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Research in hydraulic brake components and operational factors influencing the hysteresis losses

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Abstract: Up-to-date automotive brake systems place stringent requirements upon the performance, reliability, and active safety. Such advanced systems as antilock braking systems (ABS), the electronic stability programme system, and the anti-slip control system assist a driver in ensuring driving safety under many conditions. The influence of the brake components on active safety systems is mainly determined through the hysteresis loop width. Among other negative outcomes, this parameter limits the possible frequency of cyclic braking during ABS operation.

This paper presents an experimental analysis of the factors influencing the hysteresis pressure losses in a hydraulic brake system. The factors under investigations are the brake pedal stroke velocity, the gaps between the brake pads and the brake disc, and the configuration of the brake system. Experiments were carried out on the brake test equipment at the Automotive Engineering Department, Faculty for Mechanical Engineering, Technische Universität Ilmenau, Germany.

Keywords: hysteresis, hydraulic system, brake, AMESim

1 INTRODUCTION

The stability of the static and dynamic characteristics of a brake system depends in many ways on the frictional forces arising in a master cylinder, valves, pipelines, and other brake elements. The wear, gaps, and slacks on the brake devices are also caused by these inner frictional forces. Traditionally, the internal friction is seldom considered in brake dynamics except for evaluating the brake torque oscillations. However, the most critical outcome of the above influence is connected with a hysteresis phenomenon.

The hysteresis takes place by changing the sign of the frictional forces at the brake release mode and occurs because a pressing force on the friction surface of the brake pad is still retained by the time of the brake release.

A literature survey has revealed that the problems of reduction in hysteresis losses in the brake system components have been actively investigated in many respects. The analysis performed shows that the hysteresis impacts in one way or another on the following:

(a) the operation of the brake calipers and valves [**1–3**];
(b) the performance of the disc brakes, especially for heavy vehicles [**4**, **5**];
(c) the response speed of anti-lock braking systems [**6–8**].

The presented paper describes the investigations of hydraulic brake system components from the point of view of the hysteresis losses and the operational factors having an influence on the hysteresis value. The research procedures consisted of the combined application of bench testing with the subsequent simulation. The detailed description of the test bench used during the investigations has been presented in reference [**9**]. For simulation purposes, the model of an automotive brake system was created using the AMESim software environment. The general approach to the model development was

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Table 1 Test bench data

Test concept	Servohydraulic
Pedal force domain	$F = 0$–5000 N
Pedal velocity domain	$v = 0$–1000 mm/s
Maximal frequency of sensor scanning	6 kHz
Test modes	Force control; stroke control; ramp-shaped control effort; oscillating control effort
Safety measures	Emergency switch; adjustable mechanical force limiter

based on the methodologies discussed in references [**10**] to [**13**].

The general idea of the presented research lies in the definition of the share of hysteresis losses by the main components of a typical automotive brake system such as the master cylinder, brake gear, or booster. This approach allows weak points in a brake hydraulic chain to be found and recommendations are given for the development of an advanced brake system, in which the principal hysteresis-generating elements may be replaced by similar mechatronic devices.

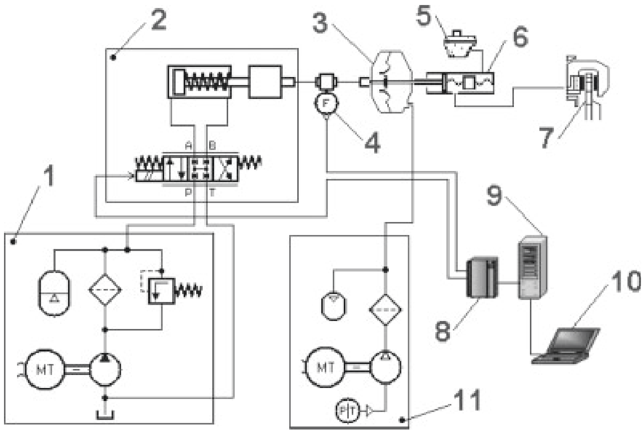

Fig. 1 Structure of the component test bench for braking systems: 1, hydraulic unit; 2, brake robot actuator; 3, vacuum booster; 4, force and displacement sensors; 5, overflow tank; 6, brake master cylinder; 7, wheel brake with disc; 8, signal converter; 9, real-time computer; 10, laptop with suitable software; 11, vacuum unit

2 TEST PROCEDURE

The experimental investigations of hydraulic brake components were performed on the special test bench configuration with a 'brake robot', allowing the dynamic and precise reconstruction of the brake pedal action. Table 1 shows the main data of the test stand.

The following brake system components and equipment were used for experiments: brake robot actuator, brake master cylinder, vacuum booster, wheel brake, signal converter, real-time computer, laptop with controlling software, force and pressure sensors, hydraulic and vacuum pump, hydraulic and vacuum tank, charging unit, storage batteries, overflow tank, and hydraulic and air pipes (Figs 1 and 2).

The research work consists of several stages.

1. The influence of the brake pedal stroke velocity on hysteresis value is estimated; next the operating modes were chosen:

   (a) steady state braking;
   (b) service braking;
   (c) emergency braking.

None of these braking modes has exact limits on the brake pedal stroke velocity. From statistical and practical data, the following intervals of brake pedal stroke velocity were taken: steady state braking, 5–15 mm/s; service braking, 50–200 mm/s; emergency braking, 1000–1300 mm/s [**14**]. Each of the velocity intervals was divided into several parts during experiments to obtain hysteresis characteristics for all subintervals.

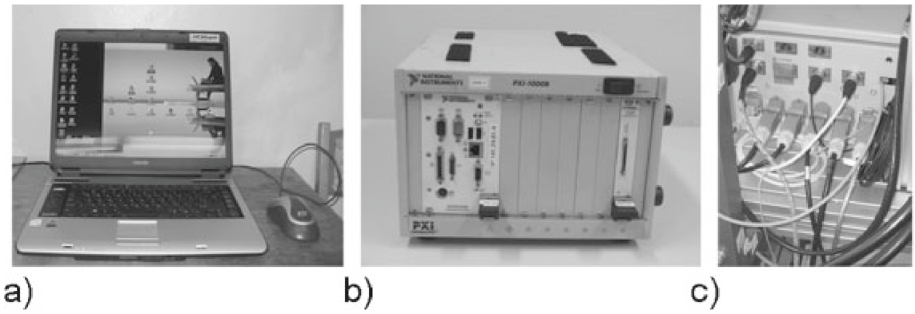

Fig. 2 Measurement system: (a) laptop with suitable software; (b) real-time computer; (c) signal converter

The simulation of the brake pedal action in the considered brake system configurations was realized using a special actuator allowing control of the rod velocity and displacement. The actuator rod is directly connected to the rod of the brake master cylinder or booster rod. Taking into consideration the pedal ratio U_{ped}, calculated on the basis of geometric parameters (Fig. 3), the values of the rod velocities for brake pedal stroke velocity during the emergency, service, and steady state braking were obtained (Table 2).

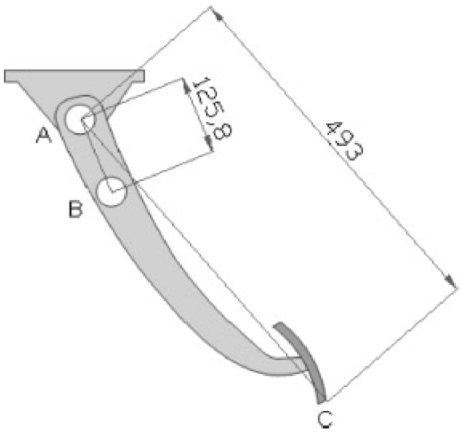

Fig. 3 Geometric parameters of the brake pedal in millimetres

Thus, the pedal ratio is

$$U_{ped} = \frac{AC}{AB} = \frac{493}{125.8} = 3.92 \quad (1)$$

2. To estimate the influence of different brake system components (vacuum booster, disc brake, brake master cylinder, and pipelines) on the value of the hysteresis losses, the sequential exclusion of these components from the hydraulic system configuration with one disc brake was carried out with subsequent repetition of all experiments:

   stage 1: full brake system (Fig. 4);
   stage 2: configuration from stage 1 without the brake booster (Fig. 5);

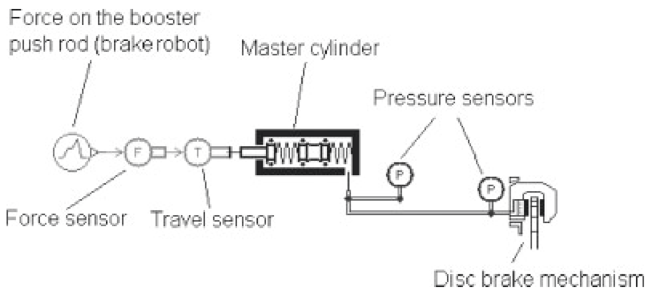

Fig. 5 The brake system configuration at stage 2

Table 2 Brake pedal stroke velocity and actuator rod velocity

	Velocity (mm/s)					
	Emergency braking		Service braking		Steady state braking	
Test	Pedal	Brake robot	Pedal	Brake robot	Pedal	Brake robot
1	1000	255	50	13	5	1
2	1100	281	75	19	10	3
3	1200	306	100	26	15	4
4	1300	332	125	32		
5			150	38		
6			175	45		
7			200	51		

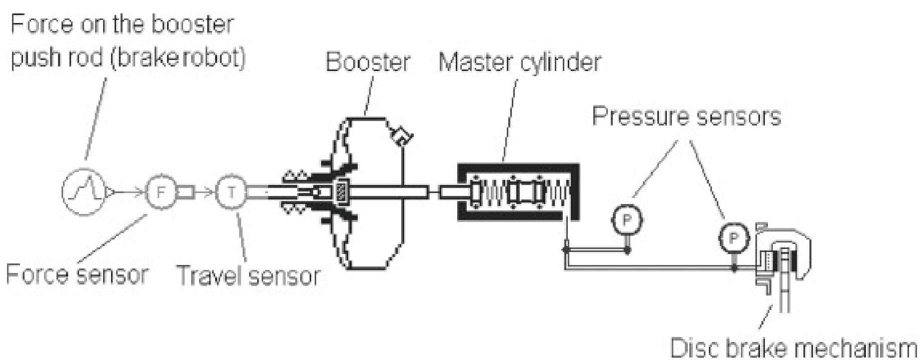

Fig. 4 The brake system configuration at stage 1

stage 3: configuration from stage 2 without the disc brake mechanism (Fig. 6);
stage 4: configuration with the hydraulic pipes only (Fig. 7).

3 ANALYSIS OF EXPERIMENTAL RESULTS

The hysteresis, as applied to the performed brake-testing procedures, is derived from the dependence between the pressure actuating on the brake gear and the brake pedal (actuator rod) displacement. Because of the strong non-linearity of this phenomenon, the method of areas limited by the hysteresis characteristic curves during the braking and brake release and abscissa axis is used for calculation of the hysteresis losses.

The hysteresis values were calculated from the area inside the build-up and release curves. First, the polynomials of quintic orders describing the experimental curves have been formed using special statistical software. Then the above-mentioned polynomials have been integrated.

The pressure in the brake gear can be successfully converted to the braking force via a reduced coefficient of conversion. The force integral over the displacement gives the work done by the brake gear to create the braking pressure; the hysteresis losses are equivalent to the parasitic work of the brake gear. In addition, the pressure integral over the displacement gives the hysteresis work reduced to the pressure (specific pressure hysteresis).

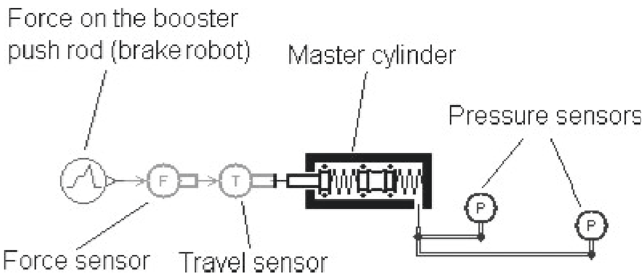

Fig. 6 The brake system configuration at stage 3

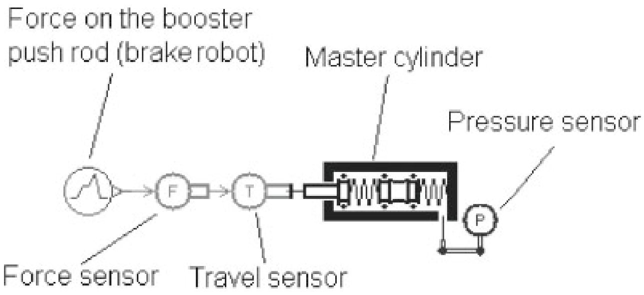

Fig. 7 The brake system configuration at stage 4

3.1 Influence of the braking pressure velocity on the hysteresis value

It may be deduced from analysis of the obtained experimental data that the character of the influence of the actuator rod velocity on the value of the hysteresis losses in the hydraulic brake system is the same during all experimental stages. The diagram (Figs 8 to 13) and numerical values below

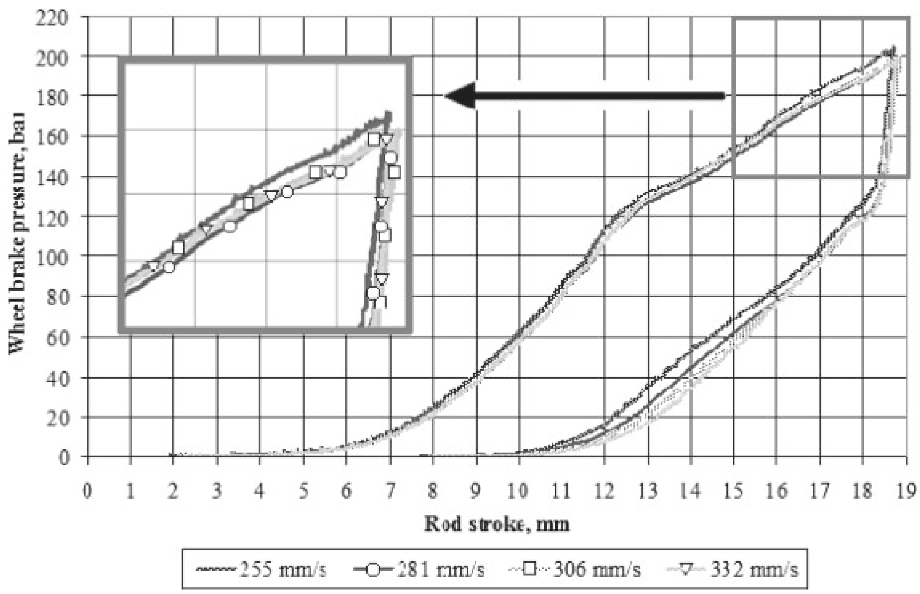

Fig. 8 Influence of the braking velocity on the hysteresis characteristic without gaps between the brake pads and disc during emergency braking at stage 1

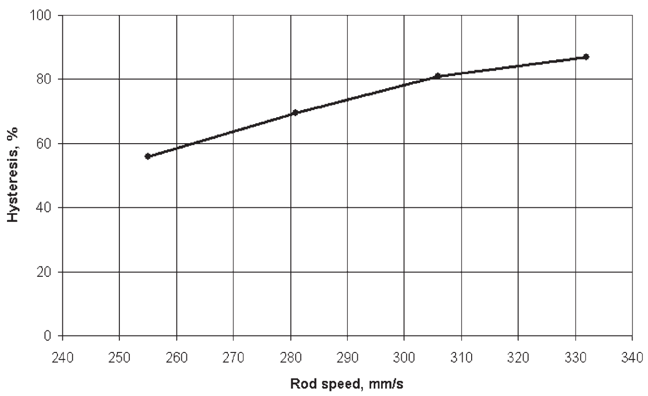

Fig. 9 Dependence of the change in the value of the hysteresis losses on the braking velocity during emergency braking at stage 1 without gaps

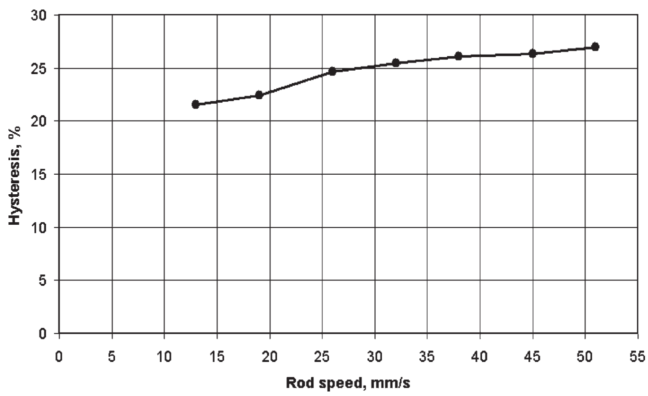

Fig. 11 Dependence of the change in the value of the hysteresis losses on the braking velocity during service braking at stage 1 without gaps

correspond to the brake system configuration in Fig. 4. Results for other configurations are shown in Appendix 2.

Table 3 shows the numerical values of the actuator rod velocity and hysteresis values during emergency braking without gaps. The same parameters for service braking are presented in Table 4. The test results for steady state braking without gaps are shown in Table 5. It was determined that the influence of the actuator rod velocity on the hysteresis characteristic is minimal in this case. From this standpoint, Fig. 12 displays the results obtained only for a rod velocity of 4 mm/s.

It can be seen as a consequence for all braking modes that the increase in the actuator rod velocity requires a quasi-proportional extension of the hysteresis value.

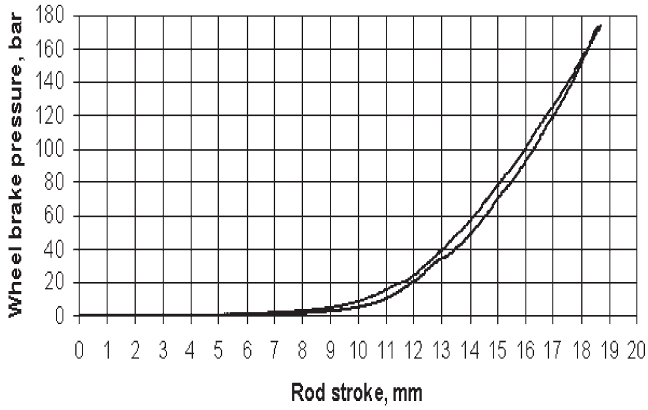

Fig. 12 Hysteresis characteristic of the brake system without gaps during steady state braking at stage 1 with a rod velocity of 4 mm/s

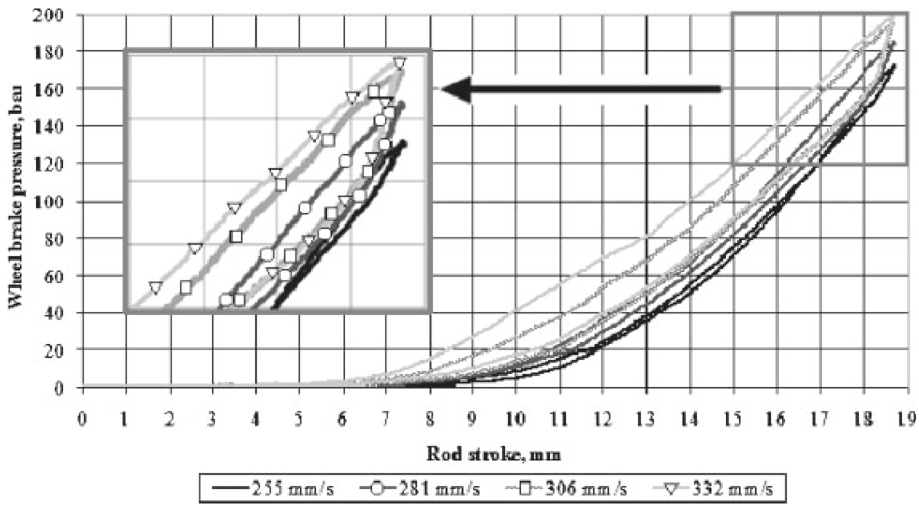

Fig. 10 Influence of the braking velocity on the hysteresis characteristic without gaps during service braking at stage 1

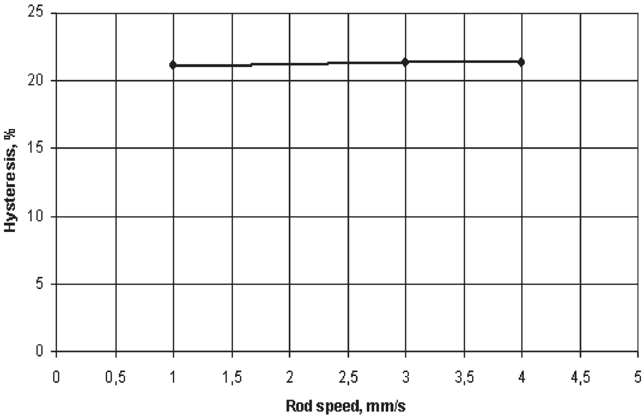

Fig. 13 Dependence of the change in the value of the hysteresis losses on the braking velocity during steady state braking at stage 1 without gaps

3.2 Influence of the gaps between the brake pads and wheel disc on the hysteresis value

In common with previous discussions, the influences of the gaps on the hysteresis value were similar for all experimental stages. The diagrams (Figs 14 to 16) and numerical values are given only for the brake system configuration at stage 1 of the experiments. The results for other tested configurations are given in Appendix 2.

Table 6 displays the actuator rod velocity and hysteresis values during emergency braking with and without gaps. It can be seen that the hysteresis losses increase, on average, by 7.16 per cent in this case. At the service braking mode this magnitude was an average of 5.65 per cent (Table 7).

The test results for the steady state braking reveal in turn the minimal influence on hysteresis (Table 8). As the braking velocity does not exceed

Table 3 Braking velocity and hysteresis losses during emergency braking at stage 1 without gaps

Velocity (mm/s)	Hysteresis value (%)
255	55.75
281	69.53
306	80.92
332	86.91

Table 4 Braking velocity and hysteresis losses during service braking at stage 1 without gaps

Velocity (mm/s)	Hysteresis value (%)
13	21.53
19	22.43
26	24.67
32	25.43
38	26.13
45	26.33
51	26.95

Table 5 Braking velocity and hysteresis losses during steady state braking at stage 1 without gaps

Velocity (mm/s)	Hysteresis value (%)
1	21.08
3	21.33
4	21.36

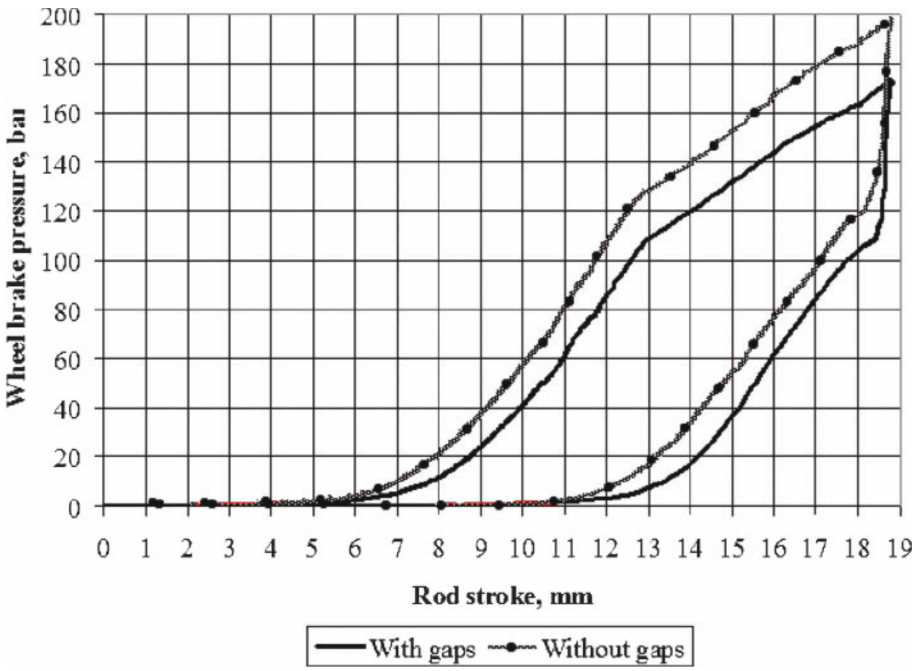

Fig. 14 Influence of the gaps on the hysteresis characteristic during emergency braking at stage 1 with a rod velocity of 332 mm/s

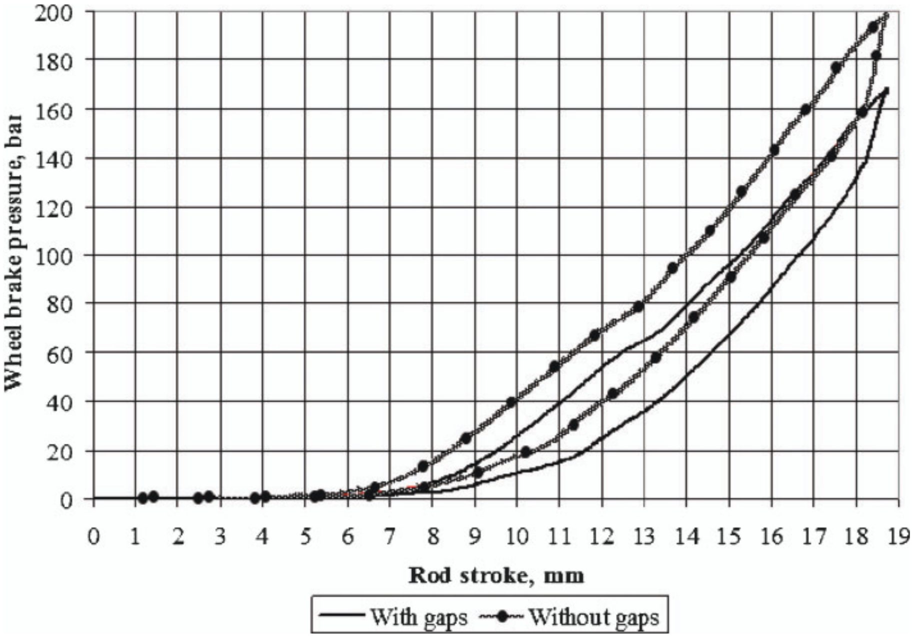

Fig. 15 Influence of the gaps on the hysteresis characteristic during service braking at stage 1 with a rod velocity of 51 mm/s

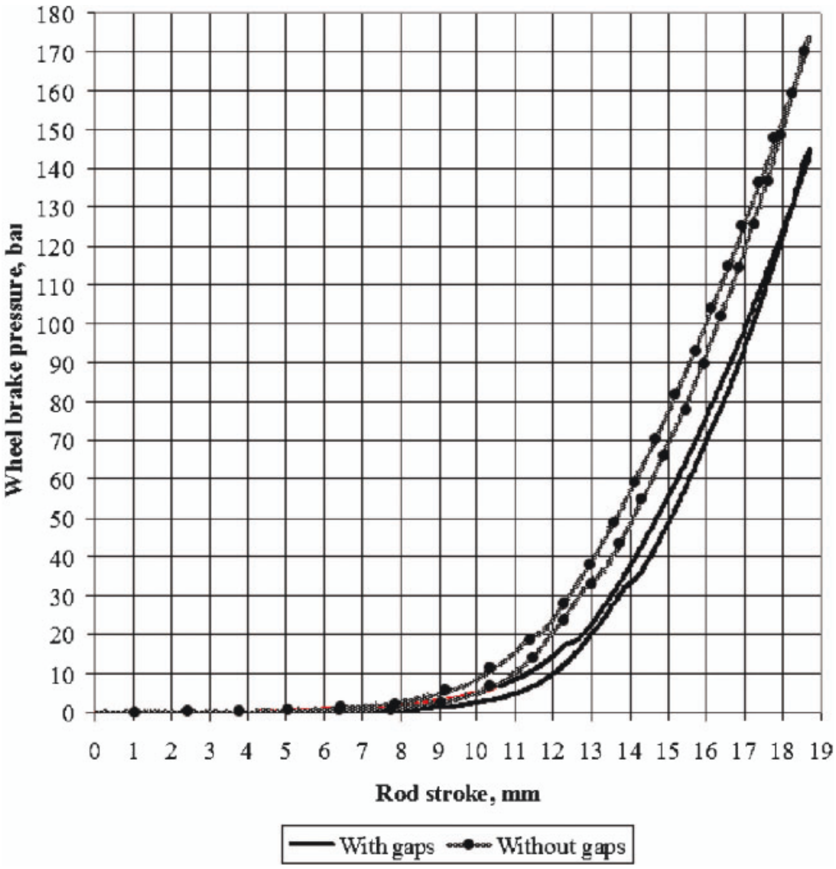

Fig. 16 Influence of the gaps on the hysteresis characteristic during steady state braking at stage 1 with a rod velocity of 4 mm/s

Table 6 Braking velocity and hysteresis losses during emergency braking with and without gaps

	Hysteresis value (%)		
Velocity (mm/s)	With gaps	Without gaps	Change in the system without gaps in comparison with the system with gaps (%)
255	61.26	55.75	5.51
281	76.88	69.53	7.35
306	88.33	80.92	7.41
332	92.78	86.91	5.87

Table 7 Braking velocity and hysteresis losses during service braking with and without gaps

	Hysteresis value (%)		
Velocity (mm/s)	With gaps	Without gaps	Change in the system without gaps in comparison with the system with gaps (%)
13	27.16	21.53	5.63
19	28.11	22.43	5.68
26	30.34	24.67	5.67
32	30.53	25.43	5.1
38	31.48	26.13	5.35
45	32.43	26.33	6.1
51	32.85	26.85	6

Table 8 Braking velocity and hysteresis losses during steady state braking with and without gaps

	Hysteresis value (%)		
Velocity (mm/s)	With gaps	Without gaps	Change in the system without gaps in comparison with the system with gaps (%)
1	25.94	21.08	4.86
3	26.06	21.33	4.73
4	26.08	21.36	4.72

4 mm/s, the hysteresis value changes are not greater than 4.8 per cent.

3.3 Theoretical justification of the pressure fluctuation in the brake pipeline at high velocities of the brake pedal stroke

Stage 3 of the experiments (without the disc brake) revealed the pressure fluctuation in the brake pipeline at emergency braking. Because of removal of the disc brake, the hydraulic pipeline was blanked off. As a result, the stiffness of the system rose sharply. It was found from a literature survey that such wave processes in pipelines have been investigated and proved in detail in references [**15**] to [**17**]. The sharp displacement of the actuator rod at a high velocity causes an elastic wave in the pipeline. This elastic wave propagates along the full length of the pipeline and reflects from the bullnose at the end of the tube. Then the wave reflected from the piston returns to the beginning of the pipeline where it again reflects. A new wave, which has then formed, approaches the bullnose and reflects from it. By the interaction with the bullnose, new waves eventually occur. The described phenomenon becomes especially complicated in the case of a non-uniform pipeline.

To verify the factors influencing the nature of the apparent fluctuations in brake pressure, the model of wave processes in pipelines has been investigated using AMESim software for the ufilized configuration of hydraulic brake system (a pipe diameter of 4.5 mm and a wall thickness of 0.1 mm). The corresponding AMESim model of the pipeline is presented in Fig. 17. The AMESim pipeline model has the following distinctive features.

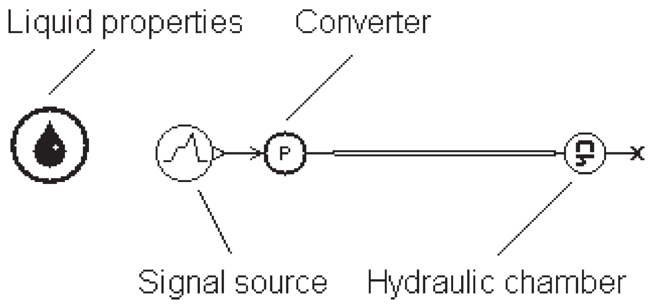

Fig. 17 Model of the pipeline in AMESim software

1. Compressibility of the fluid and expansion of the pipe wall with pressure are taken into account by using an effective bulk modulus. This can be calculated from the wall thickness and Young's modulus for the wall material.
2. Pipe friction is taken into account using a friction factor based on the Reynolds number and relative roughness. The value of absolute roughness for the simulated pipeline is 1.5 μm.
3. Inertia of the fluid is taken into account and wave dynamics equations are used.

The full system of equations describing the hydraulic components in AMESim software has been given in reference [**18**]. The main equations adapted to the investigated brake hydraulic system are cited in Appendix 3. Figure 18 gives the results of the pipeline tests in AMESim.

The length-dependent values of the amplitudes and frequencies of liquid fluctuations in the pipeline are presented in Table 9.

Under real conditions the investigated phenomenon of the pressure fluctuation in the pipelines does not have an essential influence on the brake

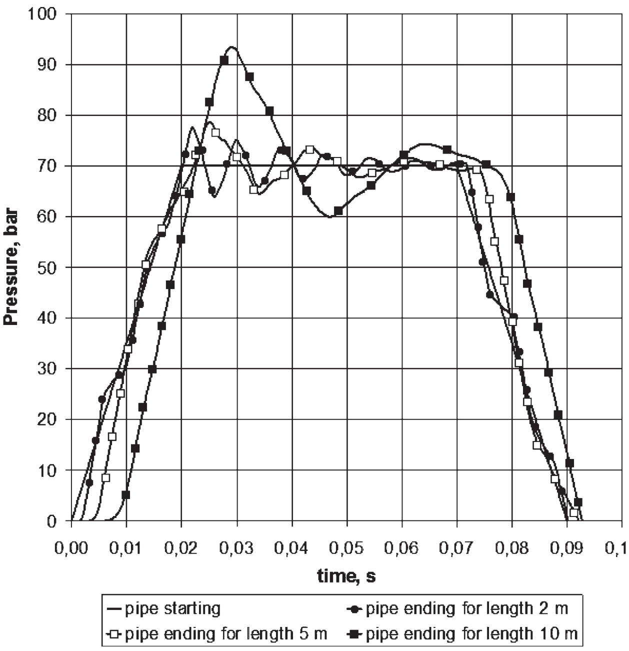

Fig. 18 Influence of the pipe length on pressure fluctuations

system operation and does not affect its reliability and working capacity. As the real system stiffness is essentially smaller than the stiffness of the considered system (with a bullnose on the pipeline end), the pressure increases with a high speed in very rare cases (emergency braking).

Hence, this information should be considered as helpful to a greater extent, but it can be useful for the calculation of length and thickness of the hydraulic pipelines for vehicle brake systems.

3.4 Influence of different brake system components on the value of the hysteresis losses

One of the most important tasks of the experimental investigations performed was to assess the influence of single components of a hydraulic brake system such as the vacuum booster, disc brake gear, brake master cylinder, and pipelines on the value of the hysteresis losses.

The test data analysis at every one of four stages allows this task to be solved because of the sequential exclusion of the above-mentioned components from the initial system configuration (see Appendix 2).

It should be noted that the greatest influences on hysteresis (from the value of the total losses) are as follows:

(a) vacuum booster, near 10.7 per cent;
(b) disc brake mechanism, near 8.7 per cent;
(c) brake master cylinder, near 4.3 per cent;
(d) pipelines, near 2.0 per cent.

4 CONCLUSIONS

From the test results it can be deduced that the component test bench for brake systems allows the efficient estimation of parameters and working capacity both for the full-length hydraulic brake system and for its single components. The original test procedure was developed for the component test bench for brake systems. To calculate the hysteresis losses, the method of areas limited by

Table 9 Amplitude and frequency of liquid fluctuations in the pipeline

Pipe length (m)	Amplitude (MPa)	Change in comparison with a pipe of 2 m length (%)	Frequency (Hz)	Change in comparison with a pipe of 2 m length (%)
2	13.57	0	125	0
5	14.06	3.61	54	56.56
10	33.36	137.27	28.4	47.41

the hysteresis characteristic curves of the brake pressure build-up and release and by the abscissa axis was used in this work.

The first investigated problem (stage 1) was the influence of the brake pedal stroke velocity (actuator rod velocity) on the hysteresis value. Regarding the analysis of the experimental data obtained, it was found that, when the actuator rod velocity increases during both the service and emergency braking, the hysteresis value increases nearly in proportion to this velocity growth (Tables 3 to 5).

The second area of the investigation (stage 2) covered the influence of the gaps between the brake pads and wheel disc on the hysteresis value. It was determined that the hysteresis losses in the hydraulic brake system with gaps between the brake pads and wheel disc increased on average by 7.16 per cent during emergency braking and by 5.65 per cent during service braking. During steady state braking, the influence of the gaps was minimal.

The research stage 3, which consisted of the experimental work (without the disc brake) together with the AMESim simulation for emergency braking, revealed a pressure fluctuation in the brake pipeline. Here the correlation between the pipeline length and the amplitude and frequency for fluctuations was found.

The last part of the presented work (stage 4) deals with an analysis of the influences of different braking system components on the hysteresis value. The corresponding effects were evaluated for each of the brake components.

The results obtained allow the formulation of recommendations for the design of long-term brake systems with reduced hysteresis losses and small response time.

The research performed made it possible to indicate 'trouble spots' in the vehicle hydraulic brake system from the viewpoint of hysteresis losses, i.e. the brake devices, which should be eliminated first through their replacement by 'brake-by-wire' components.

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APPENDIX 1

Notation

E	resulting elasticity modulus of the liquid–pipeline (or cylinder) complex
p	pressure of the liquid
t	time
x	coordinate along the pipeline axis

APPENDIX 2

The experimental results are given in Table 10.

APPENDIX 3

The used submodel of a pipeline is described by six internal pressure state variables and five internal flow rate state variables forming a staggered grid as follows: — C — $(R$ — $C)_{5x}$ —, where C is the equivalent chamber volume, R is the equivalent resistance in a pipe section; $5x$ indicates that a pipe is represented in the form of five sections with identical volume and resistance.

The following system of equations [**17**] was used for hydraulic brake system simulations in the AMESim environment. Some standard AMESim calculation procedures have been adapted to make a comprehensive analysis of wave processes in brake pipelines.

1. The volumetric compliance w_{comp} of a pipe is

$$w_{\mathrm{comp}} = \frac{2}{E}\left[\frac{r_{\mathrm{o}}^2 + r_{\mathrm{i}}^2}{r_{\mathrm{o}}^2 - r_{\mathrm{i}}^2} + \nu\left(1 - \frac{r_{\mathrm{i}}^2}{r_{\mathrm{o}}^2 + r_{\mathrm{i}}^2}\right)\right] \tag{2}$$

where r_{o} is the outer radius of a pipe, r_{i} is the inner radius of a pipe, $\nu = 0.3$ is Poisson's ratio, E is Young's modulus of the wall material of the pipe.

2. The volume V of the pipe is

$$V = V_0\left(1 + w_{\mathrm{comp}}p\right) \tag{3}$$

where V_0 is the volume of a pipe at a pressure of 0 Pa gauge, and p is the current pressure.

3. The effective bulk modulus of the brake liquid–pipe combination is

$$B_{\mathrm{eff}} = \frac{1}{1/B_{\mathrm{L}} + w_{\mathrm{comp}}} \tag{4}$$

where B_{L} is the bulk modulus of the brake liquid with the density ρ and is given by

$$B_{\mathrm{L}} = \rho\frac{\mathrm{d}p}{\mathrm{d}\rho} \tag{5}$$

4. The derivative of pressure is

$$\frac{\mathrm{d}p}{\mathrm{d}t} = \frac{B_{\mathrm{eff}}Q}{V} \tag{6}$$

where Q is the net flowrate into the volume.

5. The general derivative of the pressure at the internal pressure nodes is

$$\frac{\partial p}{\partial t} = -\frac{B_{\mathrm{eff}}}{A}\frac{\partial Q}{\partial x} \tag{7}$$

where A is the cross-sectional area of the pipe.

6. The general flowrate is

$$\frac{\partial Q}{\partial t} = \frac{A}{\rho}\frac{\partial p}{\partial x} - 9.81A\sin\theta - V\frac{\partial Q}{\partial x} - \frac{fQ^2\,\mathrm{sgn}(Q)}{2dA} \tag{8}$$

where d is the diameter of the section of pipe, θ is its inclination, and f is the friction factor.

Table 10 Experimental results

Stage	With or without gaps	Braking	Test	Stroke velocity (mm/s)	Hysteresis losses (%)
1	With gaps	Emergency braking	1	255	61,26
			2	281	76,88
			3	306	88,33
			4	332	92,78
		Service braking	1	13	27,16
			2	19	28,11
			3	26	30,34
			4	32	30,53
			5	38	31,48
			6	45	32,43
			7	51	32,85
		Steady state braking	1	1	25,94
			2	3	26,06
			3	4	26,08
	Without gaps	Emergency braking	1	255	55,75
			2	281	69,53
			3	306	80,92
			4	332	86,91
		Service braking	1	13	21,53
			2	19	22,43
			3	26	24,67
			4	32	25,43
			5	38	26,13
			6	45	26,33
			7	51	26,95
		Steady state braking	1	1	21,08
			2	3	21,33
			3	4	21,36
2	With gaps	Emergency braking	1	255	48,95
			2	281	58,84
			3	306	67,05
			4	332	81,21
		Service braking	1	13	17,20
			2	19	17,98
			3	26	18,75
			4	32	19,53
			5	39	20,31
			6	45	21,08
			7	51	21,86
		Steady state braking	1	1	17,03
			2	3	17,19
			3	4	17,25
	Without gaps	Emergency braking	1	255	43,84
			2	281	58,30
			3	306	64,75
			4	332	75,21
		Service braking	1	13	12,33
			2	19	13,07
			3	26	14,32
			4	32	15,06
			5	38	16,49
			6	45	17,23
			7	51	16,75
		Steady state braking	1	1	11,98
			2	3	12,03
			3	4	12,16

Table 10 (continued)

Stage	With or without gaps	Braking	Test	Stroke velocity (mm/s)	Hysteresis losses (%)
3	Without gaps	Emergency braking	1	255	33,32
			2	281	45,54
			3	306	57,76
			4	332	63,98
		Service braking	1	13	5,93
			2	19	6,48
			3	26	6,77
			4	32	7,05
			5	38	7,28
			6	45	7,57
			7	51	7,65
		Steady state braking	1	1	5,64
			2	3	5,69
			3	4	5,73
4	Without gaps	Emergency braking	1	255	30,56
			2	281	43,75
			3	306	56,94
			4	332	61,14
		Service braking	1	13	4,59
			2	19	4,84
			3	26	5,08
			4	32	5,33
			5	38	5,43
			6	45	5,68
			7	51	6,07
		Steady state braking	1	1	4,25*
			2	3	4,29*
			3	4	4,33*

*These data were used for the calculation of the average hysteresis losses value for the brake master cylinder.