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APPROACHES ON MATERIAL ANALYSIS AND MODELING OF BOUNCING PUTTY

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ABSTRACT

In this contribution the mechanical properties of Bouncing Putty are investigated and the material parameters are determined by a rheological model. The Bouncing Putty is a silicone polymer, which shows viscoelastic behavior and it is characterized as a Non-Newtonian fluid with dilatancy. If it is thrown on a surface, it bounces back like a spring. But it flows like a viscous fluid, if it is simply put on a desk. The reason why it behaves either like an elastic material or like a viscous liquid is the velocity of the deformation. For dilatant also named shear-thickening materials the viscosity increases, if the strain rate (velocity of the deformation) rises. Because of the non-linear stiffness and damping behavior, this material is very interesting as a smart material in modern engineering, especially to handle disturbing vibrations and dynamic effects. Hence, to use this material for applications in engineering the non-linear spring-damping behavior has to be assessed by the functional relations of the mechanical parameters. But, determining elasticity and viscosity is not trivial, because the non-constant values are based on experiments and they depend on the theoretical model, which is applied. Due to this, an appropriate rheological model to map the effect of shear-thickening is developed and a testing method, based on a deformation test under a constant velocity of deformation using a servo-hydraulic testing machine, is presented. The recorded testing data is fitted to the rheological model and this yields the values of the material parameters according to the model. As a result, the elasticity and the viscosity parameters of the dilatant Bouncing Putty are quantified for different deformation velocities.

Index Terms - Bouncing Putty, Non-Newtonian fluid, viscoelastic, shear-thickening, dilatant, rheological model, material testing, spring-damper behavior

1. INTRODUCTION

In mechanical engineering springs are used as machine elements to store mechanical energy [1]. The characteristic of a spring, presented in Figure 1 (a), is either progressive or degressive or linear, depending on the material and especially on the shape of the spring. Due to a low internal damping they are often combined with damper elements in spring damper systems for dynamic applications to reduce the effects of vibrations and particularly impacts. The damping characteristic for frequently used dashpots is based on linear viscous damping shown in Figure 1 (b). For several applications in mechanical and automotive engineering rubbers are utilized because of their viscoelastic properties. Force-deflection characteristic of a rubber element is non-linear and due to the internal damping the unloading curve is beneath loading curve as Figure 1 (c) illustrates [2, 3]. An application-oriented design of rubber elements realizes specific force-deflection characteristics. To achieve highly adapted elastic and viscous properties for special and maybe changing environmental conditions for example in artificial limbs, active controlled systems with actuators and sensors are requested [4, 5]. Contrary to these active mechatronic spring-damper devices, energy-efficient smart materials such as the dilatant Bouncing Putty are more and more in the focus of research.

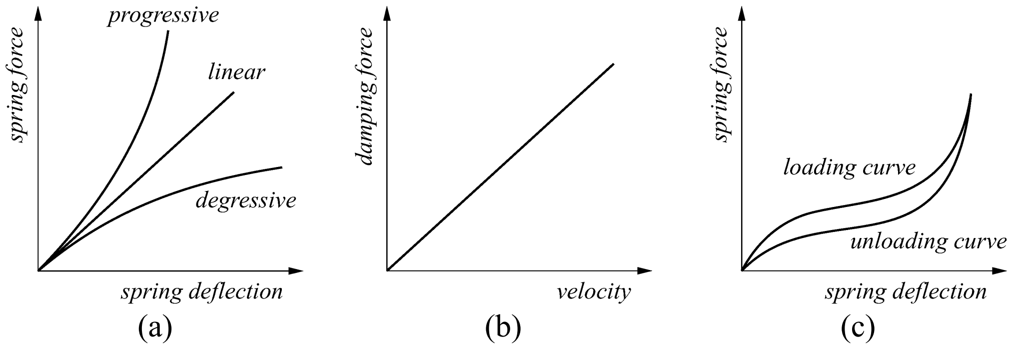
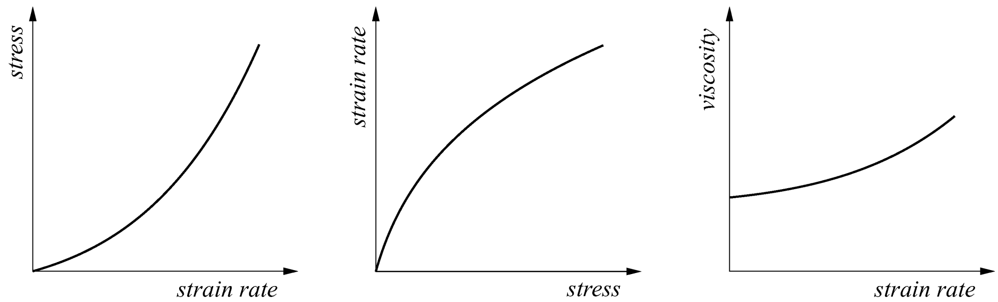


Figure 1: Spring and damping characteristics [3]; (a) – force-deflection characteristic of springs; (b) – linear damping behavior of a viscous damper; (c) – force-deflection characteristic of a rubber element

Several publications describe spring-damper devices based on smart materials [6, 7]. By a research project at FH Aachen the usage of dilatant materials as damping system was investigated [8]. As a commercial application, the dilatant material D3O® is used for impact protection in ski and motor sport. Protectors filled with this material minimize the risk of injury because the material is energy-absorbing and it hardens in case of impact. The D3O®-gel and the protectors are presented in Figure 2.

Figure 2: D3O®-material and application in sport protectors; (a) – energy-absorbing D3O® gel [9]; (b) – back Protector with D3O® [10]; (c) – glove with D3O® [11]

There is a big potential to realize an adaptable and energy-efficient spring-damping behavior without any actuators, sensors and controllers using dilatant materials such as the Bouncing Putty. The Bouncing Putty is a silicone polymer showing viscoelastic material properties. Depending on the strain rate, represented by the velocity of deformation, it is either elastic like a spring or viscous like a damper. Under the effect of gravity the material behaves like a highly viscous liquid and it flows due to the very small strain rate. The flow is initiated without any required initial stress. A sample of Bouncing Putty subjected to stress at a moderate velocity the material is deformable and reacts like a plastic solid. For high velocities of applied stress the Bouncing Putty gets more and more elastic and it is bounceable, hence it is characterized as an elastic solid. This effect of Non-Newtonian fluids is known as dilatancy or shear-thickening without a stress threshold. So, the Bouncing Putty is a fluid, despite it is similar to a solid for high strain rates. One parameter to characterize viscous media is the viscosity, which is a function of velocity for Non-Newtonian fluids [12]. Figure 3 summarizes this effect in different diagrams, but all representing a dilatant fluid without a threshold of stress. The dilatant behavior of Bouncing Putty is well known in a qualitative sense as illustrated in Figure 3 [13]. But for applications in engineering it is important to determine parameters and to quantify values by experiments. This work is directed to approaches of material analysis and modeling of elastic and viscous properties of Bouncing Putty.



(a) (b) (c)

Figure 3: Dilatant material behavior in qualitative sense [12]; (a) – stress-strain rate diagram; (b) – strain rate-stress diagram; (c) – viscosity-strain rate diagram

2. THEORETICAL MODEL

The theoretical model is the basis to quantify the values of elasticity and viscosity of the Bouncing Putty according to the deformation velocity. Because the phenomenological effects, how the material behaves are well known [8, 9], the theoretical model has to map the non-linear elastic and viscous behavior, especially the effect of shear-thickening. But only measuring data give hints, which model is practical to the material, meaning that the selection of the model is influenced by the measuring data.
Generally, the relations between load and deformation of bodies are given by mathematical equations of material theory [14]. Measuring and describing the material behavior under external loads and deformations is also a part of rheology [14]. Rheological models realize a mechanical description of main effects of materials in accordance with experiments. A physical description of material behavior can be achieved using rheological features, which are given in Table 1.

Table 1: Overview of linear rheological bodies and their features [12, 15]

rheological body	ideal elastic body	ideal viscous body	ideal plastic body
rheological feature	elasticity	viscosity	plasticity
basic law	HOOK's law	NEWTON's law	ST.-VENANT's law
Illustration	spring	dashpot	stick-slip element
symbol of parameter	E	η	σ_V

A combination of these rheological features yields a rheological model, which is more or less detailed to fit the model to reality and measurements. The two-parameter MAXWELL model, shown in Figure 4 (a) is a simple rheological model to map viscoelasticity. But, with a look at the results in section 4 this series of a spring and a dashpot is impractical to describe the Bouncing Putty, because of the difference between the testing data and the model-based data in Figure 9. Another often used rheological model for viscoelastic material is the linear rheological model consisting of a spring parallel arranged with a series of another spring and a dashpot, shown in Figure 4 (b) [13]. But in contrast to [13], this is not an appropriate model for the viscoelastic Bouncing Putty because the material satisfying this model behaves like a solid body and not like a fluid. The solid character of the viscoelastic model shown in Figure 4 (b) is set by the spring E_2. Due to the ideal elastic behavior of this spring, the material mapped by this model is not able to flow as the Bouncing Putty does. A better model, consisting of two parallel arranged series of a spring and a dashpot is presented in Figure 4 (c). Because there is a dashpot in each branch, the material subjected to this model is able to flow and actually it is a fluid. So, the four-parameter viscoelastic fluid model is focused.

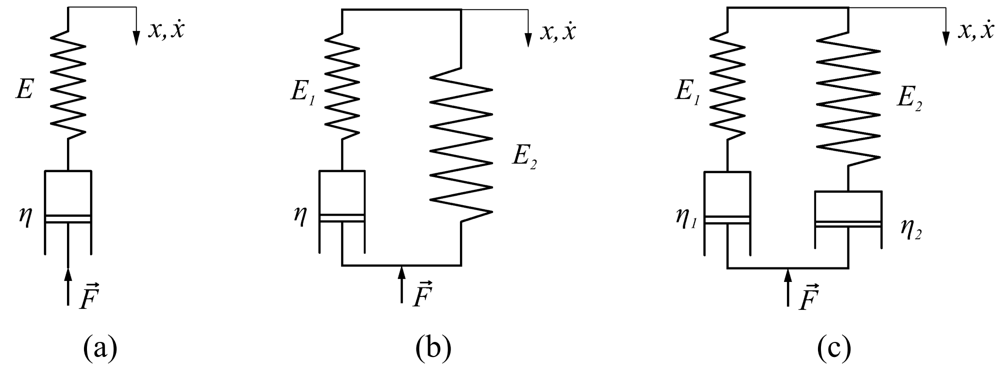


Figure 4: Rheological models for viscoelastic materials [12, 13, 15]; (a) – MAXWELL model; (b) – three-parameter model for a viscoelastic solid; (c) – four-parameter model for a viscoelastic fluid

The relations between force and displacement of this linear rheological model in Figure 4 (c) are given by the following consecutive differential equation (dots over the symbols represent derivates with respect to time t):

$$F + \frac{E_1\eta_2 + E_2\eta_1}{E_1E_2}\dot{F} + \frac{\eta_1\eta_2}{E_1E_2}\ddot{F} = (\eta_1 + \eta_2)\,\dot{x} + \frac{\eta_1\eta_2(E_1 + E_2)}{E_1E_2}\,\ddot{x} \tag{1}$$

Dependent on the experiments, there are different solutions of this linear differential equation (1). To analyze linear viscoelastic materials the German standard DIN 13343 introduces basic experiments, where either the stress or the deformation is applied as load to the material [16]. In the residual deformation test, which is the basic experiment in this work, the deformation velocity on the probe is set as step function and the force reaction is measured [16]. The measured force reaction contains information to characterize the viscoelastic material behavior. A detailed description of the test is documented in section 3.

Solving the differential equation (1) requires setting the deformation velocity to a constant value v_0 at time $t > 0$ s according to the residual deformation test. The velocity and the displacement as input of the system are qualitatively presented in Figure 5 (a) and (b). Using the Heaviside-Function yields the condition:

$$\dot{x}(t) = v_0\,H(t) \quad \dot{x}(0) = 0 \tag{2}$$

Under assumption of only positive time values equation (1) is now:

$$F + \frac{E_1\eta_2 + E_2\eta_1}{E_1E_2}\dot{F} + \frac{\eta_1\eta_2}{E_1E_2}\ddot{F} = (\eta_1 + \eta_2)\,v_0 \tag{3}$$

The solution of equation (3) is the step response for one certain velocity v_0, as follows:

$$F(t)|_{v_0=\text{const.}} = v_0\,\eta_1\left(1 - e^{-(E_1/\eta_1)\,t}\right) + v_0\,\eta_2\left(1 - e^{-(E_2/\eta_2)\,t}\right) \tag{4}$$

Additionally to the force-time function of the four-parameter fluid model (Figure 4 (c)) in qualitative sense, the characteristic of the simple MAXWELL model (Figure 4 (b)) is also presented in Figure 5 (c).

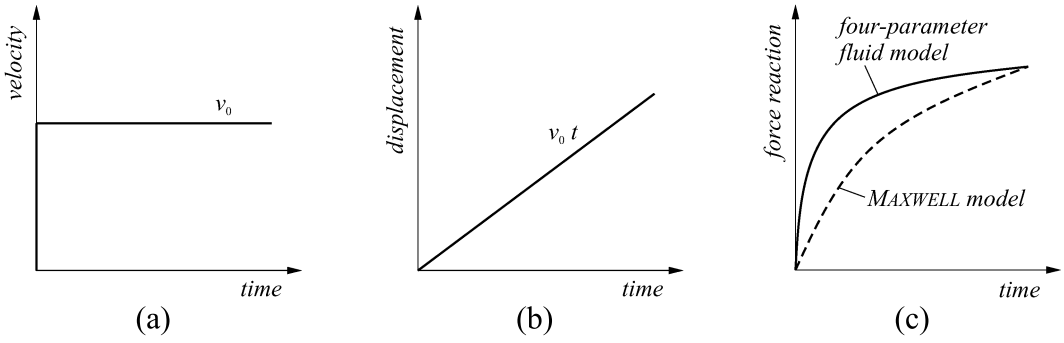


Figure 5: Qualitative input and output curves of the rheological models; (a) –input deformation velocity; (b) – input displacement; (c) – output force reaction for MAXWELL and four-parameter fluid model

The force-time functions of the MAXWELL and the four-parameter fluid model in Figure 5 (c) are different. Curve of four-parameter model (equation (4)), has two exponential functions gaining an intensively rising section at the beginning followed by a less rising section. Contrary, the force-time function of the MAXWELL model rises continuously because of only one exponential function. In section 4, the reason why the four-parameter model is used for the Bouncing Putty instead of the simple MAXWELL model is shown.
The elasticity and the viscosity of the Bouncing Putty are represented by the model parameters E_1, E_2, η_1 and η_2. But these parameters are unknown and it is not possible to measure them directly. Noticing that the force and the time data are measured due to experiments (section 3), a curve fitting of the theoretical function $F(t)|_{v0\,=\,\mathrm{const.}}$ and the testing data f_i is performed. Using least squares method yields the unknown parameters for any constant velocity.

$$\sum_{i=1}^{N}(F(t_i, E_1, E_2, \eta_1, \eta_2) - f_i)^2 \quad \longrightarrow \quad \mathrm{Min} \quad \Rightarrow \quad E_1, E_2, \eta_1, \eta_2\, |_{v_0=\mathrm{const.}} \tag{5}$$

As a result of equation (5) the parameters E_1, E_2, η_1 and η_2 are determined for one constant velocity v_0. But, remembering the dilatancy or shear-thickening as a velocity-dependent effect, the evaluation of one set E_1, E_2, η_1, η_2 is not enough to describe the elastic and viscous properties of Bouncing Putty. Only a variety of experiments with different velocities provides the velocity dependency of the rheological properties. This dependency will be presented in section 4.

3. EXPERIMENT AND MEASUREMENT

As pointed out in section 2, quantifying values of the four model parameters E_1, E_2, η_1 and η_2 requires experimental data of the time-dependent force reaction $F(t)|_{v0\,=\,\mathrm{const.}}$. Experimental data in this work are limited to the residual deformation test, based on the basic experiments in [16]. All experiments are performed using a servo-hydraulic material-testing machine (Zwick/Roell HB 100) equipped with a measurement setup, shown in Figure 6 (a). At initial position the piston is pressed into the 100 g sample of Bouncing Putty. During the test, the testing machine presses the piston 2 mm into the Bouncing Putty at room temperature. A dynamic force sensor measures the time-dependent force reaction, which is transmitted by the Bouncing Putty. The test is performed with a "super soft" and a "strong" mixture of Bouncing Putty [17] and velocities of 0.6 mm/s, 2 mm/s and 8 mm/s.

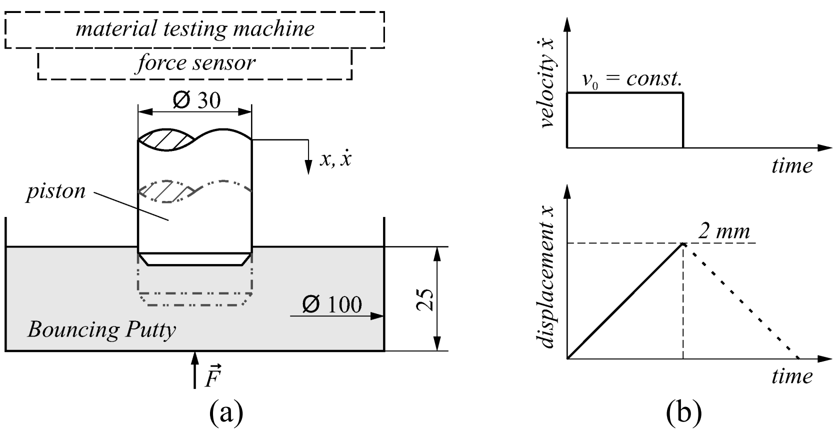
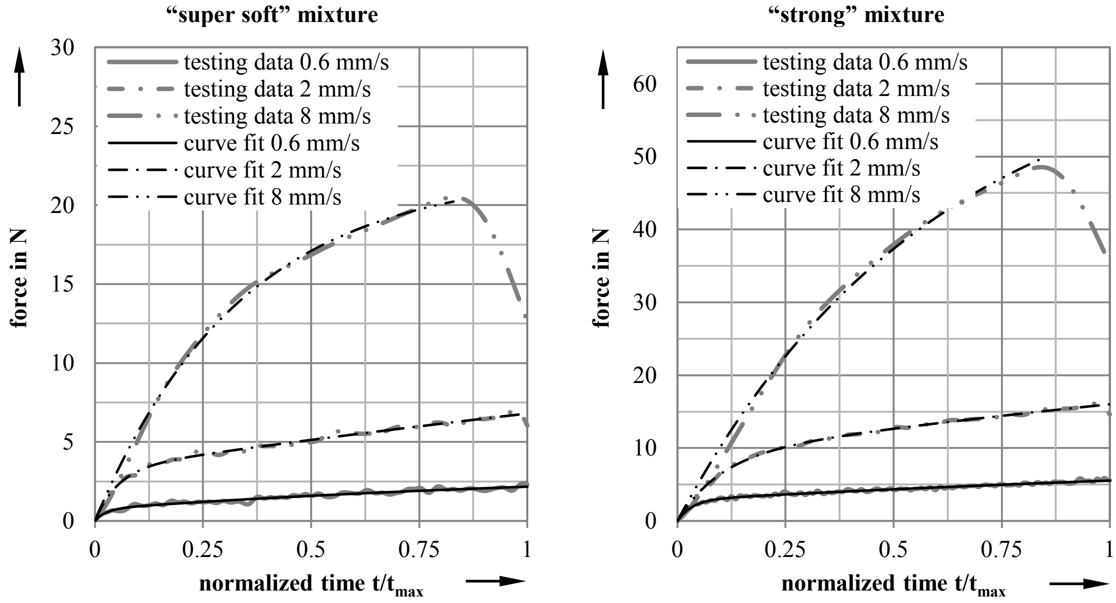


Figure 6: Measurement setup for servo-hydraulic material-testing machine and testing conditions; (a) – Setup consisting of a cylindrical piston and a container filled with the Bouncing Putty; (b) – input curves of load regime applied by the testing machine

4. RESULTS

In this section the results of the residual deformation test and the curve fitting according to the four-parameter viscoelastic fluid model are presented. The testing data and the curve fits of the “super soft” and the “strong” mixture for 0.6 mm/s, 2 mm/s and 8 mm/s are illustrated in Figure 7 (a) and (b).



(a) (b)

Figure 7: Testing data and curve fits of time-dependent force reaction with normalized time axis for different velocities; (a) – “super soft” mixture of Bouncing Putty [17]; (b) – “strong” mixture of Bouncing Putty [17]

Because of the different velocities but the constant residual deformation of 2 mm for all tests the recording time changes. To simplify the comparison of the results, the recording time is normalized with the time maximum of each test, which is the residual deformation divided by the actual velocity v_0. So, in the diagrams in Figure 7 the time axis is scaled from zero to one. All testing data curves in Figure 7 (gray color) start in the origin and they run with a positive slope. Comparing the slope of the testing curves in Figure 7, the slope near by the origin is always greater than far from the origin. Especially the curves in Figure 7 (a) and (b) related to velocities of 0.6 mm/s and 2 mm/s verify this trend. But regarding the curves related to 8 mm/s, there is an exception of this trend for both mixtures. The residual deformation test requires a constant velocity over the whole recording time but the controller of the servo-hydraulic material testing machine brakes the motion of the piston prior the maximal displacement. This is the reason of the decreasing of the force-time curve, which is significantly marked for high velocities (8 mm/s). In consequence, the conditions of the test are not strictly observed, but for low velocities the effect can be neglected. Nevertheless the shape of the testing curves is similar for the different velocities and the two mixtures. After an intensively rising section at the beginning a less rising section follows. As it is illustrated in Figure 7 (a) increasing the velocity will generate higher forces. The maximum forces of the “super soft” mixture are about 2.5 N if v_0 is 0.6 mm/s, 7 N if v_0 is 2 mm/s and 21 N if v_0 is 8 mm/s. As the name of the mixture suggests, the “strong” mixture provides higher forces than the “super soft” mixture. For “strong” mixture the force maximums are 6 N if v_0 is 0.6 mm/s, 16 N if v_0 is 2 mm/s and 48 N if v_0 is 8 mm/s. A comparison of the mixtures yields a factor between the force maximums of “super soft” and “strong” of approximately 2.5. Not only the force maximum but also other force values differ by this factor.

Using equation (5), the testing data curves are fitted by the theoretical model-based function (4) to determine the elasticity and viscosity parameters. The four-parameter fluid model, presented in Figure 4 (c), contains two elasticity parameters (E_1, E_2) and two viscosity parameters (η_1, η_2). For each of the six testing data curves (two mixtures and each tested by three velocities) there will be one fit providing the parameters E_1, E_2, η_1 and η_2 of the linear rheological model. The parameters are evaluated by optimization of the sum of least squares and mentioned in Table 2.

Table 2: Elasticity and viscosity parameters of "super soft" and "strong" mixture of Bouncing Putty [17] due to model-based optimization for different velocities

mixture	parameter	v_0 = 0.6 mm/s	v_0 = 2 mm/s	v_0 = 8 mm/s
"super soft"	E_1	28.77	30.59	32.14
	E_2	1.06	1.87	2.35
	η_1	1.40	1.65	2.38
	η_2	5.49	13.73	29.39
"strong"	E_1	42.43	52.25	55.62
	E_2	1.57	2.11	5.91
	η_1	4.83	5.49	6.04
	η_2	15.4	17.36	36.25

Using these parameters achieves a well accordance between the testing data and the theoretical four-parameter viscoelastic fluid model Figure 4 (c). In the diagrams of Figure 7 (a) and (b) there are only small offsets of the gray (testing data) and black (model-based) curves. Hence, the four-parameter viscoelastic fluid model is practical to map the material behavior of Bouncing Putty under assumption of a constant deformation velocity. The plots in Figure 8 demonstrate the dependency of the velocity on the elasticity and viscosity parameters.

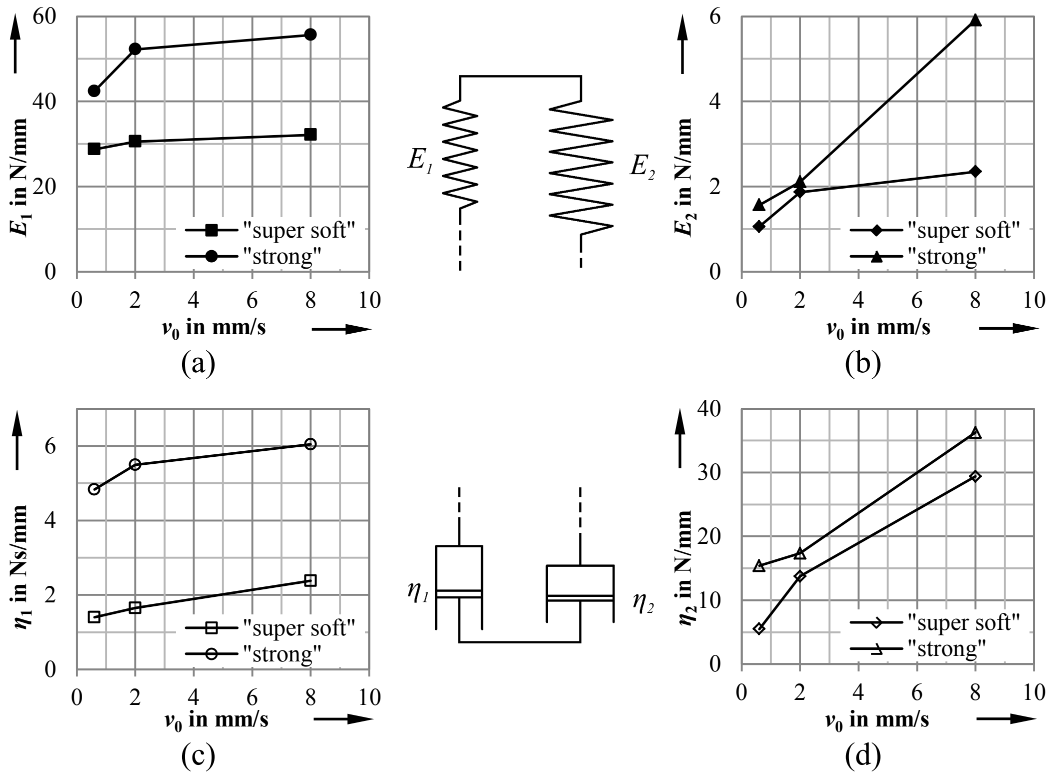
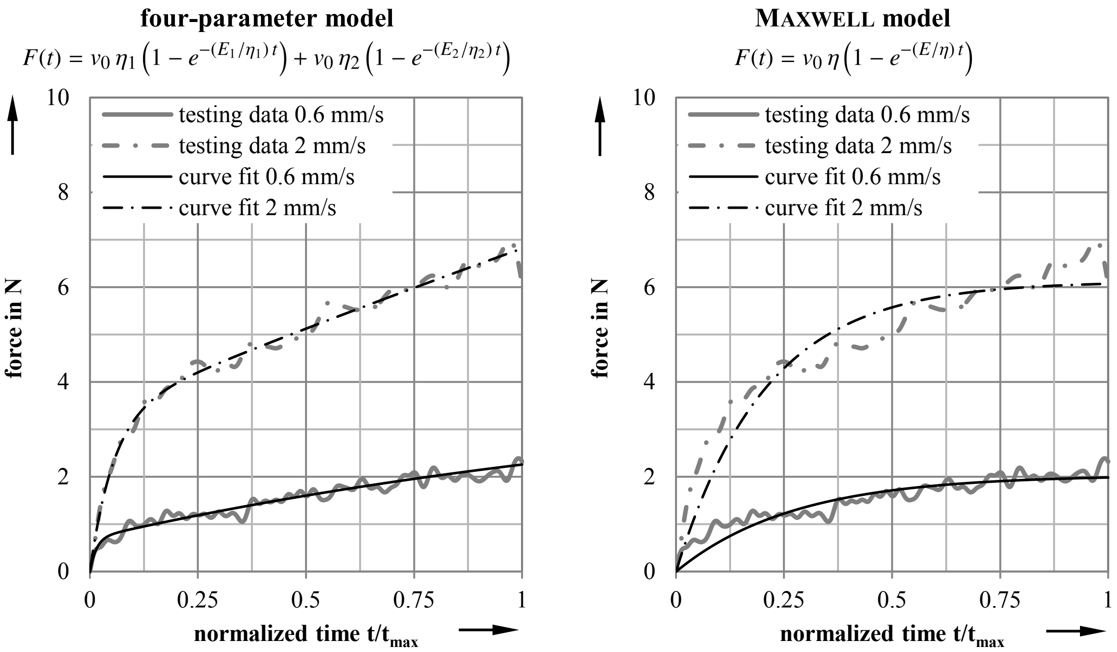


Figure 8: Velocity dependency of the elasticity and viscosity parameters of four-parameter fluid model; (a) – elasticity parameter E_1; (b) – elasticity parameter E_2; (c) – viscosity parameter η_1; (d) – viscosity parameter η_2

The elasticity parameters (E_1, E_2) and the viscosity parameters (η_1, η_2) rise with an increasing deformation velocity. Comparing the elasticity parameters, E_1 is much bigger than E_2 for all velocities v_0. In a reversed order, the viscosity parameter η_2 is greater than η_1. So, a hard spring and a low damper build one branch of the rheological model. The other branch is built by a weak spring and a strong damper. To determine a functional tendency of the values is very difficult. Assuming a linear relation between the deformation velocity and the parameters, the slopes of the elasticity E_1 and the viscosity η_2 are much higher than the slopes of the corresponding parameters E_2 and η_1.
As mentioned in section 2, the MAXWELL model is impractical to map the viscoelastic behavior of Bouncing Putty. This is demonstrated by a comparison of the fitted curves of the “super soft” mixture. Especially the curves related to v_0 = 0.6 mm/s and v_0 = 2 mm/s show the differences because of the intensively rising at the beginning followed by the less rising curve segment. In comparison to the four-parameter model curve fits in Figure 9 (a), where testing data and curve fits coincide, the curve fits of the MAXWELL model in Figure 9 (b) mismatch to the testing data.



(a) (b)

Figure 9: Comparison of model-based curve fitting of “super soft” mixture; (a) – four-parameter model; (b) – MAXWELL model

5. DISCUSSION AND CONCLUSION

The residual deformation test, where a constant deformation velocity is applied to a sample of material and the force reaction is measured, is one method to analyze the Bouncing Putty. Due to the characteristics of the testing data curves the Bouncing Putty is characterized as a viscoelastic material, meaning it exhibits elastic and viscous properties. For a constant deformation velocity the force-time behavior in qualitative sense is presented in Figure 10 (a). To describe the viscoelastic behavior in detail a four-parameter rheological model is used. This model (Figure 4 (c)) is able to map the testing data and provides rheological parameters quantifying the elasticity and the viscosity of Bouncing Putty. Because there are two elasticity and two viscosity parameters, the effect of dilatancy, as illustrated in Figure 3, is not equally recognized in the results. Instead, the dependency of the deformation velocity on the parameters is shown in Figure 8. The rise of all material parameters with an increasing deformation velocity demonstrates the shear-thickening effect. A schematic chart in Figure 10 (b) summarizes the dilatant behavior of the Bouncing Putty according to the four-parameter model, noticing that velocity dependency is assumed to be linear. Determining the shape of the function exactly requires more testing data, especially for higher velocities. But due to the braking of the piston, the residual deformation test performed by the testing machine will cause wrong testing data for high velocities. This will generate curve fits and respectively rheological parameters with errors. Consequently a new measuring configuration will be obligatory for higher deformation velocities.

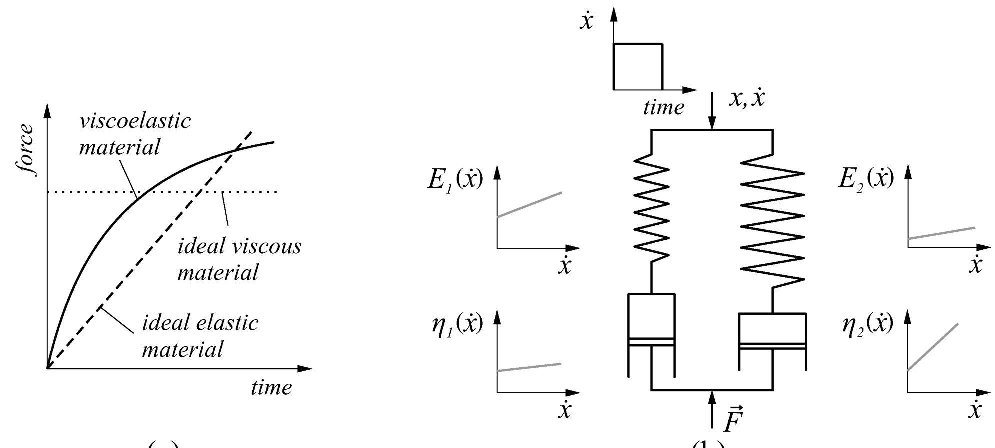


Figure 10: Illustration of viscoelastic material behavior; (a) – overview on force-time characteristics of elastic, viscous and viscoelastic material; (b) – dilatancy of Bouncing Putty according to four-parameter fluid model

The evaluated parameters confirm the phenomenological observations of the material behavior of Bouncing Putty. A sample of Bouncing Putty flows, if no load is applied, because it behaves initially like a liquid. This requires a zero-viscosity [12, 18], which is the resulting viscosity of η_1 and η_2. In reality the flow ends after a certain time, which is represented by the springs in the model. Deforming the sample with an increasing velocity the Bouncing Putty hardens and becomes more elastic until it is a pseudo-solid material. The rising of all parameters, but especially η_2, is a model-based explanation of this hardening. Finally, the four-parameter model is practical to describe the real behavior of the Bouncing Putty and to quantify elasticity and viscosity values. In general, the Bouncing Putty is a viscoelastic fluid, which has more elastic and less viscous properties for high strain rates.

6. SUMMARY AND OUTLOOK

The elastic and viscous properties of Bouncing Putty are investigated and determined in this work. A residual deformation test with several constant deformation velocities is performed, using a servo-hydraulic material-testing machine, and the time-dependent force, which is transmitted by the material, is measured. According to the testing data an appropriate rheological model is used to fit the data in functional relations and to quantify the model parameters. This model provides four velocity-depending parameters for elasticity and viscosity. As a result of the evaluation, the model is practical to explain the phenomenological effects of the dilatant Bouncing Putty. Further work is directed to new experiments to have more detailed functional relations to describe the velocity dependency of the parameters.

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