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ACCURACY IMPROVEMENT IN DIAMTER MEASUREMENT OF MICRO-SPHERE BASED ON WHISPERING GALLERY MODE

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ABSTRACT

We have been proposed the new method to measure a dimeter of a micro-sphere on a basis of whispering gallery mode resonance. Resonant wavelengths and mode numbers are necessary quantities to calculate the diameter. In this paper, we proposed and discussed the method to determine the radial mode number of WGMs. Experiments showed the well agreed diameters from various resonant wavelengths, which implies that the radial mode numbers can be successfully estimated by means of the proposed method.

Index Terms **–** Microsphere, Diameter, Whispering gallery mode, Micro-cavity

1. INTRODUCTION

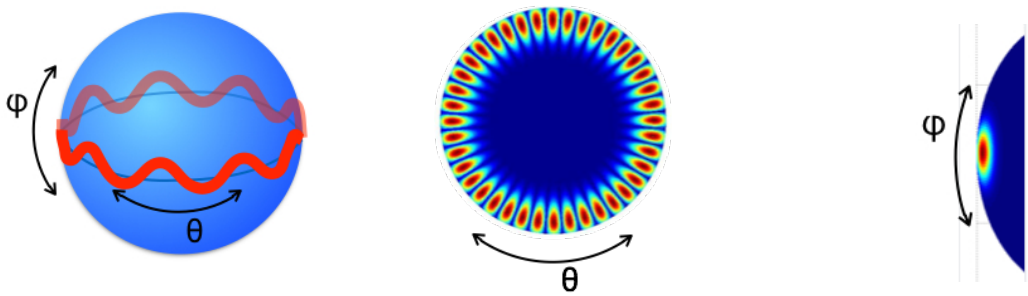
With miniaturization of products and parts, the 3-dimensional (3D) metrology has to be expanded to the micrometer scale [1]. In decade, instruments evaluating the 3D geometrical quantities such as dimension, size, tolerance for small parts have been developed [2] such as micro-coordinate measuring machine (micro-CMM) [3], micro-X ray computed tomography (micro-XCT) [4] and optical-CMM [5]. Micro-CMM is able to measure 3D geometry of small parts. Currently measurement uncertainly of the micro-CMMs is less than 100 nm. Micro-XCT, which is highly developed recently, can image total 3D structure of the parts including the inside structures. Crucial drawback of the micro-XCT is the resolution that is still on the order to 100 nm. Optical-CMM based on focus variation method is recently proposed for 3D metrology.

Calibration is necessary process for the dimensional metrology. A sphere is frequently used as a reference for calibration in the 3-dimensional metrology because of its isotropic shape. Assuring an accuracy of the reference sphere is responsible for measurement uncertainty of 3D metrology, which means sphericity and diameter of the reference sphere need to be guaranteed. For micro-scale 3D metrology, size of the reference sphere is also micro-scale from several hundred micrometers to a few tens of micrometers due to the measurable range of the mentioned instruments, and which has to be measured with accuracy of better than 10 nm. A macro-scaled-large sphere can be measured by means of interferometric technique with high accuracy of a few nanometers because the surface of the sphere can be treat as flat surfaces as against optical light wavelength. For the interferometric method, however, it is difficult to determine the diameter of the micro-scale sphere due to the curvature of sphere surface, although still sphericity might be evaluated. Therefore, nowadays, measurement of a diameter of the micro-scale sphere is important issue to be addressed. We proposed the new measurement principle of a diameter for the micro-scale sphere and aim to achieve 10 nm of the measurement accuracy. In this paper, it is investigated deterministic method for the radial mode number of WGMs, which is essential to estimate the microsphere diameter accurately.

2. MEASUREMENT PRINCIPLE

2.1 Whispering gallery mode

We have proposed the new method to measure a diameter of a sphere on a basis of whispering gallery mode (WGM) resonance [6]. WGM is the mode that a light resonates near-surface-layer of the sphere such as shown in Fig. 1(a). This is also known as morphological resonance. Conditions to excite WGMs are mainly related to the wavelength of a light, the refractive index and the diameter of a sphere, so the diameter can be estimated based on an analysis of the wavelengths to excite WGM (WGM wavelengths) if the refractive index is known. It should be noted that there are three types in WGM; angular mode (see Fig. 1(b)), azimuthal mode and radial mode. Depending on these mode numbers, the WGM wavelengths are different. For azimuthal and radial mode, the spatial distribution of the electromagnetic field is also dependent with the mode numbers as shown in Fig. 2.



(a) Whispering gallery wave. (b) Cross-sectional view along θ. (c) Cross-sectional view along φ.

Fig 1 Whispering gallery mode resonance.

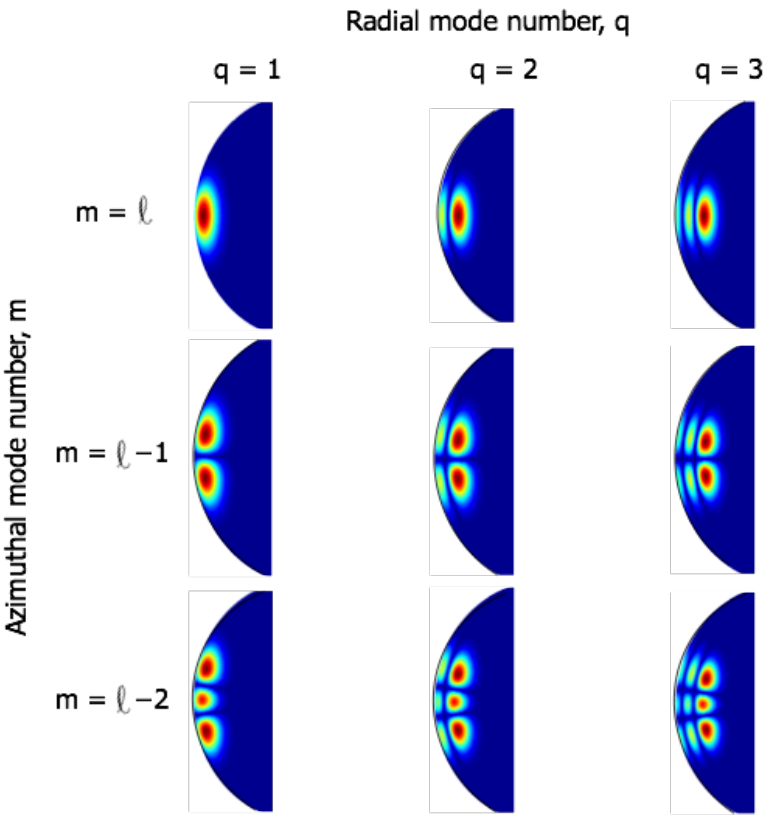


Fig 2 Distribution of electric field with different azimuthal and radial mode numbers.

2.2 Diameter measurement

In experiment, WGM wavelengths can be measured, therefore relation between the diameter, the WGM wavelength, and the angular mode number is important, which can be obtained by solving the following dispersion equation.

$$\alpha\left(1+\frac{\rho_1}{j_\ell(\rho_1)}\frac{\partial j_\ell(\rho_1)}{\partial \rho_1}\right)=1+\frac{\rho_2}{h_\ell^{(1)}(\rho_2)}\frac{\partial h_\ell^{(1)}(\rho_2)}{\partial \rho_2} \tag{1}$$

where, ρ is the size parameter, $2\pi n_s/\lambda$, , λ is the WGM wavelength, n_s is the refractive index of the sphere, l is the angular mode number, α is the polarization factor, n_s^2 for TE, 1 for TM, j is the spherical Bessel function, $h^{(1)}$ is the spherical Hankel function of first kind. Subscript 1, 2 means the inside and outside of sphere. The material dispersion for the refractive index of the sphere was considered using Sellmeier's equation. Equation (1) shows that the diameter can be calculated using the measured WGM wavelength with mode numbers In the proposed measurement principle, a diameter is determined based on the optical path length of the circumference using WGMs. Therefore, this technique is applicable for high quality sphere and transparent materials such as glass.

WGM wavelength can be experimentally obtained, so angular, radial and azimuthal mode numbers are necessary to be determined to calculate the diameter using Equation (1). However, this is not just straightforward. If the mode numbers are wrong, critical measurement error is caused. In order to measure the dimeter ultra-accurately, the mode numbers have to be determine with high accuracy. Previously the method to determine the angular mode number was discussed [7], in this paper, method to determine the radial mode number are addressed.

3. NUMERICAL SIMULATION

3.1 Coupling coefficient

To excite and measure the WGM in a sphere, a light is introduced into the sphere through a tapered optical fiber. A portion of this tapered fiber is stretched such that its diameter is reduced to a few microns. In the tapered section, the optical fiber core is also stretched, allowing light to propagate in the clad. Through total internal reflection at the interface with the air, the evanescent light is thus localized along the tapered portion of the fiber. When the sphere is exposed to the evanescent light (see Fig. 3), the light is coupled into the sphere at wavelengths that correspond to the WGMs. To determine these WGM wavelengths, the

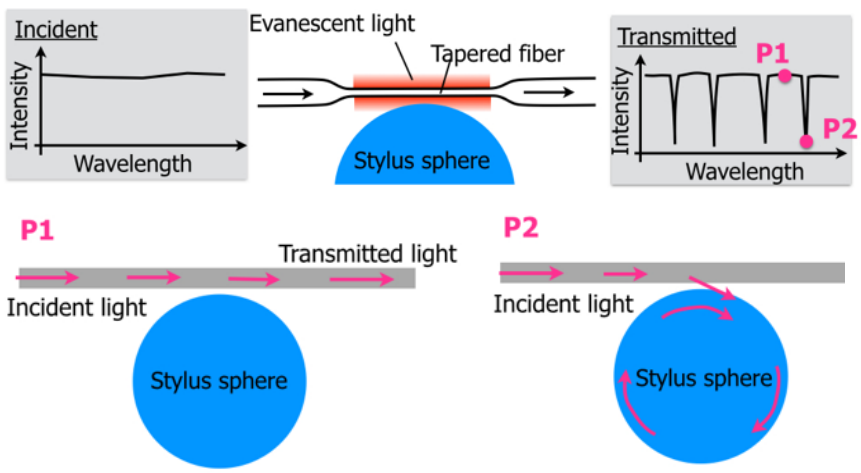


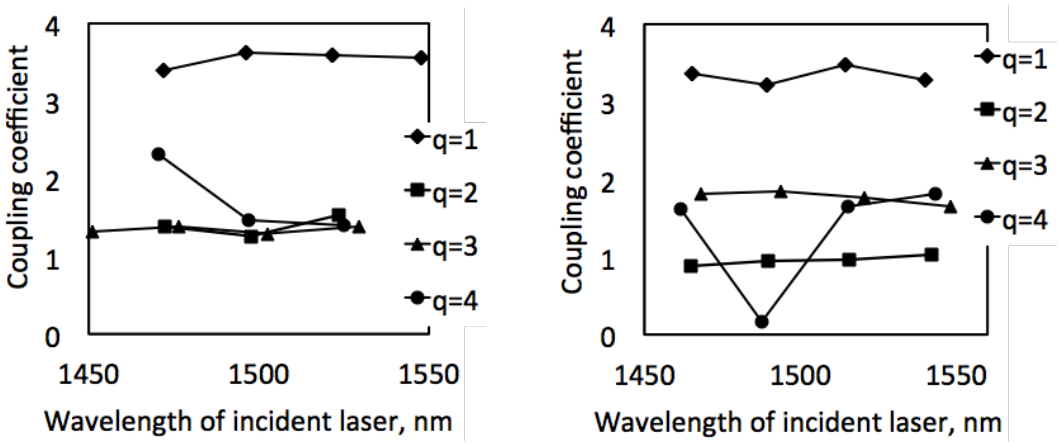
Fig 3 Light coupling scheme into sphere.

amount of light transmitted past the tapered portion of the optical fiber is observed as a function of the wavelength. A drop in the measured transmitted light should correspond to light coupling into the sphere and thus the identification of a WGM wavelength.

To measure the sphere diameter, the light coupling is important to understand. Therefore, light coupling efficiency was numerically analyzed for various radial mode numbers, and the degree of light coupling was determined by the coupling coefficient κ, given by

$$\kappa = \frac{\omega\varepsilon_0}{4P}\left(n_1^2 - n_2^2\right)\iiint \left(E_f \cdot E_s^*\right)\exp\left[(\beta_s - \beta_f)r\theta\right]drd\theta d\phi \tag{2}$$

where ω is the angular frequency of light; ε_0, the permittivity of vacuum, P, the power of the incident laser; φ and θ, the angles defined in Fig. 1; r, the radial distance from the sphere center; E_f, the electric field generated by a tapered fiber; E_s*, the complex conjugate of the electric field generated by a WGM of a sphere; and β_f, the propagation constant of the light in the tapered fiber. The electric fields at the tapered fiber and the sphere were calculated based on Maxwell's equations. The resonant wavelengths of the WGMs were obtained using Equation (1), and the following parameters were used in the numerical analysis. The microsphere diameter was set to 20 μm, mostly owing to computer memory limitations. The tapered fiber diameter was set to 1 μm. The refractive index of both the sphere and fiber was 1.501. The microsphere and tapered fiber were in contact. The power of the incident laser was 10 mW. The coupling coefficients for various radial modes are shown in Fig. 4. The azimuthal and angular mode numbers are set to 54 ($m = l = 54$), and the coupling coefficients were calculated for different wavelengths (1450–1550 nm). The results show that the largest coupling coefficients occur at $q = 1$. This indicates that the light from the tapered fiber will couple more strongly at the fundamental WGMs. Therefore, it is expected that at the resonant wavelengths of $q = 1$, the intensity of the transmitted light will be the lowest ideally.



(a) TE mode. (b) TM mode.

Fig. 4 Whispering gallery mode resonance.

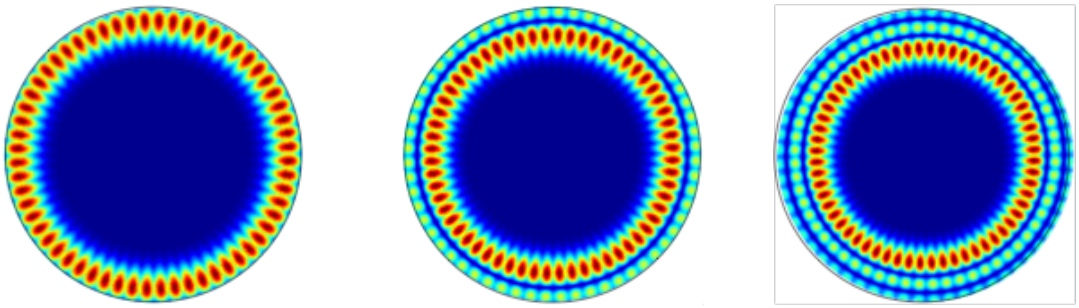
3.2 Free spectrum range

Light coupling efficiency was influenced by experimental condition, morphological distortion and also some noises, so believing only light coupling strength is not reliable to determine the radial mode number. Here, free spectrum range (FSR) is introduced to recognize the radial mode number with more high accuracy.

Example field distributions of different radial mode number are shown in Fig.5. The electric fields are extended more inner side of a sphere with increasing the radial mode

number, which implies that the resonant wavelength of a larger radial mode number becomes similar to one of a smaller sphere. In general, the smaller sphere has the wider FSR of WGM wavelengths. Now the theoretical resonant wavelengths are calculated using Equation (1). Suppose the fused silica sphere with a diameter of 100 μm. The calculated resonant wavelengths against angular mode numbers are shown in Fig.6. The resonant wavelengths with same radial mode numbers are connected with dot lines. Circle and squire plots indicate the polarization of TE and TM, respectively. FSR, which I mentioned above, is the wavelength differences between neighboring angular mode numbers, as shown in Fig.6. Averaged FSR for each radial mode number in a range of 1540 nm ± 40 nm are shown in Table 1. As explained, the smallest FSR is shown in q=1, and the FSR becomes larger with increasing the radial mode number.

As a result, in order to determine the radial mode numbers for the measured WGM wavelength, firstly, the WGM wavelengths with strong light coupling are found. Secondary, FSRs are compered among them. Then a set of the WGM wavelengths with the smallest FSR are the smallest radial mode number, namely q=1.



(a) q=1. (b) q=2. (c) q=3.

Fig. 5 Cross-sectional view of electric field distribution with different radial mode number, q.

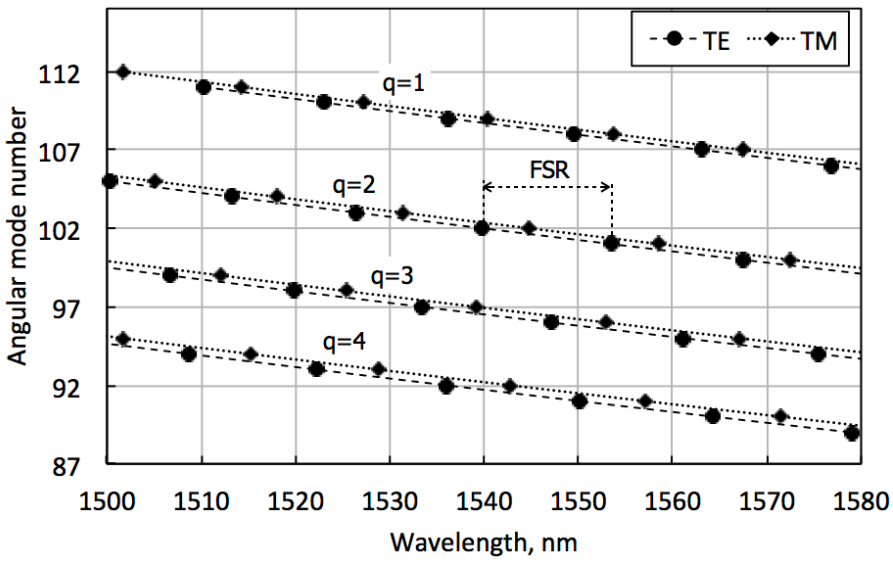


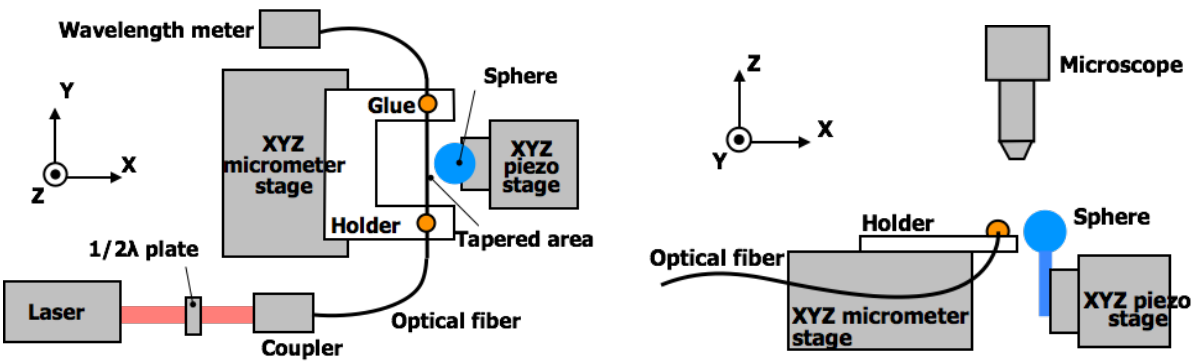
Fig.6 Theoretical resonant wavelengths for different mode numbers.

Table 1 Averaged free spectrum range for different radial mode numbers.

	Radial mode number			
	1	2	3	4
TE	13.30 nm	13.41 nm	13.77 nm	14.05 nm
TM	13.38 nm	13.51 nm	13.90 nm	14.23 nm

4. EXPERIMENTAL SETUP

The experimental setup is shown in Fig. 7. The light source is a tunable external cavity laser diode with an average output power of 15 mW. The laser wavelength can be tuned in 0.01 nm increments (1520–1570 nm). The laser line width is 100 kHz. The laser input was coupled to the tapered fiber (minimum diameter of the tapered portion is 1 μm) after the polarization controller. The fiber's other end was connected to a wavelength meter to measure the transmitted light intensity and wavelength. Accuracy of the wavelength meter is guaranteed ± 1 pm. The tapered fiber was fixed to a holder attached to the micrometer stage. The microsphere to be measured was fixed to a holder attached to the XYZ piezo-stage (9 nm resolution). The tapered fiber and a micro-sphere were observed using a microscope. The glass microsphere was made by heating a single mode optical fiber with a focused CO_2 laser, which diameter was estimated approximately 55 μm by optical microscope. The material of the sphere was pure fused silica glass. For measurements, the separated distance between the micro-sphere and the fiber must be maintained to excite WGM. For that, the transmitted power via the tapered fiber was monitored. When the sphere is close enough to the tapered fiber within the area of the evanescent light, the transmitted power is decreased. Thus the separation distance is adjusted.



(a) Top view. (b) side view.

Fig. 7 Experimental setup.

5. EXPERIMENTAL RESULT

Measuring WGM wavelengths were implemented and the measured result is shown in Fig.8. The measured transmitted light intensity was seemed periodic, therefore the data was divided into four and shown in same figure for ease to compare. The inserted numbers in the figure indicate the start wavelength for respective graph, and horizontal axis is a normalized wavelength. The dip indicates the WGM wavelength. When strongly light coupled, the strong peaks are appeared. As procedures explained in chapter 3, firstly, the four strong peaks ware

found, which was marked as “A” to “D” in bottom data (1527 nm) of Fig.8. Similar peaks can be found in other data. Then the FSRs of these peaks were compared. Averaged FSR of these peaks are 10.38 nm (A), 10.46 nm (B), 10.22 nm (C) and 10.63 nm (D), respectively. This result indicates that the peak C is corresponding for q=1, A is for q=2, B is for q=3 and D is for q=4. If only the coupling strength was taken to determine the radial mode number, the peak A seems to be selected for q=1. By introducing the FSR, the radial mode numbers can be determined accurately. To confirm the validity of this determination method, the diameter was estimated using allocated radial mode numbers. For each peak, averaged diameters are calculated as shown in Table 2. Although only the peak B (q=3) is deviated around 60 nm, mostly the diameters are well agreed on the order of 10 nm, which can proof the radial mode number was successfully determined.

Finally, the diameter of the measured sphere was estimated. WGM wavelengths of the radial mode number of 1 were used. For TE and TM, the WGM wavelengths and calculated diameter are listed with the allocated angular mode number in Table 3. The measured diameter was 52.18 μm and the maximum variation was 23 nm. Fluctuation of the measured diameter was only 3 nm. As shown, the performance of the proposed measurement technique is adequately high for the reference sphere in the micro-scale 3 dimensional metrology. As future, the measurement uncertainly has to be investigated.

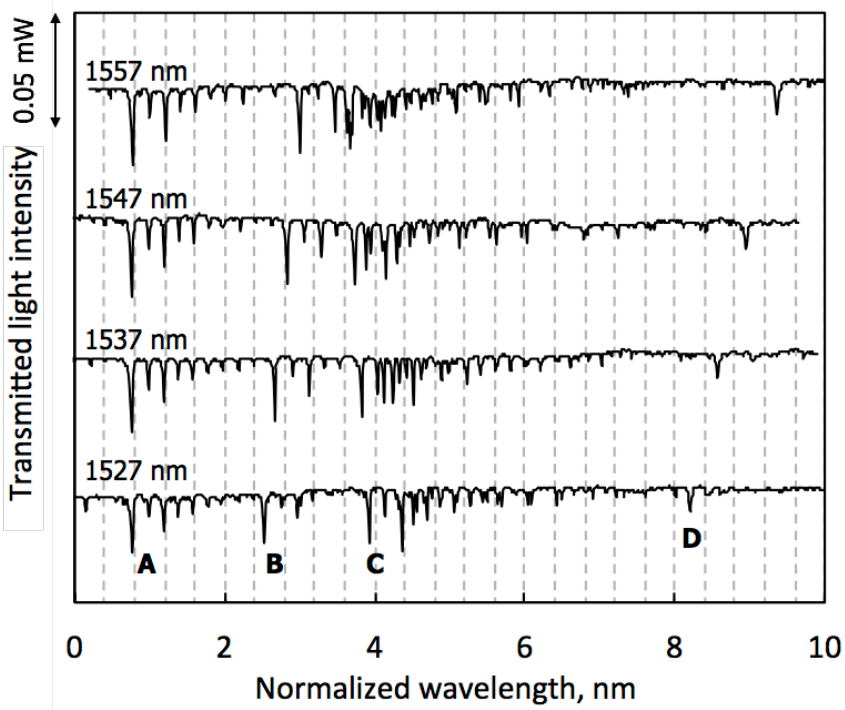


Fig. 8 Measured WGM spectrum.

Table 2 Measured diameter with different radial mode number.

A	B	C	D
52.201 nm	52.258 nm	52.191 nm	52.186 nm

Table 3 Measured diameter using radial mode number of 1.

Angular mode number	TE		TM	
	Resonant wavelength, nm	Diameter, μm	Resonant wavelength, nm	Diameter, μm
146	1527.07	52.1669	1530.83	52.1893
145	1537.08	52.1669	1540.92	52.1905
144	1547.23	52.1671	1551.12	52.1908
143	1557.52	52.1675	1561.49	52.1922
142	1567.95	52.1680		
Average		52.1673		52.1907
Variation (max − min)		0.0012		0.0028

6. CONCLUSION

We have been developing the measurement technique of a diameter of microsphere based on the whispering gallery mode resonances. To estimate the diameter accurately, the mode numbers of resonated light wave is essential in the proposed method. In this paper, we discussed the method to determine the radial mode number of WGM. With numerical analysis, the smallest radial mode number shows the highest coupling efficiency. With FSR analysis, the smaller radial mode number, the narrower the FSR. By using these two criteria, the radial mode number can be determined successfully in experiment. As a result, the diameter measurement was performed with high accuracy.

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