, and construction resource management.

For a particular construction project, there are often many feasible construction plans. To be competitive, the contractor must be able to evaluate and compare their performance. The important attributes for appraising the performance of construction options are cost and time. Construction estimating is a systematic process for forecasting construction project cost and time before the physical performance of the work actually begins to allow for prudent decision making. A complete estimate should take into consideration all necessary resources to create the facility and their impact on cost and time. However, it should be represented only at a level of detail that is useful for the contractor's decisions (Carr 1989). All estimates can be determined by one of the two main approaches: direct encoding or modeling (detailed estimating).

The direct encoding approach, such as unit price estimating, is typically adopted to prepare the pre-design and conceptual estimates for various purposes, including setting up the project's budgeted cost, comparing different design alternatives during the design phase, and verifying bid prices. In contrast, contractors need to produce more accurate estimates closer to the final project cost. Detailed cost estimating is therefore their preferred estimating approach. In detailed estimating, all work items in the project are initially categorized into divisions by using some forms of work breakdown structure (WBS). Costs associated with different divisions are estimated by using different levels of work breakdown such as the operation level, the activity level, and the process level. This decision primarily depends on available information, and the complexity and uncertainty associated with the work in a particular division. Finally, the construction plans and costs for all divisions are integrated to attain the construction plan and the total cost for the entire project.

3 Underground construction

Underground construction is a complex form of construction project. Underground structures, such as tunnels, can solve problems of difficult terrains, limited surface space, and increased demand for transportation. At the same time, however, they are expensive options where a variety of risks are encountered in every phase of the project delivery process.

Similar to other types of construction, efficient underground construction planning requires that contractors establish the optimal construction plan based on available information. It however presents a great challenge to all contractors because of the complexity and uncertainty associated with underground projects. The primary factors that significantly influence underground construction planning are geologic uncertainty, geologic variability, uncertainty in construction productivity, and the contractor's risk sensitivity (Likhitruangsilp 2003).

3.1 Geologic uncertainty

The selection of construction methods depends primarily on the anticipated geologic conditions. Regardless of the number and extent of subsurface explorations undertaken, the geology cannot be perfectly known prior to construction. Even though several practices have been adopted to mitigate geologic uncertainty, including the observational method, they cannot entirely eliminate this uncertainty from underground construction planning.

3.2 Geologic variability

Several underground projects (e.g., tunnels) traverse a variety of geologic conditions, the locations and extents of which are impossible to define with certainty. For the projects with significant geologic variability, the selected construction methods must be adaptable to all anticipated geologic conditions without significantly interrupting construction progress. Thus, underground construction is dynamic in nature.

3.3 Uncertainty in construction productivity

Another important factor results from uncertainty in the productivity of construction processes. This uncertainty stems from the variation of construction machine performance, the variation of worker outputs, and unexpected events such as accidents. Since this uncertainty exists even if geologic conditions are known, its impact on construction planning must be addressed separately.

3.4 Risk sensitivity

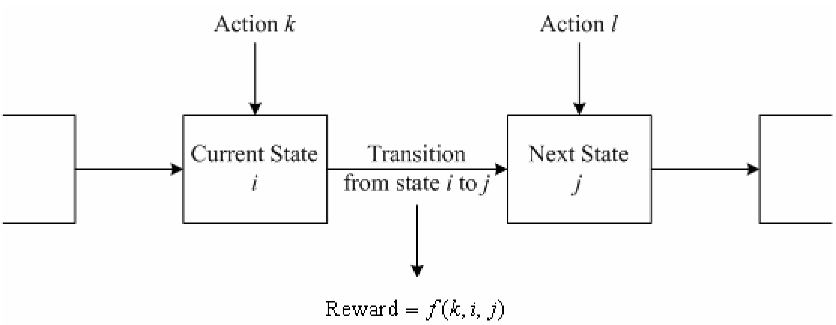
Individual valuation of benefits and costs for decisions involving risk (e.g., underground project planning) is often nonlinear because these decisions are not based on the maximization of the expected monetary value (EMV). In other words, when making decisions under uncertainty, a decision maker is typically sensitive to risk, either risk averse or risk preferring. An individual's risk sensitivity (risk preference) is influenced by such factor as the person's current net asset position. Typically, as a person's net asset position increases, the less risk-averse his behavior toward the same risk.

A contractor's risk aversion and his degree of risk exposure can have a major influence on construction plans and the necessary amount of risk premium or contingency embedded in a contractor's price in order to undertake the work. A more risk-averse contractor adopts a more conservative plan and includes a higher allowance as contingencies in his bid than a less risk-averse contractor does (Ioannou 1988). Thus, it is necessary to incorporate risk sensitivity into underground construction planning.

By considering all these factors, underground project planning and estimating can be considered a risk-sensitive dynamic probabilistic decision process.

4 Dynamic probabilistic decision process

A dynamic decision under uncertainty problem can be structured as a probabilistic sequential decision model, which can be symbolically represented by:



Symbolic Representation of Probabilistic Sequential Decision Problems

At a particular decision epoch, a decision maker observes the current state of the system. Based on this state, the decision maker chooses an action from the set of available actions for that state. An action leads to two consequences: (1) the system evolves to a possibly different state at a new epoch based on the transition probabilities of the system (here the system evolves from state *i* to *j*), and (2) the decision maker receives a reward (or a cost is incurred). Both the transition probabilities and reward may depend on the decision epoch, the choice of action, the current and next system states (i.e., the transition). At the subsequent epoch, the decision maker encounters a similar problem, but now the system may be in a different state, and there may be a different set of actions to choose from. As this process evolves through time, the decision maker receives a sequence of rewards (Puterman 1994).

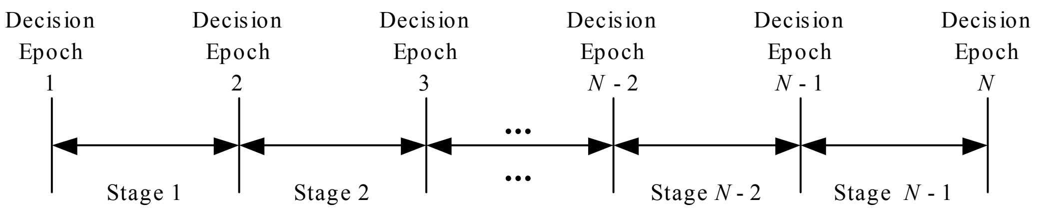
An individual is often faced with decision problems in which the state evolves through time (or space). The decision maker's goal is to choose a sequence of actions that optimizes the output of the system as expressed by a predetermined performance criterion. For a system whose state in the future depends on the decision made at the current epoch, decisions should not be made myopically, but rather the decision maker should take into account the rewards associated with future system states as well. A Markov decision process is a powerful stochastic model that is particularly applicable to this type of problems.

5 Risk-sensitive Markov decision process

5.1 Model components

A Markov decision process consists of five basic elements: (1) decision epochs, (2) system states, (3) actions, (4) transition structure, and (5) reward structure.

Decision epochs are points of time (or locations where) decisions are made. The set of decision epochs *T* can be classified as either a discrete state set or a continuum, or as a finite or an infinite set. In a discrete-time problem, time is divided into stages or periods, and a decision is made at the beginning of every stage.



Decision Epochs and Stages

At each decision epoch, the system occupies a state out of a set *S* of possible system states. Once the decision maker has observed the system state $s \in S$ at that epoch, he chooses action *a* from the set of allowable actions in state *s*, A_s.

Choosing action $a \in A_s$ at decision epoch *n* leads to two consequences: the system changes to some state at the next epoch determined by a transition probability distribution and the decision maker receives a reward. The transition of the system state is defined by the transition probability function, $P_{ij}^{a}(n)$, which denotes the probability that the system will be in state *j* at decision epoch $n+1$ given that the decision maker chooses an action *a* as the system occupies state *i* at decision epoch *n*.

The real-valued reward function $R_{ij}^{a}(n)$ defined for $i, j \in S$ and $a \in A_s$ denotes the value of the reward received if the decision maker chooses an action *a* and the system makes a transition from state *i* at the current epoch *n* to state *j* at the next epoch $n+1$. A positive $R_{ij}^{a}(n)$ is regarded as an income, and a negative $R_{ij}^{a}(n)$ is regarded as a cost.

Once all elements of the Markov decision process have been defined, a decision rule for the system can be determined based on a predetermined optimality criterion. A decision rule provides procedures for choosing the optimal action for each state at a specified epoch. A sequence of decision rules to be used at all epochs is called a policy π : $\pi = \left(d_1, d_2, ..., d_{N-1}\right)$, where d_i is the decision rule at epoch *i* as a function of state *s*. Thus, a policy provides the decision maker with a prescription for selecting the best action for any possible system states at any decision epochs.

5.2 Model formulation and solution

Consider a Markov decision process with a finite set of decision epochs $T = (1, 2, ..., N)$. Let S denote the finite set of system states, and the finite set of actions is defined by A. The state and the action at decision epoch n are represented by s_n and a_n, respectively.

For $n < N$, if the system occupies state i at decision epoch n and action k is chosen, a transition to some state at decision epoch $n+1$ can be determined by the transition probability

$$P_{ij}^{k}(n) = P(s_{n+1} = j \mid s_n = i, a_n = k) \quad (1)$$

Resulting from this transition, the decision maker earns the reward at decision epoch n, $R_{ij}^{k}(n)$. A policy π is a decision procedure that specifies a decision for each state at a given epoch. Let $\pi(n,i)$ denote any decision made according to policy π given that the system occupies state i at decision epoch n. Assume that the decision maker has a linear utility function; that is, he is risk neutral and wants to maximize the total expected reward. Thus, the optimal decision policy can be obtained by determining a policy that maximizes the total expected reward among the finite set of all possible policies.

Given that $k = \pi(n,i)$ and let $f_i(n)$ be the maximum expected reward earned from stages n through the end given that the system occupies state i at decision epoch n. According to the principle of optimality, $f_i(n)$ uniquely exists for all i and n and can be computed by the following optimality equation (Denardo 1982):

$$f_i(n) = \max_k \sum_j P_{ij}^{k}(n)\left[R_{ij}^{k}(n) + f_j(n+1)\right] \quad (2)$$

The calculation begins with the boundary condition, $f_i(N)$, for all states i. Equation (2) is applied recursively to determine the maximum expected rewards from epoch $N-1$ through the end, from epoch $N-2$ through the end, and so on. Finally, the maximum expected reward $f_i(1)$ for any beginning state i can be determined. This computational procedure is called backward recursive fixing or stochastic dynamic programming. Solving this equation also provides the optimal policy π^* for the system:

$$\pi^*(n,i) \in \arg\max_k \sum_j P_{ij}^{k}(n)\left[R_{ij}^{k}(n) + f_j(n+1)\right]; \forall i, n \quad (3)$$

5.3 Modeling risk sensitivity

The risk sensitivity of a decision maker can be encoded by a unique utility function (Howard 1977). A utility function $u(v)$ assigns a real number u in an ordinal scale to each of the possible outcomes v of an uncertain proposition (termed *lottery*). According to the properties of the utility function, the decision maker's preference in ranking alternative lotteries with uncertain outcomes can be quantified by the expected utility value (EUV), $E[u(v)]$, of each lottery (Luce and Raiffa 1957). If the larger value of v is preferred, then $u(v)$ is a monotonically increasing function of v. When the decision maker has to choose between several lotteries, the one with the greatest EUV is the most desirable choice.

The certain equivalent (CE) of a lottery $\tilde{v}$ is defined as the value of the outcome that has the same utility as the EUV of the lottery.

$$u(\tilde{v}) = E[u(v)] \quad (4)$$

The decision maker is indifferent between facing the uncertain outcomes of a lottery and receiving the CE with certainty. The CE of a lottery must be interpreted algebraically. For example, the CE (selling price) of a lottery involving monetary loss (e.g., construction costs) is negative, which represents the amount of money the decision maker is willing to pay (e.g., subcontract) in order to sell the risk of that lottery to other parties (e.g., subcontractors) (Ioannou 1989).

5.4 Integrating risk sensitivity into Markov decision process

A risk-sensitive Markov decision process can be formulated by integrating utility theory with the general Markov decision process presented above. It requires an additional assumption known as the delta property (Howard 1977). A significant implication of this assumption is that a multistage (sequential) decision problem can be broken down into single-stage problems that are easier to solve.

Since the utility function of a decision maker who accepts the delta property is restricted to either linear or exponential, the exponential utility function, which is the general case, is used in this paper to construct the cost structure.

$$u(v) = -(\text{sign } \gamma)e^{-\gamma v} \tag{5}$$

where the parameter γ is the risk aversion coefficient and (sign γ) is the sign of γ. A positive γ means the decision maker is risk averse, whereas a negative γ means the decision maker prefers risk.

By integrating utility theory and the assumption concerning risk preference presented above, the optimality equation for a risk-sensitive Markov decision process, parallel to equation (2), can be written as:

$$u^*(\tilde{v}_i(n)) = \max_k \sum_j P_{ij}^k(n) \times u[R_{ij}^k(n) + \tilde{v}_j(n+1)] \tag{6}$$

The term $u^*(\tilde{v}_i(n))$ represents the maximum expected utility of the reward earned from decision epochs *n* through the end given that the system occupies state *i* at epoch *n*. The transition from state *i* at epoch *n* to state *j* at epoch $n+1$ and action *k* is chosen leads to reward $R_{ij}^k(n)$ and the remaining transitions whose the certain equivalent is $\tilde{v}_j(n+1)$.

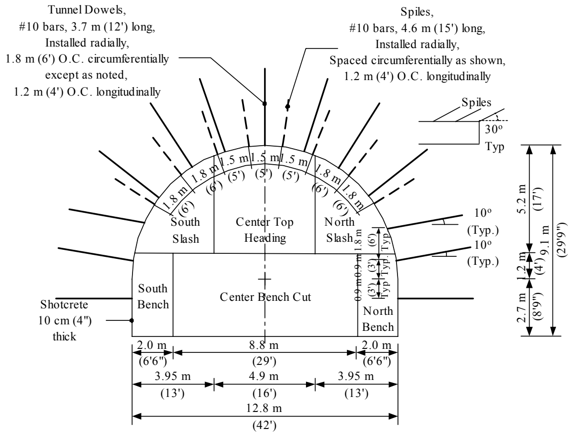
Similar to equation (2), equation (6) can be solved by stochastic dynamic programming. The problem is solved backward, starting at the end of the problem (i.e., epoch *N*) and solving the problem at epochs $N-1$, $N-2$, and so on. Given that the optimal decision rules for epochs $n+1$, $n+2$, ..., and $N-1$ are known, the action that maximizes the expected utility for each state *i* at epoch *n*, $u^*(\tilde{v}_i(n))$, can be determined. Once the maximum expected utility of the system for the beginning state *i*, $u^*(\tilde{v}_i(1))$, has been determined, its certain equivalent, $\tilde{v}_i(1)$, can be calculated using the inverse function of equation (5). Parallel to equation (3), the optimal policy for the system, π^*, can be determined by the following relationship:

$$\pi^*(n,i) \in \arg\max_k \sum_j P_{ij}^k(n) \times u[R_{ij}^k(n) + \tilde{v}_j(n+1)]; \forall n,i \tag{7}$$

6 Application to Tunneling—The Hanging Lake Tunnel Project

In this section, we apply the proposed risk-sensitive Markov decision process to determine the optimal construction plan and risk-adjusted cost for the Hanging Lake Tunnel Project, a twin-bore highway tunnel in the state of Colorado, USA. Here, we focus on the part of this rock tunneling project excavated by multiple-drift drill and blast methods, consisting of two major segments: the west segment with the total length of 753 m (2,470 ft) and the east segment with the total length of 347 m (1,139 ft).

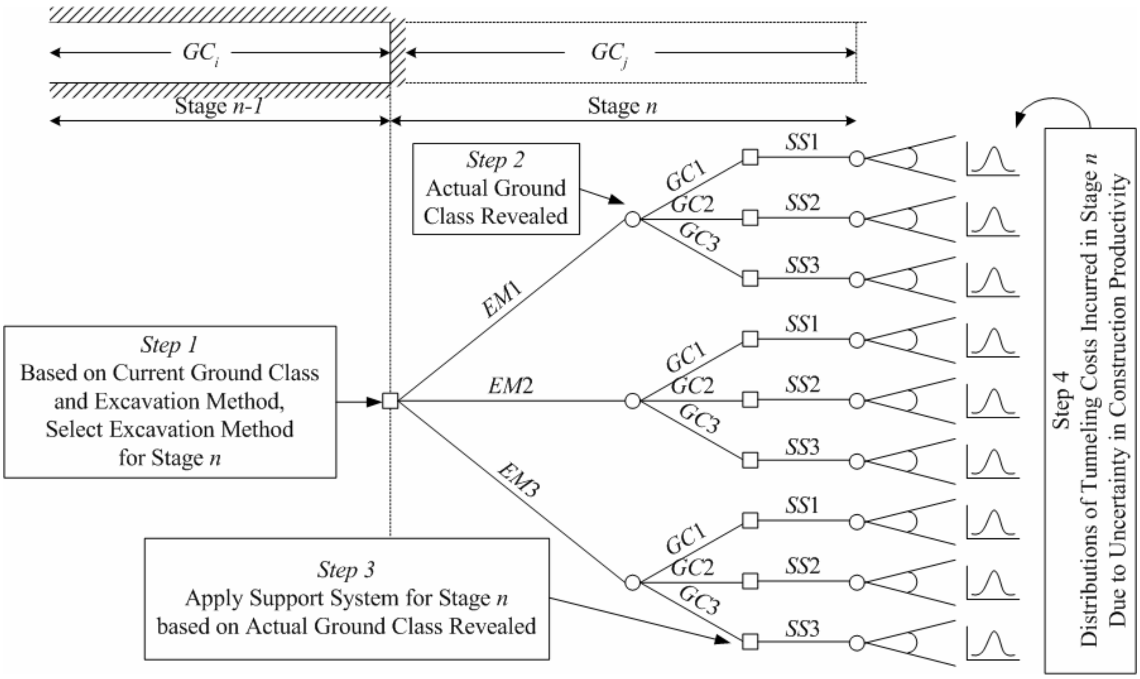
The geologic conditions were classified by using rock mass classification systems into three ground classes: *GC*1 (best), *GC*2 (medium), and *GC*3 (worst). Three excavation methods (*EM*1, *EM*2, *EM*3) and three primary support systems (*SS*1, *SS*2, *SS*3) were designed corresponding to the three ground classes. For example, taken together, *EM*3 and *SS*3 comprise the most economical and structurally adequate tunneling method for *GC*3. This project required sequential excavation of six drifts and rock reinforcement systems consisting of rock dowels, spiles, and shotcrete, as shown below:



Tunnel Cross Section and Primary Support Type 3

Detailed descriptions of the ground class classification and the specifications of excavation and support methods can be found in Leeds, Hills, and Jewett (1981); Scotese and Ackerman (1992); and Essex et al. (1993).

The risk-sensitive Markov decision process for this project is formulated by analyzing the tunneling decision process performed by the contractor in each round. The following figure shows a decision tree illustrating the tunneling decision process.



Decision Tree Illustrating Tunneling Decision Process

As can be seen, the excavation method for the next stage *n* is selected based on the current ground class and the excavation method being used. After applying the selected excavation method, the actual ground class is revealed. The contractor then chooses the primary support system corresponding to the revealed ground class. The distributions of tunneling costs incurred in stage *n*, shown as end-node values, represent randomness due to uncertainty in construction productivity, not geologic uncertainty.

The basic elements of the Markov decision process for this problem are summarized below. It is very important to notice that the system states are not just the three possible current ground classes, but rather all nine combinations of possible current ground classes and excavation methods.

Components of the Markov Decision Process for the Presented Tunneling Problem

Model Components	Tunneling Decision Process
(1) Decision epochs	Beginnings of tunneling stages (e.g., round)
(2) States	Current ground class and excavation method
(3) Actions	Excavation methods and primary support systems
(4) Transition structure	Ground class transition probabilities
(5) Cost structure	Probability distribution of tunneling costs

The optimality equation for this example can then be written as:

$$u^*(\tilde{v}(n,i,k)) = \max_{k' \in K} \sum_{j \in J} P_{ij}^{GC}(n) \times u[CM(k,k') + C(k',j) + \tilde{v}(n+1,j,k')] \quad (8)$$

The term $u^*(\tilde{v}(n,i,k))$ represents the maximum expected utility of tunneling costs incurred from the current epoch *n* through the end of tunnel given that the current state is defined as the combination of currently being in ground class *i* and using excavation method *k*.

$P_{ij}^{GC}(n)$ is the probability that the ground class makes a transition from state *i* at the current epoch *n* to state *j* at the subsequent epoch $n+1$. It should be noted that this ground class transition probability is a function of the tunnel location, not the selected tunneling method.

$CM(k,k')$ is the total cost resulting from adapting the excavation method at epoch *n*. These costs are functions of the excavation method currently used (*k*) and the method selected for this stage (k'). Thus, $CM(k,k') = 0$ when $k = k'$ (i.e., keep using the same excavation method).

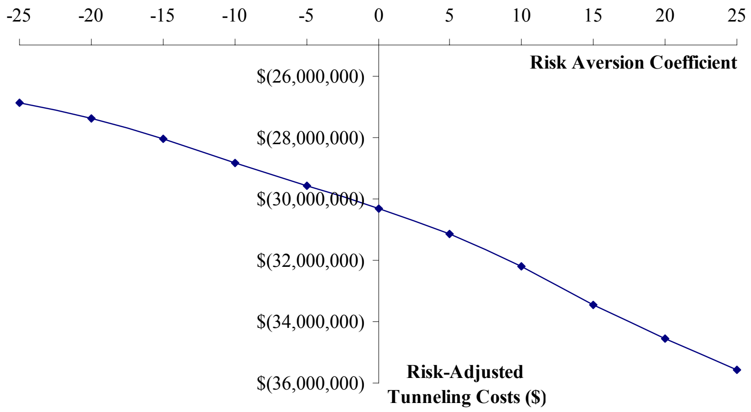
$C(k',j)$ is tunneling cost incurred if excavation method k' is chosen and the actual ground class after blasting is *j*. This term also includes costs resulting from choosing a wrong excavation method (e.g., underbreak or excessive overbreak cases).

$\tilde{v}(n+1,j,k')$ is the certain equivalent of tunneling costs incurred from epochs $n+1$ through the end of tunnel given that the ground class at the next epoch is *j*, and the excavation method selected for the current stage *n* is k'.

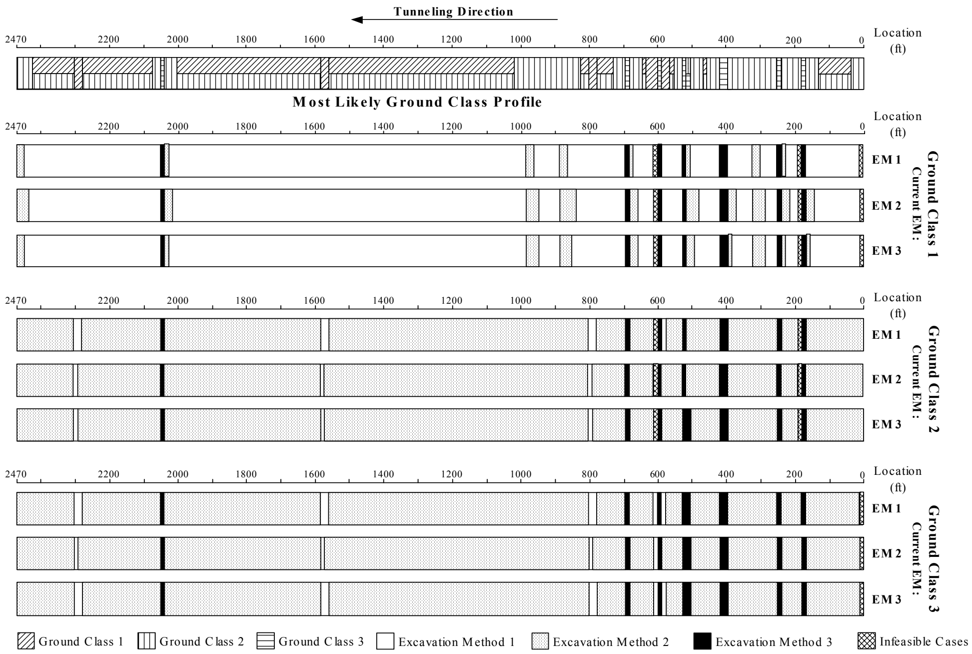
J is the set of possible ground classes in this project; $i, j \in J$. K is the set of excavation methods used in the project; $k, k' \in K$. *N* is the set of tunnel locations where the decision is made (i.e., the beginning of each round); $n \in N$.

A detailed discussion about the calculation of ground class transition probabilities and these cost components can found in Likhitruangsilp (2003).

Solving equation (8) by applying stochastic dynamic programming provides the risk-adjusted tunneling costs (certain equivalents) and the optimal tunneling sequence for different degrees of risk sensitivity (i.e., different values of γ):



Risk Adjusted Tunneling Costs for Different Risk Aversion Coefficients (γ)



Optimal Tunneling Sequence for $\gamma = 5$ (Risk-Averse Contractors)

As can be seen, the expected tunneling cost for this project $(\gamma = 0)$ is approximately $30.3M. As the risk aversion coefficient increases (i.e., a contractor is more risk averse), the risk-adjusted tunneling cost increases almost linearly. In contrast, as the risk aversion coefficient decreases (i.e., a contractor is more risk preferring), the risk-adjusted tunneling cost decreases almost linearly.

The figure above shows the optimal tunneling sequence given that the contractor is risk averse with $\gamma = 5$. Nine bars in the figure correspond to the nine possible combinations of ground classes and excavation methods during construction. For example, given that the geologic conditions encountered at location 40.2 m (132 ft) is *GC*1, and *EM*1 was used in the previous round, the optimal decision for this contractor is to use the same method (i.e., read from the first bar). However, if the current tunnel geology is *GC*2, and *EM*1 is being used, the contractor should switch to use *EM*2 for the subsequent round (i.e., read from the fourth bar).

7 Conclusions

The proposed risk-sensitive Markov decision process illustrates the methodology for applying a dynamic probabilistic decision model for planning and estimating underground projects. The model can address all important factors associated with underground construction and can provide comprehensive construction plans and realistic construction costs, both of which reflect important characteristics of subsurface projects. This model can also be applied to other construction problems involving sequential decision makings such as multiple-phase construction projects.

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