MODELING AND INVESTIGATION OF SPRING CLIP MECHANISMS AND
APPLICATIONS IN PRECISION ENGINEERING

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ABSTRACT

In many technological fields spring clip mechanisms are used as a kind of compliant mechanism, especially as clamping and securing elements. A specially formed wire bracket is supported by two parallel arranged revolute joints with shifted axes. Due to the special support of the compliant mechanism element (spring clip) in a fixed frame, the mechanism is preloaded and so the force-displacement behavior is influenced. The size of the mechanical preload is determined by the distance between the two shifted axes. In this contribution we investigate examples of the effect of a mechanical preload on the deflection of one asymmetric spring clip mechanism using numerical and analytical methods and we suggest applications of spring clip mechanisms in precision engineering devices.

Index Terms - compliant mechanism, precision engineering, spring clip mechanism

1. INTRODUCTION

In mechanical science there have only been a few theoretical investigations of spring clip mechanisms. These compliant mechanisms can be used as securing, locking, clamping and closure elements such as carabiner clasps and linchpins [1] in different designs and styles (see Figure 1). Especially for the design of scientific and measuring instruments or highly accurate devices, such as chronometers, medical and optical apparatus, small dimensions and consequently a low mass and a low inertia are advantages of spring clip mechanisms. By combining a specially shaped spring clip (wire bracket) with bending and torsional capability with a rigid frame using two revolute joints, one gets a spring clip mechanism. The spring clip is supported by two parallel arranged revolute joints with shifted axes in the fixed frame. Hence, the compliant mechanism element is deformed (change of shape) and the mechanism is preloaded. This preload which is determined by the distance between the two shifted axes influences the force-displacement behavior of the mechanism. Because of the deflection of the spring clip accomplished by the preload on the one hand and an external force on the other hand, the mechanism is able to move.

Figure 1: Technical application of spring clip mechanisms; (a) – carabiner hook with spring clip closure [2]; (b) – linchpin standard shape [3]; (c) – linchpin for pipes [3]; (d) – specially formed linchpin [3]
As described above, a change in the shape of the spring clip allows the movement of the mechanism, so one can classify such mechanisms as compliant mechanisms [4]. The compliance in the considered mechanism is distributed over the whole of the mechanism and it is gained by the elastic material, like steel. Due to the revolute joints and the compliance in shape, this spring clip mechanism is characterized as a hybrid compliant mechanism.

2. MATERIAL AND METHODS

Analyzing spring clip mechanisms and to investigate their stiffness behavior we suggest a model to use the finite element method and a simple analytical model for the highly non-linear problem.

2.1 Modeling

An essential part in the investigation of the effect of a mechanical preload on the deflection and the mobility of a spring clip mechanism is the modeling. Generally, there are several specially formed spring clips, each adapted for an application. Because it is impossible to focus all of them, one particular spring clip is chosen for investigation. In order to do this, the following restrictions and definitions are accepted (see Figure 3) [5], [6]:

- the spring clip is built up of three bending and torsion flexible straight legs,
- the undeformed spring clip is planar, meaning that all bar axes are within one plane,
- all legs are perpendicular to each other,
- asymmetry occurs because of the different lengths of the two parallel legs ($l_1 > l_3$),
- validity of SAINT-VENANT’s principle,
- application of the linear EULER-BERNOULLI theory,
- slender legs with small and constant circular cross-sections,
- plane sections remain plane and normal to the deflected neutral axis,
- the material is isotropic and homogeneous and obeys HOOKE’s law,
- friction in all hinges is neglected.

2.2 Numerical Methods

To investigate the static structural deformation of a spring clip and the mobility of the whole mechanism, the Finite Element Method (FEM) is well-suited. Using the software package ANSYS® Workbench 14.5, it is easy to handle a large number of analyses, so different element types, boundary conditions, preloads and external force loads can be considered. In reference to the modeling of a spring clip mechanism (see section 2.1), it seems to be favorable to use beam elements for the analyses. Therefore an elastic 3D-beam element (beam4) is utilized for the simulation in order to apply the EULER-BERNOULLI theory. All hinges are modeled as cylindrical support with a rotational motion around the z-axis (revolute joints). The preload which is an essential feature of this exploration will be achieved by a displacement $\delta$ of the hinge $B$ along the y-axis in a first load step as remote displacement. In a second load step, holding the preload (displacement of hinge $B$) on the same level, an external force $F_y$ along the y-axis is applied at the free end of the spring clip. The FEM model described is shown in Figure 2. Let the parameters for the geometry and material be fixed as follows:

- length of the legs: $l_1 = 50$ mm $l_2 = 25$ mm $l_3 = 40$ mm
- cross-section area: all circular with one diameter $d = 1$ mm
- Young’s / shear modulus: $E = 206$ kN/mm² $G = 78$ kN/mm²

Due to the beam elements, all the results are generated in a short time. But there is no doubt that something went wrong. Using beam4-elements, the simulation yields a disproportionately
large deflection at the point of the applied external force ($v_F \approx 1478$ mm with $\delta = 2$ mm displacement and $F_y = 5$ N force load). Neither using another type of beam element (beam188), nor a variation of the element shape function (linear or quadratic function), nor other boundary conditions will change the quality of the results. Yet, there is no idea why such a high and probably incorrect deflection occurs.

To check the obtained results with another FEM analysis, one can try a shape discretization by solid elements. These elements are more exact and any real structure can be mapped in detail. In our case, the restrictions set out in section 2.1 are not all necessary for the solid elements and so there is a new FEM model.

![Figure 2: FEM model for analyses of an asymmetrical spring clip mechanism and parameters [7]](image)

In particular, neglecting the bending radii and all fillets is not valid for this type of simulation. Furthermore, using solid elements does not satisfy the Euler-Bernoulli theory because shear deflection is observed. As a new type of element, we use a 3D-solid tetrahedron element with a quadratic element shape function, the solid187-element. Due to the increasing amount of nodes, the required calculation time is accordingly higher. All analysis settings (e.g. boundary conditions and loads) are unchanged in comparison to the simulation in which beam elements are used (see Figure 2). Again, we have two load steps (first preload and subsequent external force load). Within these modifications in modeling, the simulation clearly yields a realistic result, as follows: $v_F \approx 14$ mm with $\delta = 2$ mm displacement and $F_y = 5$ N load. Also, another simulation with a solid185-element (tetrahedral elements with a linear element shape function) obtains nearly the same result and so the applicability of the solid elements is proven. The difference between using the solid185-elements and solid187-elements is not significant, but there is a big difference with the results of the beam4-elements.

As we pointed out, there is a mismatch between the FEM results. A simple bar approach (beam elements) on the one hand is in contradiction with an approach on the basis of continuum mechanics (solid elements) on the other hand. From the mechanical point of view and based on the mechanism theory, it should be possible to handle this problem with the linear Euler-Bernoulli theory. The spring clip mechanism satisfies all restrictions to adapt the problem with Euler-Bernoulli theory (see section 2.1) and so we continue to analyze the problem with an analytical method.

### 2.3 Analytical Methods

First of all, we should think about a strategy to solve the problem analytically. In the same way as in the FEM simulations, the mechanical preload (displacement) and the external force load have to be applied stepwise and consecutively. Otherwise, the preload will deform the spring clip, that the local bar axes are not within one plane and not straight for the next step (external force load). Hence, to solve the second load step (external force load), the curvature
of all axes must be known and the deformed bar axes of the first load step will be the new ones for the second load step. A split calculation done only with the undeformed straight axes and a superposition of the partial deflections would lead us to inaccurate results.

The primary purpose of this section is to check if a calculation of the given problem is suitable to be solved by beam theory. In fact, this would resolve the contradiction between the widely differing FEM results. Because of the reasons given above and the restrictions in section 2.1, the analytical calculations are limited to computing the effect of preload and no external force on the deflection of the asymmetric \((l_1 > l_3)\) spring clip mechanism. In general, there are several ways to investigate the problem analytically. One way is to determine the deflections respectively for tension / compression, torsion and bending by their equations, or instead one could use energy methods, such as CASTIGLIANO’s method. The aim is to get the deformed shape of the spring clip and consequently the deflection of each point of the structure. Therefore, the first way, using the classical equations, is needed and so all support reactions, the normal forces, the shear forces and the bending moments have to be computed.

![Figure 3: FEM model for analyses of an asymmetrical spring clip mechanism and parameters [8]](image)

The static equilibrium yields three scalar force equations and three scalar moment equations in a Cartesian coordinate system. For the concrete case illustrated in Figure 3, the spring clip supported by two revolute joints (fixed hinges \(A\) and \(B\)), the static equilibrium for the first load step (only the preload) generates:

\[
\begin{align*}
\begin{cases}
A_x + B_x = 0 \\
A_y + B_y = 0 \\
A_z + B_z = 0 \\
(l_1 - l_3)A_x + l_2B_x = 0
\end{cases}
\end{align*}
\]

The displacement \(\delta\) at the fixed hinge \(B\) is not in equations (1) and (2). Instead, there is the unknown support reaction \(B_y\) which cannot be calculated by (1) or (2). At this point, an energy method, namely MENABREA’s theorem, is suitable to use to obtain the unknown support reactions \(B_x\), \(B_y\) and \(B_z\) [6]:

\[
\frac{\partial W_F}{\partial B_x} = \frac{\partial W_F}{\partial B_y} = 0. \quad (3)
\]

Supporting the spring clip in the fixed frame by two parallel arranged revolute joints with shifted axes, the mechanism is preloaded, which can be considered by the displacement \(\delta\) at the fixed hinge \(B\):

\[
\frac{\partial W_F}{\partial B_y} = -\delta. \quad (4)
\]

According to Figure 3, the spring clip consists of three straight segments (legs). Concerning the local Cartesian coordinate systems, the normal force, the shear force and the bending moment can be specified for each segment. Hence, the deformation energy \(W_{F1}\) of the first leg \((0 \leq x_1 \leq l_1)\), neglecting shear effects, is given by [6]:

\[
\text{Figure 3: FEM model for analyses of an asymmetrical spring clip mechanism and parameters [8]}
\]
The calculation of the deformation energy for the other segments is done in the same way, adapting the limits \(0 \leq x_2 \leq l_2/2\), \(l_2/2 \leq x_2 \leq l_2\) and \(0 \leq x_3 \leq l_3\), forces and moments. Using equations (3) and (4) and also the adapted equations to compute the deformation energy (cf. (5)), we gain all the support reactions. Under the assumption of equal cross-section areas and equal material for all three legs \((A_1 = A_2 = A_3 = A; I_{p1} = I_{p2} = I_{p3} = I_p; I_{z1} = I_{z2} = I_{z3} = I_z; E = \text{const.}; G = \text{const.})\) it follows that:

\[
A_x = B_x = 0, \\
A_z = B_z = 0, \\
A_y = -B_y = -\frac{3\delta GI_p E I_z}{GI_p(l_1^3 + l_2^3 - l_3^3 - 3l_1^2 l_3 + 3l_2^2 l_1) + 3EI_z(l_2^2 l_1 + l_3^2 l_2)}.
\]

Now, computing the deflection of the spring clip is easy because of the calculated support reactions, the normal forces and the bending moments. Regarding all three legs, we have the following mechanical impacts summarized in Table 1.

<table>
<thead>
<tr>
<th>limits</th>
<th>reactions</th>
<th>mechanical impact</th>
</tr>
</thead>
<tbody>
<tr>
<td>leg 1</td>
<td>(0 \leq x_1 \leq l_1)</td>
<td>((Q_{y1}); M_{tx1}; M_{tz1}) torsion; bending around z</td>
</tr>
<tr>
<td>leg 2</td>
<td>(0 \leq x_2 \leq l_2/2)</td>
<td>((Q_{y2}); M_{tx2}; M_{tz2}) torsion; bending around z</td>
</tr>
<tr>
<td></td>
<td>(l_2/2 \leq x_2 \leq l_2)</td>
<td>((Q_{y3}); M_{tx3}; M_{tz3}) torsion; bending around z</td>
</tr>
<tr>
<td>leg 3</td>
<td>(0 \leq x_3 \leq l_3)</td>
<td>((Q_{y4}); M_{tz4}) bending around z</td>
</tr>
</tbody>
</table>

Two differential equations describe the slope angle \(\varphi()\) and the bending deflection \(v()\) [6]:

\[
\varphi'(s) = \frac{M_f(s)}{GI_p} \quad \text{and} \quad \varphi''(s) = \frac{M_b(s)}{EI_z}.
\]

The total deformation along the y-axis \(v(x, z)\) is a superposition of each partial deflection with regard to the boundary and matching conditions. Defining \(s\) as a new coordinate, which puts \(x_1, x_2\) and \(x_3\) together, one has \(v(s)\).

\[
v(s) = \begin{cases} 
v_1(s) & 0 \leq s \leq l_1 \\
v_2(s) + \varphi_1(l_1) \cdot s & l_1 < s \leq l_2 \\
v_3(s) + \varphi_2(l_2) \cdot s & l_2 < s \leq l_3. 
\end{cases}
\]

Equation (8) is a short form of \(v(s)\) and the authors would like to announce a detailed solution \(v(s)\) because the results are presented in chapter 3. With a look at Figure 4, presented in section 3.1, we could accomplish our purpose to solve the given problem by beam theory. In addition, an investigation of the second load step (external force) can be executed using the curvature \(v(s)\) as new bar axes. If the displacement \(\delta\) (representing the preload) is small, the obtained deflection of the spring clip should be small too and so it might be possible to approximate the total superposed deformation (displacement and external force) within equations (1) to (8). Some small errors between analytics and FEM can probably be expected, but this is not in the current focus. The equations mentioned above should also be able to handle the application of an external force instead of a displacement. We will present all the results in chapter 3.
3. RESULTS AND COMPARISON

3.1 Preload

When the planar spring clip is brought into the fixed frame with shifted axes, it will be deformed due to a displacement $\delta$ at revolute joint $B$. Utilizing numerical and analytical methods, the deflection of the spring clip (deformed shape) was calculated. See Figure 4 for a three-dimensional visualization of the evaluated deformed shape, especially $v(x, z)$ for different displacements $\delta$.

![Figure 4: Three-dimensional illustration of the deflection $v(x, z)$ (deformed shape) of the selected asymmetric spring clip mechanism with different displacements $\delta$ [8]; (a) – FEM-based results; (b) – analytical results](image)

But this figure is not well-suited for a comparison or an error estimation of the results established by numerical and analytical methods. For that, the deflection $v(s)$ both numerically and analytically (see (8)) is sketched in Figure 5.

![Figure 5: Deflection $v(s)$ of the selected asymmetric spring clip mechanism with different displacements $\delta$ [8]](image)

As one can see, there is a good accordance between the numerical and the analytical results for small displacements. When increasing the size of the displacement $\delta$, the error ensues equally. This is not a mistake in the calculation, but on the contrary it is in conformity with the assumptions in the theory.

3.2 External Force Load

Before the mechanical preload and the external force are put together as one combined load to the spring clip mechanism, it might be of benefit to look at what is affected when only an external force at the free end of the spring clip, without any displacement of the joint $B$, is applied. Solving this task numerically is done without any problems by using the FEM. Adapting equations (1) and (4) and using the other equations, it is probably possible to handle
this task analytically because only the boundary conditions change. In Figure 6, one can see that there are giant errors between the analytical (beam theory) and FEM-based results (continuum mechanics). Not only does the magnitude of the results vary, but also the course of the characteristics is quite different. This large gap is certainly the same as the one between the numerical results of the beam4- and the solid187-elements, as mentioned in section 2.2.

![Figure 6: Deflection v(s) of the selected asymmetric spring clip mechanism without any displacements δ but with different external forces Fy][8]

3.3 Preload and External Force Load

In relation to the quite different results from beam theory versus continuum mechanics, the following results for a preload and an external force load are limited to FEM-based results of solid187-elements.

![Figure 7: FEM-based deflection v(x, z) (deformed shape) of the selected asymmetric spring clip mechanism with different displacements δ and different external forces Fy][8]
For all simulations we used three negative (against the positive y-axis) displacements $\delta$ ($\delta = -2 \text{ mm}$; $\delta = -4 \text{ mm}$; $\delta = -6 \text{ mm}$) and respectively two positive and two negative external forces $F_y$. Figure 7 illustrates the results (deformed shape) of the whole mechanism. For negative displacements $\delta$ and positive external forces, the deflection $v(x, z)$ increases in the positive y-direction, and otherwise (negative forces) $v(x, z)$ decreases (negative y-direction). Even small positive forces generate such an increase, so the mechanism is more sensitive to external forces than to the imprinted displacement. If no displacement ($\delta = 0$) is applied, the deflection is only a result of the external force load (see Figure 6).

4. DISCUSSION AND CONCLUSION

A mechanical preload which is accomplished by a displacement $\delta$ of the revolute joint $B$ has an effect on the force-displacement behavior of the considered spring clip mechanism. Due to this displacement $\delta$, the spring clip is deformed while the maximum of deflection is around the default displacement. The deflection of a spring clip mechanism chosen as an example (see section 2.1 and 2.2) was calculated by the usage of numerical and analytical methods. As one can see in Figure 4 and Figure 5 there is only a small error in the results and so both the numerical and the analytical method are well-suited to solve the problem. An external force load applied to a non-preloaded spring clip mechanism causes also a deflection. This case was also handled numerically and analytically, but as we established in Figure 6, there is a difference in the magnitude and even in the characteristics of the results. But how can the deflection due to displacement be evaluated exactly, and why is there such a distinction for an external force load? The spring clip is a slender structure and it satisfies all the restrictions of beam theory. With a focus on that, it does not matter, whether the problem is solved by the FEM with beam elements (beam4 und beam188) or analytically. In fact, all FEM simulations using beam elements failed to provide the correct results either for only a displacement $\delta$, or for only an external force $F_y$, or for a combination of both. However, the deflection of the spring clip mechanism under the effect of a displacement $\delta$ was analytically dealt with, using the EULER-BERNOULLI theory and the results were sufficiently precise (see Figure 4 and Figure 5). Even if the combined load case ($\delta$ and $F_y$) cannot be handled, at least the simple case of only a displacement $\delta$ is. In contrast to the analytical way, in which the loads are sharply applied from one moment to another, the FEM uses a linear rising of the load. Maybe there was an instability at the beginning of the simulation that the FEM could not deal with and so the wrong results were generated.

It is shown in Figure 6 that an analytical evaluation of the deflection due to an external force $F_y$ produces inaccurate results. The reason for this has to be in the support of the spring clip in the frame. Because of the two revolute joints, there might be a bifurcation of the longest leg ($l_1$), which means that its curvature could be positive or negative so that there is no unique state. Fixed supports instead of revolute joints would generate the right results.

Additionally, it is not possible to compute the deflection of the structure in consequence of a combined load (preload due to displacement and external force load) by the methods of beam theory. The reason for the error in this case might be in addition to the bifurcation sought in the curvature of the bar axes. None of the reasons suggested can was proven by the authors and so these conclusions are only valid with reservations.

Nonetheless, using the methods of continuum mechanics yields good results compared with, e.g. the results of beam theory. Modeling the spring clip by solid elements (solid187) provides displacements only at the nodes and there is no twisting of the elements. Hence, the revolute joints are modeled by surfaces consisting of several nodes and not only of one node as in beam theory. In summary, these changes in the boundary conditions might be a reason for realistic results.
In the following, we deal only with the results based on the FEM simulations with \textit{solid187}-elements. Generally, the correlation of the external force $F_y$ and the deflection $v(x, z)$ remains linear for different displacements $\delta$, which means that a displacement $\delta$ at joint $B$ has an effect only on the magnitude of the deflection and not on its characteristic. To gain a non-linear force-displacement behavior, one could use non-constant cross-section areas, like $A(s) \neq \text{const.}; I_p(s) \neq \text{const.}; I_z(s) \neq \text{const.}$ In Figure 7 one can clearly recognize a rotation of the spring clip in revolute joint $A$ because of the slope. Due to the dimensions and the preload of the mechanism, the rotation at revolute joint $B$ is very small, so it looks like a fixed support.

Up to this moment, we have focused on the deflection of the mechanism caused by a preload and/or an external force. Now we change the point of view, a given deflection at a certain point generates a force referring to the fixed frame. This is a typical application such as those required for securing, locking, clamping and closure elements (see Figure 1). The magnitude of the force depends on the geometry (length of the legs, cross-section area, area moment of inertia), the material, the displacement $\delta$ of joint $B$ and the given deflection. To vary the magnitude of the gained force for one chosen spring clip, one can change the distance of the two shifted axes, namely the displacement $\delta$. The utilization of such compliant mechanisms provides big opportunities in application, making it possible to reduce the number of parts and the effort of assembly.

5. APPLICATIONS

In several precision engineering devices escapement and ratchet mechanisms are used to keep an element in a non-operating state, Figure 8. Therefor a ratchet is locked by a pawl, which is usually pivoted and spring-loaded. Also in cam mechanisms springs are used to keep the mechanism elements in contact, Figure 8. The spring in each application can be a coil spring or a bending spring. For a bending spring like a cantilever beam the range of motion is strictly limited because of the mechanical stress. To realize a wide range of motion a pivoted rocker arm is loaded by tension or spiral springs but these are additional parts in construction.

![Figure 8: Escapement and ratchet mechanisms with spring-loaded pawls, cam mechanism with different spring loaded rocker arms, linchpins as examples for spring clip mechanisms [9, 10, 11]](image)

Downsizing of parts and elements is recommended in precision engineering and this is a reason for integration of functions and a decrease in number of parts. Additionally,
manufacturing and assembling of miniaturized and function integrated components is more
difficult in comparison to macro machine elements. When the motion of parts and assemblies
in precision engineering is characterized by high velocities and accelerations dynamic effects
can be reduced by an exceptionally low inertia [9, 10, 11]. To reach a high precision and a
minor inertia, the components and machine elements of such devices have small dimensions
and consequently a low mass. With a look at mechanisms with pivotally mounted rockers,
especially escapement and ratchet mechanism, we introduce a spring clip mechanism as a
rocker arm with spring capability for precision engineering applications. Hence, this yields
some advantages, like a reduced number of parts, a functional integration of rocker arm and
spring capability. Due to many different shapes of the spring clip it is very easy to adapt the
structure to different applications in precision engineering. Of course scaling the geometry
and miniaturization of such mechanisms to adapt them to special applications is possible and
their manufacturing is easy. Figure 9 visualizes some applications for these mechanisms.

![Application of spring clip mechanisms](image)

(a)  (b)  (c)

*Figure 9: Application of spring clip mechanisms in precision engineering: (a) – cam mechanism with spring clip mechanism as rocker arm; (b) – spring clip mechanism as pawl in a ratchet mechanism; (c) – parallel spring with a wide range of motion due to spring clip mechanisms [7]*

### 6. SUMMARY

This contribution is about the effect of a mechanical preload on the deflection of an
asymmetrical spring clip mechanism, analyzed by numerical and analytical methods.
Regarding the slender shape of the structure, it is valid to investigate the problem by the use
of beam theory. But, as we pointed out, there are giant errors in some load cases in analytical
and even in numerical approaches using beam theory. The usage of beam elements is not an
appropriate way to compute even one load case by simulation, yet actually the analytical
approach works in the load case of only a displacement. Generally, it is recommended to
consider the deformable structure as a continuum for all load cases and to work with solid
elements in the FEM analyses.

The displacement leads to a change in the shape of the spring clip and this affects the force-
displacement behavior of the spring clip mechanism. Due to the imprinted displacement,
the magnitude of the deflection is influenced but the characteristic of stiffness and the linear
force-displacement behavior is not affected. Because of the great variety in geometry,
material and preload, these kinds of compliant mechanisms are suitable for clamping and
securing systems for several applications in precision engineering.
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