

Ilmenau University of Technology
Faculty of Computer Science and Automation

A System of Autonomously Flying Helicopters for Load Transportation

Additional Equations

Markus Bernard

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Doctoral Adviser: Prof. Dr.-Ing. Armin Zimmermann
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1 Blade angle calculation, including the Bell-Hiller-Bar

The following equations describe the relation between the swash plate angle α_{fl}^{SP} , the Bell-Hiller-Bar angle γ_{mr1}^{BHB} and the main-rotor blade angle α_{mr1}^{BL} :

$$h_0 = -l_3 - l_7$$

$$h_1 = -l_7$$

$$h_2 = 0$$

$$x_{1,3} = x_1 - x_3$$

$$y_{1,3} = y_1 - y_3$$

$$x_0 = \sin(\alpha_{fl}^{SP}) (l_1 + l_2)$$

$$y_0 = \cos(\alpha_{fl}^{SP}) (l_1 + l_2) - l_3 - l_7$$

$$x_1 = \sin(\alpha_{fl}^{SP}) l_1$$

$$y_1 = \cos(\alpha_{fl}^{SP}) l_1 - l_3 - l_7$$

$$x_2 = \left(-l_3^2 x_{1,3}^2 + (x_{1,3} l_5^2 + 2l_6 x_{1,3} l_5 + l_6^2 x_{1,3} + (x_1 + x_3) (x_{1,3}^2 + y_{1,3}^2)) x_{1,3} + \sqrt{-x_{1,3}^2 (l_3^4 - 2((l_5 + l_6)^2 + x_{1,3}^2 + y_{1,3}^2) l_3^2 + ((l_5 + l_6)^2 + x_{1,3}^2 + y_{1,3}^2)^2} (y_3 - y_1) \right) / (2x_{1,3}^3 + 2x_{1,3} y_{1,3}^2)$$

$$y_2 = \left((y_3 - y_1) l_3^2 + l_5^2 y_{1,3} + l_6^2 y_{1,3} + 2l_5 l_6 y_{1,3} + (y_1 + y_3) (x_{1,3}^2 + y_{1,3}^2) + \sqrt{-x_{1,3}^2 (l_3^4 - 2((l_5 + l_6)^2 + x_{1,3}^2 + y_{1,3}^2) l_3^2 + ((l_5 + l_6)^2 + x_{1,3}^2 + y_{1,3}^2)^2} \right) / (2x_{1,3}^2 + 2y_{1,3}^2)$$

1 Blade angle calculation, including the Bell-Hiller-Bar

$$\begin{aligned}
 x_3 &= \sin(\gamma_{\text{mr1}}^{\text{BHB}}) (l_4 + l_5 + l_6) \\
 y_3 &= \cos(\gamma_{\text{mr1}}^{\text{BHB}}) (l_4 + l_5 + l_6) - l_7 \\
 x_4 &= (l_6 x_2 + l_5 x_3) / (l_5 + l_6) \\
 y_4 &= (l_6 y_2 + l_5 y_3) / (l_5 + l_6) \\
 x_5 &= - \left(y_4 \sqrt{-x_4^2 (l_7^4 - 2 (l_8^2 + x_4^2 + y_4^2) l_7^2 + (-l_8^2 + x_4^2 + y_4^2)^2)} + \right. \\
 &\quad \left. l_7^2 x_4^2 - (l_8^2 + x_4^2 + y_4^2) x_4^2 \right) / (2 x_4^3 + 2 x_4 y_4^2) \\
 y_5 &= \left(\sqrt{-x_4^2 (l_7^4 - 2 (l_8^2 + x_4^2 + y_4^2) l_7^2 + (-l_8^2 + x_4^2 + y_4^2)^2)} + \right. \\
 &\quad \left. y_4 (-l_7^2 + l_8^2 + x_4^2 + y_4^2) \right) / (2 x_4^2 + 2 y_4^2) \\
 \alpha_{\text{mr1}}^{\text{BL}} &= \arctan2(-y_5, x_5)
 \end{aligned}$$

The constants l_x are described in the thesis main document [Ber13, pages 38ff.]. The coordinates (x_u, y_u) belong to intermediate points taken relative to a local coordinate system.

2 Dynamic Equations of a Single Helicopter

In the following, the complete dynamic equations of a single helicopter are presented. The variables z_{xyz} are intermediate results, which are generated automatically. The result of the Kane method is the following set of equations:

$$\begin{aligned}
 0 &= z_{50} + z_{74} \dot{w} - z_{79} - z_{73} \dot{r} - z_{75} \dot{q} - z_{76} \dot{u} - z_{77} \dot{v} - z_{78} \dot{p} \\
 0 &= z_{51} + z_{81} \dot{w} - z_{85} - z_{77} \dot{u} - z_{80} \dot{r} - z_{82} \dot{q} - z_{83} \dot{v} - z_{84} \dot{p} \\
 0 &= z_{52} + z_{74} \dot{u} + z_{81} \dot{v} - z_{90} - z_{86} \dot{r} - z_{87} \dot{w} - z_{88} \dot{q} - z_{89} \dot{p} \\
 0 &= z_{55} - z_{94} - z_{91} \dot{p} - z_{92} \dot{r} - z_{93} \dot{q} - z_{78} \dot{u} - z_{84} \dot{v} - z_{89} \dot{w} \\
 0 &= z_{57} - z_{97} - z_{93} \dot{p} - z_{95} \dot{q} - z_{96} \dot{r} - z_{75} \dot{u} - z_{82} \dot{v} - z_{88} \dot{w} \\
 0 &= z_{58} - z_{99} - z_{92} \dot{p} - z_{96} \dot{q} - z_{98} \dot{r} - z_{73} \dot{u} - z_{80} \dot{v} - z_{86} \dot{w}
 \end{aligned}$$

The equations need to be solved for the desired variables (\dot{u} , \dot{v} , \dot{w} , \dot{p} , \dot{q} , \dot{r}). It is possible to use Gaussian elimination to get an explicit solution, since the equations are linear with respect to the desired variables. It should be noted that result may contain singularities, since e.g. a division through a z_{xyz} variable might become a division through zero. Solving the equations numerically during simulation, using e.g. Singular Value Decomposition (SVD), is another possibility, which avoids those singularities. The following equations describe the z_{xyz} variables, which had not been defined until now:

$$\begin{aligned}
 z_1 &= \pi + \psi \\
 z_2 &= \pi + \theta \\
 z_3 &= \cos(z_1) \\
 z_4 &= \cos(z_2) \\
 z_5 &= z_3 z_4 \\
 z_6 &= \sin(\varphi) \\
 z_7 &= \sin(z_2) \\
 z_8 &= \cos(\varphi) \\
 z_9 &= \sin(z_1)
 \end{aligned}$$

2 Dynamic Equations of a Single Helicopter

$$\begin{aligned}
z_{10} &= z_3 z_6 z_7 - z_8 z_9 \\
z_{11} &= z_6 z_9 + z_3 z_7 z_8 \\
z_{12} &= z_4 z_9 \\
z_{13} &= z_3 z_8 + z_6 z_7 z_9 \\
z_{14} &= z_7 z_8 z_9 - z_3 z_6 \\
z_{15} &= z_4 z_6 \\
z_{16} &= z_4 z_8 \\
z_{23} &= \omega_{MR} + r \\
z_{24} &= q(r - z_{23}) \\
z_{25} &= p(r - z_{23}) \\
z_{26} &= m_F + m_{MR} \\
z_{27} &= (dO Fo_1 m_F + dOM Ro_1 m_{MR}) / z_{26} \\
z_{28} &= (dO Fo_2 m_F + dOM Ro_2 m_{MR}) / z_{26} \\
z_{29} &= (dO Fo_3 m_F + dOM Ro_3 m_{MR}) / z_{26} \\
z_{30} &= dOM Ro_1 - z_{27} \\
z_{31} &= dOM Ro_2 - z_{28} \\
z_{32} &= dOM Ro_3 - z_{29} \\
z_{33} &= dO Fo_1 - z_{27} \\
z_{34} &= dO Fo_2 - z_{28} \\
z_{35} &= dO Fo_3 - z_{29} \\
z_{36} &= dOT_1 - z_{27} \\
z_{38} &= dOT_3 - z_{29} \\
z_{39} &= -q(z_{30} q - z_{31} p) - r(z_{30} r - z_{32} p) \\
z_{40} &= p(z_{30} q - z_{31} p) - r(z_{31} r - z_{32} q) \\
z_{41} &= p(z_{30} r - z_{32} p) + q(z_{31} r - z_{32} q) \\
z_{42} &= -q(z_{33} q - z_{34} p) - r(z_{33} r - z_{35} p) \\
z_{43} &= p(z_{33} q - z_{34} p) - r(z_{34} r - z_{35} q) \\
z_{44} &= p(z_{33} r - z_{35} p) + q(z_{34} r - z_{35} q) \\
z_{48} &= g m_F \\
z_{49} &= g m_{MR} \\
z_{50} &= F_3^{MR} z_{11} + F_2^{TR} z_{10} + z_{49} z_5 z_7 + z_{48} (z_5 z_7 - z_{10} z_{15} - z_{11} z_{16}) - z_{49} z_{10} z_{15} - z_{49} z_{11} z_{16}
\end{aligned}$$

2 Dynamic Equations of a Single Helicopter

$$\begin{aligned}
 z_{51} &= F_3^{MR} z_{14} + F_2^{TR} z_{13} + z_{49} z_7 z_{12} \\
 &\quad + z_{48} (z_7 z_{12} - z_{13} z_{15} - z_{14} z_{16}) - z_{49} z_{13} z_{15} - z_{49} z_{14} z_{16} \\
 z_{52} &= F_3^{MR} z_{16} + F_2^{TR} z_{15} - z_{49} z_7^2 - z_{49} z_{15}^2 - z_{49} z_{16}^2 - z_{48} (z_7^2 + z_{15}^2 + z_{16}^2) \\
 z_{53} &= z_{32} z_{49} \\
 z_{54} &= z_{31} z_{49} \\
 z_{55} &= T_1^{MR} + z_{31} F_3^{MR} + z_{53} z_{15} - z_{38} F_2^{TR} - z_{54} z_{16} - z_{48} (z_{34} z_{16} - z_{35} z_{15}) \\
 z_{56} &= z_{30} z_{49} \\
 z_{57} &= T_2^{MR} + T_2^{TR} + z_{53} z_7 + z_{56} z_{16} + z_{48} (z_{33} z_{16} + z_{35} z_7) - z_{30} F_3^{MR} \\
 z_{58} &= T_3^{MR} + z_{36} F_2^{TR} - z_{48} (z_{33} z_{15} + z_{34} z_7) - z_{49} (z_{30} z_{15} + z_{31} z_7) \\
 z_{59} &= I_{11}^F p \\
 z_{60} &= I_{22}^F q \\
 z_{61} &= I_{33}^F r \\
 z_{62} &= p z_{60} - q z_{59} \\
 z_{63} &= r z_{59} - p z_{61} \\
 z_{64} &= q z_{61} - r z_{60} \\
 z_{65} &= I_{11}^{MR} p \\
 z_{66} &= I_{22}^{MR} q \\
 z_{67} &= I_{33}^{MR} z_{23} \\
 z_{68} &= I_{11}^{MR} z_{24} \\
 z_{69} &= I_{22}^{MR} z_{25} \\
 z_{70} &= p z_{66} - q z_{65} \\
 z_{71} &= z_{23} z_{65} - p z_{67} \\
 z_{72} &= q z_{67} - z_{23} z_{66} \\
 z_{73} &= m_F (z_{33} z_{10} - z_{34} z_5) + m_{MR} (z_{30} z_{10} - z_{31} z_5) \\
 z_{74} &= z_{26} (z_5 z_7 - z_{10} z_{15} - z_{11} z_{16}) \\
 z_{75} &= -m_F (z_{33} z_{11} - z_{35} z_5) - m_{MR} (z_{30} z_{11} - z_{32} z_5) \\
 z_{76} &= z_{26} (z_5^2 + z_{10}^2 + z_{11}^2) \\
 z_{77} &= z_{26} (z_5 z_{12} + z_{10} z_{13} + z_{11} z_{14}) \\
 z_{78} &= m_F (z_{34} z_{11} - z_{35} z_{10}) + m_{MR} (z_{31} z_{11} - z_{32} z_{10})
 \end{aligned}$$

2 Dynamic Equations of a Single Helicopter

$$\begin{aligned}
z_{79} &= m_F(z_5 z_{42} + z_{10} z_{43} + z_{11} z_{44}) + m_{MR}(z_5 z_{39} + z_{10} z_{40} + z_{11} z_{41}) \\
z_{80} &= m_F(z_{33} z_{13} - z_{34} z_{12}) + m_{MR}(z_{30} z_{13} - z_{31} z_{12}) \\
z_{81} &= z_{26} (z_7 z_{12} - z_{13} z_{15} - z_{14} z_{16}) \\
z_{82} &= -m_F(z_{33} z_{14} - z_{35} z_{12}) - m_{MR}(z_{30} z_{14} - z_{32} z_{12}) \\
z_{83} &= z_{26} (z_{12}^2 + z_{13}^2 + z_{14}^2) \\
z_{84} &= m_F(z_{34} z_{14} - z_{35} z_{13}) + m_{MR}(z_{31} z_{14} - z_{32} z_{13}) \\
z_{85} &= m_F(z_{12} z_{42} + z_{13} z_{43} + z_{14} z_{44}) + m_{MR}(z_{12} z_{39} + z_{13} z_{40} + z_{14} z_{41}) \\
z_{86} &= m_F(z_{33} z_{15} + z_{34} z_7) + m_{MR}(z_{30} z_{15} + z_{31} z_7) \\
z_{87} &= z_{26} (z_7^2 + z_{15}^2 + z_{16}^2) \\
z_{88} &= -m_F(z_{33} z_{16} + z_{35} z_7) - m_{MR}(z_{30} z_{16} + z_{32} z_7) \\
z_{89} &= m_F(z_{34} z_{16} - z_{35} z_{15}) + m_{MR}(z_{31} z_{16} - z_{32} z_{15}) \\
z_{90} &= -m_F(z_7 z_{42} - z_{15} z_{43} - z_{16} z_{44}) - m_{MR}(z_7 z_{39} - z_{15} z_{40} - z_{16} z_{41}) \\
z_{91} &= I_{11}^F + I_{11}^{MR} + m_F(z_{34}^2 + z_{35}^2) + m_{MR}(z_{31}^2 + z_{32}^2) \\
z_{92} &= -m_F z_{33} z_{35} - m_{MR} z_{30} z_{32} \\
z_{93} &= -m_F z_{33} z_{34} - m_{MR} z_{30} z_{31} \\
z_{94} &= z_{64} + z_{72} + m_F(z_{34} z_{44} - z_{35} z_{43}) + m_{MR}(z_{31} z_{41} - z_{32} z_{40}) - z_{68} \\
z_{95} &= I_{22}^F + I_{22}^{MR} + m_F(z_{33}^2 + z_{35}^2) + m_{MR}(z_{30}^2 + z_{32}^2) \\
z_{96} &= -m_F z_{34} z_{35} - m_{MR} z_{31} z_{32} \\
z_{97} &= z_{63} + z_{69} + z_{71} - m_F(z_{33} z_{44} - z_{35} z_{42}) - m_{MR}(z_{30} z_{41} - z_{32} z_{39}) \\
z_{98} &= I_{33}^F + I_{33}^{MR} + m_F(z_{33}^2 + z_{34}^2) + m_{MR}(z_{30}^2 + z_{31}^2) \\
z_{99} &= z_{62} + z_{70} + m_F(z_{33} z_{43} - z_{34} z_{42}) + m_{MR}(z_{30} z_{40} - z_{31} z_{39})
\end{aligned}$$

3 Dynamic Equations of the Single-Lift Model

The dynamic equations of the single-lift model are presented, which includes both model variants, see the thesis main document [Ber13, pages 55ff.]. The variables z_{xyz} represent intermediate results, which are generated automatically.

3.1 First Variant - Frame R Relative to Frame F

For the first model variant, the result of the Kane method is the following set of equations:

$$\begin{aligned}
0 &= z_{115} + F_3^{MR} z_{11} + F_2^{TR} z_{10} + z_{141} \dot{w} - z_{148} - z_{140} \dot{r} \\
&\quad - z_{142} \dot{q} - z_{143} \dot{u} - z_{144} \dot{v} - z_{145} \dot{p} - z_{146} \dot{q}_r^F - z_{147} \dot{p}_r^F \\
0 &= z_{116} + F_3^{MR} z_{14} + F_2^{TR} z_{13} + z_{150} \dot{w} - z_{156} - z_{144} \dot{u} \\
&\quad - z_{149} \dot{r} - z_{151} \dot{q} - z_{152} \dot{v} - z_{153} \dot{p} - z_{154} \dot{q}_r^F - z_{155} \dot{p}_r^F \\
0 &= z_{117} + F_3^{MR} z_{16} + F_2^{TR} z_{15} + z_{141} \dot{u} + z_{150} \dot{v} - z_{163} \\
&\quad - z_{157} \dot{r} - z_{158} \dot{w} - z_{159} \dot{q} - z_{160} \dot{p} - z_{161} \dot{q}_r^F - z_{162} \dot{p}_r^F \\
0 &= T_1^{MR} + z_{31} F_3^{MR} + z_{120} + z_{176} \dot{q}_r^F + z_{177} \dot{p}_r^F - z_{38} F_2^{TR} - z_{178} \\
&\quad - z_{145} \dot{u} - z_{153} \dot{v} - z_{160} \dot{w} - z_{167} \dot{p} - z_{171} \dot{r} - z_{175} \dot{q} \\
0 &= T_2^{MR} + T_2^{TR} + z_{122} - z_{30} F_3^{MR} - z_{187} - z_{142} \dot{u} - z_{151} \dot{v} \\
&\quad - z_{159} \dot{w} - z_{175} \dot{p} - z_{181} \dot{q} - z_{184} \dot{r} - z_{185} \dot{q}_r^F - z_{186} \dot{p}_r^F \\
0 &= T_3^{MR} + z_{36} F_2^{TR} + z_{123} + z_{191} \dot{q}_r^F + z_{192} \dot{p}_r^F - z_{193} \\
&\quad - z_{140} \dot{u} - z_{149} \dot{v} - z_{157} \dot{w} - z_{171} \dot{p} - z_{184} \dot{q} - z_{190} \dot{r} \\
0 &= z_{177} \dot{p} + z_{192} \dot{r} - z_{124} - z_{196} - z_{147} \dot{u} - z_{155} \dot{v} - z_{162} \dot{w} - z_{186} \dot{q} - z_{194} \dot{q}_r^F - z_{195} \dot{p}_r^F \\
0 &= z_{176} \dot{p} + z_{191} \dot{r} - z_{125} - z_{198} - z_{146} \dot{u} - z_{154} \dot{v} - z_{161} \dot{w} - z_{185} \dot{q} - z_{194} \dot{p}_r^F - z_{197} \dot{q}_r^F
\end{aligned}$$

The equations need to be solved for the desired variables (\dot{u} , \dot{v} , \dot{w} , \dot{p} , \dot{q} , \dot{r} , \dot{p}_r^F , \dot{q}_r^F) to get the explicit equations.

The following equations define the remaining z_{xyz} variables:

$$\begin{aligned}
z_1 &= \pi + \psi \\
z_2 &= \pi + \theta
\end{aligned}$$

3 Dynamic Equations of the Single-Lift Model

$$\begin{aligned}
z_3 &= \cos(z_1) \\
z_4 &= \cos(z_2) \\
z_5 &= z_3 z_4 \\
z_6 &= \sin(\varphi) \\
z_7 &= \sin(z_2) \\
z_8 &= \cos(\varphi) \\
z_9 &= \sin(z_1) \\
z_{10} &= z_3 z_6 z_7 - z_8 z_9 \\
z_{11} &= z_6 z_9 + z_3 z_7 z_8 \\
z_{12} &= z_4 z_9 \\
z_{13} &= z_3 z_8 + z_6 z_7 z_9 \\
z_{14} &= z_7 z_8 z_9 - z_3 z_6 \\
z_{15} &= z_4 z_6 \\
z_{16} &= z_4 z_8 \\
z_{23} &= \omega_{MR} + r \\
z_{24} &= q(r - z_{23}) \\
z_{25} &= p(r - z_{23}) \\
z_{26} &= m_F + m_{MR} \\
z_{27} &= (dOFo_1 m_F + dOMRo_1 m_{MR}) / z_{26} \\
z_{28} &= (dOFo_2 m_F + dOMRo_2 m_{MR}) / z_{26} \\
z_{29} &= (dOFo_3 m_F + dOMRo_3 m_{MR}) / z_{26} \\
z_{30} &= dOMRo_1 - z_{27} \\
z_{31} &= dOMRo_2 - z_{28} \\
z_{32} &= dOMRo_3 - z_{29} \\
z_{33} &= dOFo_1 - z_{27} \\
z_{34} &= dOFo_2 - z_{28} \\
z_{35} &= dOFo_3 - z_{29} \\
z_{36} &= dOT_1 - z_{27} \\
z_{38} &= dOT_3 - z_{29} \\
z_{39} &= dORo_1 - z_{27} \\
z_{40} &= dORo_2 - z_{28}
\end{aligned}$$

3 Dynamic Equations of the Single-Lift Model

$$\begin{aligned}
z_{41} &= dORo_3 - z_{29} \\
z_{42} &= -q(z_{30}q - z_{31}p) - r(z_{30}r - z_{32}p) \\
z_{43} &= p(z_{30}q - z_{31}p) - r(z_{31}r - z_{32}q) \\
z_{44} &= p(z_{30}r - z_{32}p) + q(z_{31}r - z_{32}q) \\
z_{45} &= -q(z_{33}q - z_{34}p) - r(z_{33}r - z_{35}p) \\
z_{46} &= p(z_{33}q - z_{34}p) - r(z_{34}r - z_{35}q) \\
z_{47} &= p(z_{33}r - z_{35}p) + q(z_{34}r - z_{35}q) \\
z_{54} &= \cos(\theta_r^F) \\
z_{55} &= \sin(\varphi_r^F) \\
z_{56} &= \sin(\theta_r^F) \\
z_{57} &= z_{55} z_{56} \\
z_{58} &= \cos(\varphi_r^F) \\
z_{59} &= z_{56} z_{58} \\
z_{60} &= z_{54} z_{55} \\
z_{61} &= z_{54} z_{58} \\
z_{62} &= z_{59} z_{60} - z_{57} z_{61} \\
z_{63} &= z_{55}^2 + z_{59}^2 + z_{61}^2 \\
z_{64} &= z_{54} z_{61} + z_{56} z_{59} \\
z_{65} &= z_{56} z_{60} - z_{54} z_{57} \\
z_{66} &= -z_{54} z_{60} - z_{56} z_{57} \\
z_{67} &= z_5 z_{54} - z_{11} z_{56} \\
z_{68} &= z_{12} z_{54} - z_{14} z_{56} \\
z_{69} &= -z_7 z_{54} - z_{16} z_{56} \\
z_{70} &= z_5 z_{57} + z_{10} z_{58} + z_{11} z_{60} \\
z_{71} &= z_{12} z_{57} + z_{13} z_{58} + z_{14} z_{60} \\
z_{72} &= z_{15} z_{58} + z_{16} z_{60} - z_7 z_{57} \\
z_{73} &= z_5 z_{59} + z_{11} z_{61} - z_{10} z_{55} \\
z_{74} &= z_{12} z_{59} + z_{14} z_{61} - z_{13} z_{55} \\
z_{75} &= z_{16} z_{61} - z_7 z_{59} - z_{15} z_{55} \\
z_{76} &= l_{\eta p} z_{57} \\
z_{77} &= l_{\eta p} z_{58}
\end{aligned}$$

3 Dynamic Equations of the Single-Lift Model

$$\begin{aligned}
z_{78} &= l_{\eta} z_{60} \\
z_{79} &= l_{\eta} z_{64} \\
z_{80} &= l_{\eta} z_{65} \\
z_{81} &= l_{\eta} z_{56} \\
z_{82} &= l_{\eta} z_{54} \\
z_{83} &= l_{\eta} z_{62} \\
z_{84} &= l_{\eta} z_{63} \\
z_{85} &= z_{41} q - z_{40} r \\
z_{86} &= z_{39} r - z_{41} p \\
z_{87} &= z_{40} p - z_{39} q \\
z_{88} &= z_{76} p + z_{77} q + z_{78} r + z_{79} q_r^F + z_{80} p_r^F \\
z_{89} &= z_{81} r - z_{82} p - z_{83} q_r^F - z_{84} p_r^F \\
z_{90} &= z_{54} z_{55} q_r^F + z_{56} z_{58} p_r^F \\
z_{91} &= l_{\eta} z_{55} p_r^F \\
z_{92} &= z_{54} z_{58} p_r^F - z_{55} z_{56} q_r^F \\
z_{93} &= -z_{54} z_{55} p_r^F - z_{56} z_{58} q_r^F \\
z_{94} &= z_{54} z_{58} q_r^F - z_{55} z_{56} p_r^F \\
z_{95} &= z_{54} z_{59} q_r^F + z_{54} z_{93} + z_{56} z_{94} - z_{56} z_{61} q_r^F \\
z_{96} &= z_{54} z_{60} q_r^F + z_{56} z_{57} q_r^F + z_{56} z_{92} - z_{54} z_{90} \\
z_{97} &= l_{\eta} p z_{90} + l_{\eta} r z_{92} + l_{\eta} p_r^F z_{96} + l_{\eta} q_r^F z_{95} - q z_{91} \\
z_{98} &= z_{54} p + z_{62} q_r^F + z_{63} p_r^F - z_{56} r \\
z_{99} &= z_{57} p + z_{58} q + z_{60} r + z_{64} q_r^F + z_{65} p_r^F \\
z_{100} &= z_{59} p + z_{61} r + z_{66} q_r^F - z_{55} q \\
z_{101} &= l_{\eta} z_{54} q_r^F \\
z_{102} &= l_{\eta} z_{56} q_r^F \\
z_{103} &= z_{59} z_{92} + z_{60} z_{94} - z_{57} z_{93} - z_{61} z_{90} \\
z_{104} &= 2z_{55} z_{58} p_r^F + 2z_{59} z_{94} + 2z_{61} z_{93}
\end{aligned}$$

3 Dynamic Equations of the Single-Lift Model

$$\begin{aligned}
z_{105} &= p z_{102} + r z_{101} - l_p p_r^F z_{104} - l_p q_r^F z_{103} \\
z_{106} &= q z_{87} - r z_{86} \\
z_{107} &= r z_{85} - p z_{87} \\
z_{108} &= p z_{86} - q z_{85} \\
z_{109} &= z_{97} - z_{89} z_{100} \\
z_{110} &= z_{105} + z_{88} z_{100} \\
z_{111} &= z_{89} z_{98} - z_{88} z_{99} \\
z_{112} &= g m_F \\
z_{113} &= g l_p \\
z_{114} &= g m_{MR} \\
z_{115} &= z_{114} z_5 z_7 + z_{112} (z_5 z_7 - z_{10} z_{15} - z_{11} z_{16}) - z_{114} z_{10} z_{15} - z_{114} z_{11} z_{16} \\
z_{116} &= z_{114} z_7 z_{12} + z_{112} (z_7 z_{12} - z_{13} z_{15} - z_{14} z_{16}) - z_{114} z_{13} z_{15} - z_{114} z_{14} z_{16} \\
z_{117} &= -z_{113} - z_{114} z_7^2 - z_{114} z_{15}^2 - z_{114} z_{16}^2 - z_{112} (z_7^2 + z_{15}^2 + z_{16}^2) \\
z_{118} &= z_{32} z_{114} \\
z_{119} &= z_{31} z_{114} \\
z_{120} &= z_{118} z_{15} - z_{119} z_{16} - z_{112} (z_{34} z_{16} - z_{35} z_{15}) \\
&\quad - z_{113} (z_{40} z_{16} + z_{69} z_{76} - z_{41} z_{15} - z_{72} z_{82}) \\
z_{121} &= z_{30} z_{114} \\
z_{122} &= z_{118} z_7 + z_{121} z_{16} + z_{112} (z_{33} z_{16} + z_{35} z_7) + z_{113} (z_{39} z_{16} + z_{41} z_7 - z_{69} z_{77}) \\
z_{123} &= -z_{112} (z_{33} z_{15} + z_{34} z_7) - z_{114} (z_{30} z_{15} + z_{31} z_7) \\
&\quad - z_{113} (z_{39} z_{15} + z_{40} z_7 + z_{69} z_{78} + z_{72} z_{81}) \\
z_{124} &= z_{113} (z_{69} z_{80} - z_{72} z_{84}) \\
z_{125} &= z_{113} (z_{69} z_{79} - z_{72} z_{83}) \\
z_{126} &= I_{11}^F p \\
z_{127} &= I_{22}^F q \\
z_{128} &= I_{33}^F r \\
z_{129} &= p z_{127} - q z_{126} \\
z_{130} &= r z_{126} - p z_{128} \\
z_{131} &= q z_{128} - r z_{127} \\
z_{132} &= I_{11}^{MR} p \\
z_{133} &= I_{22}^{MR} q
\end{aligned}$$

3 Dynamic Equations of the Single-Lift Model

$$\begin{aligned}
z_{134} &= I_{33}^{MR} z_{23} \\
z_{135} &= I_{11}^{MR} z_{24} \\
z_{136} &= I_{22}^{MR} z_{25} \\
z_{137} &= p z_{133} - q z_{132} \\
z_{138} &= z_{23} z_{132} - p z_{134} \\
z_{139} &= q z_{134} - z_{23} z_{133} \\
z_{140} &= m_F (z_{33} z_{10} - z_{34} z_5) + m_{MR} (z_{30} z_{10} - z_{31} z_5) \\
&\quad - l_p (z_{40} z_5 - z_{39} z_{10} - z_{67} z_{78} - z_{70} z_{81}) \\
z_{141} &= z_{26} (z_5 z_7 - z_{10} z_{15} - z_{11} z_{16}) \\
z_{142} &= -m_F (z_{33} z_{11} - z_{35} z_5) - m_{MR} (z_{30} z_{11} - z_{32} z_5) - l_p (z_{39} z_{11} - z_{41} z_5 - z_{67} z_{77}) \\
z_{143} &= l_p + m_F (z_5^2 + z_{10}^2 + z_{11}^2) + m_{MR} (z_5^2 + z_{10}^2 + z_{11}^2) \\
z_{144} &= z_{26} (z_5 z_{12} + z_{10} z_{13} + z_{11} z_{14}) \\
z_{145} &= m_F (z_{34} z_{11} - z_{35} z_{10}) + m_{MR} (z_{31} z_{11} - z_{32} z_{10}) \\
&\quad + l_p (z_{40} z_{11} + z_{67} z_{76} - z_{41} z_{10} - z_{70} z_{82}) \\
z_{146} &= l_p (z_{67} z_{79} - z_{70} z_{83}) \\
z_{147} &= l_p (z_{67} z_{80} - z_{70} z_{84}) \\
z_{148} &= m_F (z_5 z_{45} + z_{10} z_{46} + z_{11} z_{47}) + m_{MR} (z_5 z_{42} + z_{10} z_{43} + z_{11} z_{44}) \\
&\quad + l_p (z_5 z_{106} + z_{10} z_{107} + z_{11} z_{108} + z_{67} z_{109} + z_{70} z_{110} + z_{73} z_{111}) \\
z_{149} &= m_F (z_{33} z_{13} - z_{34} z_{12}) + m_{MR} (z_{30} z_{13} - z_{31} z_{12}) \\
&\quad - l_p (z_{40} z_{12} - z_{39} z_{13} - z_{68} z_{78} - z_{71} z_{81}) \\
z_{150} &= z_{26} (z_7 z_{12} - z_{13} z_{15} - z_{14} z_{16}) \\
z_{151} &= -m_F (z_{33} z_{14} - z_{35} z_{12}) - m_{MR} (z_{30} z_{14} - z_{32} z_{12}) - l_p (z_{39} z_{14} - z_{41} z_{12} - z_{68} z_{77}) \\
z_{152} &= l_p + m_F (z_{12}^2 + z_{13}^2 + z_{14}^2) + m_{MR} (z_{12}^2 + z_{13}^2 + z_{14}^2) \\
z_{153} &= m_F (z_{34} z_{14} - z_{35} z_{13}) + m_{MR} (z_{31} z_{14} - z_{32} z_{13}) \\
&\quad + l_p (z_{40} z_{14} + z_{68} z_{76} - z_{41} z_{13} - z_{71} z_{82}) \\
z_{154} &= l_p (z_{68} z_{79} - z_{71} z_{83}) \\
z_{155} &= l_p (z_{68} z_{80} - z_{71} z_{84}) \\
z_{156} &= m_F (z_{12} z_{45} + z_{13} z_{46} + z_{14} z_{47}) + m_{MR} (z_{12} z_{42} + z_{13} z_{43} + z_{14} z_{44}) \\
&\quad + l_p (z_{12} z_{106} + z_{13} z_{107} + z_{14} z_{108} + z_{68} z_{109} + z_{71} z_{110} + z_{74} z_{111}) \\
z_{157} &= m_F (z_{33} z_{15} + z_{34} z_7) + m_{MR} (z_{30} z_{15} + z_{31} z_7) \\
&\quad + l_p (z_{39} z_{15} + z_{40} z_7 + z_{69} z_{78} + z_{72} z_{81})
\end{aligned}$$

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$$\begin{aligned}
z_{158} &= l_p + m_F (z_7^2 + z_{15}^2 + z_{16}^2) + m_{MR} (z_7^2 + z_{15}^2 + z_{16}^2) \\
z_{159} &= -m_F (z_{33} z_{16} + z_{35} z_7) - m_{MR} (z_{30} z_{16} + z_{32} z_7) - l_p (z_{39} z_{16} + z_{41} z_7 - z_{69} z_{77}) \\
z_{160} &= m_F (z_{34} z_{16} - z_{35} z_{15}) + m_{MR} (z_{31} z_{16} - z_{32} z_{15}) \\
&\quad + l_p (z_{40} z_{16} + z_{69} z_{76} - z_{41} z_{15} - z_{72} z_{82}) \\
z_{161} &= l_p (z_{69} z_{79} - z_{72} z_{83}) \\
z_{162} &= l_p (z_{69} z_{80} - z_{72} z_{84}) \\
z_{163} &= -m_F (z_7 z_{45} - z_{15} z_{46} - z_{16} z_{47}) - m_{MR} (z_7 z_{42} - z_{15} z_{43} - z_{16} z_{44}) \\
&\quad - l_p (z_7 z_{106} - z_{15} z_{107} - z_{16} z_{108} - z_{69} z_{109} - z_{72} z_{110} - z_{75} z_{111}) \\
z_{164} &= I_{11}^F + I_{11}^{MR} + m_F (z_{34}^2 + z_{35}^2) + m_{MR} (z_{31}^2 + z_{32}^2) \\
z_{165} &= z_{40}^2 \\
z_{166} &= z_{41}^2 \\
z_{167} &= z_{164} - l_p (2z_{40} z_{56} z_{76} + 2z_{40} z_{60} z_{82} - z_{165} - z_{166} - z_{76}^2 - z_{82}^2 - 2z_{41} z_{58} z_{82}) \\
z_{168} &= m_F z_{33} z_{35} \\
z_{169} &= m_{MR} z_{30} z_{32} \\
z_{170} &= z_{39} z_{41} \\
z_{171} &= l_p (z_{76} z_{78} + z_{40} z_{57} z_{82} + z_{40} z_{60} z_{81} - z_{170} - z_{81} z_{82} - z_{39} z_{58} z_{82} - z_{40} z_{54} z_{76} \\
&\quad - z_{40} z_{56} z_{78} - z_{41} z_{58} z_{81}) - z_{168} - z_{169} \\
z_{172} &= m_F z_{33} z_{34} \\
z_{173} &= m_{MR} z_{30} z_{31} \\
z_{174} &= z_{39} z_{40} \\
z_{175} &= l_p (z_{76} z_{77} + z_{39} z_{56} z_{76} + z_{39} z_{60} z_{82} + z_{41} z_{54} z_{76} - z_{174} - z_{40} z_{56} z_{77} - z_{41} z_{57} z_{82}) \\
&\quad - z_{172} - z_{173} \\
z_{176} &= l_p (z_{40} z_{56} z_{79} + z_{40} z_{60} z_{83} - z_{76} z_{79} - z_{82} z_{83} - z_{41} z_{58} z_{83}) \\
z_{177} &= l_p (z_{40} z_{56} z_{80} + z_{40} z_{60} z_{84} - z_{76} z_{80} - z_{82} z_{84} - z_{41} z_{58} z_{84}) \\
z_{178} &= z_{131} + z_{139} + m_F (z_{34} z_{47} - z_{35} z_{46}) + m_{MR} (z_{31} z_{44} - z_{32} z_{43}) \\
&\quad + l_p (z_{40} z_{108} + z_{76} z_{109} + z_{40} z_{60} z_{110} + z_{40} z_{61} z_{111} + z_{41} z_{55} z_{111} + z_{54} z_{76} z_{106} \\
&\quad - z_{41} z_{107} - z_{82} z_{110} - z_{40} z_{56} z_{109} - z_{41} z_{58} z_{110} - z_{56} z_{76} z_{108} - z_{57} z_{82} z_{106} \\
&\quad - z_{58} z_{82} z_{107} - z_{60} z_{82} z_{108}) - z_{135} \\
z_{179} &= I_{22}^F + I_{22}^{MR} + m_F (z_{33}^2 + z_{35}^2) + m_{MR} (z_{30}^2 + z_{32}^2) \\
z_{180} &= z_{39}^2 + z_{41}^2 \\
z_{181} &= z_{179} + l_p (z_{180} + z_{77}^2 + 2z_{39} z_{56} z_{77} + 2z_{41} z_{54} z_{77})
\end{aligned}$$

3 Dynamic Equations of the Single-Lift Model

$$\begin{aligned}
z_{182} &= -m_F z_{34} z_{35} - m_{MR} z_{31} z_{32} \\
z_{183} &= z_{40} z_{41} \\
z_{184} &= z_{182} \\
&\quad - l_p (z_{183} + z_{39} z_{60} z_{81} + z_{40} z_{54} z_{77} - z_{77} z_{78} - z_{39} z_{56} z_{78} - z_{41} z_{54} z_{78} - z_{41} z_{57} z_{81}) \\
z_{185} &= l_p (z_{77} z_{79} + z_{39} z_{56} z_{79} + z_{39} z_{60} z_{83} + z_{41} z_{54} z_{79} - z_{41} z_{57} z_{83}) \\
z_{186} &= l_p (z_{77} z_{80} + z_{39} z_{56} z_{80} + z_{39} z_{60} z_{84} + z_{41} z_{54} z_{80} - z_{41} z_{57} z_{84}) \\
z_{187} &= z_{130} + z_{136} + z_{138} + l_p (z_{41} z_{106} + z_{77} z_{109} + z_{39} z_{56} z_{109} + z_{41} z_{54} z_{109} + z_{41} z_{57} z_{110} \\
&\quad + z_{41} z_{59} z_{111} + z_{54} z_{77} z_{106} - z_{39} z_{108} - z_{39} z_{60} z_{110} - z_{39} z_{61} z_{111} - z_{56} z_{77} z_{108}) \\
&\quad - m_F (z_{33} z_{47} - z_{35} z_{45}) - m_{MR} (z_{30} z_{44} - z_{32} z_{42}) \\
z_{188} &= I_{33}^F + I_{33}^{MR} + m_F (z_{33}^2 + z_{34}^2) + m_{MR} (z_{30}^2 + z_{31}^2) \\
z_{189} &= z_{39}^2 + z_{40}^2 \\
z_{190} &= z_{188} + l_p (z_{189} + z_{78}^2 + z_{81}^2 + 2z_{39} z_{58} z_{81} - 2z_{40} z_{54} z_{78} - 2z_{40} z_{57} z_{81}) \\
z_{191} &= l_p (z_{81} z_{83} + z_{39} z_{58} z_{83} + z_{40} z_{54} z_{79} - z_{78} z_{79} - z_{40} z_{57} z_{83}) \\
z_{192} &= l_p (z_{81} z_{84} + z_{39} z_{58} z_{84} + z_{40} z_{54} z_{80} - z_{78} z_{80} - z_{40} z_{57} z_{84}) \\
z_{193} &= z_{129} + z_{137} + m_F (z_{33} z_{46} - z_{34} z_{45}) + m_{MR} (z_{30} z_{43} - z_{31} z_{42}) \\
&\quad - l_p (z_{40} z_{106} + z_{39} z_{55} z_{111} + z_{40} z_{54} z_{109} + z_{40} z_{57} z_{110} + z_{40} z_{59} z_{111} \\
&\quad + z_{56} z_{78} z_{108} - z_{39} z_{107} - z_{78} z_{109} - z_{81} z_{110} - z_{39} z_{58} z_{110} - z_{54} z_{78} z_{106} \\
&\quad - z_{57} z_{81} z_{106} - z_{58} z_{81} z_{107} - z_{60} z_{81} z_{108}) \\
z_{194} &= l_p (z_{79} z_{80} + z_{83} z_{84}) \\
z_{195} &= l_p (z_{80}^2 + z_{84}^2) \\
z_{196} &= l_p (z_{80} z_{109} + z_{54} z_{80} z_{106} - z_{84} z_{110} - z_{56} z_{80} z_{108} - z_{57} z_{84} z_{106} - z_{58} z_{84} z_{107} \\
&\quad - z_{60} z_{84} z_{108}) \\
z_{197} &= l_p (z_{79}^2 + z_{83}^2) \\
z_{198} &= l_p (z_{79} z_{109} + z_{54} z_{79} z_{106} - z_{83} z_{110} - z_{56} z_{79} z_{108} - z_{57} z_{83} z_{106} - z_{58} z_{83} z_{107} \\
&\quad - z_{60} z_{83} z_{108})
\end{aligned}$$

3.2 Second Variant - Frame R Relative to Frame N

For the second model variant, the result of the Kane method is the following set of equations:

$$\begin{aligned}
0 &= z_{99} + F_3^{MR} z_{11} + F_2^{TR} z_{10} + z_{134} \dot{w} + z_{139} \dot{q}_r^N + z_{140} \dot{p}_r^N \\
&\quad - z_{141} - z_{133} \dot{r} - z_{135} \dot{q} - z_{136} \dot{u} - z_{137} \dot{v} - z_{138} \dot{p}
\end{aligned}$$

3 Dynamic Equations of the Single-Lift Model

$$\begin{aligned}
0 &= z_{100} + F_3^{MR} z_{14} + F_2^{TR} z_{13} + z_{143} \dot{w} - z_{149} - z_{137} \dot{u} \\
&\quad - z_{142} \dot{r} - z_{144} \dot{q} - z_{145} \dot{v} - z_{146} \dot{p} - z_{147} \dot{q}_r^N - z_{148} \dot{p}_r^N \\
0 &= z_{101} + F_3^{MR} z_{16} + F_2^{TR} z_{15} + z_{134} \dot{u} + z_{143} \dot{v} + z_{154} \dot{q}_r^N \\
&\quad + z_{155} \dot{p}_r^N - z_{156} - z_{150} \dot{r} - z_{151} \dot{w} - z_{152} \dot{q} - z_{153} \dot{p} \\
0 &= T_1^{MR} + z_{31} F_3^{MR} + z_{104} + z_{160} \dot{q}_r^N + z_{161} \dot{p}_r^N - z_{38} F_2^{TR} \\
&\quad - z_{162} - z_{157} \dot{p} - z_{158} \dot{r} - z_{159} \dot{q} - z_{138} \dot{u} - z_{146} \dot{v} - z_{153} \dot{w} \\
0 &= T_2^{MR} + T_2^{TR} + z_{106} - z_{30} F_3^{MR} - z_{167} - z_{159} \dot{p} - z_{163} \dot{q} \\
&\quad - z_{164} \dot{r} - z_{135} \dot{u} - z_{144} \dot{v} - z_{152} \dot{w} - z_{165} \dot{q}_r^N - z_{166} \dot{p}_r^N \\
0 &= T_3^{MR} + z_{36} F_2^{TR} + z_{107} + z_{169} \dot{q}_r^N + z_{170} \dot{p}_r^N - z_{171} \\
&\quad - z_{158} \dot{p} - z_{164} \dot{q} - z_{168} \dot{r} - z_{133} \dot{u} - z_{142} \dot{v} - z_{150} \dot{w} \\
0 &= z_{108} + z_{140} \dot{u} + z_{155} \dot{w} + z_{161} \dot{p} + z_{170} \dot{r} - z_{174} - z_{148} \dot{v} - z_{166} \dot{q} - z_{172} \dot{q}_r^N - z_{173} \dot{p}_r^N \\
0 &= z_{109} + z_{139} \dot{u} + z_{154} \dot{w} + z_{160} \dot{p} + z_{169} \dot{r} - z_{176} - z_{147} \dot{v} - z_{165} \dot{q} - z_{172} \dot{p}_r^N - z_{175} \dot{q}_r^N
\end{aligned}$$

The equations need to be solved for the desired variables $(\dot{u}, \dot{v}, \dot{w}, \dot{p}, \dot{q}, \dot{r}, \dot{p}_r^N, \dot{q}_r^N)$ to get the explicit equations.

The following equations define the remaining z_{xyz} variables:

$$\begin{aligned}
z_1 &= \pi + \psi \\
z_2 &= \pi + \theta \\
z_3 &= \cos(z_1) \\
z_4 &= \cos(z_2) \\
z_5 &= z_3 z_4 \\
z_6 &= \sin(\varphi) \\
z_7 &= \sin(z_2) \\
z_8 &= \cos(\varphi) \\
z_9 &= \sin(z_1) \\
z_{10} &= z_3 z_6 z_7 - z_8 z_9 \\
z_{11} &= z_6 z_9 + z_3 z_7 z_8 \\
z_{12} &= z_4 z_9 \\
z_{13} &= z_3 z_8 + z_6 z_7 z_9 \\
z_{14} &= z_7 z_8 z_9 - z_3 z_6 \\
z_{15} &= z_4 z_6 \\
z_{16} &= z_4 z_8
\end{aligned}$$

3 Dynamic Equations of the Single-Lift Model

$$\begin{aligned}
z_{23} &= \omega_{MR} + r \\
z_{24} &= q(r - z_{23}) \\
z_{25} &= p(r - z_{23}) \\
z_{26} &= m_F + m_{MR} \\
z_{27} &= (dOFo_1 m_F + dOMRo_1 m_{MR}) / z_{26} \\
z_{28} &= (dOFo_2 m_F + dOMRo_2 m_{MR}) / z_{26} \\
z_{29} &= (dOFo_3 m_F + dOMRo_3 m_{MR}) / z_{26} \\
z_{30} &= dOMRo_1 - z_{27} \\
z_{31} &= dOMRo_2 - z_{28} \\
z_{32} &= dOMRo_3 - z_{29} \\
z_{33} &= dFO_1 - z_{27} \\
z_{34} &= dFO_2 - z_{28} \\
z_{35} &= dFO_3 - z_{29} \\
z_{36} &= dOT_1 - z_{27} \\
z_{38} &= dOT_3 - z_{29} \\
z_{39} &= dORo_1 - z_{27} \\
z_{40} &= dORo_2 - z_{28} \\
z_{41} &= dORo_3 - z_{29} \\
z_{42} &= -q(z_{30}q - z_{31}p) - r(z_{30}r - z_{32}p) \\
z_{43} &= p(z_{30}q - z_{31}p) - r(z_{31}r - z_{32}q) \\
z_{44} &= p(z_{30}r - z_{32}p) + q(z_{31}r - z_{32}q) \\
z_{45} &= -q(z_{33}q - z_{34}p) - r(z_{33}r - z_{35}p) \\
z_{46} &= p(z_{33}q - z_{34}p) - r(z_{34}r - z_{35}q) \\
z_{47} &= p(z_{33}r - z_{35}p) + q(z_{34}r - z_{35}q) \\
z_{54} &= \pi + \theta_r^N \\
z_{55} &= \cos(z_{54}) \\
z_{56} &= \sin(\varphi_r^N) \\
z_{57} &= \sin(z_{54}) \\
z_{58} &= z_{56} z_{57} \\
z_{59} &= \cos(\varphi_r^N)
\end{aligned}$$

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$$\begin{aligned}
z_{60} &= z_{57} z_{59} \\
z_{61} &= z_{55} z_{56} \\
z_{62} &= z_{55} z_{59} \\
z_{63} &= z_{60} z_{61} - z_{58} z_{62} \\
z_{64} &= z_{56}^2 + z_{60}^2 + z_{62}^2 \\
z_{65} &= z_{55} z_{62} + z_{57} z_{60} \\
z_{66} &= z_{57} z_{61} - z_{55} z_{58} \\
z_{67} &= -z_{55} z_{61} - z_{57} z_{58} \\
z_{68} &= l_{\eta} z_{65} \\
z_{69} &= l_{\eta} z_{66} \\
z_{70} &= l_{\eta} z_{63} \\
z_{71} &= l_{\eta} z_{64} \\
z_{72} &= z_{41} q - z_{40} r \\
z_{73} &= z_{39} r - z_{41} p \\
z_{74} &= z_{40} p - z_{39} q \\
z_{75} &= z_{68} q_r^N + z_{69} p_r^N \\
z_{76} &= -z_{70} q_r^N - z_{71} p_r^N \\
z_{77} &= -z_{55} z_{56} p_r^N - z_{57} z_{59} q_r^N \\
z_{78} &= z_{55} z_{59} q_r^N - z_{56} z_{57} p_r^N \\
z_{79} &= z_{55} z_{60} q_r^N + z_{55} z_{77} + z_{57} z_{78} - z_{57} z_{62} q_r^N \\
z_{80} &= z_{55} z_{59} p_r^N - z_{56} z_{57} q_r^N \\
z_{81} &= z_{55} z_{56} q_r^N + z_{57} z_{59} p_r^N \\
z_{82} &= z_{55} z_{61} q_r^N + z_{57} z_{58} q_r^N + z_{57} z_{80} - z_{55} z_{81} \\
z_{83} &= l_{\eta} (p_r^N z_{82} + q_r^N z_{79}) \\
z_{84} &= z_{63} q_r^N + z_{64} p_r^N \\
z_{85} &= z_{65} q_r^N + z_{66} p_r^N \\
z_{86} &= z_{67} q_r^N \\
z_{87} &= z_{60} z_{80} + z_{61} z_{78} - z_{58} z_{77} - z_{62} z_{81} \\
z_{88} &= 2z_{56} z_{59} p_r^N + 2z_{60} z_{78} + 2z_{62} z_{77}
\end{aligned}$$

3 Dynamic Equations of the Single-Lift Model

$$z_{89} = l_p (p_r^N z_{88} + q_r^N z_{87})$$

$$z_{90} = q z_{74} - r z_{73}$$

$$z_{91} = r z_{72} - p z_{74}$$

$$z_{92} = p z_{73} - q z_{72}$$

$$z_{93} = z_{83} - z_{76} z_{86}$$

$$z_{94} = z_{75} z_{86} - z_{89}$$

$$z_{95} = z_{76} z_{84} - z_{75} z_{85}$$

$$z_{96} = g m_F$$

$$z_{97} = g l_p$$

$$z_{98} = g m_{MR}$$

$$z_{99} = z_{98} z_5 z_7 + z_{96} (z_5 z_7 - z_{10} z_{15} - z_{11} z_{16}) - z_{98} z_{10} z_{15} - z_{98} z_{11} z_{16}$$

$$z_{100} = z_{98} z_7 z_{12} + z_{96} (z_7 z_{12} - z_{13} z_{15} - z_{14} z_{16}) - z_{98} z_{13} z_{15} - z_{98} z_{14} z_{16}$$

$$z_{101} = -z_{97} - z_{98} z_7^2 - z_{98} z_{15}^2 - z_{98} z_{16}^2 - z_{96} (z_7^2 + z_{15}^2 + z_{16}^2)$$

$$z_{102} = z_{32} z_{98}$$

$$z_{103} = z_{31} z_{98}$$

$$z_{104} = z_{102} z_{15} - z_{103} z_{16} - z_{96} (z_{34} z_{16} - z_{35} z_{15}) - z_{97} (z_{40} z_{16} - z_{41} z_{15})$$

$$z_{105} = z_{30} z_{98}$$

$$z_{106} = z_{102} z_7 + z_{105} z_{16} + z_{96} (z_{33} z_{16} + z_{35} z_7) + z_{97} (z_{39} z_{16} + z_{41} z_7)$$

$$z_{107} = -z_{96} (z_{33} z_{15} + z_{34} z_7) - z_{97} (z_{39} z_{15} + z_{40} z_7) - z_{98} (z_{30} z_{15} + z_{31} z_7)$$

$$z_{108} = z_{97} (z_{57} z_{69} + z_{61} z_{71})$$

$$z_{109} = z_{97} (z_{57} z_{68} + z_{61} z_{70})$$

$$z_{110} = I_{11}^F p$$

$$z_{111} = I_{22}^F q$$

$$z_{112} = I_{33}^F r$$

$$z_{113} = p z_{111} - q z_{110}$$

$$z_{114} = r z_{110} - p z_{112}$$

$$z_{115} = q z_{112} - r z_{111}$$

$$z_{116} = I_{11}^{MR} p$$

$$z_{117} = I_{22}^{MR} q$$

$$z_{118} = I_{33}^{MR} z_{23}$$

3 Dynamic Equations of the Single-Lift Model

$$\begin{aligned}
z_{119} &= I_{11}^{MR} z_{24} \\
z_{120} &= I_{22}^{MR} z_{25} \\
z_{121} &= p z_{117} - q z_{116} \\
z_{122} &= z_{23} z_{116} - p z_{118} \\
z_{123} &= q z_{118} - z_{23} z_{117} \\
z_{124} &= z_7 z_{57} - z_5 z_{55} \\
z_{125} &= -z_5 z_{58} - z_7 z_{61} - z_{12} z_{59} \\
z_{126} &= z_{12} z_{56} - z_5 z_{60} - z_7 z_{62} \\
z_{127} &= -z_{10} z_{55} - z_{15} z_{57} \\
z_{128} &= z_{15} z_{61} - z_{10} z_{58} - z_{13} z_{59} \\
z_{129} &= z_{13} z_{56} + z_{15} z_{62} - z_{10} z_{60} \\
z_{130} &= -z_{11} z_{55} - z_{16} z_{57} \\
z_{131} &= z_{16} z_{61} - z_{11} z_{58} - z_{14} z_{59} \\
z_{132} &= z_{14} z_{56} + z_{16} z_{62} - z_{11} z_{60} \\
z_{133} &= m_F (z_{33} z_{10} - z_{34} z_5) + l_{\mathcal{P}} (z_{39} z_{10} - z_{40} z_5) + m_{MR} (z_{30} z_{10} - z_{31} z_5) \\
z_{134} &= z_{26} (z_5 z_7 - z_{10} z_{15} - z_{11} z_{16}) \\
z_{135} &= -m_F (z_{33} z_{11} - z_{35} z_5) - l_{\mathcal{P}} (z_{39} z_{11} - z_{41} z_5) - m_{MR} (z_{30} z_{11} - z_{32} z_5) \\
z_{136} &= l_{\mathcal{P}} + m_F (z_5^2 + z_{10}^2 + z_{11}^2) + m_{MR} (z_5^2 + z_{10}^2 + z_{11}^2) \\
z_{137} &= z_{26} (z_5 z_{12} + z_{10} z_{13} + z_{11} z_{14}) \\
z_{138} &= m_F (z_{34} z_{11} - z_{35} z_{10}) + l_{\mathcal{P}} (z_{40} z_{11} - z_{41} z_{10}) + m_{MR} (z_{31} z_{11} - z_{32} z_{10}) \\
z_{139} &= l_{\mathcal{P}} (z_{55} z_{68} - z_{58} z_{70}) \\
z_{140} &= l_{\mathcal{P}} (z_{55} z_{69} - z_{58} z_{71}) \\
z_{141} &= m_F (z_5 z_{45} + z_{10} z_{46} + z_{11} z_{47}) + m_{MR} (z_5 z_{42} + z_{10} z_{43} + z_{11} z_{44}) \\
&\quad + l_{\mathcal{P}} (z_5 z_{90} + z_{10} z_{91} + z_{11} z_{92} - z_{55} z_{93} - z_{58} z_{94} - z_{60} z_{95}) \\
z_{142} &= m_F (z_{33} z_{13} - z_{34} z_{12}) + l_{\mathcal{P}} (z_{39} z_{13} - z_{40} z_{12}) + m_{MR} (z_{30} z_{13} - z_{31} z_{12}) \\
z_{143} &= z_{26} (z_7 z_{12} - z_{13} z_{15} - z_{14} z_{16}) \\
z_{144} &= -m_F (z_{33} z_{14} - z_{35} z_{12}) - l_{\mathcal{P}} (z_{39} z_{14} - z_{41} z_{12}) - m_{MR} (z_{30} z_{14} - z_{32} z_{12}) \\
z_{145} &= l_{\mathcal{P}} + m_F (z_{12}^2 + z_{13}^2 + z_{14}^2) + m_{MR} (z_{12}^2 + z_{13}^2 + z_{14}^2) \\
z_{146} &= m_F (z_{34} z_{14} - z_{35} z_{13}) + l_{\mathcal{P}} (z_{40} z_{14} - z_{41} z_{13}) + m_{MR} (z_{31} z_{14} - z_{32} z_{13}) \\
z_{147} &= l_{\mathcal{P}} z_{59} z_{70}
\end{aligned}$$

3 Dynamic Equations of the Single-Lift Model

$$\begin{aligned}
z_{148} &= l_{\mathcal{P}} z_{59} z_{71} \\
z_{149} &= m_F (z_{12} z_{45} + z_{13} z_{46} + z_{14} z_{47}) + m_{MR} (z_{12} z_{42} + z_{13} z_{43} + z_{14} z_{44}) \\
&\quad + l_{\mathcal{P}} (z_{12} z_{90} + z_{13} z_{91} + z_{14} z_{92} + z_{56} z_{95} - z_{59} z_{94}) \\
z_{150} &= m_F (z_{33} z_{15} + z_{34} z_{16}) + l_{\mathcal{P}} (z_{39} z_{15} + z_{40} z_{16}) + m_{MR} (z_{30} z_{15} + z_{31} z_{16}) \\
z_{151} &= l_{\mathcal{P}} + m_F (z_7^2 + z_{15}^2 + z_{16}^2) + m_{MR} (z_7^2 + z_{15}^2 + z_{16}^2) \\
z_{152} &= -m_F (z_{33} z_{16} + z_{35} z_{17}) - l_{\mathcal{P}} (z_{39} z_{16} + z_{41} z_{17}) - m_{MR} (z_{30} z_{16} + z_{32} z_{17}) \\
z_{153} &= m_F (z_{34} z_{16} - z_{35} z_{15}) + l_{\mathcal{P}} (z_{40} z_{16} - z_{41} z_{15}) + m_{MR} (z_{31} z_{16} - z_{32} z_{15}) \\
z_{154} &= l_{\mathcal{P}} (z_{57} z_{68} + z_{61} z_{70}) \\
z_{155} &= l_{\mathcal{P}} (z_{57} z_{69} + z_{61} z_{71}) \\
z_{156} &= -m_F (z_7 z_{45} - z_{15} z_{46} - z_{16} z_{47}) - m_{MR} (z_7 z_{42} - z_{15} z_{43} - z_{16} z_{44}) \\
&\quad - l_{\mathcal{P}} (z_7 z_{90} + z_{57} z_{93} - z_{15} z_{91} - z_{16} z_{92} - z_{61} z_{94} - z_{62} z_{95}) \\
z_{157} &= I_{11}^F + I_{11}^{MR} + m_F (z_{34}^2 + z_{35}^2) + l_{\mathcal{P}} (z_{40}^2 + z_{41}^2) + m_{MR} (z_{31}^2 + z_{32}^2) \\
z_{158} &= -m_F z_{33} z_{35} - l_{\mathcal{P}} z_{39} z_{41} - m_{MR} z_{30} z_{32} \\
z_{159} &= -m_F z_{33} z_{34} - l_{\mathcal{P}} z_{39} z_{40} - m_{MR} z_{30} z_{31} \\
z_{160} &= l_{\mathcal{P}} (z_{40} z_{70} z_{131} + z_{41} z_{68} z_{127} - z_{40} z_{68} z_{130} - z_{41} z_{70} z_{128}) \\
z_{161} &= l_{\mathcal{P}} (z_{40} z_{71} z_{131} + z_{41} z_{69} z_{127} - z_{40} z_{69} z_{130} - z_{41} z_{71} z_{128}) \\
z_{162} &= z_{115} + z_{123} + m_F (z_{34} z_{47} - z_{35} z_{46}) + m_{MR} (z_{31} z_{44} - z_{32} z_{43}) \\
&\quad + l_{\mathcal{P}} (z_{40} z_{92} + z_{40} z_{130} z_{93} + z_{40} z_{131} z_{94} + z_{40} z_{132} z_{95} - z_{41} z_{91} - z_{41} z_{127} z_{93} \\
&\quad\quad\quad - z_{41} z_{128} z_{94} - z_{41} z_{129} z_{95}) - z_{119} \\
z_{163} &= I_{22}^F + I_{22}^{MR} + m_F (z_{33}^2 + z_{35}^2) + l_{\mathcal{P}} (z_{39}^2 + z_{41}^2) + m_{MR} (z_{30}^2 + z_{32}^2) \\
z_{164} &= -m_F z_{34} z_{35} - l_{\mathcal{P}} z_{40} z_{41} - m_{MR} z_{31} z_{32} \\
z_{165} &= l_{\mathcal{P}} (z_{39} z_{70} z_{131} + z_{41} z_{68} z_{124} - z_{39} z_{68} z_{130} - z_{41} z_{70} z_{125}) \\
z_{166} &= l_{\mathcal{P}} (z_{39} z_{71} z_{131} + z_{41} z_{69} z_{124} - z_{39} z_{69} z_{130} - z_{41} z_{71} z_{125}) \\
z_{167} &= z_{114} + z_{120} + z_{122} - m_F (z_{33} z_{47} - z_{35} z_{45}) - m_{MR} (z_{30} z_{44} - z_{32} z_{42}) \\
&\quad - l_{\mathcal{P}} (z_{39} z_{92} + z_{39} z_{130} z_{93} + z_{39} z_{131} z_{94} + z_{39} z_{132} z_{95} - z_{41} z_{90} - z_{41} z_{124} z_{93} \\
&\quad\quad\quad - z_{41} z_{125} z_{94} - z_{41} z_{126} z_{95}) \\
z_{168} &= I_{33}^F + I_{33}^{MR} + m_F (z_{33}^2 + z_{34}^2) + l_{\mathcal{P}} (z_{39}^2 + z_{40}^2) + m_{MR} (z_{30}^2 + z_{31}^2) \\
z_{169} &= l_{\mathcal{P}} (z_{39} z_{70} z_{128} + z_{40} z_{68} z_{124} - z_{39} z_{68} z_{127} - z_{40} z_{70} z_{125}) \\
z_{170} &= l_{\mathcal{P}} (z_{39} z_{71} z_{128} + z_{40} z_{69} z_{124} - z_{39} z_{69} z_{127} - z_{40} z_{71} z_{125})
\end{aligned}$$

3 Dynamic Equations of the Single-Lift Model

$$z_{171} = z_{113} + z_{121} + m_F (z_{33} z_{46} - z_{34} z_{45}) + m_{MR} (z_{30} z_{43} - z_{31} z_{42}) \\ + l_{\mathcal{P}} (z_{39} z_{91} + z_{39} z_{127} z_{93} + z_{39} z_{128} z_{94} + z_{39} z_{129} z_{95} - z_{40} z_{90} - z_{40} z_{124} z_{93} \\ - z_{40} z_{125} z_{94} - z_{40} z_{126} z_{95})$$

$$z_{172} = l_{\mathcal{P}} (z_{68} z_{69} + z_{70} z_{71})$$

$$z_{173} = l_{\mathcal{P}} (z_{69}^2 + z_{71}^2)$$

$$z_{174} = l_{\mathcal{P}} (z_{69} z_{93} + z_{69} z_{124} z_{90} + z_{69} z_{127} z_{91} + z_{69} z_{130} z_{92} - z_{71} z_{94} - z_{71} z_{125} z_{90} \\ - z_{71} z_{128} z_{91} - z_{71} z_{131} z_{92})$$

$$z_{175} = l_{\mathcal{P}} (z_{68}^2 + z_{70}^2)$$

$$z_{176} = l_{\mathcal{P}} (z_{68} z_{93} + z_{68} z_{124} z_{90} + z_{68} z_{127} z_{91} + z_{68} z_{130} z_{92} - z_{70} z_{94} - z_{70} z_{125} z_{90} \\ - z_{70} z_{128} z_{91} - z_{70} z_{131} z_{92})$$

4 Dynamic Equations of the Dual-Lift Model

The following equations describe the dynamics of the multi-lift model, presented in the thesis main document [Ber13, pages 76ff.], for two coupled helicopters (dual-lift configuration). The variables z_{xyz} represent intermediate results, which are generated automatically.

$$0 = z_{644} - z_{414} \dot{u}_{h1} - z_{415} \dot{v}_{h1} - z_{416} \dot{w}_{h1} - z_{417} \dot{p}_{h1} - z_{418} \dot{q}_{h1} - z_{419} \dot{r}_{h1} \\ - z_{420} \dot{u}_{h2} - z_{421} \dot{v}_{h2} - z_{422} \dot{w}_{h2} - z_{423} \dot{p}_{h2} - z_{424} \dot{q}_{h2} - z_{425} \dot{r}_{h2} - z_{426} \dot{v}_l$$

$$0 = z_{645} - z_{432} \dot{u}_{h1} - z_{433} \dot{v}_{h1} - z_{434} \dot{w}_{h1} - z_{435} \dot{p}_{h1} - z_{436} \dot{q}_{h1} - z_{437} \dot{r}_{h1} \\ - z_{438} \dot{u}_{h2} - z_{439} \dot{v}_{h2} - z_{440} \dot{w}_{h2} - z_{441} \dot{p}_{h2} - z_{442} \dot{q}_{h2} - z_{443} \dot{r}_{h2} - z_{444} \dot{v}_l$$

$$0 = z_{646} - z_{450} \dot{u}_{h1} - z_{451} \dot{v}_{h1} - z_{452} \dot{w}_{h1} - z_{453} \dot{p}_{h1} - z_{454} \dot{q}_{h1} - z_{455} \dot{r}_{h1} \\ - z_{456} \dot{u}_{h2} - z_{457} \dot{v}_{h2} - z_{458} \dot{w}_{h2} - z_{459} \dot{p}_{h2} - z_{460} \dot{q}_{h2} - z_{461} \dot{r}_{h2} - z_{462} \dot{v}_l$$

$$0 = z_{647} - z_{468} \dot{u}_{h1} - z_{469} \dot{v}_{h1} - z_{470} \dot{w}_{h1} - z_{471} \dot{p}_{h1} - z_{472} \dot{q}_{h1} - z_{473} \dot{r}_{h1} \\ - z_{474} \dot{u}_{h2} - z_{475} \dot{v}_{h2} - z_{476} \dot{w}_{h2} - z_{477} \dot{p}_{h2} - z_{478} \dot{q}_{h2} - z_{479} \dot{r}_{h2} - z_{480} \dot{v}_l$$

$$0 = z_{648} - z_{486} \dot{u}_{h1} - z_{487} \dot{v}_{h1} - z_{488} \dot{w}_{h1} - z_{489} \dot{p}_{h1} - z_{490} \dot{q}_{h1} - z_{491} \dot{r}_{h1} \\ - z_{492} \dot{u}_{h2} - z_{493} \dot{v}_{h2} - z_{494} \dot{w}_{h2} - z_{495} \dot{p}_{h2} - z_{496} \dot{q}_{h2} - z_{497} \dot{r}_{h2} - z_{498} \dot{v}_l$$

$$0 = z_{649} - z_{504} \dot{u}_{h1} - z_{505} \dot{v}_{h1} - z_{506} \dot{w}_{h1} - z_{507} \dot{p}_{h1} - z_{508} \dot{q}_{h1} - z_{509} \dot{r}_{h1} \\ - z_{510} \dot{u}_{h2} - z_{511} \dot{v}_{h2} - z_{512} \dot{w}_{h2} - z_{513} \dot{p}_{h2} - z_{514} \dot{q}_{h2} - z_{515} \dot{r}_{h2} - z_{516} \dot{v}_l$$

$$0 = z_{650} - z_{522} \dot{u}_{h1} - z_{523} \dot{v}_{h1} - z_{524} \dot{w}_{h1} - z_{525} \dot{p}_{h1} - z_{526} \dot{q}_{h1} - z_{527} \dot{r}_{h1} \\ - z_{528} \dot{u}_{h2} - z_{529} \dot{v}_{h2} - z_{530} \dot{w}_{h2} - z_{531} \dot{p}_{h2} - z_{532} \dot{q}_{h2} - z_{533} \dot{r}_{h2} - z_{534} \dot{v}_l$$

$$0 = z_{651} - z_{540} \dot{u}_{h1} - z_{541} \dot{v}_{h1} - z_{542} \dot{w}_{h1} - z_{543} \dot{p}_{h1} - z_{544} \dot{q}_{h1} - z_{545} \dot{r}_{h1} \\ - z_{546} \dot{u}_{h2} - z_{547} \dot{v}_{h2} - z_{548} \dot{w}_{h2} - z_{549} \dot{p}_{h2} - z_{550} \dot{q}_{h2} - z_{551} \dot{r}_{h2} - z_{552} \dot{v}_l$$

$$0 = z_{652} - z_{558} \dot{u}_{h1} - z_{559} \dot{v}_{h1} - z_{560} \dot{w}_{h1} - z_{561} \dot{p}_{h1} - z_{562} \dot{q}_{h1} - z_{563} \dot{r}_{h1} \\ - z_{564} \dot{u}_{h2} - z_{565} \dot{v}_{h2} - z_{566} \dot{w}_{h2} - z_{567} \dot{p}_{h2} - z_{568} \dot{q}_{h2} - z_{569} \dot{r}_{h2} - z_{570} \dot{v}_l$$

$$0 = z_{653} - z_{576} \dot{u}_{h1} - z_{577} \dot{v}_{h1} - z_{578} \dot{w}_{h1} - z_{579} \dot{p}_{h1} - z_{580} \dot{q}_{h1} - z_{581} \dot{r}_{h1} \\ - z_{582} \dot{u}_{h2} - z_{583} \dot{v}_{h2} - z_{584} \dot{w}_{h2} - z_{585} \dot{p}_{h2} - z_{586} \dot{q}_{h2} - z_{587} \dot{r}_{h2} - z_{588} \dot{v}_l$$

$$0 = z_{654} - z_{594} \dot{u}_{h1} - z_{595} \dot{v}_{h1} - z_{596} \dot{w}_{h1} - z_{597} \dot{p}_{h1} - z_{598} \dot{q}_{h1} - z_{599} \dot{r}_{h1} \\ - z_{600} \dot{u}_{h2} - z_{601} \dot{v}_{h2} - z_{602} \dot{w}_{h2} - z_{603} \dot{p}_{h2} - z_{604} \dot{q}_{h2} - z_{605} \dot{r}_{h2} - z_{606} \dot{v}_l$$

$$0 = z_{655} - z_{612} \dot{u}_{h1} - z_{613} \dot{v}_{h1} - z_{614} \dot{w}_{h1} - z_{615} \dot{p}_{h1} - z_{616} \dot{q}_{h1} - z_{617} \dot{r}_{h1} \\ - z_{618} \dot{u}_{h2} - z_{619} \dot{v}_{h2} - z_{620} \dot{w}_{h2} - z_{621} \dot{p}_{h2} - z_{622} \dot{q}_{h2} - z_{623} \dot{r}_{h2} - z_{624} \dot{v}_l$$

4 Dynamic Equations of the Dual-Lift Model

$$0 = z_{656} - z_{630} \dot{u}_{h1} - z_{631} \dot{v}_{h1} - z_{632} \dot{w}_{h1} - z_{633} \dot{p}_{h1} - z_{634} \dot{q}_{h1} - z_{635} \dot{r}_{h1} \\ - z_{636} \dot{u}_{h2} - z_{637} \dot{v}_{h2} - z_{638} \dot{w}_{h2} - z_{639} \dot{p}_{h2} - z_{640} \dot{q}_{h2} - z_{641} \dot{r}_{h2} - z_{642} \dot{v}_l$$

The equations need to be solved for the desired variables $(\dot{u}_{hx}, \dot{v}_{hx}, \dot{w}_{hx}, \dot{p}_{hx}, \dot{q}_{hx}, \dot{r}_{hx}, \dot{v}_l)$, with $x \in \{1, 2, 3\}$, to get the explicit equations.

The following equations define the z_{xyz} variables:

$$z_6 = \sin(\varphi_{h1})$$

$$z_{36} = \tan(\theta_{h1})$$

$$z_{37} = z_6 z_{36}$$

$$z_8 = \cos(\varphi_{h1})$$

$$z_{38} = z_8 z_{36}$$

$$\dot{\varphi}_{h1} = p_{h1} + z_{37} q_{h1} + z_{38} r_{h1}$$

$$\dot{\theta}_{h1} = z_8 q_{h1} - z_6 r_{h1}$$

$$z_{33} = \cos(\theta_{h1})$$

$$z_{34} = z_6 / z_{33}$$

$$z_{35} = z_8 / z_{33}$$

$$\dot{\psi}_{h1} = z_{34} q_{h1} + z_{35} r_{h1}$$

$$z_{22} = \sin(\varphi_{h2})$$

$$z_{42} = \tan(\theta_{h2})$$

$$z_{43} = z_{22} z_{42}$$

$$z_{24} = \cos(\varphi_{h2})$$

$$z_{44} = z_{24} z_{42}$$

$$\dot{\varphi}_{h2} = p_{h2} + z_{43} q_{h2} + z_{44} r_{h2}$$

$$\dot{\theta}_{h2} = z_{24} q_{h2} - z_{22} r_{h2}$$

$$z_{39} = \cos(\theta_{h2})$$

$$z_{40} = z_{22} / z_{39}$$

$$z_{41} = z_{24} / z_{39}$$

$$\dot{\psi}_{h2} = z_{40} q_{h2} + z_{41} r_{h2}$$

$$z_{55} = x_{h2} - x_{h1}$$

$$z_{56} = y_{h2} - y_{h1}$$

$$z_{57} = z_{h2} - z_{h1}$$

4 Dynamic Equations of the Dual-Lift Model

$$\begin{aligned}
 z_{58} &= z_{55}^2 + z_{56}^2 + z_{57}^2 \\
 z_{59} &= z_{58}^{0.5} \\
 z_{60} &= z_{55} / z_{59} \\
 z_{129} &= z_{h1} - z_l \\
 z_{65} &= z_l - z_{h1} \\
 z_{63} &= x_l - x_{h1} \\
 z_{64} &= y_l - y_{h1} \\
 z_{66} &= z_{63}^2 + z_{64}^2 + z_{65}^2 \\
 z_{67} &= z_{66}^{0.5} \\
 z_{70} &= z_{65} / z_{67} \\
 z_{62} &= z_{57} / z_{59} \\
 z_{68} &= z_{63} / z_{67} \\
 z_{61} &= z_{56} / z_{59} \\
 z_{69} &= z_{64} / z_{67} \\
 z_{51} &= m_F + m_{MR} \\
 z_{53} &= (dO Fo_2 m_F + dOM Ro_2 m_{MR}) / z_{51} \\
 z_{98} &= dOR o_2 - z_{53} \\
 z_2 &= \pi + \theta_{h1} \\
 z_4 &= \cos(z_2) \\
 z_{15} &= z_4 z_6 \\
 z_{54} &= (dO Fo_3 m_F + dOM Ro_3 m_{MR}) / z_{51} \\
 z_{99} &= dOR o_3 - z_{54} \\
 z_{16} &= z_4 z_8 \\
 z_{127} &= x_{h1} - x_l \\
 z_{52} &= (dO Fo_1 m_F + dOM Ro_1 m_{MR}) / z_{51} \\
 z_{97} &= dOR o_1 - z_{52} \\
 z_1 &= \pi + \psi_{h1} \\
 z_3 &= \cos(z_1) \\
 z_5 &= z_3 z_4 \\
 z_7 &= \sin(z_2)
 \end{aligned}$$

4 Dynamic Equations of the Dual-Lift Model

$$\begin{aligned}
z_9 &= \sin(z_1) \\
z_{10} &= z_3 z_6 z_7 - z_8 z_9 \\
z_{11} &= z_6 z_9 + z_3 z_7 z_8 \\
z_{128} &= y_{h1} - y_l \\
z_{12} &= z_4 z_9 \\
z_{13} &= z_3 z_8 + z_6 z_7 z_9 \\
z_{14} &= z_7 z_8 z_9 - z_3 z_6 \\
z_{160} &= 2z_{129} (z_{60} (z_{60} z_{70} - z_{62} z_{68}) + z_{61} (z_{61} z_{70} - z_{62} z_{69})) \\
&\quad + 2z_{98} z_{15} (z_{60} (z_{60} z_{70} - z_{62} z_{68}) + z_{61} (z_{61} z_{70} - z_{62} z_{69})) \\
&\quad + 2z_{99} z_{16} (z_{60} (z_{60} z_{70} - z_{62} z_{68}) + z_{61} (z_{61} z_{70} - z_{62} z_{69})) - 2z_{62} z_{127} - 2z_{97} z_5 z_{62} \\
&\quad - 2z_{98} z_{10} z_{62} - 2z_{99} z_{11} z_{62} - 2z_{128} (z_{60} z_{69} - z_{61} z_{68}) - 2z_{97} z_{12} (z_{60} z_{69} - z_{61} z_{68}) \\
&\quad - 2z_{98} z_{13} (z_{60} z_{69} - z_{61} z_{68}) - 2z_{99} z_{14} (z_{60} z_{69} - z_{61} z_{68}) \\
&\quad - 2z_{97} z_7 (z_{60} (z_{60} z_{70} - z_{62} z_{68}) + z_{61} (z_{61} z_{70} - z_{62} z_{69})) \\
z_{18} &= \pi + \theta_{h2} \\
z_{20} &= \cos(z_{18}) \\
z_{143} &= z_{h2} - z_l \\
z_{142} &= y_{h2} - y_l \\
z_{17} &= \pi + \psi_{h2} \\
z_{19} &= \cos(z_{17}) \\
z_{21} &= z_{19} z_{20} \\
z_{23} &= \sin(z_{18}) \\
z_{25} &= \sin(z_{17}) \\
z_{150} &= z_{23} z_{25} \\
z_{153} &= z_{22} z_{23} \\
z_{32} &= z_{20} z_{24} \\
z_{141} &= x_{h2} - x_l \\
z_{144} &= z_{19} z_{23} \\
z_{28} &= z_{20} z_{25} \\
z_{154} &= z_{23} z_{24} \\
z_{31} &= z_{20} z_{22} \\
z_{145} &= z_{19} z_{20} z_{22} \\
z_{27} &= z_{22} z_{25} + z_{19} z_{23} z_{24} \\
z_{146} &= -z_{19} z_{24} - z_{22} z_{23} z_{25} \\
z_{151} &= z_{20} z_{22} z_{25}
\end{aligned}$$

4 Dynamic Equations of the Dual-Lift Model

$$\begin{aligned}
z_{26} &= z_{19} z_{22} z_{23} - z_{24} z_{25} \\
z_{30} &= z_{23} z_{24} z_{25} - z_{19} z_{22} \\
z_{149} &= z_{19} z_{20} z_{24} \\
z_{148} &= z_{19} z_{22} - z_{23} z_{24} z_{25} \\
z_{147} &= z_{24} z_{25} - z_{19} z_{22} z_{23} \\
z_{152} &= z_{20} z_{24} z_{25} \\
z_{164} &= 2z_{97} z_{20} z_{22} z_{143} + 2z_{97} z_{142} (z_{21} z_{41} + z_{22} z_{150}) + 2z_{98} z_{143} (z_{22} z_{153} + z_{32} z_{44}) \\
&\quad + 2z_{97} z_{141} (z_{22} z_{144} - z_{28} z_{41}) + 2z_{99} z_{143} (z_{22} z_{154} - z_{31} z_{44}) \\
&\quad - 2z_{98} z_{141} (z_{22} z_{145} - z_{27} z_{44} - z_{41} z_{146}) - 2z_{98} z_{142} (z_{22} z_{151} - z_{26} z_{41} - z_{30} z_{44}) \\
&\quad - 2z_{99} z_{141} (z_{22} z_{149} - z_{41} z_{148} - z_{44} z_{147}) - 2z_{99} z_{142} (z_{22} z_{152} - z_{27} z_{41} - z_{44} z_{146}) \\
z_{158} &= 2z_{97} z_7 (z_{61} (z_{60} z_{69} - z_{61} z_{68}) + z_{62} (z_{60} z_{70} - z_{62} z_{68})) - 2z_{60} z_{127} \\
&\quad - 2z_{97} z_5 z_{60} - 2z_{98} z_{10} z_{60} - 2z_{99} z_{11} z_{60} - 2z_{128} (z_{61} z_{70} - z_{62} z_{69}) \\
&\quad - 2z_{97} z_{12} (z_{61} z_{70} - z_{62} z_{69}) - 2z_{98} z_{13} (z_{61} z_{70} - z_{62} z_{69}) \\
&\quad - 2z_{99} z_{14} (z_{61} z_{70} - z_{62} z_{69}) - 2z_{129} (z_{61} (z_{60} z_{69} - z_{61} z_{68}) + z_{62} (z_{60} z_{70} - z_{62} z_{68})) \\
&\quad - 2z_{98} z_{15} (z_{61} (z_{60} z_{69} - z_{61} z_{68}) + z_{62} (z_{60} z_{70} - z_{62} z_{68})) \\
&\quad - 2z_{99} z_{16} (z_{61} (z_{60} z_{69} - z_{61} z_{68}) + z_{62} (z_{60} z_{70} - z_{62} z_{68})) \\
z_{29} &= z_{19} z_{24} + z_{22} z_{23} z_{25} \\
z_{169} &= 2z_{143} (z_{60} (z_{60} z_{70} - z_{62} z_{68}) + z_{61} (z_{61} z_{70} - z_{62} z_{69})) \\
&\quad + 2z_{98} z_{31} (z_{60} (z_{60} z_{70} - z_{62} z_{68}) + z_{61} (z_{61} z_{70} - z_{62} z_{69})) \\
&\quad + 2z_{99} z_{32} (z_{60} (z_{60} z_{70} - z_{62} z_{68}) + z_{61} (z_{61} z_{70} - z_{62} z_{69})) - 2z_{62} z_{141} - 2z_{97} z_{21} z_{62} \\
&\quad - 2z_{98} z_{26} z_{62} - 2z_{99} z_{27} z_{62} - 2z_{142} (z_{60} z_{69} - z_{61} z_{68}) - 2z_{97} z_{28} (z_{60} z_{69} - z_{61} z_{68}) \\
&\quad - 2z_{98} z_{29} (z_{60} z_{69} - z_{61} z_{68}) - 2z_{99} z_{30} (z_{60} z_{69} - z_{61} z_{68}) \\
&\quad - 2z_{97} z_{23} (z_{60} (z_{60} z_{70} - z_{62} z_{68}) + z_{61} (z_{61} z_{70} - z_{62} z_{69})) \\
z_{167} &= 2z_{97} z_{23} (z_{61} (z_{60} z_{69} - z_{61} z_{68}) + z_{62} (z_{60} z_{70} - z_{62} z_{68})) - 2z_{60} z_{141} \\
&\quad - 2z_{97} z_{21} z_{60} - 2z_{98} z_{26} z_{60} - 2z_{99} z_{27} z_{60} - 2z_{142} (z_{61} z_{70} - z_{62} z_{69}) \\
&\quad - 2z_{97} z_{28} (z_{61} z_{70} - z_{62} z_{69}) - 2z_{98} z_{29} (z_{61} z_{70} - z_{62} z_{69}) \\
&\quad - 2z_{99} z_{30} (z_{61} z_{70} - z_{62} z_{69}) - 2z_{143} (z_{61} (z_{60} z_{69} - z_{61} z_{68}) + z_{62} (z_{60} z_{70} - z_{62} z_{68})) \\
&\quad - 2z_{98} z_{31} (z_{61} (z_{60} z_{69} - z_{61} z_{68}) + z_{62} (z_{60} z_{70} - z_{62} z_{68})) \\
&\quad - 2z_{99} z_{32} (z_{61} (z_{60} z_{69} - z_{61} z_{68}) + z_{62} (z_{60} z_{70} - z_{62} z_{68})) \\
z_{173} &= z_{158} z_{169} - z_{160} z_{167} \\
z_{181} &= z_{160} z_{164} / z_{173} \\
z_{165} &= 2z_{97} z_{142} (z_{21} z_{40} - z_{24} z_{150}) + 2z_{98} z_{141} (z_{24} z_{145} + z_{27} z_{43} + z_{40} z_{146}) \\
&\quad + 2z_{98} z_{142} (z_{24} z_{151} + z_{26} z_{40} + z_{30} z_{43}) + 2z_{99} z_{141} (z_{24} z_{149} + z_{40} z_{148} + z_{43} z_{147}) \\
&\quad + 2z_{99} z_{142} (z_{24} z_{152} + z_{27} z_{40} + z_{43} z_{146}) \\
&\quad - 2z_{97} z_{20} z_{24} z_{143} - 2z_{97} z_{141} (z_{24} z_{144} + z_{28} z_{40}) \\
&\quad - 2z_{99} z_{143} (z_{24} z_{154} + z_{31} z_{43}) - 2z_{98} z_{143} (z_{24} z_{153} - z_{32} z_{43}) \\
z_{182} &= z_{160} z_{165} / z_{173}
\end{aligned}$$

4 Dynamic Equations of the Dual-Lift Model

$$\begin{aligned}
z_{166} &= 2z_{141} + 2z_{97} z_{21} + 2z_{98} z_{26} + 2z_{99} z_{27} \\
z_{183} &= z_{160} z_{166} / z_{173} \\
z_{170} &= 2z_{98} z_{27} z_{141} + 2z_{98} z_{30} z_{142} + 2z_{98} z_{32} z_{143} \\
&\quad + 2z_{99} z_{141} z_{147} + 2z_{99} z_{142} z_{146} - 2z_{99} z_{31} z_{143} \\
z_{184} &= z_{160} z_{170} / z_{173} \\
z_{171} &= 2z_{142} + 2z_{97} z_{28} + 2z_{98} z_{29} + 2z_{99} z_{30} \\
z_{185} &= z_{160} z_{171} / z_{173} \\
z_{172} &= 2z_{143} + 2z_{98} z_{31} + 2z_{99} z_{32} - 2z_{97} z_{23} \\
z_{186} &= z_{160} z_{172} / z_{173} \\
z_{136} &= z_7 z_9 \\
z_{139} &= z_6 z_7 \\
z_{130} &= z_3 z_7 \\
z_{140} &= z_7 z_8 \\
z_{131} &= z_3 z_4 z_6 \\
z_{132} &= -z_3 z_8 - z_6 z_7 z_9 \\
z_{137} &= z_4 z_6 z_9 \\
z_{135} &= z_3 z_4 z_8 \\
z_{134} &= z_3 z_6 - z_7 z_8 z_9 \\
z_{133} &= z_8 z_9 - z_3 z_6 z_7 \\
z_{138} &= z_4 z_8 z_9 \\
z_{155} &= 2z_{97} z_4 z_6 z_{129} + 2z_{97} z_{128} (z_5 z_{35} + z_6 z_{136}) + 2z_{98} z_{129} (z_6 z_{139} + z_{16} z_{38}) \\
&\quad + 2z_{97} z_{127} (z_6 z_{130} - z_{12} z_{35}) + 2z_{99} z_{129} (z_6 z_{140} - z_{15} z_{38}) \\
&\quad - 2z_{98} z_{127} (z_6 z_{131} - z_{11} z_{38} - z_{35} z_{132}) - 2z_{98} z_{128} (z_6 z_{137} - z_{10} z_{35} - z_{14} z_{38}) \\
&\quad - 2z_{99} z_{127} (z_6 z_{135} - z_{35} z_{134} - z_{38} z_{133}) - 2z_{99} z_{128} (z_6 z_{138} - z_{11} z_{35} - z_{38} z_{132}) \\
z_{174} &= z_{155} z_{169} / z_{173} \\
z_{156} &= 2z_{97} z_{128} (z_5 z_{34} - z_8 z_{136}) + 2z_{98} z_{127} (z_8 z_{131} + z_{11} z_{37} + z_{34} z_{132}) \\
&\quad + 2z_{98} z_{128} (z_8 z_{137} + z_{10} z_{34} + z_{14} z_{37}) + 2z_{99} z_{127} (z_8 z_{135} + z_{34} z_{134} + z_{37} z_{133}) \\
&\quad + 2z_{99} z_{128} (z_8 z_{138} + z_{11} z_{34} + z_{37} z_{132}) - 2z_{97} z_4 z_8 z_{129} - 2z_{97} z_{127} (z_8 z_{130} + z_{12} z_{34}) \\
&\quad - 2z_{99} z_{129} (z_8 z_{140} + z_{15} z_{37}) - 2z_{98} z_{129} (z_8 z_{139} - z_{16} z_{37}) \\
z_{175} &= z_{156} z_{169} / z_{173} \\
z_{157} &= 2z_{127} + 2z_{97} z_5 + 2z_{98} z_{10} + 2z_{99} z_{11} \\
z_{176} &= z_{157} z_{169} / z_{173}
\end{aligned}$$

4 Dynamic Equations of the Dual-Lift Model

$$\begin{aligned}
 z_{159} = & 2z_{128} (z_{60} z_{70} - z_{62} z_{68}) + 2z_{97} z_{12} (z_{60} z_{70} - z_{62} z_{68}) + 2z_{98} z_{13} (z_{60} z_{70} - z_{62} z_{68}) \\
 & + 2z_{99} z_{14} (z_{60} z_{70} - z_{62} z_{68}) + 2z_{129} (z_{60} (z_{60} z_{69} - z_{61} z_{68}) - z_{62} (z_{61} z_{70} - z_{62} z_{69})) \\
 & + 2z_{98} z_{15} (z_{60} (z_{60} z_{69} - z_{61} z_{68}) - z_{62} (z_{61} z_{70} - z_{62} z_{69})) \\
 & + 2z_{99} z_{16} (z_{60} (z_{60} z_{69} - z_{61} z_{68}) - z_{62} (z_{61} z_{70} - z_{62} z_{69})) - 2z_{61} z_{127} - 2z_{97} z_5 z_{61} \\
 & - 2z_{98} z_{10} z_{61} - 2z_{99} z_{11} z_{61} - 2z_{97} z_7 (z_{60} (z_{60} z_{69} - z_{61} z_{68}) - z_{62} (z_{61} z_{70} - z_{62} z_{69}))
 \end{aligned}$$

$$\begin{aligned}
 z_{168} = & 2z_{142} (z_{60} z_{70} - z_{62} z_{68}) + 2z_{97} z_{28} (z_{60} z_{70} - z_{62} z_{68}) + 2z_{98} z_{29} (z_{60} z_{70} - z_{62} z_{68}) \\
 & + 2z_{99} z_{30} (z_{60} z_{70} - z_{62} z_{68}) + 2z_{143} (z_{60} (z_{60} z_{69} - z_{61} z_{68}) - z_{62} (z_{61} z_{70} - z_{62} z_{69})) \\
 & + 2z_{98} z_{31} (z_{60} (z_{60} z_{69} - z_{61} z_{68}) - z_{62} (z_{61} z_{70} - z_{62} z_{69})) \\
 & + 2z_{99} z_{32} (z_{60} (z_{60} z_{69} - z_{61} z_{68}) - z_{62} (z_{61} z_{70} - z_{62} z_{69})) - 2z_{61} z_{141} - 2z_{97} z_{21} z_{61} \\
 & - 2z_{98} z_{26} z_{61} - 2z_{99} z_{27} z_{61} - 2z_{97} z_{23} (z_{60} (z_{60} z_{69} - z_{61} z_{68}) - z_{62} (z_{61} z_{70} - z_{62} z_{69}))
 \end{aligned}$$

$$z_{177} = (z_{159} z_{169} - z_{160} z_{168}) / z_{173}$$

$$\begin{aligned}
 z_{161} = & 2z_{98} z_{11} z_{127} + 2z_{98} z_{14} z_{128} + 2z_{98} z_{16} z_{129} \\
 & + 2z_{99} z_{127} z_{133} + 2z_{99} z_{128} z_{132} - 2z_{99} z_{15} z_{129}
 \end{aligned}$$

$$z_{178} = z_{161} z_{169} / z_{173}$$

$$z_{162} = 2z_{128} + 2z_{97} z_{12} + 2z_{98} z_{13} + 2z_{99} z_{14}$$

$$z_{179} = z_{162} z_{169} / z_{173}$$

$$z_{163} = 2z_{129} + 2z_{98} z_{15} + 2z_{99} z_{16} - 2z_{97} z_7$$

$$z_{180} = z_{163} z_{169} / z_{173}$$

$$\begin{aligned}
 u_l = & z_{181} r_{h2} + z_{182} q_{h2} + z_{183} u_{h2} + z_{184} p_{h2} + z_{185} v_{h2} + z_{186} w_{h2} - z_{174} r_{h1} \\
 & - z_{175} q_{h1} - z_{176} u_{h1} - z_{177} v_l - z_{178} p_{h1} - z_{179} v_{h1} - z_{180} w_{h1}
 \end{aligned}$$

$$z_{187} = z_{155} z_{167} / z_{173}$$

$$z_{188} = z_{156} z_{167} / z_{173}$$

$$z_{189} = z_{157} z_{167} / z_{173}$$

$$z_{191} = z_{161} z_{167} / z_{173}$$

$$z_{192} = z_{162} z_{167} / z_{173}$$

$$z_{193} = z_{163} z_{167} / z_{173}$$

$$z_{190} = (z_{158} z_{168} - z_{159} z_{167}) / z_{173}$$

$$z_{194} = z_{158} z_{164} / z_{173}$$

$$z_{195} = z_{158} z_{165} / z_{173}$$

$$z_{196} = z_{158} z_{166} / z_{173}$$

$$z_{197} = z_{158} z_{170} / z_{173}$$

$$z_{198} = z_{158} z_{171} / z_{173}$$

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$$z_{199} = z_{158} z_{172} / z_{173}$$

$$w_l = z_{187} r_{h1} + z_{188} q_{h1} + z_{189} u_{h1} + z_{191} p_{h1} + z_{192} v_{h1} + z_{193} w_{h1} - z_{190} v_l \\ - z_{194} r_{h2} - z_{195} q_{h2} - z_{196} u_{h2} - z_{197} p_{h2} - z_{198} v_{h2} - z_{199} w_{h2}$$

$$\dot{x}_l = z_{60} u_l + z_{61} v_l + z_{62} w_l$$

$$\dot{y}_l = (z_{60} z_{69} - z_{61} z_{68}) w_l + (z_{61} z_{70} - z_{62} z_{69}) u_l - (z_{60} z_{70} - z_{62} z_{68}) v_l$$

$$\dot{z}_l = (z_{61} (z_{60} z_{69} - z_{61} z_{68}) + z_{62} (z_{60} z_{70} - z_{62} z_{68})) u_l \\ - (z_{60} (z_{60} z_{70} - z_{62} z_{68}) + z_{61} (z_{61} z_{70} - z_{62} z_{69})) w_l \\ - (z_{60} (z_{60} z_{69} - z_{61} z_{68}) - z_{62} (z_{61} z_{70} - z_{62} z_{69})) v_l$$

$$z_{45} = \omega_{MR} + r_{h1}$$

$$z_{46} = q_{h1} (r_{h1} - z_{45})$$

$$z_{47} = p_{h1} (r_{h1} - z_{45})$$

$$z_{48} = \omega_{MR} + r_{h2}$$

$$z_{49} = q_{h2} (r_{h2} - z_{48})$$

$$z_{50} = p_{h2} (r_{h2} - z_{48})$$

$$z_{71} = -2z_{55} (u_{h1} - u_{h2}) - 2z_{56} (v_{h1} - v_{h2}) - 2z_{57} (w_{h1} - w_{h2})$$

$$z_{72} = (2z_{59} (u_{h1} - u_{h2}) + z_{55} z_{71} / z_{58}^{0.5}) / z_{59}^2$$

$$z_{73} = (2z_{59} (v_{h1} - v_{h2}) + z_{56} z_{71} / z_{58}^{0.5}) / z_{59}^2$$

$$z_{74} = (2z_{59} (w_{h1} - w_{h2}) + z_{57} z_{71} / z_{58}^{0.5}) / z_{59}^2$$

$$z_{75} = -0.5u_l z_{72} - 0.5v_l z_{73} - 0.5w_l z_{74}$$

$$z_{76} = -2z_{63} (u_{h1} - \dot{x}_l) - 2z_{64} (v_{h1} - \dot{y}_l) - 2z_{65} (w_{h1} - \dot{z}_l)$$

$$z_{77} = (2z_{67} (v_{h1} - \dot{y}_l) + z_{64} z_{76} / z_{66}^{0.5}) / z_{67}^2$$

$$z_{78} = (2z_{67} (u_{h1} - \dot{x}_l) + z_{63} z_{76} / z_{66}^{0.5}) / z_{67}^2$$

$$z_{79} = (2z_{67} (w_{h1} - \dot{z}_l) + z_{65} z_{76} / z_{66}^{0.5}) / z_{67}^2$$

$$z_{80} = z_{60} z_{69} - z_{61} z_{68}$$

$$z_{81} = z_{61} z_{70} - z_{62} z_{69}$$

$$z_{82} = z_{62} z_{68} - z_{60} z_{70}$$

$$z_{83} = 0.5v_l (z_{70} z_{72} + z_{60} z_{79} - z_{68} z_{74} - z_{62} z_{78}) - 0.5u_l (z_{70} z_{73} + z_{61} z_{79} - z_{69} z_{74} - z_{62} z_{77}) \\ - 0.5w_l (z_{69} z_{72} + z_{60} z_{77} - z_{68} z_{73} - z_{61} z_{78})$$

$$z_{84} = z_{61} (z_{60} z_{69} - z_{61} z_{68}) + z_{62} (z_{60} z_{70} - z_{62} z_{68})$$

$$z_{85} = -z_{60} (z_{60} z_{70} - z_{62} z_{68}) - z_{61} (z_{61} z_{70} - z_{62} z_{69})$$

4 Dynamic Equations of the Dual-Lift Model

$$\begin{aligned}
z_{86} &= z_{62} (z_{61} z_{70} - z_{62} z_{69}) - z_{60} (z_{60} z_{69} - z_{61} z_{68}) \\
z_{87} &= 0.5w_l ((z_{60} z_{70} - z_{62} z_{68})z_{72} + (z_{61} z_{70} - z_{62} z_{69})z_{73} \\
&\quad + z_{60} (z_{70} z_{72} + z_{60} z_{79} - z_{68} z_{74} - z_{62} z_{78}) \\
&\quad + z_{61} (z_{70} z_{73} + z_{61} z_{79} - z_{69} z_{74} - z_{62} z_{77})) \\
&\quad + 0.5v_l ((z_{60} z_{69} - z_{61} z_{68})z_{72} + z_{60} (z_{69} z_{72} + z_{60} z_{77} - z_{68} z_{73} - z_{61} z_{78}) \\
&\quad - (z_{61} z_{70} - z_{62} z_{69})z_{74} - z_{62} (z_{70} z_{73} + z_{61} z_{79} - z_{69} z_{74} - z_{62} z_{77})) \\
&\quad - 0.5u_l ((z_{60} z_{69} - z_{61} z_{68})z_{73} + (z_{60} z_{70} - z_{62} z_{68})z_{74} \\
&\quad + z_{61} (z_{69} z_{72} + z_{60} z_{77} - z_{68} z_{73} - z_{61} z_{78}) \\
&\quad + z_{62} (z_{70} z_{72} + z_{60} z_{79} - z_{68} z_{74} - z_{62} z_{78})) \\
z_{88} &= dOMRo_1 - z_{52} \\
z_{89} &= dOMRo_2 - z_{53} \\
z_{90} &= dOMRo_3 - z_{54} \\
z_{91} &= dOFo_1 - z_{52} \\
z_{92} &= dOFo_2 - z_{53} \\
z_{93} &= dOFo_3 - z_{54} \\
z_{94} &= dOT_1 - z_{52} \\
z_{96} &= dOT_3 - z_{54} \\
z_{100} &= -q_{h1} (z_{88} q_{h1} - z_{89} p_{h1}) - r_{h1} (z_{88} r_{h1} - z_{90} p_{h1}) \\
z_{101} &= p_{h1} (z_{88} q_{h1} - z_{89} p_{h1}) - r_{h1} (z_{89} r_{h1} - z_{90} q_{h1}) \\
z_{102} &= p_{h1} (z_{88} r_{h1} - z_{90} p_{h1}) + q_{h1} (z_{89} r_{h1} - z_{90} q_{h1}) \\
z_{103} &= -q_{h1} (z_{91} q_{h1} - z_{92} p_{h1}) - r_{h1} (z_{91} r_{h1} - z_{93} p_{h1}) \\
z_{104} &= p_{h1} (z_{91} q_{h1} - z_{92} p_{h1}) - r_{h1} (z_{92} r_{h1} - z_{93} q_{h1}) \\
z_{105} &= p_{h1} (z_{91} r_{h1} - z_{93} p_{h1}) + q_{h1} (z_{92} r_{h1} - z_{93} q_{h1}) \\
z_{112} &= -q_{h2} (z_{88} q_{h2} - z_{89} p_{h2}) - r_{h2} (z_{88} r_{h2} - z_{90} p_{h2}) \\
z_{113} &= p_{h2} (z_{88} q_{h2} - z_{89} p_{h2}) - r_{h2} (z_{89} r_{h2} - z_{90} q_{h2}) \\
z_{114} &= p_{h2} (z_{88} r_{h2} - z_{90} p_{h2}) + q_{h2} (z_{89} r_{h2} - z_{90} q_{h2}) \\
z_{115} &= -q_{h2} (z_{91} q_{h2} - z_{92} p_{h2}) - r_{h2} (z_{91} r_{h2} - z_{93} p_{h2}) \\
z_{116} &= p_{h2} (z_{91} q_{h2} - z_{92} p_{h2}) - r_{h2} (z_{92} r_{h2} - z_{93} q_{h2}) \\
z_{117} &= p_{h2} (z_{91} r_{h2} - z_{93} p_{h2}) + q_{h2} (z_{92} r_{h2} - z_{93} q_{h2}) \\
z_{124} &= g m_F \\
z_{125} &= g m_{MR} \\
z_{126} &= g l_p
\end{aligned}$$

4 Dynamic Equations of the Dual-Lift Model

$$\begin{aligned}
z_{200} &= -z_3 z_7 \dot{\theta}_{h1} - z_4 z_9 \dot{\psi}_{h1} \\
z_{201} &= \sin(\theta_{h1}) \\
z_{202} &= (z_6 z_{33} \dot{\phi}_{h1} - z_8 z_{201} \dot{\theta}_{h1})/z_{33}^2 \\
z_{203} &= z_3 z_7 \dot{\psi}_{h1} + z_4 z_9 \dot{\theta}_{h1} \\
z_{204} &= z_4 z_6 \dot{\theta}_{h1} + z_7 z_8 \dot{\phi}_{h1} \\
z_{205} &= -z_4 z_6 \dot{\phi}_{h1} - z_7 z_8 \dot{\theta}_{h1} \\
z_{206} &= z_8 \dot{\theta}_{h1}/z_{33}^2 - z_6 z_{36} \dot{\phi}_{h1} \\
z_{207} &= z_3 z_4 \dot{\theta}_{h1} - z_7 z_9 \dot{\psi}_{h1} \\
z_{208} &= z_3 z_4 \dot{\psi}_{h1} - z_7 z_9 \dot{\theta}_{h1} \\
z_{209} &= z_4 z_8 \dot{\theta}_{h1} - z_6 z_7 \dot{\phi}_{h1} \\
z_{210} &= z_4 z_8 \dot{\phi}_{h1} - z_6 z_7 \dot{\theta}_{h1} \\
z_{211} &= z_3 z_4 z_8 \dot{\phi}_{h1} - z_3 z_6 z_7 \dot{\theta}_{h1} - z_4 z_6 z_9 \dot{\psi}_{h1} \\
z_{212} &= z_3 z_6 \dot{\psi}_{h1} + z_8 z_9 \dot{\phi}_{h1} + z_3 z_4 z_8 \dot{\theta}_{h1} - z_3 z_6 z_7 \dot{\phi}_{h1} - z_7 z_8 z_9 \dot{\psi}_{h1} \\
z_{213} &= z_3 z_6 \dot{\phi}_{h1} + z_8 z_9 \dot{\psi}_{h1} - z_3 z_6 z_7 \dot{\psi}_{h1} - z_4 z_6 z_9 \dot{\theta}_{h1} - z_7 z_8 z_9 \dot{\phi}_{h1} \\
z_{214} &= z_3 z_4 z_6 \dot{\psi}_{h1} + z_4 z_8 z_9 \dot{\phi}_{h1} - z_6 z_7 z_9 \dot{\theta}_{h1} \\
z_{215} &= z_6 z_9 \dot{\phi}_{h1} + z_3 z_4 z_6 \dot{\theta}_{h1} + z_3 z_7 z_8 \dot{\phi}_{h1} - z_3 z_8 \dot{\psi}_{h1} - z_6 z_7 z_9 \dot{\psi}_{h1} \\
z_{216} &= z_6 z_9 \dot{\psi}_{h1} + z_3 z_7 z_8 \dot{\psi}_{h1} + z_4 z_8 z_9 \dot{\theta}_{h1} - z_3 z_8 \dot{\phi}_{h1} - z_6 z_7 z_9 \dot{\phi}_{h1} \\
z_{217} &= -z_3 z_4 z_6 \dot{\phi}_{h1} - z_3 z_7 z_8 \dot{\theta}_{h1} - z_4 z_8 z_9 \dot{\psi}_{h1} \\
z_{218} &= z_3 z_8 \dot{\phi}_{h1} + z_6 z_7 z_9 \dot{\phi}_{h1} - z_6 z_9 \dot{\psi}_{h1} - z_3 z_7 z_8 \dot{\psi}_{h1} - z_4 z_8 z_9 \dot{\theta}_{h1} \\
z_{219} &= z_3 z_8 \dot{\psi}_{h1} + z_6 z_7 z_9 \dot{\psi}_{h1} - z_6 z_9 \dot{\phi}_{h1} - z_3 z_4 z_6 \dot{\theta}_{h1} - z_3 z_7 z_8 \dot{\phi}_{h1} \\
z_{220} &= z_3 z_4 z_8 \dot{\psi}_{h1} - z_4 z_6 z_9 \dot{\phi}_{h1} - z_7 z_8 z_9 \dot{\theta}_{h1} \\
z_{221} &= 2z_{97} z_4 z_8 z_{129} \dot{\phi}_{h1} + 2z_{97} z_4 z_6 (w_{h1} - \dot{z}_l) \\
&\quad + 2z_{97} (z_5 z_{35} + z_6 z_{136})(v_{h1} - \dot{y}_l) + 2z_{98} (z_6 z_{139} + z_{16} z_{38})(w_{h1} - \dot{z}_l) \\
&\quad + 2z_{97} (z_6 z_{130} - z_{12} z_{35})(u_{h1} - \dot{x}_l) + 2z_{99} (z_6 z_{140} - z_{15} z_{38})(w_{h1} - \dot{z}_l) \\
&\quad + 2z_{98} z_{129} (z_8 z_{139} \dot{\phi}_{h1} + z_6 z_{204} + z_{16} z_{206} + z_{38} z_{205}) \\
&\quad + 2z_{97} z_{127} (z_8 z_{130} \dot{\phi}_{h1} + z_6 z_{207} + z_{12} z_{202} - z_{35} z_{208}) \\
&\quad + 2z_{99} z_{129} (z_8 z_{140} \dot{\phi}_{h1} + z_6 z_{209} - z_{15} z_{206} - z_{38} z_{210}) \\
&\quad + 2z_{98} z_{127} (z_{11} z_{206} + z_{35} z_{213} + z_{38} z_{212} - z_8 z_{131} \dot{\phi}_{h1} - z_6 z_{211} - z_{132} z_{202}) \\
&\quad + 2z_{99} z_{127} (z_{35} z_{218} + z_{38} z_{219} + z_{133} z_{206} - z_8 z_{135} \dot{\phi}_{h1} - z_6 z_{217} - z_{134} z_{202}) \\
&\quad - 2z_{97} z_6 z_7 z_{129} \dot{\theta}_{h1} - 2z_{98} (z_6 z_{131} - z_{11} z_{38} - z_{35} z_{132})(u_{h1} - \dot{x}_l) \\
&\quad - 2z_{98} (z_6 z_{137} - z_{10} z_{35} - z_{14} z_{38})(v_{h1} - \dot{y}_l) \\
&\quad - 2z_{99} (z_6 z_{135} - z_{35} z_{134} - z_{38} z_{133})(u_{h1} - \dot{x}_l) \\
&\quad - 2z_{99} (z_6 z_{138} - z_{11} z_{35} - z_{38} z_{132})(v_{h1} - \dot{y}_l) \\
&\quad - 2z_{97} z_{128} (z_5 z_{202} - z_8 z_{136} \dot{\phi}_{h1} - z_6 z_{203} - z_{35} z_{200}) \\
&\quad - 2z_{98} z_{128} (z_8 z_{137} \dot{\phi}_{h1} + z_6 z_{214} + z_{10} z_{202} - z_{14} z_{206} - z_{35} z_{215} - z_{38} z_{216}) \\
&\quad - 2z_{99} z_{128} (z_8 z_{138} \dot{\phi}_{h1} + z_6 z_{220} + z_{11} z_{202} - z_{35} z_{212} - z_{38} z_{213} - z_{132} z_{206})
\end{aligned}$$

4 Dynamic Equations of the Dual-Lift Model

$$\begin{aligned}
z_{222} &= (z_6 z_{201} \dot{\theta}_{h1} + z_8 z_{33} \dot{\varphi}_{h1})/z_{33}^2 \\
z_{223} &= z_8 z_{36} \dot{\varphi}_{h1} + z_6 \dot{\theta}_{h1}/z_{33}^2 \\
z_{224} &= 2z_{97} z_4 z_6 z_{129} \dot{\varphi}_{h1} + 2z_{97} z_7 z_8 z_{129} \dot{\theta}_{h1} + 2z_{97} (z_5 z_{34} - z_8 z_{136})(v_{h1} - \dot{y}_l) \\
&\quad + 2z_{98} (z_8 z_{131} + z_{11} z_{37} + z_{34} z_{132})(u_{h1} - \dot{x}_l) \\
&\quad + 2z_{98} (z_8 z_{137} + z_{10} z_{34} + z_{14} z_{37})(v_{h1} - \dot{y}_l) \\
&\quad + 2z_{99} (z_8 z_{135} + z_{34} z_{134} + z_{37} z_{133})(u_{h1} - \dot{x}_l) \\
&\quad + 2z_{99} (z_8 z_{138} + z_{11} z_{34} + z_{37} z_{132})(v_{h1} - \dot{y}_l) \\
&\quad + 2z_{97} z_{127} (z_6 z_{130} \dot{\varphi}_{h1} - z_8 z_{207} - z_{12} z_{222} - z_{34} z_{208}) \\
&\quad + 2z_{99} z_{129} (z_6 z_{140} \dot{\varphi}_{h1} - z_8 z_{209} - z_{15} z_{223} - z_{37} z_{210}) \\
&\quad - 2z_{97} z_4 z_8 (w_{h1} - \dot{z}_l) - 2z_{97} (z_8 z_{130} + z_{12} z_{34})(u_{h1} - \dot{x}_l) \\
&\quad - 2z_{99} (z_8 z_{140} + z_{15} z_{37})(w_{h1} - \dot{z}_l) - 2z_{98} (z_8 z_{139} - z_{16} z_{37})(w_{h1} - \dot{z}_l) \\
&\quad - 2z_{97} z_{128} (z_8 z_{203} - z_6 z_{136} \dot{\varphi}_{h1} - z_5 z_{222} - z_{34} z_{200}) \\
&\quad - 2z_{98} z_{129} (z_8 z_{204} - z_6 z_{139} \dot{\varphi}_{h1} - z_{16} z_{223} - z_{37} z_{205}) \\
&\quad - 2z_{98} z_{127} (z_6 z_{131} \dot{\varphi}_{h1} - z_8 z_{211} - z_{11} z_{223} - z_{34} z_{213} - z_{37} z_{212} - z_{132} z_{222}) \\
&\quad - 2z_{98} z_{128} (z_6 z_{137} \dot{\varphi}_{h1} - z_8 z_{214} - z_{10} z_{222} - z_{14} z_{223} - z_{34} z_{215} - z_{37} z_{216}) \\
&\quad - 2z_{99} z_{127} (z_6 z_{135} \dot{\varphi}_{h1} - z_8 z_{217} - z_{34} z_{218} - z_{37} z_{219} - z_{133} z_{223} - z_{134} z_{222}) \\
&\quad - 2z_{99} z_{128} (z_6 z_{138} \dot{\varphi}_{h1} - z_8 z_{220} - z_{11} z_{222} - z_{34} z_{212} - z_{37} z_{213} - z_{132} z_{223}) \\
z_{225} &= 2u_{h1} + 2z_{97} z_{200} + 2z_{98} z_{215} + 2z_{99} z_{212} - 2\dot{x}_l \\
z_{226} &= z_3 z_6 z_7 \dot{\psi}_{h1} + z_4 z_6 z_9 \dot{\theta}_{h1} + z_7 z_8 z_9 \dot{\varphi}_{h1} - z_3 z_6 \dot{\varphi}_{h1} - z_8 z_9 \dot{\psi}_{h1} \\
z_{227} &= z_{127} z_{72} + z_{97} z_5 z_{72} + z_{98} z_{10} z_{72} + z_{99} z_{11} z_{72} \\
&\quad + 2z_{97} z_4 (z_{61} (z_{60} z_{69} - z_{61} z_{68}) + z_{62} (z_{60} z_{70} - z_{62} z_{68})) \dot{\theta}_{h1} \\
&\quad + z_{128} (z_{70} z_{73} + z_{61} z_{79} - z_{69} z_{74} - z_{62} z_{77}) \\
&\quad + z_{97} z_{12} (z_{70} z_{73} + z_{61} z_{79} - z_{69} z_{74} - z_{62} z_{77}) \\
&\quad + z_{98} z_{13} (z_{70} z_{73} + z_{61} z_{79} - z_{69} z_{74} - z_{62} z_{77}) \\
&\quad + z_{99} z_{14} (z_{70} z_{73} + z_{61} z_{79} - z_{69} z_{74} - z_{62} z_{77}) + z_{129} ((z_{60} z_{69} - z_{61} z_{68}) z_{73} \\
&\quad\quad + (z_{60} z_{70} - z_{62} z_{68}) z_{74} + z_{61} (z_{69} z_{72} + z_{60} z_{77} - z_{68} z_{73} - z_{61} z_{78}) \\
&\quad\quad + z_{62} (z_{70} z_{72} + z_{60} z_{79} - z_{68} z_{74} - z_{62} z_{78})) + z_{98} z_{15} ((z_{60} z_{69} - z_{61} z_{68}) z_{73} \\
&\quad\quad + (z_{60} z_{70} - z_{62} z_{68}) z_{74} + z_{61} (z_{69} z_{72} + z_{60} z_{77} - z_{68} z_{73} - z_{61} z_{78}) \\
&\quad\quad + z_{62} (z_{70} z_{72} + z_{60} z_{79} - z_{68} z_{74} - z_{62} z_{78})) + z_{99} z_{16} ((z_{60} z_{69} - z_{61} z_{68}) z_{73} \\
&\quad\quad + (z_{60} z_{70} - z_{62} z_{68}) z_{74} + z_{61} (z_{69} z_{72} + z_{60} z_{77} - z_{68} z_{73} - z_{61} z_{78}) \\
&\quad\quad + z_{62} (z_{70} z_{72} + z_{60} z_{79} - z_{68} z_{74} - z_{62} z_{78})) \\
&\quad - 2z_{60} (u_{h1} - \dot{x}_l) - 2(z_{61} z_{70} - z_{62} z_{69})(v_{h1} - \dot{y}_l) \\
&\quad - 2(z_{61} (z_{60} z_{69} - z_{61} z_{68}) + z_{62} (z_{60} z_{70} - z_{62} z_{68}))(w_{h1} - \dot{z}_l) \\
&\quad - 2z_{97} z_{60} z_{200} - 2z_{98} z_{60} z_{215} - 2z_{99} z_{60} z_{212} - 2z_{97} (z_{61} z_{70} - z_{62} z_{69}) z_{208} \\
&\quad - 2z_{98} (z_{61} z_{70} - z_{62} z_{69}) z_{226} - 2z_{99} (z_{61} z_{70} - z_{62} z_{69}) z_{216} \\
&\quad - 2z_{98} (z_{61} (z_{60} z_{69} - z_{61} z_{68}) + z_{62} (z_{60} z_{70} - z_{62} z_{68})) z_{210} \\
&\quad - 2z_{99} (z_{61} (z_{60} z_{69} - z_{61} z_{68}) + z_{62} (z_{60} z_{70} - z_{62} z_{68})) z_{205} \\
&\quad - z_{97} z_7 ((z_{60} z_{69} - z_{61} z_{68}) z_{73} + (z_{60} z_{70} - z_{62} z_{68}) z_{74} \\
&\quad\quad + z_{61} (z_{69} z_{72} + z_{60} z_{77} - z_{68} z_{73} - z_{61} z_{78}) \\
&\quad\quad + z_{62} (z_{70} z_{72} + z_{60} z_{79} - z_{68} z_{74} - z_{62} z_{78}))
\end{aligned}$$

4 Dynamic Equations of the Dual-Lift Model

$$\begin{aligned}
z_{228} = & z_{127} z_{73} + z_{97} z_5 z_{73} + z_{98} z_{10} z_{73} + z_{99} z_{11} z_{73} + 2(z_{60} z_{70} - z_{62} z_{68})(v_{h1} - \dot{y}_l) \\
& + 2(z_{60} (z_{60} z_{69} - z_{61} z_{68}) - z_{62} (z_{61} z_{70} - z_{62} z_{69}))(w_{h1} - \dot{z}_l) \\
& + 2z_{97} (z_{60} z_{70} - z_{62} z_{68})z_{208} \\
& + 2z_{98} (z_{60} z_{70} - z_{62} z_{68})z_{226} + 2z_{99} (z_{60} z_{70} - z_{62} z_{68})z_{216} \\
& + 2z_{98} (z_{60} (z_{60} z_{69} - z_{61} z_{68}) - z_{62} (z_{61} z_{70} - z_{62} z_{69}))z_{210} \\
& + 2z_{99} (z_{60} (z_{60} z_{69} - z_{61} z_{68}) - z_{62} (z_{61} z_{70} - z_{62} z_{69}))z_{205} \\
& + z_{97} z_7 ((z_{60} z_{69} - z_{61} z_{68})z_{72} + z_{60} (z_{69} z_{72} + z_{60} z_{77} - z_{68} z_{73} - z_{61} z_{78}) \\
& - (z_{61} z_{70} - z_{62} z_{69})z_{74} - z_{62} (z_{70} z_{73} + z_{61} z_{79} - z_{69} z_{74} - z_{62} z_{77})) - 2z_{61} (u_{h1} - \dot{x}_l) \\
& - 2z_{97} z_4 (z_{60} (z_{60} z_{69} - z_{61} z_{68}) - z_{62} (z_{61} z_{70} - z_{62} z_{69}))\dot{\theta}_{h1} - 2z_{97} z_{61} z_{200} \\
& - 2z_{98} z_{61} z_{215} - 2z_{99} z_{61} z_{212} - z_{128} (z_{70} z_{72} + z_{60} z_{79} - z_{68} z_{74} - z_{62} z_{78}) \\
& - z_{97} z_{12} (z_{70} z_{72} + z_{60} z_{79} - z_{68} z_{74} - z_{62} z_{78}) \\
& - z_{98} z_{13} (z_{70} z_{72} + z_{60} z_{79} - z_{68} z_{74} - z_{62} z_{78}) \\
& - z_{99} z_{14} (z_{70} z_{72} + z_{60} z_{79} - z_{68} z_{74} - z_{62} z_{78}) \\
& - z_{129} ((z_{60} z_{69} - z_{61} z_{68})z_{72} + z_{60} (z_{69} z_{72} + z_{60} z_{77} - z_{68} z_{73} - z_{61} z_{78}) \\
& - (z_{61} z_{70} - z_{62} z_{69})z_{74} - z_{62} (z_{70} z_{73} + z_{61} z_{79} - z_{69} z_{74} - z_{62} z_{77})) \\
& - z_{98} z_{15} ((z_{60} z_{69} - z_{61} z_{68})z_{72} + z_{60} (z_{69} z_{72} + z_{60} z_{77} - z_{68} z_{73} - z_{61} z_{78}) \\
& - (z_{61} z_{70} - z_{62} z_{69})z_{74} - z_{62} (z_{70} z_{73} + z_{61} z_{79} - z_{69} z_{74} - z_{62} z_{77})) \\
& - z_{99} z_{16} ((z_{60} z_{69} - z_{61} z_{68})z_{72} + z_{60} (z_{69} z_{72} + z_{60} z_{77} - z_{68} z_{73} - z_{61} z_{78}) \\
& - (z_{61} z_{70} - z_{62} z_{69})z_{74} - z_{62} (z_{70} z_{73} + z_{61} z_{79} - z_{69} z_{74} - z_{62} z_{77}))
\end{aligned}$$

$$\begin{aligned}
z_{229} = & z_{127} z_{74} + z_{97} z_5 z_{74} + z_{98} z_{10} z_{74} + z_{99} z_{11} z_{74} \\
& + 2(z_{60} (z_{60} z_{70} - z_{62} z_{68}) + z_{61} (z_{61} z_{70} - z_{62} z_{69}))(w_{h1} - \dot{z}_l) \\
& + z_{128} (z_{69} z_{72} + z_{60} z_{77} - z_{68} z_{73} - z_{61} z_{78}) \\
& + z_{97} z_{12} (z_{69} z_{72} + z_{60} z_{77} - z_{68} z_{73} - z_{61} z_{78}) \\
& + z_{98} z_{13} (z_{69} z_{72} + z_{60} z_{77} - z_{68} z_{73} - z_{61} z_{78}) \\
& + z_{99} z_{14} (z_{69} z_{72} + z_{60} z_{77} - z_{68} z_{73} - z_{61} z_{78}) \\
& + 2z_{98} (z_{60} (z_{60} z_{70} - z_{62} z_{68}) + z_{61} (z_{61} z_{70} - z_{62} z_{69}))z_{210} \\
& + 2z_{99} (z_{60} (z_{60} z_{70} - z_{62} z_{68}) + z_{61} (z_{61} z_{70} - z_{62} z_{69}))z_{205} \\
& + z_{97} z_7 ((z_{60} z_{70} - z_{62} z_{68})z_{72} + (z_{61} z_{70} - z_{62} z_{69})z_{73} \\
& + z_{60} (z_{70} z_{72} + z_{60} z_{79} - z_{68} z_{74} - z_{62} z_{78}) \\
& + z_{61} (z_{70} z_{73} + z_{61} z_{79} - z_{69} z_{74} - z_{62} z_{77})) \\
& - 2z_{62} (u_{h1} - \dot{x}_l) - 2(z_{60} z_{69} - z_{61} z_{68})(v_{h1} - \dot{y}_l) \\
& - 2z_{97} z_4 (z_{60} (z_{60} z_{70} - z_{62} z_{68}) + z_{61} (z_{61} z_{70} - z_{62} z_{69}))\dot{\theta}_{h1} \\
& - 2z_{97} z_{62} z_{200} - 2z_{98} z_{62} z_{215} - 2z_{99} z_{62} z_{212} - 2z_{97} (z_{60} z_{69} - z_{61} z_{68})z_{208} \\
& - 2z_{98} (z_{60} z_{69} - z_{61} z_{68})z_{226} - 2z_{99} (z_{60} z_{69} - z_{61} z_{68})z_{216} \\
& - z_{129} ((z_{60} z_{70} - z_{62} z_{68})z_{72} + (z_{61} z_{70} - z_{62} z_{69})z_{73} \\
& + z_{60} (z_{70} z_{72} + z_{60} z_{79} - z_{68} z_{74} - z_{62} z_{78}) \\
& + z_{61} (z_{70} z_{73} + z_{61} z_{79} - z_{69} z_{74} - z_{62} z_{77})) - z_{98} z_{15} ((z_{60} z_{70} - z_{62} z_{68})z_{72} \\
& + (z_{61} z_{70} - z_{62} z_{69})z_{73} + z_{60} (z_{70} z_{72} + z_{60} z_{79} - z_{68} z_{74} - z_{62} z_{78}) \\
& + z_{61} (z_{70} z_{73} + z_{61} z_{79} - z_{69} z_{74} - z_{62} z_{77})) - z_{99} z_{16} ((z_{60} z_{70} - z_{62} z_{68})z_{72} \\
& + (z_{61} z_{70} - z_{62} z_{69})z_{73} + z_{60} (z_{70} z_{72} + z_{60} z_{79} - z_{68} z_{74} - z_{62} z_{78}) \\
& + z_{61} (z_{70} z_{73} + z_{61} z_{79} - z_{69} z_{74} - z_{62} z_{77}))
\end{aligned}$$

4 Dynamic Equations of the Dual-Lift Model

$$\begin{aligned}
z_{230} &= 2z_{98} z_{11} (u_{h1} - \dot{x}_l) + 2z_{98} z_{14} (v_{h1} - \dot{y}_l) + 2z_{98} z_{16} (w_{h1} - \dot{z}_l) + 2z_{99} z_{132} (v_{h1} - \dot{y}_l) \\
&\quad + 2z_{99} z_{133} (u_{h1} - \dot{x}_l) + 2z_{98} z_{127} z_{212} + 2z_{98} z_{128} z_{216} + 2z_{98} z_{129} z_{205} \\
&\quad + 2z_{99} z_{127} z_{219} + 2z_{99} z_{128} z_{213} - 2z_{99} z_{15} (w_{h1} - \dot{z}_l) - 2z_{99} z_{129} z_{210} \\
z_{231} &= 2v_{h1} + 2z_{97} z_{208} + 2z_{98} z_{226} + 2z_{99} z_{216} - 2\dot{y}_l \\
z_{232} &= 2w_{h1} + 2z_{98} z_{210} + 2z_{99} z_{205} - 2\dot{z}_l - 2z_{97} z_4 \dot{\theta}_{h1} \\
z_{233} &= u_{h1} z_{225} + u_l z_{227} + v_l z_{228} + w_l z_{229} + v_{h1} z_{231} \\
&\quad + w_{h1} z_{232} + p_{h1} z_{230} + q_{h1} z_{224} + r_{h1} z_{221} \\
z_{234} &= -z_{19} z_{23} \dot{\theta}_{h2} - z_{20} z_{25} \dot{\psi}_{h2} \\
z_{235} &= \sin(\theta_{h2}) \\
z_{236} &= (z_{22} z_{39} \dot{\varphi}_{h2} - z_{24} z_{235} \dot{\theta}_{h2})/z_{39}^2 \\
z_{237} &= z_{19} z_{23} \dot{\psi}_{h2} + z_{20} z_{25} \dot{\theta}_{h2} \\
z_{238} &= z_{20} z_{22} \dot{\theta}_{h2} + z_{23} z_{24} \dot{\varphi}_{h2} \\
z_{239} &= -z_{20} z_{22} \dot{\varphi}_{h2} - z_{23} z_{24} \dot{\theta}_{h2} \\
z_{240} &= z_{24} \dot{\theta}_{h2}/z_{39}^2 - z_{22} z_{42} \dot{\varphi}_{h2} \\
z_{241} &= z_{19} z_{20} \dot{\theta}_{h2} - z_{23} z_{25} \dot{\psi}_{h2} \\
z_{242} &= z_{19} z_{20} \dot{\psi}_{h2} - z_{23} z_{25} \dot{\theta}_{h2} \\
z_{243} &= z_{20} z_{24} \dot{\theta}_{h2} - z_{22} z_{23} \dot{\varphi}_{h2} \\
z_{244} &= z_{20} z_{24} \dot{\varphi}_{h2} - z_{22} z_{23} \dot{\theta}_{h2} \\
z_{245} &= z_{19} z_{20} z_{24} \dot{\varphi}_{h2} - z_{19} z_{22} z_{23} \dot{\theta}_{h2} - z_{20} z_{22} z_{25} \dot{\psi}_{h2} \\
z_{246} &= z_{19} z_{22} \dot{\psi}_{h2} + z_{24} z_{25} \dot{\varphi}_{h2} + z_{19} z_{20} z_{24} \dot{\theta}_{h2} - z_{19} z_{22} z_{23} \dot{\varphi}_{h2} - z_{23} z_{24} z_{25} \dot{\psi}_{h2} \\
z_{247} &= z_{19} z_{22} \dot{\varphi}_{h2} + z_{24} z_{25} \dot{\psi}_{h2} - z_{19} z_{22} z_{23} \dot{\psi}_{h2} - z_{20} z_{22} z_{25} \dot{\theta}_{h2} - z_{23} z_{24} z_{25} \dot{\varphi}_{h2} \\
z_{248} &= z_{19} z_{20} z_{22} \dot{\psi}_{h2} + z_{20} z_{24} z_{25} \dot{\varphi}_{h2} - z_{22} z_{23} z_{25} \dot{\theta}_{h2} \\
z_{249} &= z_{22} z_{25} \dot{\varphi}_{h2} + z_{19} z_{20} z_{22} \dot{\theta}_{h2} + z_{19} z_{23} z_{24} \dot{\varphi}_{h2} - z_{19} z_{24} \dot{\psi}_{h2} - z_{22} z_{23} z_{25} \dot{\psi}_{h2} \\
z_{250} &= z_{22} z_{25} \dot{\psi}_{h2} + z_{19} z_{23} z_{24} \dot{\psi}_{h2} + z_{20} z_{24} z_{25} \dot{\theta}_{h2} - z_{19} z_{24} \dot{\varphi}_{h2} - z_{22} z_{23} z_{25} \dot{\varphi}_{h2} \\
z_{251} &= -z_{19} z_{20} z_{22} \dot{\varphi}_{h2} - z_{19} z_{23} z_{24} \dot{\theta}_{h2} - z_{20} z_{24} z_{25} \dot{\psi}_{h2} \\
z_{252} &= z_{19} z_{24} \dot{\varphi}_{h2} + z_{22} z_{23} z_{25} \dot{\varphi}_{h2} - z_{22} z_{25} \dot{\psi}_{h2} - z_{19} z_{23} z_{24} \dot{\psi}_{h2} - z_{20} z_{24} z_{25} \dot{\theta}_{h2} \\
z_{253} &= z_{19} z_{24} \dot{\psi}_{h2} + z_{22} z_{23} z_{25} \dot{\psi}_{h2} - z_{22} z_{25} \dot{\varphi}_{h2} - z_{19} z_{20} z_{22} \dot{\theta}_{h2} - z_{19} z_{23} z_{24} \dot{\varphi}_{h2} \\
z_{254} &= z_{19} z_{20} z_{24} \dot{\psi}_{h2} - z_{20} z_{22} z_{25} \dot{\varphi}_{h2} - z_{23} z_{24} z_{25} \dot{\theta}_{h2}
\end{aligned}$$

4 Dynamic Equations of the Dual-Lift Model

$$\begin{aligned}
z_{255} &= 2z_{97} z_{20} z_{24} z_{143} \dot{\varphi}_{h2} + 2z_{97} z_{20} z_{22} (w_{h2} - \dot{z}_l) \\
&\quad + 2z_{97} (z_{21} z_{41} + z_{22} z_{150})(v_{h2} - \dot{y}_l) + 2z_{98} (z_{22} z_{153} + z_{32} z_{44})(w_{h2} - \dot{z}_l) \\
&\quad + 2z_{97} (z_{22} z_{144} - z_{28} z_{41})(u_{h2} - \dot{x}_l) + 2z_{99} (z_{22} z_{154} - z_{31} z_{44})(w_{h2} - \dot{z}_l) \\
&\quad + 2z_{98} z_{143} (z_{24} z_{153} \dot{\varphi}_{h2} + z_{22} z_{238} + z_{32} z_{240} + z_{44} z_{239}) \\
&\quad + 2z_{97} z_{141} (z_{24} z_{144} \dot{\varphi}_{h2} + z_{22} z_{241} + z_{28} z_{236} - z_{41} z_{242}) \\
&\quad + 2z_{99} z_{143} (z_{24} z_{154} \dot{\varphi}_{h2} + z_{22} z_{243} - z_{31} z_{240} - z_{44} z_{244}) \\
&\quad + 2z_{98} z_{141} (z_{27} z_{240} + z_{41} z_{247} + z_{44} z_{246} - z_{24} z_{145} \dot{\varphi}_{h2} - z_{22} z_{245} - z_{146} z_{236}) \\
&\quad + 2z_{99} z_{141} (z_{41} z_{252} + z_{44} z_{253} + z_{147} z_{240} - z_{24} z_{149} \dot{\varphi}_{h2} - z_{22} z_{251} - z_{148} z_{236}) \\
&\quad - 2z_{97} z_{22} z_{23} z_{143} \dot{\theta}_{h2} - 2z_{98} (z_{22} z_{145} - z_{27} z_{44} - z_{41} z_{146})(u_{h2} - \dot{x}_l) \\
&\quad - 2z_{98} (z_{22} z_{151} - z_{26} z_{41} - z_{30} z_{44})(v_{h2} - \dot{y}_l) \\
&\quad - 2z_{99} (z_{22} z_{149} - z_{41} z_{148} - z_{44} z_{147})(u_{h2} - \dot{x}_l) \\
&\quad - 2z_{99} (z_{22} z_{152} - z_{27} z_{41} - z_{44} z_{146})(v_{h2} - \dot{y}_l) \\
&\quad - 2z_{97} z_{142} (z_{21} z_{236} - z_{24} z_{150} \dot{\varphi}_{h2} - z_{22} z_{237} - z_{41} z_{234}) \\
&\quad - 2z_{98} z_{142} (z_{24} z_{151} \dot{\varphi}_{h2} + z_{22} z_{248} + z_{26} z_{236} - z_{30} z_{240} - z_{41} z_{249} - z_{44} z_{250}) \\
&\quad - 2z_{99} z_{142} (z_{24} z_{152} \dot{\varphi}_{h2} + z_{22} z_{254} + z_{27} z_{236} - z_{41} z_{246} - z_{44} z_{247} - z_{146} z_{240}) \\
z_{256} &= (z_{22} z_{235} \dot{\theta}_{h2} + z_{24} z_{39} \dot{\varphi}_{h2})/z_{39}^2 \\
z_{257} &= z_{24} z_{42} \dot{\varphi}_{h2} + z_{22} \dot{\theta}_{h2}/z_{39}^2 \\
z_{258} &= 2z_{97} z_{20} z_{22} z_{143} \dot{\varphi}_{h2} + 2z_{97} z_{23} z_{24} z_{143} \dot{\theta}_{h2} + 2z_{97} (z_{21} z_{40} - z_{24} z_{150})(v_{h2} - \dot{y}_l) \\
&\quad + 2z_{98} (z_{24} z_{145} + z_{27} z_{43} + z_{40} z_{146})(u_{h2} - \dot{x}_l) \\
&\quad + 2z_{98} (z_{24} z_{151} + z_{26} z_{40} + z_{30} z_{43})(v_{h2} - \dot{y}_l) \\
&\quad + 2z_{99} (z_{24} z_{149} + z_{40} z_{148} + z_{43} z_{147})(u_{h2} - \dot{x}_l) \\
&\quad + 2z_{99} (z_{24} z_{152} + z_{27} z_{40} + z_{43} z_{146})(v_{h2} - \dot{y}_l) \\
&\quad + 2z_{97} z_{141} (z_{22} z_{144} \dot{\varphi}_{h2} - z_{24} z_{241} - z_{28} z_{256} - z_{40} z_{242}) \\
&\quad + 2z_{99} z_{143} (z_{22} z_{154} \dot{\varphi}_{h2} - z_{24} z_{243} - z_{31} z_{257} - z_{43} z_{244}) \\
&\quad - 2z_{97} z_{20} z_{24} (w_{h2} - \dot{z}_l) - 2z_{97} (z_{24} z_{144} + z_{28} z_{40})(u_{h2} - \dot{x}_l) \\
&\quad - 2z_{99} (z_{24} z_{154} + z_{31} z_{43})(w_{h2} - \dot{z}_l) - 2z_{98} (z_{24} z_{153} - z_{32} z_{43})(w_{h2} - \dot{z}_l) \\
&\quad - 2z_{97} z_{142} (z_{24} z_{237} - z_{22} z_{150} \dot{\varphi}_{h2} - z_{21} z_{256} - z_{40} z_{234}) \\
&\quad - 2z_{98} z_{143} (z_{24} z_{238} - z_{22} z_{153} \dot{\varphi}_{h2} - z_{32} z_{257} - z_{43} z_{239}) \\
&\quad - 2z_{98} z_{141} (z_{22} z_{145} \dot{\varphi}_{h2} - z_{24} z_{245} - z_{27} z_{257} - z_{40} z_{247} - z_{43} z_{246} - z_{146} z_{256}) \\
&\quad - 2z_{98} z_{142} (z_{22} z_{151} \dot{\varphi}_{h2} - z_{24} z_{248} - z_{26} z_{256} - z_{30} z_{257} - z_{40} z_{249} - z_{43} z_{250}) \\
&\quad - 2z_{99} z_{141} (z_{22} z_{149} \dot{\varphi}_{h2} - z_{24} z_{251} - z_{40} z_{252} - z_{43} z_{253} - z_{147} z_{257} - z_{148} z_{256}) \\
&\quad - 2z_{99} z_{142} (z_{22} z_{152} \dot{\varphi}_{h2} - z_{24} z_{254} - z_{27} z_{256} - z_{40} z_{246} - z_{43} z_{247} - z_{146} z_{257}) \\
z_{259} &= 2u_{h2} + 2z_{97} z_{234} + 2z_{98} z_{249} + 2z_{99} z_{246} - 2\dot{x}_l \\
z_{260} &= z_{19} z_{22} z_{23} \dot{\psi}_{h2} + z_{20} z_{22} z_{25} \dot{\theta}_{h2} + z_{23} z_{24} z_{25} \dot{\varphi}_{h2} - z_{19} z_{22} \dot{\varphi}_{h2} - z_{24} z_{25} \dot{\psi}_{h2}
\end{aligned}$$

4 Dynamic Equations of the Dual-Lift Model

$$\begin{aligned}
z_{261} = & z_{141} z_{72} + z_{97} z_{21} z_{72} + z_{98} z_{26} z_{72} + z_{99} z_{27} z_{72} \\
& + 2z_{97} z_{20} (z_{61} (z_{60} z_{69} - z_{61} z_{68}) + z_{62} (z_{60} z_{70} - z_{62} z_{68})) \dot{\theta}_{h2} \\
& + z_{142} (z_{70} z_{73} + z_{61} z_{79} - z_{69} z_{74} - z_{62} z_{77}) \\
& + z_{97} z_{28} (z_{70} z_{73} + z_{61} z_{79} - z_{69} z_{74} - z_{62} z_{77}) \\
& + z_{98} z_{29} (z_{70} z_{73} + z_{61} z_{79} - z_{69} z_{74} - z_{62} z_{77}) \\
& + z_{99} z_{30} (z_{70} z_{73} + z_{61} z_{79} - z_{69} z_{74} - z_{62} z_{77}) + z_{143} ((z_{60} z_{69} - z_{61} z_{68}) z_{73} \\
& \quad + (z_{60} z_{70} - z_{62} z_{68}) z_{74} + z_{61} (z_{69} z_{72} + z_{60} z_{77} - z_{68} z_{73} - z_{61} z_{78})) \\
& + z_{62} (z_{70} z_{72} + z_{60} z_{79} - z_{68} z_{74} - z_{62} z_{78}) + z_{98} z_{31} ((z_{60} z_{69} - z_{61} z_{68}) z_{73} \\
& \quad + (z_{60} z_{70} - z_{62} z_{68}) z_{74} + z_{61} (z_{69} z_{72} + z_{60} z_{77} - z_{68} z_{73} - z_{61} z_{78})) \\
& + z_{62} (z_{70} z_{72} + z_{60} z_{79} - z_{68} z_{74} - z_{62} z_{78}) + z_{99} z_{32} ((z_{60} z_{69} - z_{61} z_{68}) z_{73} \\
& \quad + (z_{60} z_{70} - z_{62} z_{68}) z_{74} + z_{61} (z_{69} z_{72} + z_{60} z_{77} - z_{68} z_{73} - z_{61} z_{78})) \\
& \quad + z_{62} (z_{70} z_{72} + z_{60} z_{79} - z_{68} z_{74} - z_{62} z_{78})) \\
& - 2z_{60} (u_{h2} - \dot{x}_l) - 2(z_{61} z_{70} - z_{62} z_{69})(v_{h2} - \dot{y}_l) \\
& - 2(z_{61} (z_{60} z_{69} - z_{61} z_{68}) + z_{62} (z_{60} z_{70} - z_{62} z_{68}))(w_{h2} - \dot{z}_l) \\
& - 2z_{97} z_{60} z_{234} - 2z_{98} z_{60} z_{249} - 2z_{99} z_{60} z_{246} - 2z_{97} (z_{61} z_{70} - z_{62} z_{69}) z_{242} \\
& - 2z_{98} (z_{61} z_{70} - z_{62} z_{69}) z_{260} - 2z_{99} (z_{61} z_{70} - z_{62} z_{69}) z_{250} \\
& - 2z_{98} (z_{61} (z_{60} z_{69} - z_{61} z_{68}) + z_{62} (z_{60} z_{70} - z_{62} z_{68})) z_{244} \\
& - 2z_{99} (z_{61} (z_{60} z_{69} - z_{61} z_{68}) + z_{62} (z_{60} z_{70} - z_{62} z_{68})) z_{239} \\
& - z_{97} z_{23} ((z_{60} z_{69} - z_{61} z_{68}) z_{73} + (z_{60} z_{70} - z_{62} z_{68}) z_{74} \\
& \quad + z_{61} (z_{69} z_{72} + z_{60} z_{77} - z_{68} z_{73} - z_{61} z_{78})) \\
& \quad + z_{62} (z_{70} z_{72} + z_{60} z_{79} - z_{68} z_{74} - z_{62} z_{78}))
\end{aligned}$$

$$\begin{aligned}
z_{262} = & z_{141} z_{73} + z_{97} z_{21} z_{73} + z_{98} z_{26} z_{73} + z_{99} z_{27} z_{73} + 2(z_{60} z_{70} - z_{62} z_{68})(v_{h2} - \dot{y}_l) \\
& + 2(z_{60} (z_{60} z_{69} - z_{61} z_{68}) - z_{62} (z_{61} z_{70} - z_{62} z_{69}))(w_{h2} - \dot{z}_l) \\
& + 2z_{97} (z_{60} z_{70} - z_{62} z_{68}) z_{242} \\
& + 2z_{98} (z_{60} z_{70} - z_{62} z_{68}) z_{260} + 2z_{99} (z_{60} z_{70} - z_{62} z_{68}) z_{250} \\
& + 2z_{98} (z_{60} (z_{60} z_{69} - z_{61} z_{68}) - z_{62} (z_{61} z_{70} - z_{62} z_{69})) z_{244} \\
& + 2z_{99} (z_{60} (z_{60} z_{69} - z_{61} z_{68}) - z_{62} (z_{61} z_{70} - z_{62} z_{69})) z_{239} \\
& + z_{97} z_{23} ((z_{60} z_{69} - z_{61} z_{68}) z_{72} + z_{60} (z_{69} z_{72} + z_{60} z_{77} - z_{68} z_{73} - z_{61} z_{78})) \\
& \quad - (z_{61} z_{70} - z_{62} z_{69}) z_{74} - z_{62} (z_{70} z_{73} + z_{61} z_{79} - z_{69} z_{74} - z_{62} z_{77})) - 2z_{61} (u_{h2} - \dot{x}_l) \\
& - 2z_{97} z_{20} (z_{60} (z_{60} z_{69} - z_{61} z_{68}) - z_{62} (z_{61} z_{70} - z_{62} z_{69})) \dot{\theta}_{h2} - 2z_{97} z_{61} z_{234} \\
& - 2z_{98} z_{61} z_{249} - 2z_{99} z_{61} z_{246} - z_{142} (z_{70} z_{72} + z_{60} z_{79} - z_{68} z_{74} - z_{62} z_{78}) \\
& - z_{97} z_{28} (z_{70} z_{72} + z_{60} z_{79} - z_{68} z_{74} - z_{62} z_{78}) \\
& - z_{98} z_{29} (z_{70} z_{72} + z_{60} z_{79} - z_{68} z_{74} - z_{62} z_{78}) \\
& - z_{99} z_{30} (z_{70} z_{72} + z_{60} z_{79} - z_{68} z_{74} - z_{62} z_{78}) \\
& - z_{143} ((z_{60} z_{69} - z_{61} z_{68}) z_{72} + z_{60} (z_{69} z_{72} + z_{60} z_{77} - z_{68} z_{73} - z_{61} z_{78})) \\
& \quad - (z_{61} z_{70} - z_{62} z_{69}) z_{74} - z_{62} (z_{70} z_{73} + z_{61} z_{79} - z_{69} z_{74} - z_{62} z_{77})) \\
& - z_{98} z_{31} ((z_{60} z_{69} - z_{61} z_{68}) z_{72} + z_{60} (z_{69} z_{72} + z_{60} z_{77} - z_{68} z_{73} - z_{61} z_{78})) \\
& \quad - (z_{61} z_{70} - z_{62} z_{69}) z_{74} - z_{62} (z_{70} z_{73} + z_{61} z_{79} - z_{69} z_{74} - z_{62} z_{77})) \\
& - z_{99} z_{32} ((z_{60} z_{69} - z_{61} z_{68}) z_{72} + z_{60} (z_{69} z_{72} + z_{60} z_{77} - z_{68} z_{73} - z_{61} z_{78})) \\
& \quad - (z_{61} z_{70} - z_{62} z_{69}) z_{74} - z_{62} (z_{70} z_{73} + z_{61} z_{79} - z_{69} z_{74} - z_{62} z_{77}))
\end{aligned}$$

4 Dynamic Equations of the Dual-Lift Model

$$\begin{aligned}
z_{263} &= z_{141} z_{74} + z_{97} z_{21} z_{74} + z_{98} z_{26} z_{74} + z_{99} z_{27} z_{74} \\
&\quad + 2(z_{60} (z_{60} z_{70} - z_{62} z_{68}) + z_{61} (z_{61} z_{70} - z_{62} z_{69}))(w_{h2} - \dot{z}_l) \\
&\quad + z_{142} (z_{69} z_{72} + z_{60} z_{77} - z_{68} z_{73} - z_{61} z_{78}) \\
&\quad + z_{97} z_{28} (z_{69} z_{72} + z_{60} z_{77} - z_{68} z_{73} - z_{61} z_{78}) \\
&\quad + z_{98} z_{29} (z_{69} z_{72} + z_{60} z_{77} - z_{68} z_{73} - z_{61} z_{78}) \\
&\quad + z_{99} z_{30} (z_{69} z_{72} + z_{60} z_{77} - z_{68} z_{73} - z_{61} z_{78}) \\
&\quad + 2z_{98} (z_{60} (z_{60} z_{70} - z_{62} z_{68}) + z_{61} (z_{61} z_{70} - z_{62} z_{69}))z_{244} \\
&\quad + 2z_{99} (z_{60} (z_{60} z_{70} - z_{62} z_{68}) + z_{61} (z_{61} z_{70} - z_{62} z_{69}))z_{239} \\
&\quad + z_{97} z_{23} ((z_{60} z_{70} - z_{62} z_{68})z_{72} + (z_{61} z_{70} - z_{62} z_{69})z_{73} \\
&\quad\quad\quad + z_{60} (z_{70} z_{72} + z_{60} z_{79} - z_{68} z_{74} - z_{62} z_{78}) \\
&\quad\quad\quad + z_{61} (z_{70} z_{73} + z_{61} z_{79} - z_{69} z_{74} - z_{62} z_{77})) \\
&\quad - 2z_{62} (u_{h2} - \dot{x}_l) - 2(z_{60} z_{69} - z_{61} z_{68})(v_{h2} - \dot{y}_l) \\
&\quad - 2z_{97} z_{20} (z_{60} (z_{60} z_{70} - z_{62} z_{68}) + z_{61} (z_{61} z_{70} - z_{62} z_{69}))\dot{\theta}_{h2} \\
&\quad - 2z_{97} z_{62} z_{234} - 2z_{98} z_{62} z_{249} - 2z_{99} z_{62} z_{246} - 2z_{97} (z_{60} z_{69} - z_{61} z_{68})z_{242} \\
&\quad - 2z_{98} (z_{60} z_{69} - z_{61} z_{68})z_{260} - 2z_{99} (z_{60} z_{69} - z_{61} z_{68})z_{250} \\
&\quad - z_{143} ((z_{60} z_{70} - z_{62} z_{68})z_{72} + (z_{61} z_{70} - z_{62} z_{69})z_{73} \\
&\quad\quad\quad + z_{60} (z_{70} z_{72} + z_{60} z_{79} - z_{68} z_{74} - z_{62} z_{78}) \\
&\quad\quad\quad + z_{61} (z_{70} z_{73} + z_{61} z_{79} - z_{69} z_{74} - z_{62} z_{77})) - z_{98} z_{31} ((z_{60} z_{70} - z_{62} z_{68})z_{72} \\
&\quad\quad\quad + (z_{61} z_{70} - z_{62} z_{69})z_{73} + z_{60} (z_{70} z_{72} + z_{60} z_{79} - z_{68} z_{74} - z_{62} z_{78}) \\
&\quad\quad\quad + z_{61} (z_{70} z_{73} + z_{61} z_{79} - z_{69} z_{74} - z_{62} z_{77})) - z_{99} z_{32} ((z_{60} z_{70} - z_{62} z_{68})z_{72} \\
&\quad\quad\quad + (z_{61} z_{70} - z_{62} z_{69})z_{73} + z_{60} (z_{70} z_{72} + z_{60} z_{79} - z_{68} z_{74} - z_{62} z_{78}) \\
&\quad\quad\quad + z_{61} (z_{70} z_{73} + z_{61} z_{79} - z_{69} z_{74} - z_{62} z_{77})) \\
z_{264} &= 2z_{98} z_{27} (u_{h2} - \dot{x}_l) + 2z_{98} z_{30} (v_{h2} - \dot{y}_l) + 2z_{98} z_{32} (w_{h2} - \dot{z}_l) + 2z_{99} z_{146} (v_{h2} - \dot{y}_l) \\
&\quad + 2z_{99} z_{147} (u_{h2} - \dot{x}_l) + 2z_{98} z_{141} z_{246} + 2z_{98} z_{142} z_{250} + 2z_{98} z_{143} z_{239} \\
&\quad + 2z_{99} z_{141} z_{253} + 2z_{99} z_{142} z_{247} - 2z_{99} z_{31} (w_{h2} - \dot{z}_l) - 2z_{99} z_{143} z_{244} \\
z_{265} &= 2v_{h2} + 2z_{97} z_{242} + 2z_{98} z_{260} + 2z_{99} z_{250} - 2\dot{y}_l \\
z_{266} &= 2w_{h2} + 2z_{98} z_{244} + 2z_{99} z_{239} - 2\dot{z}_l - 2z_{97} z_{20} \dot{\theta}_{h2} \\
z_{267} &= p_{h2} z_{264} + q_{h2} z_{258} + r_{h2} z_{255} + u_l z_{261} + v_l z_{262} \\
&\quad + w_l z_{263} + u_{h2} z_{259} + v_{h2} z_{265} + w_{h2} z_{266} \\
z_{268} &= (z_{160} z_{267} - z_{169} z_{233})/z_{173} \\
z_{269} &= (z_{158} z_{267} - z_{167} z_{233})/z_{173} \\
z_{272} &= -z_{176} (z_{61} (z_{60} z_{69} - z_{61} z_{68}) + z_{62} (z_{60} z_{70} - z_{62} z_{68})) \\
&\quad - z_{189} (z_{60} (z_{60} z_{70} - z_{62} z_{68}) + z_{61} (z_{61} z_{70} - z_{62} z_{69})) \\
z_{275} &= -z_{179} (z_{61} (z_{60} z_{69} - z_{61} z_{68}) + z_{62} (z_{60} z_{70} - z_{62} z_{68})) \\
&\quad - z_{192} (z_{60} (z_{60} z_{70} - z_{62} z_{68}) + z_{61} (z_{61} z_{70} - z_{62} z_{69})) \\
z_{278} &= -z_{180} (z_{61} (z_{60} z_{69} - z_{61} z_{68}) + z_{62} (z_{60} z_{70} - z_{62} z_{68})) \\
&\quad - z_{193} (z_{60} (z_{60} z_{70} - z_{62} z_{68}) + z_{61} (z_{61} z_{70} - z_{62} z_{69}))
\end{aligned}$$

4 Dynamic Equations of the Dual-Lift Model

$$\begin{aligned}
z_{281} &= -z_{178} (z_{61} (z_{60} z_{69} - z_{61} z_{68}) + z_{62} (z_{60} z_{70} - z_{62} z_{68})) \\
&\quad - z_{191} (z_{60} (z_{60} z_{70} - z_{62} z_{68}) + z_{61} (z_{61} z_{70} - z_{62} z_{69})) \\
z_{284} &= -z_{175} (z_{61} (z_{60} z_{69} - z_{61} z_{68}) + z_{62} (z_{60} z_{70} - z_{62} z_{68})) \\
&\quad - z_{188} (z_{60} (z_{60} z_{70} - z_{62} z_{68}) + z_{61} (z_{61} z_{70} - z_{62} z_{69})) \\
z_{287} &= -z_{174} (z_{61} (z_{60} z_{69} - z_{61} z_{68}) + z_{62} (z_{60} z_{70} - z_{62} z_{68})) \\
&\quad - z_{187} (z_{60} (z_{60} z_{70} - z_{62} z_{68}) + z_{61} (z_{61} z_{70} - z_{62} z_{69})) \\
z_{290} &= z_{183} (z_{61} (z_{60} z_{69} - z_{61} z_{68}) + z_{62} (z_{60} z_{70} - z_{62} z_{68})) \\
&\quad + z_{196} (z_{60} (z_{60} z_{70} - z_{62} z_{68}) + z_{61} (z_{61} z_{70} - z_{62} z_{69})) \\
z_{293} &= z_{185} (z_{61} (z_{60} z_{69} - z_{61} z_{68}) + z_{62} (z_{60} z_{70} - z_{62} z_{68})) \\
&\quad + z_{198} (z_{60} (z_{60} z_{70} - z_{62} z_{68}) + z_{61} (z_{61} z_{70} - z_{62} z_{69})) \\
z_{296} &= z_{186} (z_{61} (z_{60} z_{69} - z_{61} z_{68}) + z_{62} (z_{60} z_{70} - z_{62} z_{68})) \\
&\quad + z_{199} (z_{60} (z_{60} z_{70} - z_{62} z_{68}) + z_{61} (z_{61} z_{70} - z_{62} z_{69})) \\
z_{299} &= z_{184} (z_{61} (z_{60} z_{69} - z_{61} z_{68}) + z_{62} (z_{60} z_{70} - z_{62} z_{68})) \\
&\quad + z_{197} (z_{60} (z_{60} z_{70} - z_{62} z_{68}) + z_{61} (z_{61} z_{70} - z_{62} z_{69})) \\
z_{302} &= z_{182} (z_{61} (z_{60} z_{69} - z_{61} z_{68}) + z_{62} (z_{60} z_{70} - z_{62} z_{68})) \\
&\quad + z_{195} (z_{60} (z_{60} z_{70} - z_{62} z_{68}) + z_{61} (z_{61} z_{70} - z_{62} z_{69})) \\
z_{305} &= z_{181} (z_{61} (z_{60} z_{69} - z_{61} z_{68}) + z_{62} (z_{60} z_{70} - z_{62} z_{68})) \\
&\quad + z_{194} (z_{60} (z_{60} z_{70} - z_{62} z_{68}) + z_{61} (z_{61} z_{70} - z_{62} z_{69})) \\
z_{308} &= z_{62} (z_{61} z_{70} - z_{62} z_{69}) + z_{190} (z_{60} (z_{60} z_{70} - z_{62} z_{68}) + z_{61} (z_{61} z_{70} - z_{62} z_{69})) \\
&\quad - z_{60} (z_{60} z_{69} - z_{61} z_{68}) - z_{177} (z_{61} (z_{60} z_{69} - z_{61} z_{68}) + z_{62} (z_{60} z_{70} - z_{62} z_{68})) \\
z_{309} &= F_{h1,3}^{MR} z_{11} + F_{h1,2}^{TR} z_{10} + z_{125} z_5 z_7 + z_{124} (z_5 z_7 - z_{10} z_{15} - z_{11} z_{16}) \\
&\quad - z_{126} z_{272} - z_{125} z_{10} z_{15} - z_{125} z_{11} z_{16} \\
z_{310} &= F_{h1,3}^{MR} z_{14} + F_{h1,2}^{TR} z_{13} + z_{125} z_7 z_{12} + z_{124} (z_7 z_{12} - z_{13} z_{15} - z_{14} z_{16}) \\
&\quad - z_{126} z_{275} - z_{125} z_{13} z_{15} - z_{125} z_{14} z_{16} \\
z_{311} &= F_{h1,3}^{MR} z_{16} + F_{h1,2}^{TR} z_{15} - z_{126} z_{278} - z_{125} z_7^2 - z_{125} z_{15}^2 - z_{125} z_{16}^2 - z_{124} (z_7^2 + z_{15}^2 + z_{16}^2) \\
z_{312} &= z_{90} z_{125} \\
z_{313} &= z_{89} z_{125} \\
z_{314} &= T_{h1,1}^{MR} + z_{89} F_{h1,3}^{MR} + z_{312} z_{15} - z_{96} F_{h1,2}^{TR} - z_{126} z_{281} - z_{313} z_{16} - z_{124} (z_{92} z_{16} - z_{93} z_{15}) \\
z_{315} &= z_{88} z_{125} \\
z_{316} &= T_{h1,2}^{MR} + T_{h1,2}^{TR} + z_{312} z_7 + z_{315} z_{16} + z_{124} (z_{91} z_{16} + z_{93} z_7) - z_{88} F_{h1,3}^{MR} - z_{126} z_{284} \\
z_{317} &= T_{h1,3}^{MR} + z_{94} F_{h1,2}^{TR} - z_{126} z_{287} - z_{124} (z_{91} z_{15} + z_{92} z_7) - z_{125} (z_{88} z_{15} + z_{89} z_7) \\
z_{318} &= F_{h2,3}^{MR} z_{27} + F_{h2,2}^{TR} z_{26} + z_{125} z_{21} z_{23} + z_{124} (z_{21} z_{23} - z_{26} z_{31} - z_{27} z_{32}) \\
&\quad - z_{126} z_{290} - z_{125} z_{26} z_{31} - z_{125} z_{27} z_{32}
\end{aligned}$$

4 Dynamic Equations of the Dual-Lift Model

$$\begin{aligned}
z_{319} &= F_{h2,3}^{MR} z_{30} + F_{h2,2}^{TR} z_{29} + z_{125} z_{23} z_{28} + z_{124} (z_{23} z_{28} - z_{29} z_{31} - z_{30} z_{32}) \\
&\quad - z_{126} z_{293} - z_{125} z_{29} z_{31} - z_{125} z_{30} z_{32} \\
z_{320} &= F_{h2,3}^{MR} z_{32} + F_{h2,2}^{TR} z_{31} - z_{126} z_{296} - z_{125} z_{23}^2 - z_{125} z_{31}^2 - z_{125} z_{32}^2 - z_{124} (z_{23}^{2+} z_{31}^{2+} z_{32}^2) \\
z_{321} &= T_{h2,1}^{MR} + z_{89} F_{h2,3}^{MR} + z_{312} z_{31} - z_{96} F_{h2,2}^{TR} - z_{126} z_{299} - z_{313} z_{32} - z_{124} (z_{92} z_{32} - z_{93} z_{31}) \\
z_{322} &= T_{h2,2}^{MR} + T_{h2,2}^{TR} + z_{312} z_{23} + z_{315} z_{32} + z_{124} (z_{91} z_{32} + z_{93} z_{23}) - z_{88} F_{h2,3}^{MR} - z_{126} z_{302} \\
z_{323} &= T_{h2,3}^{MR} + z_{94} F_{h2,2}^{TR} - z_{126} z_{305} - z_{124} (z_{91} z_{31} + z_{92} z_{23}) - z_{125} (z_{88} z_{31} + z_{89} z_{23}) \\
z_{324} &= z_{126} z_{308} \\
z_{325} &= I_{11}^F p_{h1} \\
z_{326} &= I_{22}^F q_{h1} \\
z_{327} &= I_{33}^F r_{h1} \\
z_{328} &= p_{h1} z_{326} - q_{h1} z_{325} \\
z_{329} &= r_{h1} z_{325} - p_{h1} z_{327} \\
z_{330} &= q_{h1} z_{327} - r_{h1} z_{326} \\
z_{331} &= I_{11}^{MR} p_{h1} \\
z_{332} &= I_{22}^{MR} q_{h1} \\
z_{333} &= I_{33}^{MR} z_{45} \\
z_{334} &= I_{11}^{MR} z_{46} \\
z_{335} &= I_{22}^{MR} z_{47} \\
z_{336} &= p_{h1} z_{332} - q_{h1} z_{331} \\
z_{337} &= z_{45} z_{331} - p_{h1} z_{333} \\
z_{338} &= q_{h1} z_{333} - z_{45} z_{332} \\
z_{339} &= I_{11}^F p_{h2} \\
z_{340} &= I_{22}^F q_{h2} \\
z_{341} &= I_{33}^F r_{h2} \\
z_{342} &= p_{h2} z_{340} - q_{h2} z_{339} \\
z_{343} &= r_{h2} z_{339} - p_{h2} z_{341} \\
z_{344} &= q_{h2} z_{341} - r_{h2} z_{340} \\
z_{345} &= I_{11}^{MR} p_{h2}
\end{aligned}$$

4 Dynamic Equations of the Dual-Lift Model

$$\begin{aligned}
z_{346} &= I_{22}^{MR} q_{h2} \\
z_{347} &= I_{33}^{MR} z_{48} \\
z_{348} &= I_{11}^{MR} z_{49} \\
z_{349} &= I_{22}^{MR} z_{50} \\
z_{350} &= p_{h2} z_{346} - q_{h2} z_{345} \\
z_{351} &= z_{48} z_{345} - p_{h2} z_{347} \\
z_{352} &= q_{h2} z_{347} - z_{48} z_{346} \\
z_{353} &= m_F (z_{91} z_{10} - z_{92} z_5) + m_{MR} (z_{88} z_{10} - z_{89} z_5) \\
z_{354} &= z_{51} (z_5 z_7 - z_{10} z_{15} - z_{11} z_{16}) \\
z_{355} &= -m_F (z_{91} z_{11} - z_{93} z_5) - m_{MR} (z_{88} z_{11} - z_{90} z_5) \\
z_{356} &= z_{51} (z_5^{2+} z_{10}^{2+} z_{11}^2) \\
z_{357} &= z_{51} (z_5 z_{12} + z_{10} z_{13} + z_{11} z_{14}) \\
z_{358} &= m_F (z_{92} z_{11} - z_{93} z_{10}) + m_{MR} (z_{89} z_{11} - z_{90} z_{10}) \\
z_{359} &= m_F (z_5 z_{103} + z_{10} z_{104} + z_{11} z_{105}) + m_{MR} (z_5 z_{100} + z_{10} z_{101} + z_{11} z_{102}) \\
z_{360} &= m_F (z_{91} z_{13} - z_{92} z_{12}) + m_{MR} (z_{88} z_{13} - z_{89} z_{12}) \\
z_{361} &= z_{51} (z_7 z_{12} - z_{13} z_{15} - z_{14} z_{16}) \\
z_{362} &= -m_F (z_{91} z_{14} - z_{93} z_{12}) - m_{MR} (z_{88} z_{14} - z_{90} z_{12}) \\
z_{363} &= z_{51} (z_{12}^{2+} z_{13}^{2+} z_{14}^2) \\
z_{364} &= m_F (z_{92} z_{14} - z_{93} z_{13}) + m_{MR} (z_{89} z_{14} - z_{90} z_{13}) \\
z_{365} &= m_F (z_{12} z_{103} + z_{13} z_{104} + z_{14} z_{105}) + m_{MR} (z_{12} z_{100} + z_{13} z_{101} + z_{14} z_{102}) \\
z_{366} &= m_F (z_{91} z_{15} + z_{92} z_7) + m_{MR} (z_{88} z_{15} + z_{89} z_7) \\
z_{367} &= z_{51} (z_7^{2+} z_{15}^{2+} z_{16}^2) \\
z_{368} &= -m_F (z_{91} z_{16} + z_{93} z_7) - m_{MR} (z_{88} z_{16} + z_{90} z_7) \\
z_{369} &= m_F (z_{92} z_{16} - z_{93} z_{15}) + m_{MR} (z_{89} z_{16} - z_{90} z_{15}) \\
z_{370} &= -m_F (z_7 z_{103} - z_{15} z_{104} - z_{16} z_{105}) - m_{MR} (z_7 z_{100} - z_{15} z_{101} - z_{16} z_{102}) \\
z_{371} &= I_{11}^F + I_{11}^{MR} + m_F (z_{92}^{2+} z_{93}^2) + m_{MR} (z_{89}^{2+} z_{90}^2) \\
z_{372} &= -m_F z_{91} z_{93} - m_{MR} z_{88} z_{90} \\
z_{373} &= -m_F z_{91} z_{92} - m_{MR} z_{88} z_{89}
\end{aligned}$$

4 Dynamic Equations of the Dual-Lift Model

$$\begin{aligned}
z_{374} &= z_{330} + z_{338} + m_F (z_{92} z_{105} - z_{93} z_{104}) + m_{MR} (z_{89} z_{102} - z_{90} z_{101}) - z_{334} \\
z_{375} &= I_{22}^F + I_{22}^{MR} + m_F (z_{91}^{2+} z_{93}^2) + m_{MR} (z_{88}^{2+} z_{90}^2) \\
z_{376} &= -m_F z_{92} z_{93} - m_{MR} z_{89} z_{90} \\
z_{377} &= z_{329} + z_{335} + z_{337} - m_F (z_{91} z_{105} - z_{93} z_{103}) - m_{MR} (z_{88} z_{102} - z_{90} z_{100}) \\
z_{378} &= I_{33}^F + I_{33}^{MR} + m_F (z_{91}^{2+} z_{92}^2) + m_{MR} (z_{88}^{2+} z_{89}^2) \\
z_{379} &= z_{328} + z_{336} + m_F (z_{91} z_{104} - z_{92} z_{103}) + m_{MR} (z_{88} z_{101} - z_{89} z_{100}) \\
z_{380} &= m_F (z_{91} z_{26} - z_{92} z_{21}) + m_{MR} (z_{88} z_{26} - z_{89} z_{21}) \\
z_{381} &= z_{51} (z_{21} z_{23} - z_{26} z_{31} - z_{27} z_{32}) \\
z_{382} &= -m_F (z_{91} z_{27} - z_{93} z_{21}) - m_{MR} (z_{88} z_{27} - z_{90} z_{21}) \\
z_{383} &= z_{51} (z_{21}^{2+} z_{26}^{2+} z_{27}^2) \\
z_{384} &= z_{51} (z_{21} z_{28} + z_{26} z_{29} + z_{27} z_{30}) \\
z_{385} &= m_F (z_{92} z_{27} - z_{93} z_{26}) + m_{MR} (z_{89} z_{27} - z_{90} z_{26}) \\
z_{386} &= m_F (z_{21} z_{115} + z_{26} z_{116} + z_{27} z_{117}) + m_{MR} (z_{21} z_{112} + z_{26} z_{113} + z_{27} z_{114}) \\
z_{387} &= m_F (z_{91} z_{29} - z_{92} z_{28}) + m_{MR} (z_{88} z_{29} - z_{89} z_{28}) \\
z_{388} &= z_{51} (z_{23} z_{28} - z_{29} z_{31} - z_{30} z_{32}) \\
z_{389} &= -m_F (z_{91} z_{30} - z_{93} z_{28}) - m_{MR} (z_{88} z_{30} - z_{90} z_{28}) \\
z_{390} &= z_{51} (z_{28}^{2+} z_{29}^{2+} z_{30}^2) \\
z_{391} &= m_F (z_{92} z_{30} - z_{93} z_{29}) + m_{MR} (z_{89} z_{30} - z_{90} z_{29}) \\
z_{392} &= m_F (z_{28} z_{115} + z_{29} z_{116} + z_{30} z_{117}) + m_{MR} (z_{28} z_{112} + z_{29} z_{113} + z_{30} z_{114}) \\
z_{393} &= m_F (z_{91} z_{31} + z_{92} z_{23}) + m_{MR} (z_{88} z_{31} + z_{89} z_{23}) \\
z_{394} &= z_{51} (z_{23}^{2+} z_{31}^{2+} z_{32}^2) \\
z_{395} &= -m_F (z_{91} z_{32} + z_{93} z_{23}) - m_{MR} (z_{88} z_{32} + z_{90} z_{23}) \\
z_{396} &= m_F (z_{92} z_{32} - z_{93} z_{31}) + m_{MR} (z_{89} z_{32} - z_{90} z_{31}) \\
z_{397} &= -m_F (z_{23} z_{115} - z_{31} z_{116} - z_{32} z_{117}) - m_{MR} (z_{23} z_{112} - z_{31} z_{113} - z_{32} z_{114}) \\
z_{398} &= z_{344} + z_{352} + m_F (z_{92} z_{117} - z_{93} z_{116}) + m_{MR} (z_{89} z_{114} - z_{90} z_{113}) - z_{348} \\
z_{399} &= z_{343} + z_{349} + z_{351} - m_F (z_{91} z_{117} - z_{93} z_{115}) - m_{MR} (z_{88} z_{114} - z_{90} z_{112}) \\
z_{400} &= z_{342} + z_{350} + m_F (z_{91} z_{116} - z_{92} z_{115}) + m_{MR} (z_{88} z_{113} - z_{89} z_{112}) \\
z_{401} &= l_p (z_{60} z_{61} + z_{81} z_{82} + z_{84} z_{86})
\end{aligned}$$

4 Dynamic Equations of the Dual-Lift Model

$$\begin{aligned}
 z_{402} &= l_{\mathcal{P}} (z_{61}^{2+} z_{82}^{2+} z_{86}^2) \\
 z_{403} &= l_{\mathcal{P}} (z_{61} z_{62} + z_{80} z_{82} + z_{85} z_{86}) \\
 z_{404} &= l_{\mathcal{P}} (z_{61} z_{75} + z_{82} z_{83} + z_{86} z_{87}) \\
 z_{405} &= l_{\mathcal{P}} (z_{60}^{2+} z_{81}^{2+} z_{84}^2) \\
 z_{406} &= l_{\mathcal{P}} (z_{60} z_{62} + z_{80} z_{81} + z_{84} z_{85}) \\
 z_{407} &= l_{\mathcal{P}} (z_{60} z_{75} + z_{81} z_{83} + z_{84} z_{87}) \\
 z_{408} &= l_{\mathcal{P}} (z_{62}^{2+} z_{80}^{2+} z_{85}^2) \\
 z_{409} &= l_{\mathcal{P}} (z_{62} z_{75} + z_{80} z_{83} + z_{85} z_{87}) \\
 z_{410} &= z_{189} z_{406} - z_{176} z_{405} \\
 z_{411} &= z_{189} z_{403} - z_{176} z_{401} \\
 z_{412} &= z_{189} z_{408} - z_{176} z_{406} \\
 z_{413} &= z_{359} + z_{189} z_{409} - z_{176} z_{407} \\
 z_{414} &= z_{356} + z_{189} z_{412} - z_{176} z_{410} \\
 z_{415} &= z_{357} + z_{192} z_{412} - z_{179} z_{410} \\
 z_{416} &= z_{193} z_{412} - z_{354} - z_{180} z_{410} \\
 z_{417} &= z_{358} + z_{191} z_{412} - z_{178} z_{410} \\
 z_{418} &= z_{355} + z_{188} z_{412} - z_{175} z_{410} \\
 z_{419} &= z_{353} + z_{187} z_{412} - z_{174} z_{410} \\
 z_{420} &= z_{183} z_{410} - z_{196} z_{412} \\
 z_{421} &= z_{185} z_{410} - z_{198} z_{412} \\
 z_{422} &= z_{186} z_{410} - z_{199} z_{412} \\
 z_{423} &= z_{184} z_{410} - z_{197} z_{412} \\
 z_{424} &= z_{182} z_{410} - z_{195} z_{412} \\
 z_{425} &= z_{181} z_{410} - z_{194} z_{412} \\
 z_{426} &= z_{411} - z_{177} z_{410} - z_{190} z_{412} \\
 z_{427} &= z_{413} + z_{410} z_{268} - z_{412} z_{269} \\
 z_{428} &= z_{192} z_{406} - z_{179} z_{405} \\
 z_{429} &= z_{192} z_{403} - z_{179} z_{401} \\
 z_{430} &= z_{192} z_{408} - z_{179} z_{406} \\
 z_{431} &= z_{365} + z_{192} z_{409} - z_{179} z_{407} \\
 z_{432} &= z_{357} + z_{189} z_{430} - z_{176} z_{428}
 \end{aligned}$$

4 Dynamic Equations of the Dual-Lift Model

$$\begin{aligned}z_{433} &= z_{363} + z_{192} z_{430} - z_{179} z_{428} \\z_{434} &= z_{193} z_{430} - z_{361} - z_{180} z_{428} \\z_{435} &= z_{364} + z_{191} z_{430} - z_{178} z_{428} \\z_{436} &= z_{362} + z_{188} z_{430} - z_{175} z_{428} \\z_{437} &= z_{360} + z_{187} z_{430} - z_{174} z_{428} \\z_{438} &= z_{183} z_{428} - z_{196} z_{430} \\z_{439} &= z_{185} z_{428} - z_{198} z_{430} \\z_{440} &= z_{186} z_{428} - z_{199} z_{430} \\z_{441} &= z_{184} z_{428} - z_{197} z_{430} \\z_{442} &= z_{182} z_{428} - z_{195} z_{430} \\z_{443} &= z_{181} z_{428} - z_{194} z_{430} \\z_{444} &= z_{429} - z_{177} z_{428} - z_{190} z_{430} \\z_{445} &= z_{431} + z_{428} z_{268} - z_{430} z_{269} \\z_{446} &= z_{193} z_{406} - z_{180} z_{405} \\z_{447} &= z_{193} z_{403} - z_{180} z_{401} \\z_{448} &= z_{193} z_{408} - z_{180} z_{406} \\z_{449} &= z_{370} + z_{193} z_{409} - z_{180} z_{407} \\z_{450} &= z_{189} z_{448} - z_{354} - z_{176} z_{446} \\z_{451} &= z_{192} z_{448} - z_{361} - z_{179} z_{446} \\z_{452} &= z_{367} + z_{193} z_{448} - z_{180} z_{446} \\z_{453} &= z_{369} + z_{191} z_{448} - z_{178} z_{446} \\z_{454} &= z_{368} + z_{188} z_{448} - z_{175} z_{446} \\z_{455} &= z_{366} + z_{187} z_{448} - z_{174} z_{446} \\z_{456} &= z_{183} z_{446} - z_{196} z_{448} \\z_{457} &= z_{185} z_{446} - z_{198} z_{448} \\z_{458} &= z_{186} z_{446} - z_{199} z_{448} \\z_{459} &= z_{184} z_{446} - z_{197} z_{448} \\z_{460} &= z_{182} z_{446} - z_{195} z_{448} \\z_{461} &= z_{181} z_{446} - z_{194} z_{448} \\z_{462} &= z_{447} - z_{177} z_{446} - z_{190} z_{448} \\z_{463} &= z_{449} + z_{446} z_{268} - z_{448} z_{269} \\z_{464} &= z_{191} z_{406} - z_{178} z_{405} \\z_{465} &= z_{191} z_{403} - z_{178} z_{401}\end{aligned}$$

4 Dynamic Equations of the Dual-Lift Model

$$\begin{aligned}z_{466} &= z_{191} z_{408} - z_{178} z_{406} \\z_{467} &= z_{374} + z_{191} z_{409} - z_{178} z_{407} \\z_{468} &= z_{358} + z_{189} z_{466} - z_{176} z_{464} \\z_{469} &= z_{364} + z_{192} z_{466} - z_{179} z_{464} \\z_{470} &= z_{369} + z_{193} z_{466} - z_{180} z_{464} \\z_{471} &= z_{371} + z_{191} z_{466} - z_{178} z_{464} \\z_{472} &= z_{373} + z_{188} z_{466} - z_{175} z_{464} \\z_{473} &= z_{372} + z_{187} z_{466} - z_{174} z_{464} \\z_{474} &= z_{183} z_{464} - z_{196} z_{466} \\z_{475} &= z_{185} z_{464} - z_{198} z_{466} \\z_{476} &= z_{186} z_{464} - z_{199} z_{466} \\z_{477} &= z_{184} z_{464} - z_{197} z_{466} \\z_{478} &= z_{182} z_{464} - z_{195} z_{466} \\z_{479} &= z_{181} z_{464} - z_{194} z_{466} \\z_{480} &= z_{465} - z_{177} z_{464} - z_{190} z_{466} \\z_{481} &= z_{467} + z_{464} z_{268} - z_{466} z_{269} \\z_{482} &= z_{188} z_{406} - z_{175} z_{405} \\z_{483} &= z_{188} z_{403} - z_{175} z_{401} \\z_{484} &= z_{188} z_{408} - z_{175} z_{406} \\z_{485} &= z_{377} + z_{188} z_{409} - z_{175} z_{407} \\z_{486} &= z_{355} + z_{189} z_{484} - z_{176} z_{482} \\z_{487} &= z_{362} + z_{192} z_{484} - z_{179} z_{482} \\z_{488} &= z_{368} + z_{193} z_{484} - z_{180} z_{482} \\z_{489} &= z_{373} + z_{191} z_{484} - z_{178} z_{482} \\z_{490} &= z_{375} + z_{188} z_{484} - z_{175} z_{482} \\z_{491} &= z_{376} + z_{187} z_{484} - z_{174} z_{482} \\z_{492} &= z_{183} z_{482} - z_{196} z_{484} \\z_{493} &= z_{185} z_{482} - z_{198} z_{484} \\z_{494} &= z_{186} z_{482} - z_{199} z_{484} \\z_{495} &= z_{184} z_{482} - z_{197} z_{484} \\z_{496} &= z_{182} z_{482} - z_{195} z_{484} \\z_{497} &= z_{181} z_{482} - z_{194} z_{484} \\z_{498} &= z_{483} - z_{177} z_{482} - z_{190} z_{484}\end{aligned}$$

4 Dynamic Equations of the Dual-Lift Model

$$\begin{aligned}z_{499} &= z_{485} + z_{482} z_{268} - z_{484} z_{269} \\z_{500} &= z_{187} z_{406} - z_{174} z_{405} \\z_{501} &= z_{187} z_{403} - z_{174} z_{401} \\z_{502} &= z_{187} z_{408} - z_{174} z_{406} \\z_{503} &= z_{379} + z_{187} z_{409} - z_{174} z_{407} \\z_{504} &= z_{353} + z_{189} z_{502} - z_{176} z_{500} \\z_{505} &= z_{360} + z_{192} z_{502} - z_{179} z_{500} \\z_{506} &= z_{366} + z_{193} z_{502} - z_{180} z_{500} \\z_{507} &= z_{372} + z_{191} z_{502} - z_{178} z_{500} \\z_{508} &= z_{376} + z_{188} z_{502} - z_{175} z_{500} \\z_{509} &= z_{378} + z_{187} z_{502} - z_{174} z_{500} \\z_{510} &= z_{183} z_{500} - z_{196} z_{502} \\z_{511} &= z_{185} z_{500} - z_{198} z_{502} \\z_{512} &= z_{186} z_{500} - z_{199} z_{502} \\z_{513} &= z_{184} z_{500} - z_{197} z_{502} \\z_{514} &= z_{182} z_{500} - z_{195} z_{502} \\z_{515} &= z_{181} z_{500} - z_{194} z_{502} \\z_{516} &= z_{501} - z_{177} z_{500} - z_{190} z_{502} \\z_{517} &= z_{503} + z_{500} z_{268} - z_{502} z_{269} \\z_{518} &= z_{183} z_{405} - z_{196} z_{406} \\z_{519} &= z_{183} z_{401} - z_{196} z_{403} \\z_{520} &= z_{183} z_{406} - z_{196} z_{408} \\z_{521} &= z_{386} + z_{183} z_{407} - z_{196} z_{409} \\z_{522} &= z_{189} z_{520} - z_{176} z_{518} \\z_{523} &= z_{192} z_{520} - z_{179} z_{518} \\z_{524} &= z_{193} z_{520} - z_{180} z_{518} \\z_{525} &= z_{191} z_{520} - z_{178} z_{518} \\z_{526} &= z_{188} z_{520} - z_{175} z_{518} \\z_{527} &= z_{187} z_{520} - z_{174} z_{518} \\z_{528} &= z_{383} + z_{183} z_{518} - z_{196} z_{520} \\z_{529} &= z_{384} + z_{185} z_{518} - z_{198} z_{520} \\z_{530} &= z_{186} z_{518} - z_{381} - z_{199} z_{520} \\z_{531} &= z_{385} + z_{184} z_{518} - z_{197} z_{520}\end{aligned}$$

4 Dynamic Equations of the Dual-Lift Model

$$\begin{aligned}z_{532} &= z_{382} + z_{182} z_{518} - z_{195} z_{520} \\z_{533} &= z_{380} + z_{181} z_{518} - z_{194} z_{520} \\z_{534} &= z_{519} - z_{177} z_{518} - z_{190} z_{520} \\z_{535} &= z_{521} + z_{518} z_{268} - z_{520} z_{269} \\z_{536} &= z_{185} z_{405} - z_{198} z_{406} \\z_{537} &= z_{185} z_{401} - z_{198} z_{403} \\z_{538} &= z_{185} z_{406} - z_{198} z_{408} \\z_{539} &= z_{392} + z_{185} z_{407} - z_{198} z_{409} \\z_{540} &= z_{189} z_{538} - z_{176} z_{536} \\z_{541} &= z_{192} z_{538} - z_{179} z_{536} \\z_{542} &= z_{193} z_{538} - z_{180} z_{536} \\z_{543} &= z_{191} z_{538} - z_{178} z_{536} \\z_{544} &= z_{188} z_{538} - z_{175} z_{536} \\z_{545} &= z_{187} z_{538} - z_{174} z_{536} \\z_{546} &= z_{384} + z_{183} z_{536} - z_{196} z_{538} \\z_{547} &= z_{390} + z_{185} z_{536} - z_{198} z_{538} \\z_{548} &= z_{186} z_{536} - z_{388} - z_{199} z_{538} \\z_{549} &= z_{391} + z_{184} z_{536} - z_{197} z_{538} \\z_{550} &= z_{389} + z_{182} z_{536} - z_{195} z_{538} \\z_{551} &= z_{387} + z_{181} z_{536} - z_{194} z_{538} \\z_{552} &= z_{537} - z_{177} z_{536} - z_{190} z_{538} \\z_{553} &= z_{539} + z_{536} z_{268} - z_{538} z_{269} \\z_{554} &= z_{186} z_{405} - z_{199} z_{406} \\z_{555} &= z_{186} z_{401} - z_{199} z_{403} \\z_{556} &= z_{186} z_{406} - z_{199} z_{408} \\z_{557} &= z_{397} + z_{186} z_{407} - z_{199} z_{409} \\z_{558} &= z_{189} z_{556} - z_{176} z_{554} \\z_{559} &= z_{192} z_{556} - z_{179} z_{554} \\z_{560} &= z_{193} z_{556} - z_{180} z_{554} \\z_{561} &= z_{191} z_{556} - z_{178} z_{554} \\z_{562} &= z_{188} z_{556} - z_{175} z_{554} \\z_{563} &= z_{187} z_{556} - z_{174} z_{554} \\z_{564} &= z_{183} z_{554} - z_{381} - z_{196} z_{556}\end{aligned}$$

4 Dynamic Equations of the Dual-Lift Model

$$\begin{aligned}z_{565} &= z_{185} z_{554} - z_{388} - z_{198} z_{556} \\z_{566} &= z_{394} + z_{186} z_{554} - z_{199} z_{556} \\z_{567} &= z_{396} + z_{184} z_{554} - z_{197} z_{556} \\z_{568} &= z_{395} + z_{182} z_{554} - z_{195} z_{556} \\z_{569} &= z_{393} + z_{181} z_{554} - z_{194} z_{556} \\z_{570} &= z_{555} - z_{177} z_{554} - z_{190} z_{556} \\z_{571} &= z_{557} + z_{554} z_{268} - z_{556} z_{269} \\z_{572} &= z_{184} z_{405} - z_{197} z_{406} \\z_{573} &= z_{184} z_{401} - z_{197} z_{403} \\z_{574} &= z_{184} z_{406} - z_{197} z_{408} \\z_{575} &= z_{398} + z_{184} z_{407} - z_{197} z_{409} \\z_{576} &= z_{189} z_{574} - z_{176} z_{572} \\z_{577} &= z_{192} z_{574} - z_{179} z_{572} \\z_{578} &= z_{193} z_{574} - z_{180} z_{572} \\z_{579} &= z_{191} z_{574} - z_{178} z_{572} \\z_{580} &= z_{188} z_{574} - z_{175} z_{572} \\z_{581} &= z_{187} z_{574} - z_{174} z_{572} \\z_{582} &= z_{385} + z_{183} z_{572} - z_{196} z_{574} \\z_{583} &= z_{391} + z_{185} z_{572} - z_{198} z_{574} \\z_{584} &= z_{396} + z_{186} z_{572} - z_{199} z_{574} \\z_{585} &= z_{371} + z_{184} z_{572} - z_{197} z_{574} \\z_{586} &= z_{373} + z_{182} z_{572} - z_{195} z_{574} \\z_{587} &= z_{372} + z_{181} z_{572} - z_{194} z_{574} \\z_{588} &= z_{573} - z_{177} z_{572} - z_{190} z_{574} \\z_{589} &= z_{575} + z_{572} z_{268} - z_{574} z_{269} \\z_{590} &= z_{182} z_{405} - z_{195} z_{406} \\z_{591} &= z_{182} z_{401} - z_{195} z_{403} \\z_{592} &= z_{182} z_{406} - z_{195} z_{408} \\z_{593} &= z_{399} + z_{182} z_{407} - z_{195} z_{409} \\z_{594} &= z_{189} z_{592} - z_{176} z_{590} \\z_{595} &= z_{192} z_{592} - z_{179} z_{590} \\z_{596} &= z_{193} z_{592} - z_{180} z_{590} \\z_{597} &= z_{191} z_{592} - z_{178} z_{590}\end{aligned}$$

4 Dynamic Equations of the Dual-Lift Model

$$\begin{aligned}z_{598} &= z_{188} z_{592} - z_{175} z_{590} \\z_{599} &= z_{187} z_{592} - z_{174} z_{590} \\z_{600} &= z_{382} + z_{183} z_{590} - z_{196} z_{592} \\z_{601} &= z_{389} + z_{185} z_{590} - z_{198} z_{592} \\z_{602} &= z_{395} + z_{186} z_{590} - z_{199} z_{592} \\z_{603} &= z_{373} + z_{184} z_{590} - z_{197} z_{592} \\z_{604} &= z_{375} + z_{182} z_{590} - z_{195} z_{592} \\z_{605} &= z_{376} + z_{181} z_{590} - z_{194} z_{592} \\z_{606} &= z_{591} - z_{177} z_{590} - z_{190} z_{592} \\z_{607} &= z_{593} + z_{590} z_{268} - z_{592} z_{269} \\z_{608} &= z_{181} z_{405} - z_{194} z_{406} \\z_{609} &= z_{181} z_{401} - z_{194} z_{403} \\z_{610} &= z_{181} z_{406} - z_{194} z_{408} \\z_{611} &= z_{400} + z_{181} z_{407} - z_{194} z_{409} \\z_{612} &= z_{189} z_{610} - z_{176} z_{608} \\z_{613} &= z_{192} z_{610} - z_{179} z_{608} \\z_{614} &= z_{193} z_{610} - z_{180} z_{608} \\z_{615} &= z_{191} z_{610} - z_{178} z_{608} \\z_{616} &= z_{188} z_{610} - z_{175} z_{608} \\z_{617} &= z_{187} z_{610} - z_{174} z_{608} \\z_{618} &= z_{380} + z_{183} z_{608} - z_{196} z_{610} \\z_{619} &= z_{387} + z_{185} z_{608} - z_{198} z_{610} \\z_{620} &= z_{393} + z_{186} z_{608} - z_{199} z_{610} \\z_{621} &= z_{372} + z_{184} z_{608} - z_{197} z_{610} \\z_{622} &= z_{376} + z_{182} z_{608} - z_{195} z_{610} \\z_{623} &= z_{378} + z_{181} z_{608} - z_{194} z_{610} \\z_{624} &= z_{609} - z_{177} z_{608} - z_{190} z_{610} \\z_{625} &= z_{611} + z_{608} z_{268} - z_{610} z_{269} \\z_{626} &= z_{401} - z_{177} z_{405} - z_{190} z_{406} \\z_{627} &= z_{402} - z_{177} z_{401} - z_{190} z_{403} \\z_{628} &= z_{403} - z_{177} z_{406} - z_{190} z_{408} \\z_{629} &= z_{404} - z_{177} z_{407} - z_{190} z_{409} \\z_{630} &= z_{189} z_{628} - z_{176} z_{626}\end{aligned}$$

4 Dynamic Equations of the Dual-Lift Model

$$\begin{aligned}z_{631} &= z_{192} z_{628} - z_{179} z_{626} \\z_{632} &= z_{193} z_{628} - z_{180} z_{626} \\z_{633} &= z_{191} z_{628} - z_{178} z_{626} \\z_{634} &= z_{188} z_{628} - z_{175} z_{626} \\z_{635} &= z_{187} z_{628} - z_{174} z_{626} \\z_{636} &= z_{183} z_{626} - z_{196} z_{628} \\z_{637} &= z_{185} z_{626} - z_{198} z_{628} \\z_{638} &= z_{186} z_{626} - z_{199} z_{628} \\z_{639} &= z_{184} z_{626} - z_{197} z_{628} \\z_{640} &= z_{182} z_{626} - z_{195} z_{628} \\z_{641} &= z_{181} z_{626} - z_{194} z_{628} \\z_{642} &= z_{627} - z_{177} z_{626} - z_{190} z_{628} \\z_{643} &= z_{629} + z_{626} z_{268} - z_{628} z_{269} \\z_{644} &= z_{309} - z_{427} \\z_{645} &= z_{310} - z_{445} \\z_{646} &= z_{311} - z_{463} \\z_{647} &= z_{314} - z_{481} \\z_{648} &= z_{316} - z_{499} \\z_{649} &= z_{317} - z_{517} \\z_{650} &= z_{318} - z_{535} \\z_{651} &= z_{319} - z_{553} \\z_{652} &= z_{320} - z_{571} \\z_{653} &= z_{321} - z_{589} \\z_{654} &= z_{322} - z_{607} \\z_{655} &= z_{323} - z_{625} \\z_{656} &= -z_{324} - z_{643}\end{aligned}$$

5 Dynamic Equations of the Multi-Lift Model

The following equations describe the dynamics of the multi-lift model, presented in the thesis main document [Ber13, pages 76ff.], for three coupled helicopters (multi-lift configuration). The variables z_{xyz} represent intermediate results, which are generated automatically.

$$\begin{aligned}
0 &= z_{897} - z_{504} \dot{u}_{h1} - z_{505} \dot{v}_{h1} - z_{506} \dot{w}_{h1} - z_{507} \dot{p}_{h1} - z_{508} \dot{q}_{h1} - z_{509} \dot{r}_{h1} \\
&\quad - z_{510} \dot{u}_{h2} - z_{511} \dot{v}_{h2} - z_{512} \dot{w}_{h2} - z_{513} \dot{p}_{h2} - z_{514} \dot{q}_{h2} - z_{515} \dot{r}_{h2} \\
&\quad - z_{516} \dot{u}_{h3} - z_{517} \dot{v}_{h3} - z_{518} \dot{w}_{h3} - z_{519} \dot{p}_{h3} - z_{520} \dot{q}_{h3} - z_{521} \dot{r}_{h3} \\
0 &= z_{898} - z_{526} \dot{u}_{h1} - z_{527} \dot{v}_{h1} - z_{528} \dot{w}_{h1} - z_{529} \dot{p}_{h1} - z_{530} \dot{q}_{h1} - z_{531} \dot{r}_{h1} \\
&\quad - z_{532} \dot{u}_{h2} - z_{533} \dot{v}_{h2} - z_{534} \dot{w}_{h2} - z_{535} \dot{p}_{h2} - z_{536} \dot{q}_{h2} - z_{537} \dot{r}_{h2} \\
&\quad - z_{538} \dot{u}_{h3} - z_{539} \dot{v}_{h3} - z_{540} \dot{w}_{h3} - z_{541} \dot{p}_{h3} - z_{542} \dot{q}_{h3} - z_{543} \dot{r}_{h3} \\
0 &= z_{899} - z_{548} \dot{u}_{h1} - z_{549} \dot{v}_{h1} - z_{550} \dot{w}_{h1} - z_{551} \dot{p}_{h1} - z_{552} \dot{q}_{h1} - z_{553} \dot{r}_{h1} \\
&\quad - z_{554} \dot{u}_{h2} - z_{555} \dot{v}_{h2} - z_{556} \dot{w}_{h2} - z_{557} \dot{p}_{h2} - z_{558} \dot{q}_{h2} - z_{559} \dot{r}_{h2} \\
&\quad - z_{560} \dot{u}_{h3} - z_{561} \dot{v}_{h3} - z_{562} \dot{w}_{h3} - z_{563} \dot{p}_{h3} - z_{564} \dot{q}_{h3} - z_{565} \dot{r}_{h3} \\
0 &= z_{900} - z_{570} \dot{u}_{h1} - z_{571} \dot{v}_{h1} - z_{572} \dot{w}_{h1} - z_{573} \dot{p}_{h1} - z_{574} \dot{q}_{h1} - z_{575} \dot{r}_{h1} \\
&\quad - z_{576} \dot{u}_{h2} - z_{577} \dot{v}_{h2} - z_{578} \dot{w}_{h2} - z_{579} \dot{p}_{h2} - z_{580} \dot{q}_{h2} - z_{581} \dot{r}_{h2} \\
&\quad - z_{582} \dot{u}_{h3} - z_{583} \dot{v}_{h3} - z_{584} \dot{w}_{h3} - z_{585} \dot{p}_{h3} - z_{586} \dot{q}_{h3} - z_{587} \dot{r}_{h3} \\
0 &= z_{901} - z_{592} \dot{u}_{h1} - z_{593} \dot{v}_{h1} - z_{594} \dot{w}_{h1} - z_{595} \dot{p}_{h1} - z_{596} \dot{q}_{h1} - z_{597} \dot{r}_{h1} \\
&\quad - z_{598} \dot{u}_{h2} - z_{599} \dot{v}_{h2} - z_{600} \dot{w}_{h2} - z_{601} \dot{p}_{h2} - z_{602} \dot{q}_{h2} - z_{603} \dot{r}_{h2} \\
&\quad - z_{604} \dot{u}_{h3} - z_{605} \dot{v}_{h3} - z_{606} \dot{w}_{h3} - z_{607} \dot{p}_{h3} - z_{608} \dot{q}_{h3} - z_{609} \dot{r}_{h3} \\
0 &= z_{902} - z_{614} \dot{u}_{h1} - z_{615} \dot{v}_{h1} - z_{616} \dot{w}_{h1} - z_{617} \dot{p}_{h1} - z_{618} \dot{q}_{h1} - z_{619} \dot{r}_{h1} \\
&\quad - z_{620} \dot{u}_{h2} - z_{621} \dot{v}_{h2} - z_{622} \dot{w}_{h2} - z_{623} \dot{p}_{h2} - z_{624} \dot{q}_{h2} - z_{625} \dot{r}_{h2} \\
&\quad - z_{626} \dot{u}_{h3} - z_{627} \dot{v}_{h3} - z_{628} \dot{w}_{h3} - z_{629} \dot{p}_{h3} - z_{630} \dot{q}_{h3} - z_{631} \dot{r}_{h3} \\
0 &= z_{903} - z_{636} \dot{u}_{h1} - z_{637} \dot{v}_{h1} - z_{638} \dot{w}_{h1} - z_{639} \dot{p}_{h1} - z_{640} \dot{q}_{h1} - z_{641} \dot{r}_{h1} \\
&\quad - z_{642} \dot{u}_{h2} - z_{643} \dot{v}_{h2} - z_{644} \dot{w}_{h2} - z_{645} \dot{p}_{h2} - z_{646} \dot{q}_{h2} - z_{647} \dot{r}_{h2} \\
&\quad - z_{648} \dot{u}_{h3} - z_{649} \dot{v}_{h3} - z_{650} \dot{w}_{h3} - z_{651} \dot{p}_{h3} - z_{652} \dot{q}_{h3} - z_{653} \dot{r}_{h3} \\
0 &= z_{904} - z_{658} \dot{u}_{h1} - z_{659} \dot{v}_{h1} - z_{660} \dot{w}_{h1} - z_{661} \dot{p}_{h1} - z_{662} \dot{q}_{h1} - z_{663} \dot{r}_{h1} \\
&\quad - z_{664} \dot{u}_{h2} - z_{665} \dot{v}_{h2} - z_{666} \dot{w}_{h2} - z_{667} \dot{p}_{h2} - z_{668} \dot{q}_{h2} - z_{669} \dot{r}_{h2} \\
&\quad - z_{670} \dot{u}_{h3} - z_{671} \dot{v}_{h3} - z_{672} \dot{w}_{h3} - z_{673} \dot{p}_{h3} - z_{674} \dot{q}_{h3} - z_{675} \dot{r}_{h3} \\
0 &= z_{905} - z_{680} \dot{u}_{h1} - z_{681} \dot{v}_{h1} - z_{682} \dot{w}_{h1} - z_{683} \dot{p}_{h1} - z_{684} \dot{q}_{h1} - z_{685} \dot{r}_{h1} \\
&\quad - z_{686} \dot{u}_{h2} - z_{687} \dot{v}_{h2} - z_{688} \dot{w}_{h2} - z_{689} \dot{p}_{h2} - z_{690} \dot{q}_{h2} - z_{691} \dot{r}_{h2} \\
&\quad - z_{692} \dot{u}_{h3} - z_{693} \dot{v}_{h3} - z_{694} \dot{w}_{h3} - z_{695} \dot{p}_{h3} - z_{696} \dot{q}_{h3} - z_{697} \dot{r}_{h3}
\end{aligned}$$

5 Dynamic Equations of the Multi-Lift Model

$$\begin{aligned}
0 &= z_{906} - z_{702} \dot{u}_{h1} - z_{703} \dot{v}_{h1} - z_{704} \dot{w}_{h1} - z_{705} \dot{p}_{h1} - z_{706} \dot{q}_{h1} - z_{707} \dot{r}_{h1} \\
&\quad - z_{708} \dot{u}_{h2} - z_{709} \dot{v}_{h2} - z_{710} \dot{w}_{h2} - z_{711} \dot{p}_{h2} - z_{712} \dot{q}_{h2} - z_{713} \dot{r}_{h2} \\
&\quad - z_{714} \dot{u}_{h3} - z_{715} \dot{v}_{h3} - z_{716} \dot{w}_{h3} - z_{717} \dot{p}_{h3} - z_{718} \dot{q}_{h3} - z_{719} \dot{r}_{h3} \\
0 &= z_{907} - z_{724} \dot{u}_{h1} - z_{725} \dot{v}_{h1} - z_{726} \dot{w}_{h1} - z_{727} \dot{p}_{h1} - z_{728} \dot{q}_{h1} - z_{729} \dot{r}_{h1} \\
&\quad - z_{730} \dot{u}_{h2} - z_{731} \dot{v}_{h2} - z_{732} \dot{w}_{h2} - z_{733} \dot{p}_{h2} - z_{734} \dot{q}_{h2} - z_{735} \dot{r}_{h2} \\
&\quad - z_{736} \dot{u}_{h3} - z_{737} \dot{v}_{h3} - z_{738} \dot{w}_{h3} - z_{739} \dot{p}_{h3} - z_{740} \dot{q}_{h3} - z_{741} \dot{r}_{h3} \\
0 &= z_{908} - z_{746} \dot{u}_{h1} - z_{747} \dot{v}_{h1} - z_{748} \dot{w}_{h1} - z_{749} \dot{p}_{h1} - z_{750} \dot{q}_{h1} - z_{751} \dot{r}_{h1} \\
&\quad - z_{752} \dot{u}_{h2} - z_{753} \dot{v}_{h2} - z_{754} \dot{w}_{h2} - z_{755} \dot{p}_{h2} - z_{756} \dot{q}_{h2} - z_{757} \dot{r}_{h2} \\
&\quad - z_{758} \dot{u}_{h3} - z_{759} \dot{v}_{h3} - z_{760} \dot{w}_{h3} - z_{761} \dot{p}_{h3} - z_{762} \dot{q}_{h3} - z_{763} \dot{r}_{h3} \\
0 &= z_{909} - z_{768} \dot{u}_{h1} - z_{769} \dot{v}_{h1} - z_{770} \dot{w}_{h1} - z_{771} \dot{p}_{h1} - z_{772} \dot{q}_{h1} - z_{773} \dot{r}_{h1} \\
&\quad - z_{774} \dot{u}_{h2} - z_{775} \dot{v}_{h2} - z_{776} \dot{w}_{h2} - z_{777} \dot{p}_{h2} - z_{778} \dot{q}_{h2} - z_{779} \dot{r}_{h2} \\
&\quad - z_{780} \dot{u}_{h3} - z_{781} \dot{v}_{h3} - z_{782} \dot{w}_{h3} - z_{783} \dot{p}_{h3} - z_{784} \dot{q}_{h3} - z_{785} \dot{r}_{h3} \\
0 &= z_{910} - z_{790} \dot{u}_{h1} - z_{791} \dot{v}_{h1} - z_{792} \dot{w}_{h1} - z_{793} \dot{p}_{h1} - z_{794} \dot{q}_{h1} - z_{795} \dot{r}_{h1} \\
&\quad - z_{796} \dot{u}_{h2} - z_{797} \dot{v}_{h2} - z_{798} \dot{w}_{h2} - z_{799} \dot{p}_{h2} - z_{800} \dot{q}_{h2} - z_{801} \dot{r}_{h2} \\
&\quad - z_{802} \dot{u}_{h3} - z_{803} \dot{v}_{h3} - z_{804} \dot{w}_{h3} - z_{805} \dot{p}_{h3} - z_{806} \dot{q}_{h3} - z_{807} \dot{r}_{h3} \\
0 &= z_{911} - z_{812} \dot{u}_{h1} - z_{813} \dot{v}_{h1} - z_{814} \dot{w}_{h1} - z_{815} \dot{p}_{h1} - z_{816} \dot{q}_{h1} - z_{817} \dot{r}_{h1} \\
&\quad - z_{818} \dot{u}_{h2} - z_{819} \dot{v}_{h2} - z_{820} \dot{w}_{h2} - z_{821} \dot{p}_{h2} - z_{822} \dot{q}_{h2} - z_{823} \dot{r}_{h2} \\
&\quad - z_{824} \dot{u}_{h3} - z_{825} \dot{v}_{h3} - z_{826} \dot{w}_{h3} - z_{827} \dot{p}_{h3} - z_{828} \dot{q}_{h3} - z_{829} \dot{r}_{h3} \\
0 &= z_{912} - z_{834} \dot{u}_{h1} - z_{835} \dot{v}_{h1} - z_{836} \dot{w}_{h1} - z_{837} \dot{p}_{h1} - z_{838} \dot{q}_{h1} - z_{839} \dot{r}_{h1} \\
&\quad - z_{840} \dot{u}_{h2} - z_{841} \dot{v}_{h2} - z_{842} \dot{w}_{h2} - z_{843} \dot{p}_{h2} - z_{844} \dot{q}_{h2} - z_{845} \dot{r}_{h2} \\
&\quad - z_{846} \dot{u}_{h3} - z_{847} \dot{v}_{h3} - z_{848} \dot{w}_{h3} - z_{849} \dot{p}_{h3} - z_{850} \dot{q}_{h3} - z_{851} \dot{r}_{h3} \\
0 &= z_{913} - z_{856} \dot{u}_{h1} - z_{857} \dot{v}_{h1} - z_{858} \dot{w}_{h1} - z_{859} \dot{p}_{h1} - z_{860} \dot{q}_{h1} - z_{861} \dot{r}_{h1} \\
&\quad - z_{862} \dot{u}_{h2} - z_{863} \dot{v}_{h2} - z_{864} \dot{w}_{h2} - z_{865} \dot{p}_{h2} - z_{866} \dot{q}_{h2} - z_{867} \dot{r}_{h2} \\
&\quad - z_{868} \dot{u}_{h3} - z_{869} \dot{v}_{h3} - z_{870} \dot{w}_{h3} - z_{871} \dot{p}_{h3} - z_{872} \dot{q}_{h3} - z_{873} \dot{r}_{h3} \\
0 &= z_{914} - z_{878} \dot{u}_{h1} - z_{879} \dot{v}_{h1} - z_{880} \dot{w}_{h1} - z_{881} \dot{p}_{h1} - z_{882} \dot{q}_{h1} - z_{883} \dot{r}_{h1} \\
&\quad - z_{884} \dot{u}_{h2} - z_{885} \dot{v}_{h2} - z_{886} \dot{w}_{h2} - z_{887} \dot{p}_{h2} - z_{888} \dot{q}_{h2} - z_{889} \dot{r}_{h2} \\
&\quad - z_{890} \dot{u}_{h3} - z_{891} \dot{v}_{h3} - z_{892} \dot{w}_{h3} - z_{893} \dot{p}_{h3} - z_{894} \dot{q}_{h3} - z_{895} \dot{r}_{h3}
\end{aligned}$$

The equations need to be solved for the desired variables $(\dot{u}_{hx}, \dot{v}_{hx}, \dot{w}_{hx}, \dot{p}_{hx}, \dot{q}_{hx}, \dot{r}_{hx})$, with $x \in \{1, 2, 3\}$, to get the explicit equations.

The following equations define the z_{xyz} variables:

$$\begin{aligned}
z_6 &= \sin(\varphi_{h1}) \\
z_{52} &= \tan(\theta_{h1}) \\
z_{53} &= z_6 z_{52} \\
z_8 &= \cos(\varphi_{h1}) \\
z_{54} &= z_8 z_{52}
\end{aligned}$$

5 Dynamic Equations of the Multi-Lift Model

$$\dot{\varphi}_{h1} = p_{h1} + z_{53} q_{h1} + z_{54} r_{h1}$$

$$\dot{\theta}_{h1} = z_8 q_{h1} - z_6 r_{h1}$$

$$z_{49} = \cos(\theta_{h1})$$

$$z_{50} = z_6 / z_{49}$$

$$z_{51} = z_8 / z_{49}$$

$$\dot{\psi}_{h1} = z_{50} q_{h1} + z_{51} r_{h1}$$

$$z_{22} = \sin(\varphi_{h2})$$

$$z_{58} = \tan(\theta_{h2})$$

$$z_{59} = z_{22} z_{58}$$

$$z_{24} = \cos(\varphi_{h2})$$

$$z_{60} = z_{24} z_{58}$$

$$\dot{\varphi}_{h2} = p_{h2} + z_{59} q_{h2} + z_{60} r_{h2}$$

$$\dot{\theta}_{h2} = z_{24} q_{h2} - z_{22} r_{h2}$$

$$z_{55} = \cos(\theta_{h2})$$

$$z_{56} = z_{22} / z_{55}$$

$$z_{57} = z_{24} / z_{55}$$

$$\dot{\psi}_{h2} = z_{56} q_{h2} + z_{57} r_{h2}$$

$$z_{38} = \sin(\varphi_{h3})$$

$$z_{64} = \tan(\theta_{h3})$$

$$z_{65} = z_{38} z_{64}$$

$$z_{40} = \cos(\varphi_{h3})$$

$$z_{66} = z_{40} z_{64}$$

$$\dot{\varphi}_{h3} = p_{h3} + z_{65} q_{h3} + z_{66} r_{h3}$$

$$\dot{\theta}_{h3} = z_{40} q_{h3} - z_{38} r_{h3}$$

$$z_{61} = \cos(\theta_{h3})$$

$$z_{62} = z_{38} / z_{61}$$

$$z_{63} = z_{40} / z_{61}$$

$$\dot{\psi}_{h3} = z_{62} q_{h3} + z_{63} r_{h3}$$

$$z_{147} = z_{h2} - z_l$$

5 Dynamic Equations of the Multi-Lift Model

$$\begin{aligned}
z_{76} &= m_F + m_{MR} \\
z_{78} &= (dOFo_2m_F + dOMRo_2m_{MR})/z_{76} \\
z_{90} &= dORo_2 - z_{78} \\
z_{18} &= \pi + \theta_{h2} \\
z_{20} &= \cos(z_{18}) \\
z_{31} &= z_{20} z_{22} \\
z_{79} &= (dFO_3m_F + dOMRo_3m_{MR})/z_{76} \\
z_{91} &= dORo_3 - z_{79} \\
z_{32} &= z_{20} z_{24} \\
z_{77} &= (dFO_1m_F + dOMRo_1m_{MR})/z_{76} \\
z_{89} &= dORo_1 - z_{77} \\
z_{23} &= \sin(z_{18}) \\
z_{183} &= 2z_{147} + 2z_{90} z_{31} + 2z_{91} z_{32} - 2z_{89} z_{23} \\
z_{132} &= y_{h1} - y_l \\
z_2 &= \pi + \theta_{h1} \\
z_4 &= \cos(z_2) \\
z_1 &= \pi + \psi_{h1} \\
z_9 &= \sin(z_1) \\
z_{12} &= z_4 z_9 \\
z_3 &= \cos(z_1) \\
z_7 &= \sin(z_2) \\
z_{13} &= z_3 z_8 + z_6 z_7 z_9 \\
z_{14} &= z_7 z_8 z_9 - z_3 z_6 \\
z_{176} &= -2z_{132} - 2z_{89} z_{12} - 2z_{90} z_{13} - 2z_{91} z_{14} \\
z_{34} &= \pi + \theta_{h3} \\
z_{39} &= \sin(z_{34}) \\
z_{161} &= z_{h3} - z_l \\
z_{36} &= \cos(z_{34}) \\
z_{47} &= z_{36} z_{38} \\
z_{48} &= z_{36} z_{40} \\
z_{196} &= 2z_{89} z_{39} - 2z_{161} - 2z_{90} z_{47} - 2z_{91} z_{48}
\end{aligned}$$

5 Dynamic Equations of the Multi-Lift Model

$$\begin{aligned}
 z_{133} &= z_{h1} - z_l \\
 z_{15} &= z_4 z_6 \\
 z_{16} &= z_4 z_8 \\
 z_{177} &= 2z_{89} z_7 - 2z_{133} - 2z_{90} z_{15} - 2z_{91} z_{16} \\
 z_{160} &= y_{h3} - y_l \\
 z_{33} &= \pi + \psi_{h3} \\
 z_{41} &= \sin(z_{33}) \\
 z_{44} &= z_{36} z_{41} \\
 z_{35} &= \cos(z_{33}) \\
 z_{45} &= z_{35} z_{40} + z_{38} z_{39} z_{41} \\
 z_{46} &= z_{39} z_{40} z_{41} - z_{35} z_{38} \\
 z_{194} &= -2z_{160} - 2z_{89} z_{44} - 2z_{90} z_{45} - 2z_{91} z_{46} \\
 z_{207} &= z_{176} z_{196} - z_{177} z_{194} \\
 z_{159} &= x_{h3} - x_l \\
 z_{37} &= z_{35} z_{36} \\
 z_{42} &= z_{35} z_{38} z_{39} - z_{40} z_{41} \\
 z_{43} &= z_{38} z_{41} + z_{35} z_{39} z_{40} \\
 z_{192} &= -2z_{159} - 2z_{89} z_{37} - 2z_{90} z_{42} - 2z_{91} z_{43} \\
 z_{182} &= 2z_{89} z_{23} - 2z_{147} - 2z_{90} z_{31} - 2z_{91} z_{32} \\
 z_{146} &= y_{h2} - y_l \\
 z_{17} &= \pi + \psi_{h2} \\
 z_{25} &= \sin(z_{17}) \\
 z_{28} &= z_{20} z_{25} \\
 z_{19} &= \cos(z_{17}) \\
 z_{29} &= z_{19} z_{24} + z_{22} z_{23} z_{25} \\
 z_{30} &= z_{23} z_{24} z_{25} - z_{19} z_{22} \\
 z_{189} &= -2z_{146} - 2z_{89} z_{28} - 2z_{90} z_{29} - 2z_{91} z_{30} \\
 z_{202} &= z_{176} z_{182} - z_{177} z_{189} \\
 z_{131} &= x_{h1} - x_l \\
 z_5 &= z_3 z_4 \\
 z_{10} &= z_3 z_6 z_7 - z_8 z_9 \\
 z_{11} &= z_6 z_9 + z_3 z_7 z_8
 \end{aligned}$$

5 Dynamic Equations of the Multi-Lift Model

$$\begin{aligned}
z_{174} &= -2z_{131} - 2z_{89} z_5 - 2z_{90} z_{10} - 2z_{91} z_{11} \\
z_{145} &= x_{h2} - x_l \\
z_{21} &= z_{19} z_{20} \\
z_{26} &= z_{19} z_{22} z_{23} - z_{24} z_{25} \\
z_{27} &= z_{22} z_{25} + z_{19} z_{23} z_{24} \\
z_{187} &= -2z_{145} - 2z_{89} z_{21} - 2z_{90} z_{26} - 2z_{91} z_{27} \\
z_{200} &= z_{174} z_{189} - z_{176} z_{187} \\
z_{201} &= z_{174} z_{182} - z_{177} z_{187} \\
z_{203} &= z_{192} z_{202} + z_{196} z_{200} - z_{194} z_{201} \\
z_{216} &= z_{183} z_{207} / z_{203} \\
z_{154} &= z_{23} z_{25} \\
z_{157} &= z_{22} z_{23} \\
z_{148} &= z_{19} z_{23} \\
z_{158} &= z_{23} z_{24} \\
z_{149} &= z_{19} z_{20} z_{22} \\
z_{150} &= -z_{19} z_{24} - z_{22} z_{23} z_{25} \\
z_{155} &= z_{20} z_{22} z_{25} \\
z_{153} &= z_{19} z_{20} z_{24} \\
z_{152} &= z_{19} z_{22} - z_{23} z_{24} z_{25} \\
z_{151} &= z_{24} z_{25} - z_{19} z_{22} z_{23} \\
z_{156} &= z_{20} z_{24} z_{25} \\
z_{184} &= 2z_{89} z_{20} z_{22} z_{147} + 2z_{89} z_{146} (z_{21} z_{57} + z_{22} z_{154}) + 2z_{90} z_{147} (z_{22} z_{157} + z_{32} z_{60}) \\
&\quad + 2z_{89} z_{145} (z_{22} z_{148} - z_{28} z_{57}) + 2z_{91} z_{147} (z_{22} z_{158} - z_{31} z_{60}) \\
&\quad - 2z_{90} z_{145} (z_{22} z_{149} - z_{27} z_{60} - z_{57} z_{150}) - 2z_{90} z_{146} (z_{22} z_{155} - z_{26} z_{57} - z_{30} z_{60}) \\
&\quad - 2z_{91} z_{145} (z_{22} z_{153} - z_{57} z_{152} - z_{60} z_{151}) - 2z_{91} z_{146} (z_{22} z_{156} - z_{27} z_{57} - z_{60} z_{150}) \\
z_{217} &= z_{184} z_{207} / z_{203} \\
z_{185} &= 2z_{89} z_{146} (z_{21} z_{56} - z_{24} z_{154}) + 2z_{90} z_{145} (z_{24} z_{149} + z_{27} z_{59} + z_{56} z_{150}) \\
&\quad + 2z_{90} z_{146} (z_{24} z_{155} + z_{26} z_{56} + z_{30} z_{59}) + 2z_{91} z_{145} (z_{24} z_{153} + z_{56} z_{152} + z_{59} z_{151}) \\
&\quad + 2z_{91} z_{146} (z_{24} z_{156} + z_{27} z_{56} + z_{59} z_{150}) \\
&\quad - 2z_{89} z_{20} z_{24} z_{147} - 2z_{89} z_{145} (z_{24} z_{148} + z_{28} z_{56}) \\
&\quad - 2z_{91} z_{147} (z_{24} z_{158} + z_{31} z_{59}) - 2z_{90} z_{147} (z_{24} z_{157} - z_{32} z_{59}) \\
z_{218} &= z_{185} z_{207} / z_{203} \\
z_{186} &= 2z_{90} z_{27} z_{145} + 2z_{90} z_{30} z_{146} + 2z_{90} z_{32} z_{147} \\
&\quad + 2z_{91} z_{145} z_{151} + 2z_{91} z_{146} z_{150} - 2z_{91} z_{31} z_{147} \\
z_{219} &= z_{186} z_{207} / z_{203}
\end{aligned}$$

5 Dynamic Equations of the Multi-Lift Model

$$\begin{aligned}
z_{188} &= 2z_{145} + 2z_{89} z_{21} + 2z_{90} z_{26} + 2z_{91} z_{27} \\
z_{220} &= z_{188} z_{207} / z_{203} \\
z_{190} &= 2z_{146} + 2z_{89} z_{28} + 2z_{90} z_{29} + 2z_{91} z_{30} \\
z_{221} &= z_{190} z_{207} / z_{203} \\
z_{173} &= 2z_{131} + 2z_{89} z_5 + 2z_{90} z_{10} + 2z_{91} z_{11} \\
z_{204} &= z_{189} z_{196} - z_{182} z_{194} \\
z_{210} &= z_{173} z_{204} / z_{203} \\
z_{175} &= 2z_{132} + 2z_{89} z_{12} + 2z_{90} z_{13} + 2z_{91} z_{14} \\
z_{211} &= z_{175} z_{204} / z_{203} \\
z_{178} &= 2z_{133} + 2z_{90} z_{15} + 2z_{91} z_{16} - 2z_{89} z_7 \\
z_{212} &= z_{178} z_{204} / z_{203} \\
z_{140} &= z_7 z_9 \\
z_{143} &= z_6 z_7 \\
z_{134} &= z_3 z_7 \\
z_{144} &= z_7 z_8 \\
z_{135} &= z_3 z_4 z_6 \\
z_{136} &= -z_3 z_8 - z_6 z_7 z_9 \\
z_{141} &= z_4 z_6 z_9 \\
z_{139} &= z_3 z_4 z_8 \\
z_{138} &= z_3 z_6 - z_7 z_8 z_9 \\
z_{137} &= z_8 z_9 - z_3 z_6 z_7 \\
z_{142} &= z_4 z_8 z_9 \\
z_{179} &= 2z_{89} z_4 z_6 z_{133} + 2z_{89} z_{132} (z_5 z_{51} + z_6 z_{140}) + 2z_{90} z_{133} (z_6 z_{143} + z_{16} z_{54}) \\
&\quad + 2z_{89} z_{131} (z_6 z_{134} - z_{12} z_{51}) + 2z_{91} z_{133} (z_6 z_{144} - z_{15} z_{54}) \\
&\quad - 2z_{90} z_{131} (z_6 z_{135} - z_{11} z_{54} - z_{51} z_{136}) - 2z_{90} z_{132} (z_6 z_{141} - z_{10} z_{51} - z_{14} z_{54}) \\
&\quad - 2z_{91} z_{131} (z_6 z_{139} - z_{51} z_{138} - z_{54} z_{137}) - 2z_{91} z_{132} (z_6 z_{142} - z_{11} z_{51} - z_{54} z_{136}) \\
z_{213} &= z_{179} z_{204} / z_{203} \\
z_{180} &= 2z_{89} z_{132} (z_5 z_{50} - z_8 z_{140}) + 2z_{90} z_{131} (z_8 z_{135} + z_{11} z_{53} + z_{50} z_{136}) \\
&\quad + 2z_{90} z_{132} (z_8 z_{141} + z_{10} z_{50} + z_{14} z_{53}) + 2z_{91} z_{131} (z_8 z_{139} + z_{50} z_{138} + z_{53} z_{137}) \\
&\quad + 2z_{91} z_{132} (z_8 z_{142} + z_{11} z_{50} + z_{53} z_{136}) \\
&\quad - 2z_{89} z_4 z_8 z_{133} - 2z_{89} z_{131} (z_8 z_{134} + z_{12} z_{50}) \\
&\quad - 2z_{91} z_{133} (z_8 z_{144} + z_{15} z_{53}) - 2z_{90} z_{133} (z_8 z_{143} - z_{16} z_{53}) \\
z_{214} &= z_{180} z_{204} / z_{203}
\end{aligned}$$

5 Dynamic Equations of the Multi-Lift Model

$$\begin{aligned}
z_{181} &= 2z_{90} z_{11} z_{131} + 2z_{90} z_{14} z_{132} + 2z_{90} z_{16} z_{133} \\
&\quad + 2z_{91} z_{131} z_{137} + 2z_{91} z_{132} z_{136} - 2z_{91} z_{15} z_{133} \\
z_{215} &= z_{181} z_{204} / z_{203} \\
z_{191} &= 2z_{159} + 2z_{89} z_{37} + 2z_{90} z_{42} + 2z_{91} z_{43} \\
z_{222} &= z_{191} z_{202} / z_{203} \\
z_{193} &= 2z_{160} + 2z_{89} z_{44} + 2z_{90} z_{45} + 2z_{91} z_{46} \\
z_{223} &= z_{193} z_{202} / z_{203} \\
z_{195} &= 2z_{161} + 2z_{90} z_{47} + 2z_{91} z_{48} - 2z_{89} z_{39} \\
z_{224} &= z_{195} z_{202} / z_{203} \\
z_{168} &= z_{39} z_{41} \\
z_{171} &= z_{38} z_{39} \\
z_{162} &= z_{35} z_{39} \\
z_{172} &= z_{39} z_{40} \\
z_{163} &= z_{35} z_{36} z_{38} \\
z_{164} &= -z_{35} z_{40} - z_{38} z_{39} z_{41} \\
z_{169} &= z_{36} z_{38} z_{41} \\
z_{167} &= z_{35} z_{36} z_{40} \\
z_{166} &= z_{35} z_{38} - z_{39} z_{40} z_{41} \\
z_{165} &= z_{40} z_{41} - z_{35} z_{38} z_{39} \\
z_{170} &= z_{36} z_{40} z_{41} \\
z_{197} &= 2z_{89} z_{36} z_{38} z_{161} + 2z_{89} z_{160} (z_{37} z_{63} + z_{38} z_{168}) + 2z_{90} z_{161} (z_{38} z_{171} + z_{48} z_{66}) \\
&\quad + 2z_{89} z_{159} (z_{38} z_{162} - z_{44} z_{63}) + 2z_{91} z_{161} (z_{38} z_{172} - z_{47} z_{66}) \\
&\quad - 2z_{90} z_{159} (z_{38} z_{163} - z_{43} z_{66} - z_{63} z_{164}) - 2z_{90} z_{160} (z_{38} z_{169} - z_{42} z_{63} - z_{46} z_{66}) \\
&\quad - 2z_{91} z_{159} (z_{38} z_{167} - z_{63} z_{166} - z_{66} z_{165}) - 2z_{91} z_{160} (z_{38} z_{170} - z_{43} z_{63} - z_{66} z_{164}) \\
z_{225} &= z_{197} z_{202} / z_{203} \\
z_{198} &= 2z_{89} z_{160} (z_{37} z_{62} - z_{40} z_{168}) + 2z_{90} z_{159} (z_{40} z_{163} + z_{43} z_{65} + z_{62} z_{164}) \\
&\quad + 2z_{90} z_{160} (z_{40} z_{169} + z_{42} z_{62} + z_{46} z_{65}) + 2z_{91} z_{159} (z_{40} z_{167} + z_{62} z_{166} + z_{65} z_{165}) \\
&\quad + 2z_{91} z_{160} (z_{40} z_{170} + z_{43} z_{62} + z_{65} z_{164}) \\
&\quad - 2z_{89} z_{36} z_{40} z_{161} - 2z_{89} z_{159} (z_{40} z_{162} + z_{44} z_{62}) \\
&\quad - 2z_{91} z_{161} (z_{40} z_{172} + z_{47} z_{65}) - 2z_{90} z_{161} (z_{40} z_{171} - z_{48} z_{65}) \\
z_{226} &= z_{198} z_{202} / z_{203} \\
z_{199} &= 2z_{90} z_{43} z_{159} + 2z_{90} z_{46} z_{160} + 2z_{90} z_{48} z_{161} \\
&\quad + 2z_{91} z_{159} z_{165} + 2z_{91} z_{160} z_{164} - 2z_{91} z_{47} z_{161} \\
z_{227} &= z_{199} z_{202} / z_{203}
\end{aligned}$$

5 Dynamic Equations of the Multi-Lift Model

$$\begin{aligned}
 u_l &= z_{216} w_{h2} + z_{217} r_{h2} + z_{218} q_{h2} + z_{219} p_{h2} + z_{220} u_{h2} + z_{221} v_{h2} \\
 &\quad - z_{210} u_{h1} - z_{211} v_{h1} - z_{212} w_{h1} - z_{213} r_{h1} - z_{214} q_{h1} - z_{215} p_{h1} \\
 &\quad - z_{222} u_{h3} - z_{223} v_{h3} - z_{224} w_{h3} - z_{225} r_{h3} - z_{226} q_{h3} - z_{227} p_{h3} \\
 z_{205} &= z_{187} z_{196} - z_{182} z_{192} \\
 z_{228} &= z_{173} z_{205} / z_{203} \\
 z_{229} &= z_{175} z_{205} / z_{203} \\
 z_{230} &= z_{178} z_{205} / z_{203} \\
 z_{231} &= z_{179} z_{205} / z_{203} \\
 z_{232} &= z_{180} z_{205} / z_{203} \\
 z_{233} &= z_{181} z_{205} / z_{203} \\
 z_{240} &= z_{191} z_{201} / z_{203} \\
 z_{241} &= z_{193} z_{201} / z_{203} \\
 z_{242} &= z_{195} z_{201} / z_{203} \\
 z_{243} &= z_{197} z_{201} / z_{203} \\
 z_{244} &= z_{198} z_{201} / z_{203} \\
 z_{245} &= z_{199} z_{201} / z_{203} \\
 z_{208} &= z_{174} z_{196} - z_{177} z_{192} \\
 z_{234} &= z_{183} z_{208} / z_{203} \\
 z_{235} &= z_{184} z_{208} / z_{203} \\
 z_{236} &= z_{185} z_{208} / z_{203} \\
 z_{237} &= z_{186} z_{208} / z_{203} \\
 z_{238} &= z_{188} z_{208} / z_{203} \\
 z_{239} &= z_{190} z_{208} / z_{203} \\
 v_l &= z_{228} u_{h1} + z_{229} v_{h1} + z_{230} w_{h1} + z_{231} r_{h1} + z_{232} q_{h1} + z_{233} p_{h1} \\
 &\quad + z_{240} u_{h3} + z_{241} v_{h3} + z_{242} w_{h3} + z_{243} r_{h3} + z_{244} q_{h3} + z_{245} p_{h3} \\
 &\quad - z_{234} w_{h2} - z_{235} r_{h2} - z_{236} q_{h2} - z_{237} p_{h2} - z_{238} u_{h2} - z_{239} v_{h2} \\
 z_{209} &= z_{174} z_{194} - z_{176} z_{192} \\
 z_{252} &= z_{183} z_{209} / z_{203} \\
 z_{253} &= z_{184} z_{209} / z_{203} \\
 z_{254} &= z_{185} z_{209} / z_{203}
 \end{aligned}$$

5 Dynamic Equations of the Multi-Lift Model

$$z_{255} = z_{186} z_{209} / z_{203}$$

$$z_{256} = z_{188} z_{209} / z_{203}$$

$$z_{257} = z_{190} z_{209} / z_{203}$$

$$z_{206} = z_{187} z_{194} - z_{189} z_{192}$$

$$z_{246} = z_{173} z_{206} / z_{203}$$

$$z_{247} = z_{175} z_{206} / z_{203}$$

$$z_{248} = z_{178} z_{206} / z_{203}$$

$$z_{249} = z_{179} z_{206} / z_{203}$$

$$z_{250} = z_{180} z_{206} / z_{203}$$

$$z_{251} = z_{181} z_{206} / z_{203}$$

$$z_{258} = z_{191} z_{200} / z_{203}$$

$$z_{259} = z_{193} z_{200} / z_{203}$$

$$z_{260} = z_{195} z_{200} / z_{203}$$

$$z_{261} = z_{197} z_{200} / z_{203}$$

$$z_{262} = z_{198} z_{200} / z_{203}$$

$$z_{263} = z_{199} z_{200} / z_{203}$$

$$\begin{aligned} w_l = & z_{252} w_{h2} + z_{253} r_{h2} + z_{254} q_{h2} + z_{255} p_{h2} + z_{256} u_{h2} + z_{257} v_{h2} \\ & - z_{246} u_{h1} - z_{247} v_{h1} - z_{248} w_{h1} - z_{249} r_{h1} - z_{250} q_{h1} - z_{251} p_{h1} \\ & - z_{258} u_{h3} - z_{259} v_{h3} - z_{260} w_{h3} - z_{261} r_{h3} - z_{262} q_{h3} - z_{263} p_{h3} \end{aligned}$$

$$z_{67} = \omega_{MR} + r_{h1}$$

$$z_{68} = q_{h1} (r_{h1} - z_{67})$$

$$z_{69} = p_{h1} (r_{h1} - z_{67})$$

$$z_{70} = \omega_{MR} + r_{h2}$$

$$z_{71} = q_{h2} (r_{h2} - z_{70})$$

$$z_{72} = p_{h2} (r_{h2} - z_{70})$$

$$z_{73} = \omega_{MR} + r_{h3}$$

$$z_{74} = q_{h3} (r_{h3} - z_{73})$$

$$z_{75} = p_{h3} (r_{h3} - z_{73})$$

$$z_{80} = dOMRo_1 - z_{77}$$

$$z_{81} = dOMRo_2 - z_{78}$$

5 Dynamic Equations of the Multi-Lift Model

$$\begin{aligned}
z_{82} &= dOMRo_3 - z_{79} \\
z_{83} &= dOFo_1 - z_{77} \\
z_{84} &= dOFo_2 - z_{78} \\
z_{85} &= dOFo_3 - z_{79} \\
z_{86} &= dOT_1 - z_{77} \\
z_{88} &= dOT_3 - z_{79} \\
z_{92} &= -q_{h1} (z_{80} q_{h1} - z_{81} p_{h1}) - r_{h1} (z_{80} r_{h1} - z_{82} p_{h1}) \\
z_{93} &= p_{h1} (z_{80} q_{h1} - z_{81} p_{h1}) - r_{h1} (z_{81} r_{h1} - z_{82} q_{h1}) \\
z_{94} &= p_{h1} (z_{80} r_{h1} - z_{82} p_{h1}) + q_{h1} (z_{81} r_{h1} - z_{82} q_{h1}) \\
z_{95} &= -q_{h1} (z_{83} q_{h1} - z_{84} p_{h1}) - r_{h1} (z_{83} r_{h1} - z_{85} p_{h1}) \\
z_{96} &= p_{h1} (z_{83} q_{h1} - z_{84} p_{h1}) - r_{h1} (z_{84} r_{h1} - z_{85} q_{h1}) \\
z_{97} &= p_{h1} (z_{83} r_{h1} - z_{85} p_{h1}) + q_{h1} (z_{84} r_{h1} - z_{85} q_{h1}) \\
z_{104} &= -q_{h2} (z_{80} q_{h2} - z_{81} p_{h2}) - r_{h2} (z_{80} r_{h2} - z_{82} p_{h2}) \\
z_{105} &= p_{h2} (z_{80} q_{h2} - z_{81} p_{h2}) - r_{h2} (z_{81} r_{h2} - z_{82} q_{h2}) \\
z_{106} &= p_{h2} (z_{80} r_{h2} - z_{82} p_{h2}) + q_{h2} (z_{81} r_{h2} - z_{82} q_{h2}) \\
z_{107} &= -q_{h2} (z_{83} q_{h2} - z_{84} p_{h2}) - r_{h2} (z_{83} r_{h2} - z_{85} p_{h2}) \\
z_{108} &= p_{h2} (z_{83} q_{h2} - z_{84} p_{h2}) - r_{h2} (z_{84} r_{h2} - z_{85} q_{h2}) \\
z_{109} &= p_{h2} (z_{83} r_{h2} - z_{85} p_{h2}) + q_{h2} (z_{84} r_{h2} - z_{85} q_{h2}) \\
z_{116} &= -q_{h3} (z_{80} q_{h3} - z_{81} p_{h3}) - r_{h3} (z_{80} r_{h3} - z_{82} p_{h3}) \\
z_{117} &= p_{h3} (z_{80} q_{h3} - z_{81} p_{h3}) - r_{h3} (z_{81} r_{h3} - z_{82} q_{h3}) \\
z_{118} &= p_{h3} (z_{80} r_{h3} - z_{82} p_{h3}) + q_{h3} (z_{81} r_{h3} - z_{82} q_{h3}) \\
z_{119} &= -q_{h3} (z_{83} q_{h3} - z_{84} p_{h3}) - r_{h3} (z_{83} r_{h3} - z_{85} p_{h3}) \\
z_{120} &= p_{h3} (z_{83} q_{h3} - z_{84} p_{h3}) - r_{h3} (z_{84} r_{h3} - z_{85} q_{h3}) \\
z_{121} &= p_{h3} (z_{83} r_{h3} - z_{85} p_{h3}) + q_{h3} (z_{84} r_{h3} - z_{85} q_{h3}) \\
z_{128} &= g m_F \\
z_{129} &= g m_{MR} \\
z_{130} &= g l_\eta \\
z_{264} &= -z_3 z_7 \dot{\theta}_{h1} - z_4 z_9 \dot{\psi}_{h1} \\
z_{265} &= z_6 z_9 \dot{\varphi}_{h1} + z_3 z_4 z_6 \dot{\theta}_{h1} + z_3 z_7 z_8 \dot{\varphi}_{h1} - z_3 z_8 \dot{\psi}_{h1} - z_6 z_7 z_9 \dot{\psi}_{h1}
\end{aligned}$$

5 Dynamic Equations of the Multi-Lift Model

$$\begin{aligned}
z_{266} &= z_3 z_6 \dot{\psi}_{h1} + z_8 z_9 \dot{\phi}_{h1} + z_3 z_4 z_8 \dot{\theta}_{h1} - z_3 z_6 z_7 \dot{\phi}_{h1} - z_7 z_8 z_9 \dot{\psi}_{h1} \\
z_{267} &= 2u_{h1} + 2z_{89} z_{264} + 2z_{90} z_{265} + 2z_{91} z_{266} - 2u_l \\
z_{268} &= 2u_l - 2u_{h1} - 2z_{89} z_{264} - 2z_{90} z_{265} - 2z_{91} z_{266} \\
z_{269} &= z_3 z_4 \dot{\psi}_{h1} - z_7 z_9 \dot{\theta}_{h1} \\
z_{270} &= z_3 z_6 z_7 \dot{\psi}_{h1} + z_4 z_6 z_9 \dot{\theta}_{h1} + z_7 z_8 z_9 \dot{\phi}_{h1} - z_3 z_6 \dot{\phi}_{h1} - z_8 z_9 \dot{\psi}_{h1} \\
z_{271} &= z_6 z_9 \dot{\psi}_{h1} + z_3 z_7 z_8 \dot{\psi}_{h1} + z_4 z_8 z_9 \dot{\theta}_{h1} - z_3 z_8 \dot{\phi}_{h1} - z_6 z_7 z_9 \dot{\phi}_{h1} \\
z_{272} &= 2v_{h1} + 2z_{89} z_{269} + 2z_{90} z_{270} + 2z_{91} z_{271} - 2v_l \\
z_{273} &= 2v_l - 2v_{h1} - 2z_{89} z_{269} - 2z_{90} z_{270} - 2z_{91} z_{271} \\
z_{274} &= z_4 z_8 \dot{\phi}_{h1} - z_6 z_7 \dot{\theta}_{h1} \\
z_{275} &= -z_4 z_6 \dot{\phi}_{h1} - z_7 z_8 \dot{\theta}_{h1} \\
z_{276} &= 2w_l + 2z_{89} z_4 \dot{\theta}_{h1} - 2w_{h1} - 2z_{90} z_{274} - 2z_{91} z_{275} \\
z_{277} &= 2w_{h1} + 2z_{90} z_{274} + 2z_{91} z_{275} - 2w_l - 2z_{89} z_4 \dot{\theta}_{h1} \\
z_{278} &= \sin(\theta_{h1}) \\
z_{279} &= (z_6 z_{49} \dot{\phi}_{h1} - z_8 z_{278} \dot{\theta}_{h1})/z_{49}^2 \\
z_{280} &= z_3 z_7 \dot{\psi}_{h1} + z_4 z_9 \dot{\theta}_{h1} \\
z_{281} &= z_4 z_6 \dot{\theta}_{h1} + z_7 z_8 \dot{\phi}_{h1} \\
z_{282} &= z_8 \dot{\theta}_{h1}/z_{49}^2 - z_6 z_{52} \dot{\phi}_{h1} \\
z_{283} &= z_3 z_4 \dot{\theta}_{h1} - z_7 z_9 \dot{\psi}_{h1} \\
z_{284} &= z_4 z_8 \dot{\theta}_{h1} - z_6 z_7 \dot{\phi}_{h1} \\
z_{285} &= z_3 z_4 z_8 \dot{\phi}_{h1} - z_3 z_6 z_7 \dot{\theta}_{h1} - z_4 z_6 z_9 \dot{\psi}_{h1} \\
z_{286} &= z_3 z_6 \dot{\phi}_{h1} + z_8 z_9 \dot{\psi}_{h1} - z_3 z_6 z_7 \dot{\psi}_{h1} - z_4 z_6 z_9 \dot{\theta}_{h1} - z_7 z_8 z_9 \dot{\phi}_{h1} \\
z_{287} &= z_3 z_4 z_6 \dot{\psi}_{h1} + z_4 z_8 z_9 \dot{\phi}_{h1} - z_6 z_7 z_9 \dot{\theta}_{h1} \\
z_{288} &= -z_3 z_4 z_6 \dot{\phi}_{h1} - z_3 z_7 z_8 \dot{\theta}_{h1} - z_4 z_8 z_9 \dot{\psi}_{h1} \\
z_{289} &= z_3 z_8 \dot{\phi}_{h1} + z_6 z_7 z_9 \dot{\phi}_{h1} - z_6 z_9 \dot{\psi}_{h1} - z_3 z_7 z_8 \dot{\psi}_{h1} - z_4 z_8 z_9 \dot{\theta}_{h1} \\
z_{290} &= z_3 z_8 \dot{\psi}_{h1} + z_6 z_7 z_9 \dot{\psi}_{h1} - z_6 z_9 \dot{\phi}_{h1} - z_3 z_4 z_6 \dot{\theta}_{h1} - z_3 z_7 z_8 \dot{\phi}_{h1} \\
z_{291} &= z_3 z_4 z_8 \dot{\psi}_{h1} - z_4 z_6 z_9 \dot{\phi}_{h1} - z_7 z_8 z_9 \dot{\theta}_{h1}
\end{aligned}$$

5 Dynamic Equations of the Multi-Lift Model

$$\begin{aligned}
z_{292} = & 2z_{89} (z_5 z_{51} + z_6 z_{140})(v_{h1} - v_l) + 2z_{89} (z_6 z_{134} - z_{12} z_{51})(u_{h1} - u_l) \\
& + 2z_{89} z_4 z_8 z_{133} \dot{\varphi}_{h1} + 2z_{90} z_{133} (z_8 z_{143} \dot{\varphi}_{h1} + z_6 z_{281} + z_{16} z_{282} + z_{54} z_{275}) \\
& + 2z_{89} z_{131} (z_8 z_{134} \dot{\varphi}_{h1} + z_6 z_{283} + z_{12} z_{279} - z_{51} z_{269}) \\
& + 2z_{91} z_{133} (z_8 z_{144} \dot{\varphi}_{h1} + z_6 z_{284} - z_{15} z_{282} - z_{54} z_{274}) \\
& + 2z_{90} z_{131} (z_{11} z_{282} + z_{51} z_{286} + z_{54} z_{266} - z_8 z_{135} \dot{\varphi}_{h1} - z_6 z_{285} - z_{136} z_{279}) \\
& + 2z_{91} z_{131} (z_{51} z_{289} + z_{54} z_{290} + z_{137} z_{282} - z_8 z_{139} \dot{\varphi}_{h1} - z_6 z_{288} - z_{138} z_{279}) \\
& - 2z_{89} z_4 z_6 (w_l - w_{h1}) - 2z_{90} (z_6 z_{143} + z_{16} z_{54})(w_l - w_{h1}) \\
& - 2z_{91} (z_6 z_{144} - z_{15} z_{54})(w_l - w_{h1}) - 2z_{90} (z_6 z_{135} - z_{11} z_{54} - z_{51} z_{136})(u_{h1} - u_l) \\
& - 2z_{90} (z_6 z_{141} - z_{10} z_{51} - z_{14} z_{54})(v_{h1} - v_l) \\
& - 2z_{91} (z_6 z_{139} - z_{51} z_{138} - z_{54} z_{137})(u_{h1} - u_l) \\
& - 2z_{91} (z_6 z_{142} - z_{11} z_{51} - z_{54} z_{136})(v_{h1} - v_l) - 2z_{89} z_6 z_7 z_{133} \dot{\theta}_{h1} \\
& - 2z_{89} z_{132} (z_5 z_{279} - z_8 z_{140} \dot{\varphi}_{h1} - z_6 z_{280} - z_{51} z_{264}) \\
& - 2z_{90} z_{132} (z_8 z_{141} \dot{\varphi}_{h1} + z_6 z_{287} + z_{10} z_{279} - z_{14} z_{282} - z_{51} z_{265} - z_{54} z_{271}) \\
& - 2z_{91} z_{132} (z_8 z_{142} \dot{\varphi}_{h1} + z_6 z_{291} + z_{11} z_{279} - z_{51} z_{266} - z_{54} z_{286} - z_{136} z_{282})
\end{aligned}$$

$$z_{293} = (z_6 z_{278} \dot{\theta}_{h1} + z_8 z_{49} \dot{\varphi}_{h1})/z_{49}^2$$

$$z_{294} = z_8 z_{52} \dot{\varphi}_{h1} + z_6 \dot{\theta}_{h1}/z_{49}^2$$

$$\begin{aligned}
z_{295} = & 2z_{89} z_4 z_8 (w_l - w_{h1}) + 2z_{91} (z_8 z_{144} + z_{15} z_{53})(w_l - w_{h1}) \\
& + 2z_{89} (z_5 z_{50} - z_8 z_{140})(v_{h1} - v_l) + 2z_{90} (z_8 z_{143} - z_{16} z_{53})(w_l - w_{h1}) \\
& + 2z_{90} (z_8 z_{135} + z_{11} z_{53} + z_{50} z_{136})(u_{h1} - u_l) \\
& + 2z_{90} (z_8 z_{141} + z_{10} z_{50} + z_{14} z_{53})(v_{h1} - v_l) \\
& + 2z_{91} (z_8 z_{139} + z_{50} z_{138} + z_{53} z_{137})(u_{h1} - u_l) \\
& + 2z_{91} (z_8 z_{142} + z_{11} z_{50} + z_{53} z_{136})(v_{h1} - v_l) + 2z_{89} z_4 z_6 z_{133} \dot{\varphi}_{h1} \\
& + 2z_{89} z_7 z_8 z_{133} \dot{\theta}_{h1} + 2z_{89} z_{131} (z_6 z_{134} \dot{\varphi}_{h1} - z_8 z_{283} - z_{12} z_{293} - z_{50} z_{269}) \\
& + 2z_{91} z_{133} (z_6 z_{144} \dot{\varphi}_{h1} - z_8 z_{284} - z_{15} z_{294} - z_{53} z_{274}) \\
& - 2z_{89} (z_8 z_{134} + z_{12} z_{50})(u_{h1} - u_l) \\
& - 2z_{89} z_{132} (z_8 z_{280} - z_6 z_{140} \dot{\varphi}_{h1} - z_5 z_{293} - z_{50} z_{264}) \\
& - 2z_{90} z_{133} (z_8 z_{281} - z_6 z_{143} \dot{\varphi}_{h1} - z_{16} z_{294} - z_{53} z_{275}) \\
& - 2z_{90} z_{131} (z_6 z_{135} \dot{\varphi}_{h1} - z_8 z_{285} - z_{11} z_{294} - z_{50} z_{286} - z_{53} z_{266} - z_{136} z_{293}) \\
& - 2z_{90} z_{132} (z_6 z_{141} \dot{\varphi}_{h1} - z_8 z_{287} - z_{10} z_{293} - z_{14} z_{294} - z_{50} z_{265} - z_{53} z_{271}) \\
& - 2z_{91} z_{131} (z_6 z_{139} \dot{\varphi}_{h1} - z_8 z_{288} - z_{50} z_{289} - z_{53} z_{290} - z_{137} z_{294} - z_{138} z_{293}) \\
& - 2z_{91} z_{132} (z_6 z_{142} \dot{\varphi}_{h1} - z_8 z_{291} - z_{11} z_{293} - z_{50} z_{266} - z_{53} z_{286} - z_{136} z_{294})
\end{aligned}$$

$$\begin{aligned}
z_{296} = & 2z_{90} z_{11} (u_{h1} - u_l) + 2z_{90} z_{14} (v_{h1} - v_l) + 2z_{91} z_{15} (w_l - w_{h1}) + 2z_{91} z_{136} (v_{h1} - v_l) \\
& + 2z_{91} z_{137} (u_{h1} - u_l) + 2z_{90} z_{131} z_{266} + 2z_{90} z_{132} z_{271} + 2z_{90} z_{133} z_{275} \\
& + 2z_{91} z_{131} z_{290} + 2z_{91} z_{132} z_{286} - 2z_{90} z_{16} (w_l - w_{h1}) - 2z_{91} z_{133} z_{274}
\end{aligned}$$

$$\begin{aligned}
z_{297} = & u_{h1} z_{267} + u_l z_{268} + v_{h1} z_{272} + v_l z_{273} + w_l z_{276} \\
& + w_{h1} z_{277} + p_{h1} z_{296} + q_{h1} z_{295} + r_{h1} z_{292}
\end{aligned}$$

$$z_{298} = z_{20} z_{24} \dot{\varphi}_{h2} - z_{22} z_{23} \dot{\theta}_{h2}$$

$$z_{299} = -z_{20} z_{22} \dot{\varphi}_{h2} - z_{23} z_{24} \dot{\theta}_{h2}$$

5 Dynamic Equations of the Multi-Lift Model

$$\begin{aligned}
z_{300} &= 2w_l + 2z_{89} z_{20} \dot{\theta}_{h2} - 2w_{h2} - 2z_{90} z_{298} - 2z_{91} z_{299} \\
z_{301} &= 2w_{h2} + 2z_{90} z_{298} + 2z_{91} z_{299} - 2w_l - 2z_{89} z_{20} \dot{\theta}_{h2} \\
z_{302} &= -z_{19} z_{23} \dot{\theta}_{h2} - z_{20} z_{25} \dot{\psi}_{h2} \\
z_{303} &= \sin(\theta_{h2}) \\
z_{304} &= (z_{22} z_{55} \dot{\phi}_{h2} - z_{24} z_{303} \dot{\theta}_{h2})/z_{55}^2 \\
z_{305} &= z_{19} z_{23} \dot{\psi}_{h2} + z_{20} z_{25} \dot{\theta}_{h2} \\
z_{306} &= z_{20} z_{22} \dot{\theta}_{h2} + z_{23} z_{24} \dot{\phi}_{h2} \\
z_{307} &= z_{24} \dot{\theta}_{h2}/z_{55}^2 - z_{22} z_{58} \dot{\phi}_{h2} \\
z_{308} &= z_{19} z_{20} \dot{\theta}_{h2} - z_{23} z_{25} \dot{\psi}_{h2} \\
z_{309} &= z_{19} z_{20} \dot{\psi}_{h2} - z_{23} z_{25} \dot{\theta}_{h2} \\
z_{310} &= z_{20} z_{24} \dot{\theta}_{h2} - z_{22} z_{23} \dot{\phi}_{h2} \\
z_{311} &= z_{19} z_{20} z_{24} \dot{\phi}_{h2} - z_{19} z_{22} z_{23} \dot{\theta}_{h2} - z_{20} z_{22} z_{25} \dot{\psi}_{h2} \\
z_{312} &= z_{19} z_{22} \dot{\psi}_{h2} + z_{24} z_{25} \dot{\phi}_{h2} + z_{19} z_{20} z_{24} \dot{\theta}_{h2} - z_{19} z_{22} z_{23} \dot{\phi}_{h2} - z_{23} z_{24} z_{25} \dot{\psi}_{h2} \\
z_{313} &= z_{19} z_{22} \dot{\phi}_{h2} + z_{24} z_{25} \dot{\psi}_{h2} - z_{19} z_{22} z_{23} \dot{\psi}_{h2} - z_{20} z_{22} z_{25} \dot{\theta}_{h2} - z_{23} z_{24} z_{25} \dot{\phi}_{h2} \\
z_{314} &= z_{19} z_{20} z_{22} \dot{\psi}_{h2} + z_{20} z_{24} z_{25} \dot{\phi}_{h2} - z_{22} z_{23} z_{25} \dot{\theta}_{h2} \\
z_{315} &= z_{22} z_{25} \dot{\phi}_{h2} + z_{19} z_{20} z_{22} \dot{\theta}_{h2} + z_{19} z_{23} z_{24} \dot{\phi}_{h2} - z_{19} z_{24} \dot{\psi}_{h2} - z_{22} z_{23} z_{25} \dot{\psi}_{h2} \\
z_{316} &= z_{22} z_{25} \dot{\psi}_{h2} + z_{19} z_{23} z_{24} \dot{\psi}_{h2} + z_{20} z_{24} z_{25} \dot{\theta}_{h2} - z_{19} z_{24} \dot{\phi}_{h2} - z_{22} z_{23} z_{25} \dot{\phi}_{h2} \\
z_{317} &= -z_{19} z_{20} z_{22} \dot{\phi}_{h2} - z_{19} z_{23} z_{24} \dot{\theta}_{h2} - z_{20} z_{24} z_{25} \dot{\psi}_{h2} \\
z_{318} &= z_{19} z_{24} \dot{\phi}_{h2} + z_{22} z_{23} z_{25} \dot{\phi}_{h2} - z_{22} z_{25} \dot{\psi}_{h2} - z_{19} z_{23} z_{24} \dot{\psi}_{h2} - z_{20} z_{24} z_{25} \dot{\theta}_{h2} \\
z_{319} &= z_{19} z_{24} \dot{\psi}_{h2} + z_{22} z_{23} z_{25} \dot{\psi}_{h2} - z_{22} z_{25} \dot{\phi}_{h2} - z_{19} z_{20} z_{22} \dot{\theta}_{h2} - z_{19} z_{23} z_{24} \dot{\phi}_{h2} \\
z_{320} &= z_{19} z_{20} z_{24} \dot{\psi}_{h2} - z_{20} z_{22} z_{25} \dot{\phi}_{h2} - z_{23} z_{24} z_{25} \dot{\theta}_{h2} \\
z_{321} &= 2z_{90} (z_{22} z_{149} - z_{27} z_{60} - z_{57} z_{150})(u_l - u_{h2}) \\
&\quad + 2z_{90} (z_{22} z_{155} - z_{26} z_{57} - z_{30} z_{60})(v_l - v_{h2}) \\
&\quad + 2z_{91} (z_{22} z_{153} - z_{57} z_{152} - z_{60} z_{151})(u_l - u_{h2}) \\
&\quad + 2z_{91} (z_{22} z_{156} - z_{27} z_{57} - z_{60} z_{150})(v_l - v_{h2}) + 2z_{89} z_{20} z_{24} z_{147} \dot{\phi}_{h2} \\
&\quad + 2z_{90} z_{147} (z_{24} z_{157} \dot{\phi}_{h2} + z_{22} z_{306} + z_{32} z_{307} + z_{60} z_{299}) \\
&\quad + 2z_{89} z_{145} (z_{24} z_{148} \dot{\phi}_{h2} + z_{22} z_{308} + z_{28} z_{304} - z_{57} z_{309}) \\
&\quad + 2z_{91} z_{147} (z_{24} z_{158} \dot{\phi}_{h2} + z_{22} z_{310} - z_{31} z_{307} - z_{60} z_{298}) \\
&\quad + 2z_{90} z_{145} (z_{27} z_{307} + z_{57} z_{313} + z_{60} z_{312} - z_{24} z_{149} \dot{\phi}_{h2} - z_{22} z_{311} - z_{150} z_{304}) \\
&\quad + 2z_{91} z_{145} (z_{57} z_{318} + z_{60} z_{319} + z_{151} z_{307} - z_{24} z_{153} \dot{\phi}_{h2} - z_{22} z_{317} - z_{152} z_{304}) \\
&\quad - 2z_{89} z_{20} z_{22} (w_l - w_{h2}) - 2z_{89} (z_{21} z_{57} + z_{22} z_{154})(v_l - v_{h2}) \\
&\quad - 2z_{90} (z_{22} z_{157} + z_{32} z_{60})(w_l - w_{h2}) - 2z_{89} (z_{22} z_{148} - z_{28} z_{57})(u_l - u_{h2}) \\
&\quad - 2z_{91} (z_{22} z_{158} - z_{31} z_{60})(w_l - w_{h2}) - 2z_{89} z_{22} z_{23} z_{147} \dot{\theta}_{h2} \\
&\quad - 2z_{89} z_{146} (z_{21} z_{304} - z_{24} z_{154} \dot{\phi}_{h2} - z_{22} z_{305} - z_{57} z_{302}) \\
&\quad - 2z_{90} z_{146} (z_{24} z_{155} \dot{\phi}_{h2} + z_{22} z_{314} + z_{26} z_{304} - z_{30} z_{307} - z_{57} z_{315} - z_{60} z_{316}) \\
&\quad - 2z_{91} z_{146} (z_{24} z_{156} \dot{\phi}_{h2} + z_{22} z_{320} + z_{27} z_{304} - z_{57} z_{312} - z_{60} z_{313} - z_{150} z_{307})
\end{aligned}$$

5 Dynamic Equations of the Multi-Lift Model

$$\begin{aligned}
z_{322} &= (z_{22} z_{303} \dot{\theta}_{h2} + z_{24} z_{55} \dot{\varphi}_{h2})/z_{55}^2 \\
z_{323} &= z_{24} z_{58} \dot{\varphi}_{h2} + z_{22} \dot{\theta}_{h2}/z_{55}^2 \\
z_{324} &= 2z_{89} z_{20} z_{24} (w_l - w_{h2}) + 2z_{89} (z_{24} z_{148} + z_{28} z_{56}) (u_l - u_{h2}) \\
&\quad + 2z_{91} (z_{24} z_{158} + z_{31} z_{59}) (w_l - w_{h2}) + 2z_{90} (z_{24} z_{157} - z_{32} z_{59}) (w_l - w_{h2}) \\
&\quad + 2z_{89} z_{20} z_{22} z_{147} \dot{\varphi}_{h2} + 2z_{89} z_{23} z_{24} z_{147} \dot{\theta}_{h2} \\
&\quad + 2z_{89} z_{145} (z_{22} z_{148} \dot{\varphi}_{h2} - z_{24} z_{308} - z_{28} z_{322} - z_{56} z_{309}) \\
&\quad + 2z_{91} z_{147} (z_{22} z_{158} \dot{\varphi}_{h2} - z_{24} z_{310} - z_{31} z_{323} - z_{59} z_{298}) \\
&\quad - 2z_{89} (z_{21} z_{56} - z_{24} z_{154}) (v_l - v_{h2}) - 2z_{90} (z_{24} z_{149} + z_{27} z_{59} + z_{56} z_{150}) (u_l - u_{h2}) \\
&\quad - 2z_{90} (z_{24} z_{155} + z_{26} z_{56} + z_{30} z_{59}) (v_l - v_{h2}) \\
&\quad - 2z_{91} (z_{24} z_{153} + z_{56} z_{152} + z_{59} z_{151}) (u_l - u_{h2}) \\
&\quad - 2z_{91} (z_{24} z_{156} + z_{27} z_{56} + z_{59} z_{150}) (v_l - v_{h2}) \\
&\quad - 2z_{89} z_{146} (z_{24} z_{305} - z_{22} z_{154} \dot{\varphi}_{h2} - z_{21} z_{322} - z_{56} z_{302}) \\
&\quad - 2z_{90} z_{147} (z_{24} z_{306} - z_{22} z_{157} \dot{\varphi}_{h2} - z_{32} z_{323} - z_{59} z_{299}) \\
&\quad - 2z_{90} z_{145} (z_{22} z_{149} \dot{\varphi}_{h2} - z_{24} z_{311} - z_{27} z_{323} - z_{56} z_{313} - z_{59} z_{312} - z_{150} z_{322}) \\
&\quad - 2z_{90} z_{146} (z_{22} z_{155} \dot{\varphi}_{h2} - z_{24} z_{314} - z_{26} z_{322} - z_{30} z_{323} - z_{56} z_{315} - z_{59} z_{316}) \\
&\quad - 2z_{91} z_{145} (z_{22} z_{153} \dot{\varphi}_{h2} - z_{24} z_{317} - z_{56} z_{318} - z_{59} z_{319} - z_{151} z_{323} - z_{152} z_{322}) \\
&\quad - 2z_{91} z_{146} (z_{22} z_{156} \dot{\varphi}_{h2} - z_{24} z_{320} - z_{27} z_{322} - z_{56} z_{312} - z_{59} z_{313} - z_{150} z_{323}) \\
z_{325} &= 2z_{91} z_{31} (w_l - w_{h2}) + 2z_{90} z_{145} z_{312} + 2z_{90} z_{146} z_{316} + 2z_{90} z_{147} z_{299} \\
&\quad + 2z_{91} z_{145} z_{319} + 2z_{91} z_{146} z_{313} - 2z_{90} z_{27} (u_l - u_{h2}) - 2z_{90} z_{30} (v_l - v_{h2}) \\
&\quad - 2z_{90} z_{32} (w_l - w_{h2}) - 2z_{91} z_{150} (v_l - v_{h2}) - 2z_{91} z_{151} (u_l - u_{h2}) - 2z_{91} z_{147} z_{298} \\
z_{326} &= 2u_l - 2u_{h2} - 2z_{89} z_{302} - 2z_{90} z_{315} - 2z_{91} z_{312} \\
z_{327} &= 2u_{h2} + 2z_{89} z_{302} + 2z_{90} z_{315} + 2z_{91} z_{312} - 2u_l \\
z_{328} &= z_{19} z_{22} z_{23} \dot{\psi}_{h2} + z_{20} z_{22} z_{25} \dot{\theta}_{h2} + z_{23} z_{24} z_{25} \dot{\varphi}_{h2} - z_{19} z_{22} \dot{\varphi}_{h2} - z_{24} z_{25} \dot{\psi}_{h2} \\
z_{329} &= 2v_l - 2v_{h2} - 2z_{89} z_{309} - 2z_{90} z_{328} - 2z_{91} z_{316} \\
z_{330} &= 2v_{h2} + 2z_{89} z_{309} + 2z_{90} z_{328} + 2z_{91} z_{316} - 2v_l \\
z_{331} &= p_{h2} z_{325} + q_{h2} z_{324} + r_{h2} z_{321} + u_l z_{326} + v_l z_{329} \\
&\quad + w_l z_{300} + u_{h2} z_{327} + v_{h2} z_{330} + w_{h2} z_{301} \\
z_{332} &= -z_{35} z_{39} \dot{\theta}_{h3} - z_{36} z_{41} \dot{\psi}_{h3} \\
z_{333} &= z_{38} z_{41} \dot{\varphi}_{h3} + z_{35} z_{36} z_{38} \dot{\theta}_{h3} + z_{35} z_{39} z_{40} \dot{\varphi}_{h3} - z_{35} z_{40} \dot{\psi}_{h3} - z_{38} z_{39} z_{41} \dot{\psi}_{h3} \\
z_{334} &= z_{35} z_{38} \dot{\psi}_{h3} + z_{40} z_{41} \dot{\varphi}_{h3} + z_{35} z_{36} z_{40} \dot{\theta}_{h3} - z_{35} z_{38} z_{39} \dot{\varphi}_{h3} - z_{39} z_{40} z_{41} \dot{\psi}_{h3} \\
z_{335} &= 2u_{h3} + 2z_{89} z_{332} + 2z_{90} z_{333} + 2z_{91} z_{334} - 2u_l \\
z_{336} &= 2u_l - 2u_{h3} - 2z_{89} z_{332} - 2z_{90} z_{333} - 2z_{91} z_{334} \\
z_{337} &= z_{35} z_{36} \dot{\psi}_{h3} - z_{39} z_{41} \dot{\theta}_{h3} \\
z_{338} &= z_{35} z_{38} z_{39} \dot{\psi}_{h3} + z_{36} z_{38} z_{41} \dot{\theta}_{h3} + z_{39} z_{40} z_{41} \dot{\varphi}_{h3} - z_{35} z_{38} \dot{\varphi}_{h3} - z_{40} z_{41} \dot{\psi}_{h3}
\end{aligned}$$

5 Dynamic Equations of the Multi-Lift Model

$$\begin{aligned}
z_{339} &= z_{38} z_{41} \dot{\psi}_{h3} + z_{35} z_{39} z_{40} \dot{\psi}_{h3} + z_{36} z_{40} z_{41} \dot{\theta}_{h3} - z_{35} z_{40} \dot{\varphi}_{h3} - z_{38} z_{39} z_{41} \dot{\varphi}_{h3} \\
z_{340} &= 2v_{h3} + 2z_{89} z_{337} + 2z_{90} z_{338} + 2z_{91} z_{339} - 2v_l \\
z_{341} &= 2v_l - 2v_{h3} - 2z_{89} z_{337} - 2z_{90} z_{338} - 2z_{91} z_{339} \\
z_{342} &= z_{36} z_{40} \dot{\varphi}_{h3} - z_{38} z_{39} \dot{\theta}_{h3} \\
z_{343} &= -z_{36} z_{38} \dot{\varphi}_{h3} - z_{39} z_{40} \dot{\theta}_{h3} \\
z_{344} &= 2w_{h3} + 2z_{90} z_{342} + 2z_{91} z_{343} - 2w_l - 2z_{89} z_{36} \dot{\theta}_{h3} \\
z_{345} &= 2w_l + 2z_{89} z_{36} \dot{\theta}_{h3} - 2w_{h3} - 2z_{90} z_{342} - 2z_{91} z_{343} \\
z_{346} &= \sin(\theta_{h3}) \\
z_{347} &= (z_{38} z_{61} \dot{\varphi}_{h3} - z_{40} z_{346} \dot{\theta}_{h3})/z_{61}^2 \\
z_{348} &= z_{35} z_{39} \dot{\psi}_{h3} + z_{36} z_{41} \dot{\theta}_{h3} \\
z_{349} &= z_{36} z_{38} \dot{\theta}_{h3} + z_{39} z_{40} \dot{\varphi}_{h3} \\
z_{350} &= z_{40} \dot{\theta}_{h3}/z_{61}^2 - z_{38} z_{64} \dot{\varphi}_{h3} \\
z_{351} &= z_{35} z_{36} \dot{\theta}_{h3} - z_{39} z_{41} \dot{\psi}_{h3} \\
z_{352} &= z_{36} z_{40} \dot{\theta}_{h3} - z_{38} z_{39} \dot{\varphi}_{h3} \\
z_{353} &= z_{35} z_{36} z_{40} \dot{\varphi}_{h3} - z_{35} z_{38} z_{39} \dot{\theta}_{h3} - z_{36} z_{38} z_{41} \dot{\psi}_{h3} \\
z_{354} &= z_{35} z_{38} \dot{\varphi}_{h3} + z_{40} z_{41} \dot{\psi}_{h3} - z_{35} z_{38} z_{39} \dot{\psi}_{h3} - z_{36} z_{38} z_{41} \dot{\theta}_{h3} - z_{39} z_{40} z_{41} \dot{\varphi}_{h3} \\
z_{355} &= z_{35} z_{36} z_{38} \dot{\psi}_{h3} + z_{36} z_{40} z_{41} \dot{\varphi}_{h3} - z_{38} z_{39} z_{41} \dot{\theta}_{h3} \\
z_{356} &= -z_{35} z_{36} z_{38} \dot{\varphi}_{h3} - z_{35} z_{39} z_{40} \dot{\theta}_{h3} - z_{36} z_{40} z_{41} \dot{\psi}_{h3} \\
z_{357} &= z_{35} z_{40} \dot{\varphi}_{h3} + z_{38} z_{39} z_{41} \dot{\varphi}_{h3} - z_{38} z_{41} \dot{\psi}_{h3} - z_{35} z_{39} z_{40} \dot{\psi}_{h3} - z_{36} z_{40} z_{41} \dot{\theta}_{h3} \\
z_{358} &= z_{35} z_{40} \dot{\psi}_{h3} + z_{38} z_{39} z_{41} \dot{\psi}_{h3} - z_{38} z_{41} \dot{\varphi}_{h3} - z_{35} z_{36} z_{38} \dot{\theta}_{h3} - z_{35} z_{39} z_{40} \dot{\varphi}_{h3} \\
z_{359} &= z_{35} z_{36} z_{40} \dot{\psi}_{h3} - z_{36} z_{38} z_{41} \dot{\varphi}_{h3} - z_{39} z_{40} z_{41} \dot{\theta}_{h3} \\
z_{360} &= 2z_{89} z_{36} z_{38} (w_{h3} - w_l) + 2z_{89} (z_{37} z_{63} + z_{38} z_{168})(v_{h3} - v_l) \\
&\quad + 2z_{90} (z_{38} z_{171} + z_{48} z_{66})(w_{h3} - w_l) + 2z_{89} (z_{38} z_{162} - z_{44} z_{63})(u_{h3} - u_l) \\
&\quad + 2z_{91} (z_{38} z_{172} - z_{47} z_{66})(w_{h3} - w_l) + 2z_{89} z_{36} z_{40} z_{161} \dot{\varphi}_{h3} \\
&\quad + 2z_{90} z_{161} (z_{40} z_{171} \dot{\varphi}_{h3} + z_{38} z_{349} + z_{48} z_{350} + z_{66} z_{343}) \\
&\quad + 2z_{89} z_{159} (z_{40} z_{162} \dot{\varphi}_{h3} + z_{38} z_{351} + z_{44} z_{347} - z_{63} z_{337}) \\
&\quad + 2z_{91} z_{161} (z_{40} z_{172} \dot{\varphi}_{h3} + z_{38} z_{352} - z_{47} z_{350} - z_{66} z_{342}) \\
&\quad + 2z_{90} z_{159} (z_{43} z_{350} + z_{63} z_{354} + z_{66} z_{334} - z_{40} z_{163} \dot{\varphi}_{h3} - z_{38} z_{353} - z_{164} z_{347}) \\
&\quad + 2z_{91} z_{159} (z_{63} z_{357} + z_{66} z_{358} + z_{165} z_{350} - z_{40} z_{167} \dot{\varphi}_{h3} - z_{38} z_{356} - z_{166} z_{347}) \\
&\quad - 2z_{90} (z_{38} z_{163} - z_{43} z_{66} - z_{63} z_{164})(u_{h3} - u_l) \\
&\quad - 2z_{90} (z_{38} z_{169} - z_{42} z_{63} - z_{46} z_{66})(v_{h3} - v_l) \\
&\quad - 2z_{91} (z_{38} z_{167} - z_{63} z_{166} - z_{66} z_{165})(u_{h3} - u_l) \\
&\quad - 2z_{91} (z_{38} z_{170} - z_{43} z_{63} - z_{66} z_{164})(v_{h3} - v_l) - 2z_{89} z_{38} z_{39} z_{161} \dot{\theta}_{h3} \\
&\quad - 2z_{89} z_{160} (z_{37} z_{347} - z_{40} z_{168} \dot{\varphi}_{h3} - z_{38} z_{348} - z_{63} z_{332}) \\
&\quad - 2z_{90} z_{160} (z_{40} z_{169} \dot{\varphi}_{h3} + z_{38} z_{355} + z_{42} z_{347} - z_{46} z_{350} - z_{63} z_{333} - z_{66} z_{339}) \\
&\quad - 2z_{91} z_{160} (z_{40} z_{170} \dot{\varphi}_{h3} + z_{38} z_{359} + z_{43} z_{347} - z_{63} z_{334} - z_{66} z_{354} - z_{164} z_{350})
\end{aligned}$$

5 Dynamic Equations of the Multi-Lift Model

$$\begin{aligned}
z_{361} &= (z_{38} z_{346} \dot{\theta}_{h3} + z_{40} z_{61} \dot{\varphi}_{h3})/z_{61}^2 \\
z_{362} &= z_{40} z_{64} \dot{\varphi}_{h3} + z_{38} \dot{\theta}_{h3}/z_{61}^2 \\
z_{363} &= 2z_{89} (z_{37} z_{62} - z_{40} z_{168})(v_{h3} - v_l) + 2z_{90} (z_{40} z_{163} + z_{43} z_{65} + z_{62} z_{164})(u_{h3} - u_l) \\
&\quad + 2z_{90} (z_{40} z_{169} + z_{42} z_{62} + z_{46} z_{65})(v_{h3} - v_l) \\
&\quad + 2z_{91} (z_{40} z_{167} + z_{62} z_{166} + z_{65} z_{165})(u_{h3} - u_l) \\
&\quad + 2z_{91} (z_{40} z_{170} + z_{43} z_{62} + z_{65} z_{164})(v_{h3} - v_l) + 2z_{89} z_{36} z_{38} z_{161} \dot{\varphi}_{h3} \\
&\quad + 2z_{89} z_{39} z_{40} z_{161} \dot{\theta}_{h3} + 2z_{89} z_{159} (z_{38} z_{162} \dot{\varphi}_{h3} - z_{40} z_{351} - z_{44} z_{361} - z_{62} z_{337}) \\
&\quad + 2z_{91} z_{161} (z_{38} z_{172} \dot{\varphi}_{h3} - z_{40} z_{352} - z_{47} z_{362} - z_{65} z_{342}) \\
&\quad - 2z_{89} z_{36} z_{40} (w_{h3} - w_l) - 2z_{89} (z_{40} z_{162} + z_{44} z_{62})(u_{h3} - u_l) \\
&\quad - 2z_{91} (z_{40} z_{172} + z_{47} z_{65})(w_{h3} - w_l) - 2z_{90} (z_{40} z_{171} - z_{48} z_{65})(w_{h3} - w_l) \\
&\quad - 2z_{89} z_{160} (z_{40} z_{348} - z_{38} z_{168} \dot{\varphi}_{h3} - z_{37} z_{361} - z_{62} z_{332}) \\
&\quad - 2z_{90} z_{161} (z_{40} z_{349} - z_{38} z_{171} \dot{\varphi}_{h3} - z_{48} z_{362} - z_{65} z_{343}) \\
&\quad - 2z_{90} z_{159} (z_{38} z_{163} \dot{\varphi}_{h3} - z_{40} z_{353} - z_{43} z_{362} - z_{62} z_{354} - z_{65} z_{334} - z_{164} z_{361}) \\
&\quad - 2z_{90} z_{160} (z_{38} z_{169} \dot{\varphi}_{h3} - z_{40} z_{355} - z_{42} z_{361} - z_{46} z_{362} - z_{62} z_{333} - z_{65} z_{339}) \\
&\quad - 2z_{91} z_{159} (z_{38} z_{167} \dot{\varphi}_{h3} - z_{40} z_{356} - z_{62} z_{357} - z_{65} z_{358} - z_{165} z_{362} - z_{166} z_{361}) \\
&\quad - 2z_{91} z_{160} (z_{38} z_{170} \dot{\varphi}_{h3} - z_{40} z_{359} - z_{43} z_{361} - z_{62} z_{334} - z_{65} z_{354} - z_{164} z_{362}) \\
z_{364} &= 2z_{90} z_{43} (u_{h3} - u_l) + 2z_{90} z_{46} (v_{h3} - v_l) + 2z_{90} z_{48} (w_{h3} - w_l) + 2z_{91} z_{164} (v_{h3} - v_l) \\
&\quad + 2z_{91} z_{165} (u_{h3} - u_l) + 2z_{90} z_{159} z_{334} + 2z_{90} z_{160} z_{339} + 2z_{90} z_{161} z_{343} \\
&\quad + 2z_{91} z_{159} z_{358} + 2z_{91} z_{160} z_{354} - 2z_{91} z_{47} (w_{h3} - w_l) - 2z_{91} z_{161} z_{342} \\
z_{365} &= u_{h3} z_{335} + v_{h3} z_{340} + w_{h3} z_{344} + p_{h3} z_{364} + q_{h3} z_{363} \\
&\quad + r_{h3} z_{360} + u_l z_{336} + v_l z_{341} + w_l z_{345} \\
z_{366} &= (z_{202} z_{365} + z_{204} z_{297} - z_{207} z_{331})/z_{203} \\
z_{367} &= (z_{201} z_{365} + z_{205} z_{297} - z_{208} z_{331})/z_{203} \\
z_{368} &= (z_{200} z_{365} + z_{206} z_{297} - z_{209} z_{331})/z_{203} \\
z_{369} &= z_{130} z_{246} + F_{h1,3}^{MR} z_{11} + F_{h1,2}^{TR} z_{10} + z_{129} z_5 z_7 \\
&\quad + z_{128} (z_5 z_7 - z_{10} z_{15} - z_{11} z_{16}) - z_{129} z_{10} z_{15} - z_{129} z_{11} z_{16} \\
z_{370} &= z_{130} z_{247} + F_{h1,3}^{MR} z_{14} + F_{h1,2}^{TR} z_{13} + z_{129} z_7 z_{12} \\
&\quad + z_{128} (z_7 z_{12} - z_{13} z_{15} - z_{14} z_{16}) - z_{129} z_{13} z_{15} - z_{129} z_{14} z_{16} \\
z_{371} &= z_{130} z_{248} + F_{h1,3}^{MR} z_{16} + F_{h1,2}^{TR} z_{15} - z_{129} z_7^2 - z_{129} z_{15}^2 - z_{129} z_{16}^2 - z_{128} (z_7^2 + z_{15}^2 + z_{16}^2) \\
z_{372} &= z_{82} z_{129} \\
z_{373} &= z_{81} z_{129} \\
z_{374} &= T_{h1,1}^{MR} + z_{81} F_{h1,3}^{MR} + z_{130} z_{251} + z_{372} z_{15} - z_{88} F_{h1,2}^{TR} - z_{373} z_{16} - z_{128} (z_{84} z_{16} - z_{85} z_{15}) \\
z_{375} &= z_{80} z_{129} \\
z_{376} &= T_{h1,2}^{MR} + T_{h1,2}^{TR} + z_{130} z_{250} + z_{372} z_7 + z_{375} z_{16} + z_{128} (z_{83} z_{16} + z_{85} z_7) - z_{80} F_{h1,3}^{MR}
\end{aligned}$$

5 Dynamic Equations of the Multi-Lift Model

$$\begin{aligned}
z_{377} &= T_{h1,3}^{MR} + z_{86} F_{h1,2}^{TR} + z_{130} z_{249} - z_{128} (z_{83} z_{15} + z_{84} z_7) - z_{129} (z_{80} z_{15} + z_{81} z_7) \\
z_{378} &= F_{h2,3}^{MR} z_{27} + F_{h2,2}^{TR} z_{26} + z_{129} z_{21} z_{23} + z_{128} (z_{21} z_{23} - z_{26} z_{31} - z_{27} z_{32}) \\
&\quad - z_{130} z_{256} - z_{129} z_{26} z_{31} - z_{129} z_{27} z_{32} \\
z_{379} &= F_{h2,3}^{MR} z_{30} + F_{h2,2}^{TR} z_{29} + z_{129} z_{23} z_{28} + z_{128} (z_{23} z_{28} - z_{29} z_{31} - z_{30} z_{32}) \\
&\quad - z_{130} z_{257} - z_{129} z_{29} z_{31} - z_{129} z_{30} z_{32} \\
z_{380} &= F_{h2,3}^{MR} z_{32} + F_{h2,2}^{TR} z_{31} - z_{130} z_{252} - z_{129} z_{23}^2 - z_{129} z_{31}^2 - z_{129} z_{32}^2 - z_{128} (z_{23}^2 + z_{31}^2 + z_{32}^2) \\
z_{381} &= T_{h2,1}^{MR} + z_{81} F_{h2,3}^{MR} + z_{372} z_{31} - z_{88} F_{h2,2}^{TR} - z_{130} z_{255} - z_{373} z_{32} - z_{128} (z_{84} z_{32} - z_{85} z_{31}) \\
z_{382} &= T_{h2,2}^{MR} + T_{h2,2}^{TR} + z_{372} z_{23} + z_{375} z_{32} + z_{128} (z_{83} z_{32} + z_{85} z_{23}) - z_{80} F_{h2,3}^{MR} - z_{130} z_{254} \\
z_{383} &= T_{h2,3}^{MR} + z_{86} F_{h2,2}^{TR} - z_{130} z_{253} - z_{128} (z_{83} z_{31} + z_{84} z_{23}) - z_{129} (z_{80} z_{31} + z_{81} z_{23}) \\
z_{384} &= z_{130} z_{258} + F_{h3,3}^{MR} z_{43} + F_{h3,2}^{TR} z_{42} + z_{129} z_{37} z_{39} \\
&\quad + z_{128} (z_{37} z_{39} - z_{42} z_{47} - z_{43} z_{48}) - z_{129} z_{42} z_{47} - z_{129} z_{43} z_{48} \\
z_{385} &= z_{130} z_{259} + F_{h3,3}^{MR} z_{46} + F_{h3,2}^{TR} z_{45} + z_{129} z_{39} z_{44} \\
&\quad + z_{128} (z_{39} z_{44} - z_{45} z_{47} - z_{46} z_{48}) - z_{129} z_{45} z_{47} - z_{129} z_{46} z_{48} \\
z_{386} &= z_{130} z_{260} + F_{h3,3}^{MR} z_{48} + F_{h3,2}^{TR} z_{47} - z_{129} z_{39}^2 - z_{129} z_{47}^2 - z_{129} z_{48}^2 - z_{128} (z_{39}^2 + z_{47}^2 + z_{48}^2) \\
z_{387} &= T_{h3,1}^{MR} + z_{81} F_{h3,3}^{MR} + z_{130} z_{263} + z_{372} z_{47} - z_{88} F_{h3,2}^{TR} - z_{373} z_{48} - z_{128} (z_{84} z_{48} - z_{85} z_{47}) \\
z_{388} &= T_{h3,2}^{MR} + T_{h3,2}^{TR} + z_{130} z_{262} + z_{372} z_{39} + z_{375} z_{48} + z_{128} (z_{83} z_{48} + z_{85} z_{39}) - z_{80} F_{h3,3}^{MR} \\
z_{389} &= T_{h3,3}^{MR} + z_{86} F_{h3,2}^{TR} + z_{130} z_{261} - z_{128} (z_{83} z_{47} + z_{84} z_{39}) - z_{129} (z_{80} z_{47} + z_{81} z_{39}) \\
z_{390} &= I_{11}^F p_{h1} \\
z_{391} &= I_{22}^F q_{h1} \\
z_{392} &= I_{33}^F r_{h1} \\
z_{393} &= p_{h1} z_{391} - q_{h1} z_{390} \\
z_{394} &= r_{h1} z_{390} - p_{h1} z_{392} \\
z_{395} &= q_{h1} z_{392} - r_{h1} z_{391} \\
z_{396} &= I_{11}^{MR} p_{h1} \\
z_{397} &= I_{22}^{MR} q_{h1} \\
z_{398} &= I_{33}^{MR} z_{67} \\
z_{399} &= I_{11}^{MR} z_{68} \\
z_{400} &= I_{22}^{MR} z_{69}
\end{aligned}$$

5 Dynamic Equations of the Multi-Lift Model

$$z_{401} = p_{h1} z_{397} - q_{h1} z_{396}$$

$$z_{402} = z_{67} z_{396} - p_{h1} z_{398}$$

$$z_{403} = q_{h1} z_{398} - z_{67} z_{397}$$

$$z_{404} = I_{11}^F p_{h2}$$

$$z_{405} = I_{22}^F q_{h2}$$

$$z_{406} = I_{33}^F r_{h2}$$

$$z_{407} = p_{h2} z_{405} - q_{h2} z_{404}$$

$$z_{408} = r_{h2} z_{404} - p_{h2} z_{406}$$

$$z_{409} = q_{h2} z_{406} - r_{h2} z_{405}$$

$$z_{410} = I_{11}^{MR} p_{h2}$$

$$z_{411} = I_{22}^{MR} q_{h2}$$

$$z_{412} = I_{33}^{MR} z_{70}$$

$$z_{413} = I_{11}^{MR} z_{71}$$

$$z_{414} = I_{22}^{MR} z_{72}$$

$$z_{415} = p_{h2} z_{411} - q_{h2} z_{410}$$

$$z_{416} = z_{70} z_{410} - p_{h2} z_{412}$$

$$z_{417} = q_{h2} z_{412} - z_{70} z_{411}$$

$$z_{418} = I_{11}^F p_{h3}$$

$$z_{419} = I_{22}^F q_{h3}$$

$$z_{420} = I_{33}^F r_{h3}$$

$$z_{421} = p_{h3} z_{419} - q_{h3} z_{418}$$

$$z_{422} = r_{h3} z_{418} - p_{h3} z_{420}$$

$$z_{423} = q_{h3} z_{420} - r_{h3} z_{419}$$

$$z_{424} = I_{11}^{MR} p_{h3}$$

$$z_{425} = I_{22}^{MR} q_{h3}$$

$$z_{426} = I_{33}^{MR} z_{73}$$

$$z_{427} = I_{11}^{MR} z_{74}$$

$$z_{428} = I_{22}^{MR} z_{75}$$

$$z_{429} = p_{h3} z_{425} - q_{h3} z_{424}$$

5 Dynamic Equations of the Multi-Lift Model

$$\begin{aligned}
z_{430} &= z_{73} z_{424} - p_{h3} z_{426} \\
z_{431} &= q_{h3} z_{426} - z_{73} z_{425} \\
z_{432} &= m_F (z_{83} z_{10} - z_{84} z_5) + m_{MR} (z_{80} z_{10} - z_{81} z_5) \\
z_{433} &= z_{76} (z_5 z_7 - z_{10} z_{15} - z_{11} z_{16}) \\
z_{434} &= -m_F (z_{83} z_{11} - z_{85} z_5) - m_{MR} (z_{80} z_{11} - z_{82} z_5) \\
z_{435} &= z_{76} (z_5^2 + z_{10}^2 + z_{11}^2) \\
z_{436} &= z_{76} (z_5 z_{12} + z_{10} z_{13} + z_{11} z_{14}) \\
z_{437} &= m_F (z_{84} z_{11} - z_{85} z_{10}) + m_{MR} (z_{81} z_{11} - z_{82} z_{10}) \\
z_{438} &= m_F (z_5 z_{95} + z_{10} z_{96} + z_{11} z_{97}) + m_{MR} (z_5 z_{92} + z_{10} z_{93} + z_{11} z_{94}) \\
z_{439} &= m_F (z_{83} z_{13} - z_{84} z_{12}) + m_{MR} (z_{80} z_{13} - z_{81} z_{12}) \\
z_{440} &= z_{76} (z_7 z_{12} - z_{13} z_{15} - z_{14} z_{16}) \\
z_{441} &= -m_F (z_{83} z_{14} - z_{85} z_{12}) - m_{MR} (z_{80} z_{14} - z_{82} z_{12}) \\
z_{442} &= z_{76} (z_{12}^2 + z_{13}^2 + z_{14}^2) \\
z_{443} &= m_F (z_{84} z_{14} - z_{85} z_{13}) + m_{MR} (z_{81} z_{14} - z_{82} z_{13}) \\
z_{444} &= m_F (z_{12} z_{95} + z_{13} z_{96} + z_{14} z_{97}) + m_{MR} (z_{12} z_{92} + z_{13} z_{93} + z_{14} z_{94}) \\
z_{445} &= m_F (z_{83} z_{15} + z_{84} z_7) + m_{MR} (z_{80} z_{15} + z_{81} z_7) \\
z_{446} &= z_{76} (z_7^2 + z_{15}^2 + z_{16}^2) \\
z_{447} &= -m_F (z_{83} z_{16} + z_{85} z_7) - m_{MR} (z_{80} z_{16} + z_{82} z_7) \\
z_{448} &= m_F (z_{84} z_{16} - z_{85} z_{15}) + m_{MR} (z_{81} z_{16} - z_{82} z_{15}) \\
z_{449} &= -m_F (z_7 z_{95} - z_{15} z_{96} - z_{16} z_{97}) - m_{MR} (z_7 z_{92} - z_{15} z_{93} - z_{16} z_{94}) \\
z_{450} &= I_{11}^F + I_{11}^{MR} + m_F (z_{84}^2 + z_{85}^2) + m_{MR} (z_{81}^2 + z_{82}^2) \\
z_{451} &= -m_F z_{83} z_{85} - m_{MR} z_{80} z_{82} \\
z_{452} &= -m_F z_{83} z_{84} - m_{MR} z_{80} z_{81} \\
z_{453} &= z_{395} + z_{403} + m_F (z_{84} z_{97} - z_{85} z_{96}) + m_{MR} (z_{81} z_{94} - z_{82} z_{93}) - z_{399} \\
z_{454} &= I_{22}^F + I_{22}^{MR} + m_F (z_{83}^2 + z_{85}^2) + m_{MR} (z_{80}^2 + z_{82}^2) \\
z_{455} &= -m_F z_{84} z_{85} - m_{MR} z_{81} z_{82} \\
z_{456} &= z_{394} + z_{400} + z_{402} - m_F (z_{83} z_{97} - z_{85} z_{95}) - m_{MR} (z_{80} z_{94} - z_{82} z_{92}) \\
z_{457} &= I_{33}^F + I_{33}^{MR} + m_F (z_{83}^2 + z_{84}^2) + m_{MR} (z_{80}^2 + z_{81}^2) \\
z_{458} &= z_{393} + z_{401} + m_F (z_{83} z_{96} - z_{84} z_{95}) + m_{MR} (z_{80} z_{93} - z_{81} z_{92})
\end{aligned}$$

5 Dynamic Equations of the Multi-Lift Model

$$\begin{aligned}
z_{459} &= m_F (z_{83} z_{26} - z_{84} z_{21}) + m_{MR} (z_{80} z_{26} - z_{81} z_{21}) \\
z_{460} &= z_{76} (z_{21} z_{23} - z_{26} z_{31} - z_{27} z_{32}) \\
z_{461} &= -m_F (z_{83} z_{27} - z_{85} z_{21}) - m_{MR} (z_{80} z_{27} - z_{82} z_{21}) \\
z_{462} &= z_{76} (z_{21}^2 + z_{26}^2 + z_{27}^2) \\
z_{463} &= z_{76} (z_{21} z_{28} + z_{26} z_{29} + z_{27} z_{30}) \\
z_{464} &= m_F (z_{84} z_{27} - z_{85} z_{26}) + m_{MR} (z_{81} z_{27} - z_{82} z_{26}) \\
z_{465} &= m_F (z_{21} z_{107} + z_{26} z_{108} + z_{27} z_{109}) + m_{MR} (z_{21} z_{104} + z_{26} z_{105} + z_{27} z_{106}) \\
z_{466} &= m_F (z_{83} z_{29} - z_{84} z_{28}) + m_{MR} (z_{80} z_{29} - z_{81} z_{28}) \\
z_{467} &= z_{76} (z_{23} z_{28} - z_{29} z_{31} - z_{30} z_{32}) \\
z_{468} &= -m_F (z_{83} z_{30} - z_{85} z_{28}) - m_{MR} (z_{80} z_{30} - z_{82} z_{28}) \\
z_{469} &= z_{76} (z_{28}^2 + z_{29}^2 + z_{30}^2) \\
z_{470} &= m_F (z_{84} z_{30} - z_{85} z_{29}) + m_{MR} (z_{81} z_{30} - z_{82} z_{29}) \\
z_{471} &= m_F (z_{28} z_{107} + z_{29} z_{108} + z_{30} z_{109}) + m_{MR} (z_{28} z_{104} + z_{29} z_{105} + z_{30} z_{106}) \\
z_{472} &= m_F (z_{83} z_{31} + z_{84} z_{23}) + m_{MR} (z_{80} z_{31} + z_{81} z_{23}) \\
z_{473} &= z_{76} (z_{23}^2 + z_{31}^2 + z_{32}^2) \\
z_{474} &= -m_F (z_{83} z_{32} + z_{85} z_{23}) - m_{MR} (z_{80} z_{32} + z_{82} z_{23}) \\
z_{475} &= m_F (z_{84} z_{32} - z_{85} z_{31}) + m_{MR} (z_{81} z_{32} - z_{82} z_{31}) \\
z_{476} &= -m_F (z_{23} z_{107} - z_{31} z_{108} - z_{32} z_{109}) - m_{MR} (z_{23} z_{104} - z_{31} z_{105} - z_{32} z_{106}) \\
z_{477} &= z_{409} + z_{417} + m_F (z_{84} z_{109} - z_{85} z_{108}) + m_{MR} (z_{81} z_{106} - z_{82} z_{105}) - z_{413} \\
z_{478} &= z_{408} + z_{414} + z_{416} - m_F (z_{83} z_{109} - z_{85} z_{107}) - m_{MR} (z_{80} z_{106} - z_{82} z_{104}) \\
z_{479} &= z_{407} + z_{415} + m_F (z_{83} z_{108} - z_{84} z_{107}) + m_{MR} (z_{80} z_{105} - z_{81} z_{104}) \\
z_{480} &= m_F (z_{83} z_{42} - z_{84} z_{37}) + m_{MR} (z_{80} z_{42} - z_{81} z_{37}) \\
z_{481} &= z_{76} (z_{37} z_{39} - z_{42} z_{47} - z_{43} z_{48}) \\
z_{482} &= -m_F (z_{83} z_{43} - z_{85} z_{37}) - m_{MR} (z_{80} z_{43} - z_{82} z_{37}) \\
z_{483} &= z_{76} (z_{37}^2 + z_{42}^2 + z_{43}^2) \\
z_{484} &= z_{76} (z_{37} z_{44} + z_{42} z_{45} + z_{43} z_{46}) \\
z_{485} &= m_F (z_{84} z_{43} - z_{85} z_{42}) + m_{MR} (z_{81} z_{43} - z_{82} z_{42}) \\
z_{486} &= m_F (z_{37} z_{119} + z_{42} z_{120} + z_{43} z_{121}) + m_{MR} (z_{37} z_{116} + z_{42} z_{117} + z_{43} z_{118})
\end{aligned}$$

5 Dynamic Equations of the Multi-Lift Model

$$\begin{aligned}
z_{487} &= m_F (z_{83} z_{45} - z_{84} z_{44}) + m_{MR} (z_{80} z_{45} - z_{81} z_{44}) \\
z_{488} &= z_{76} (z_{39} z_{44} - z_{45} z_{47} - z_{46} z_{48}) \\
z_{489} &= -m_F (z_{83} z_{46} - z_{85} z_{44}) - m_{MR} (z_{80} z_{46} - z_{82} z_{44}) \\
z_{490} &= z_{76} (z_{44}^2 + z_{45}^2 + z_{46}^2) \\
z_{491} &= m_F (z_{84} z_{46} - z_{85} z_{45}) + m_{MR} (z_{81} z_{46} - z_{82} z_{45}) \\
z_{492} &= m_F (z_{44} z_{119} + z_{45} z_{120} + z_{46} z_{121}) + m_{MR} (z_{44} z_{116} + z_{45} z_{117} + z_{46} z_{118}) \\
z_{493} &= m_F (z_{83} z_{47} + z_{84} z_{39}) + m_{MR} (z_{80} z_{47} + z_{81} z_{39}) \\
z_{494} &= z_{76} (z_{39}^2 + z_{47}^2 + z_{48}^2) \\
z_{495} &= -m_F (z_{83} z_{48} + z_{85} z_{39}) - m_{MR} (z_{80} z_{48} + z_{82} z_{39}) \\
z_{496} &= m_F (z_{84} z_{48} - z_{85} z_{47}) + m_{MR} (z_{81} z_{48} - z_{82} z_{47}) \\
z_{497} &= -m_F (z_{39} z_{119} - z_{47} z_{120} - z_{48} z_{121}) - m_{MR} (z_{39} z_{116} - z_{47} z_{117} - z_{48} z_{118}) \\
z_{498} &= z_{423} + z_{431} + m_F (z_{84} z_{121} - z_{85} z_{120}) + m_{MR} (z_{81} z_{118} - z_{82} z_{117}) - z_{427} \\
z_{499} &= z_{422} + z_{428} + z_{430} - m_F (z_{83} z_{121} - z_{85} z_{119}) - m_{MR} (z_{80} z_{118} - z_{82} z_{116}) \\
z_{500} &= z_{421} + z_{429} + m_F (z_{83} z_{120} - z_{84} z_{119}) + m_{MR} (z_{80} z_{117} - z_{81} z_{116}) \\
z_{501} &= l_p z_{210} \\
z_{502} &= l_p z_{228} \\
z_{503} &= l_p z_{246} \\
z_{504} &= z_{435} + z_{210} z_{501} + z_{228} z_{502} + z_{246} z_{503} \\
z_{505} &= z_{436} + z_{211} z_{501} + z_{229} z_{502} + z_{247} z_{503} \\
z_{506} &= z_{212} z_{501} + z_{230} z_{502} + z_{248} z_{503} - z_{433} \\
z_{507} &= z_{437} + z_{215} z_{501} + z_{233} z_{502} + z_{251} z_{503} \\
z_{508} &= z_{434} + z_{214} z_{501} + z_{232} z_{502} + z_{250} z_{503} \\
z_{509} &= z_{432} + z_{213} z_{501} + z_{231} z_{502} + z_{249} z_{503} \\
z_{510} &= -z_{220} z_{501} - z_{238} z_{502} - z_{256} z_{503} \\
z_{511} &= -z_{221} z_{501} - z_{239} z_{502} - z_{257} z_{503} \\
z_{512} &= -z_{216} z_{501} - z_{234} z_{502} - z_{252} z_{503} \\
z_{513} &= -z_{219} z_{501} - z_{237} z_{502} - z_{255} z_{503} \\
z_{514} &= -z_{218} z_{501} - z_{236} z_{502} - z_{254} z_{503} \\
z_{515} &= -z_{217} z_{501} - z_{235} z_{502} - z_{253} z_{503} \\
z_{516} &= z_{222} z_{501} + z_{240} z_{502} + z_{258} z_{503}
\end{aligned}$$

5 Dynamic Equations of the Multi-Lift Model

$$\begin{aligned}
z_{517} &= z_{223} z_{501} + z_{241} z_{502} + z_{259} z_{503} \\
z_{518} &= z_{224} z_{501} + z_{242} z_{502} + z_{260} z_{503} \\
z_{519} &= z_{227} z_{501} + z_{245} z_{502} + z_{263} z_{503} \\
z_{520} &= z_{226} z_{501} + z_{244} z_{502} + z_{262} z_{503} \\
z_{521} &= z_{225} z_{501} + z_{243} z_{502} + z_{261} z_{503} \\
z_{522} &= z_{438} + z_{501} z_{366} + z_{502} z_{367} + z_{503} z_{368} \\
z_{523} &= l_{\eta} z_{211} \\
z_{524} &= l_{\eta} z_{229} \\
z_{525} &= l_{\eta} z_{247} \\
z_{526} &= z_{436} + z_{210} z_{523} + z_{228} z_{524} + z_{246} z_{525} \\
z_{527} &= z_{442} + z_{211} z_{523} + z_{229} z_{524} + z_{247} z_{525} \\
z_{528} &= z_{212} z_{523} + z_{230} z_{524} + z_{248} z_{525} - z_{440} \\
z_{529} &= z_{443} + z_{215} z_{523} + z_{233} z_{524} + z_{251} z_{525} \\
z_{530} &= z_{441} + z_{214} z_{523} + z_{232} z_{524} + z_{250} z_{525} \\
z_{531} &= z_{439} + z_{213} z_{523} + z_{231} z_{524} + z_{249} z_{525} \\
z_{532} &= -z_{220} z_{523} - z_{238} z_{524} - z_{256} z_{525} \\
z_{533} &= -z_{221} z_{523} - z_{239} z_{524} - z_{257} z_{525} \\
z_{534} &= -z_{216} z_{523} - z_{234} z_{524} - z_{252} z_{525} \\
z_{535} &= -z_{219} z_{523} - z_{237} z_{524} - z_{255} z_{525} \\
z_{536} &= -z_{218} z_{523} - z_{236} z_{524} - z_{254} z_{525} \\
z_{537} &= -z_{217} z_{523} - z_{235} z_{524} - z_{253} z_{525} \\
z_{538} &= z_{222} z_{523} + z_{240} z_{524} + z_{258} z_{525} \\
z_{539} &= z_{223} z_{523} + z_{241} z_{524} + z_{259} z_{525} \\
z_{540} &= z_{224} z_{523} + z_{242} z_{524} + z_{260} z_{525} \\
z_{541} &= z_{227} z_{523} + z_{245} z_{524} + z_{263} z_{525} \\
z_{542} &= z_{226} z_{523} + z_{244} z_{524} + z_{262} z_{525} \\
z_{543} &= z_{225} z_{523} + z_{243} z_{524} + z_{261} z_{525} \\
z_{544} &= z_{444} + z_{523} z_{366} + z_{524} z_{367} + z_{525} z_{368} \\
z_{545} &= l_{\eta} z_{212} \\
z_{546} &= l_{\eta} z_{230} \\
z_{547} &= l_{\eta} z_{248} \\
z_{548} &= z_{210} z_{545} + z_{228} z_{546} + z_{246} z_{547} - z_{433}
\end{aligned}$$

5 Dynamic Equations of the Multi-Lift Model

$$\begin{aligned}
z_{549} &= z_{211} z_{545} + z_{229} z_{546} + z_{247} z_{547} - z_{440} \\
z_{550} &= z_{446} + z_{212} z_{545} + z_{230} z_{546} + z_{248} z_{547} \\
z_{551} &= z_{448} + z_{215} z_{545} + z_{233} z_{546} + z_{251} z_{547} \\
z_{552} &= z_{447} + z_{214} z_{545} + z_{232} z_{546} + z_{250} z_{547} \\
z_{553} &= z_{445} + z_{213} z_{545} + z_{231} z_{546} + z_{249} z_{547} \\
z_{554} &= -z_{220} z_{545} - z_{238} z_{546} - z_{256} z_{547} \\
z_{555} &= -z_{221} z_{545} - z_{239} z_{546} - z_{257} z_{547} \\
z_{556} &= -z_{216} z_{545} - z_{234} z_{546} - z_{252} z_{547} \\
z_{557} &= -z_{219} z_{545} - z_{237} z_{546} - z_{255} z_{547} \\
z_{558} &= -z_{218} z_{545} - z_{236} z_{546} - z_{254} z_{547} \\
z_{559} &= -z_{217} z_{545} - z_{235} z_{546} - z_{253} z_{547} \\
z_{560} &= z_{222} z_{545} + z_{240} z_{546} + z_{258} z_{547} \\
z_{561} &= z_{223} z_{545} + z_{241} z_{546} + z_{259} z_{547} \\
z_{562} &= z_{224} z_{545} + z_{242} z_{546} + z_{260} z_{547} \\
z_{563} &= z_{227} z_{545} + z_{245} z_{546} + z_{263} z_{547} \\
z_{564} &= z_{226} z_{545} + z_{244} z_{546} + z_{262} z_{547} \\
z_{565} &= z_{225} z_{545} + z_{243} z_{546} + z_{261} z_{547} \\
z_{566} &= z_{449} + z_{545} z_{366} + z_{546} z_{367} + z_{547} z_{368} \\
z_{567} &= l_{\eta} z_{215} \\
z_{568} &= l_{\eta} z_{233} \\
z_{569} &= l_{\eta} z_{251} \\
z_{570} &= z_{437} + z_{210} z_{567} + z_{228} z_{568} + z_{246} z_{569} \\
z_{571} &= z_{443} + z_{211} z_{567} + z_{229} z_{568} + z_{247} z_{569} \\
z_{572} &= z_{448} + z_{212} z_{567} + z_{230} z_{568} + z_{248} z_{569} \\
z_{573} &= z_{450} + z_{215} z_{567} + z_{233} z_{568} + z_{251} z_{569} \\
z_{574} &= z_{452} + z_{214} z_{567} + z_{232} z_{568} + z_{250} z_{569} \\
z_{575} &= z_{451} + z_{213} z_{567} + z_{231} z_{568} + z_{249} z_{569} \\
z_{576} &= -z_{220} z_{567} - z_{238} z_{568} - z_{256} z_{569} \\
z_{577} &= -z_{221} z_{567} - z_{239} z_{568} - z_{257} z_{569} \\
z_{578} &= -z_{216} z_{567} - z_{234} z_{568} - z_{252} z_{569} \\
z_{579} &= -z_{219} z_{567} - z_{237} z_{568} - z_{255} z_{569} \\
z_{580} &= -z_{218} z_{567} - z_{236} z_{568} - z_{254} z_{569} \\
z_{581} &= -z_{217} z_{567} - z_{235} z_{568} - z_{253} z_{569}
\end{aligned}$$

5 Dynamic Equations of the Multi-Lift Model

$$\begin{aligned}
 z_{582} &= z_{222} z_{567} + z_{240} z_{568} + z_{258} z_{569} \\
 z_{583} &= z_{223} z_{567} + z_{241} z_{568} + z_{259} z_{569} \\
 z_{584} &= z_{224} z_{567} + z_{242} z_{568} + z_{260} z_{569} \\
 z_{585} &= z_{227} z_{567} + z_{245} z_{568} + z_{263} z_{569} \\
 z_{586} &= z_{226} z_{567} + z_{244} z_{568} + z_{262} z_{569} \\
 z_{587} &= z_{225} z_{567} + z_{243} z_{568} + z_{261} z_{569} \\
 z_{588} &= z_{453} + z_{567} z_{366} + z_{568} z_{367} + z_{569} z_{368} \\
 z_{589} &= l_{\eta} z_{214} \\
 z_{590} &= l_{\eta} z_{232} \\
 z_{591} &= l_{\eta} z_{250} \\
 z_{592} &= z_{434} + z_{210} z_{589} + z_{228} z_{590} + z_{246} z_{591} \\
 z_{593} &= z_{441} + z_{211} z_{589} + z_{229} z_{590} + z_{247} z_{591} \\
 z_{594} &= z_{447} + z_{212} z_{589} + z_{230} z_{590} + z_{248} z_{591} \\
 z_{595} &= z_{452} + z_{215} z_{589} + z_{233} z_{590} + z_{251} z_{591} \\
 z_{596} &= z_{454} + z_{214} z_{589} + z_{232} z_{590} + z_{250} z_{591} \\
 z_{597} &= z_{455} + z_{213} z_{589} + z_{231} z_{590} + z_{249} z_{591} \\
 z_{598} &= -z_{220} z_{589} - z_{238} z_{590} - z_{256} z_{591} \\
 z_{599} &= -z_{221} z_{589} - z_{239} z_{590} - z_{257} z_{591} \\
 z_{600} &= -z_{216} z_{589} - z_{234} z_{590} - z_{252} z_{591} \\
 z_{601} &= -z_{219} z_{589} - z_{237} z_{590} - z_{255} z_{591} \\
 z_{602} &= -z_{218} z_{589} - z_{236} z_{590} - z_{254} z_{591} \\
 z_{603} &= -z_{217} z_{589} - z_{235} z_{590} - z_{253} z_{591} \\
 z_{604} &= z_{222} z_{589} + z_{240} z_{590} + z_{258} z_{591} \\
 z_{605} &= z_{223} z_{589} + z_{241} z_{590} + z_{259} z_{591} \\
 z_{606} &= z_{224} z_{589} + z_{242} z_{590} + z_{260} z_{591} \\
 z_{607} &= z_{227} z_{589} + z_{245} z_{590} + z_{263} z_{591} \\
 z_{608} &= z_{226} z_{589} + z_{244} z_{590} + z_{262} z_{591} \\
 z_{609} &= z_{225} z_{589} + z_{243} z_{590} + z_{261} z_{591} \\
 z_{610} &= z_{456} + z_{589} z_{366} + z_{590} z_{367} + z_{591} z_{368} \\
 z_{611} &= l_{\eta} z_{213} \\
 z_{612} &= l_{\eta} z_{231} \\
 z_{613} &= l_{\eta} z_{249}
 \end{aligned}$$

5 Dynamic Equations of the Multi-Lift Model

$$\begin{aligned}
z_{614} &= z_{432} + z_{210} z_{611} + z_{228} z_{612} + z_{246} z_{613} \\
z_{615} &= z_{439} + z_{211} z_{611} + z_{229} z_{612} + z_{247} z_{613} \\
z_{616} &= z_{445} + z_{212} z_{611} + z_{230} z_{612} + z_{248} z_{613} \\
z_{617} &= z_{451} + z_{215} z_{611} + z_{233} z_{612} + z_{251} z_{613} \\
z_{618} &= z_{455} + z_{214} z_{611} + z_{232} z_{612} + z_{250} z_{613} \\
z_{619} &= z_{457} + z_{213} z_{611} + z_{231} z_{612} + z_{249} z_{613} \\
z_{620} &= -z_{220} z_{611} - z_{238} z_{612} - z_{256} z_{613} \\
z_{621} &= -z_{221} z_{611} - z_{239} z_{612} - z_{257} z_{613} \\
z_{622} &= -z_{216} z_{611} - z_{234} z_{612} - z_{252} z_{613} \\
z_{623} &= -z_{219} z_{611} - z_{237} z_{612} - z_{255} z_{613} \\
z_{624} &= -z_{218} z_{611} - z_{236} z_{612} - z_{254} z_{613} \\
z_{625} &= -z_{217} z_{611} - z_{235} z_{612} - z_{253} z_{613} \\
z_{626} &= z_{222} z_{611} + z_{240} z_{612} + z_{258} z_{613} \\
z_{627} &= z_{223} z_{611} + z_{241} z_{612} + z_{259} z_{613} \\
z_{628} &= z_{224} z_{611} + z_{242} z_{612} + z_{260} z_{613} \\
z_{629} &= z_{227} z_{611} + z_{245} z_{612} + z_{263} z_{613} \\
z_{630} &= z_{226} z_{611} + z_{244} z_{612} + z_{262} z_{613} \\
z_{631} &= z_{225} z_{611} + z_{243} z_{612} + z_{261} z_{613} \\
z_{632} &= z_{458} + z_{611} z_{366} + z_{612} z_{367} + z_{613} z_{368} \\
z_{633} &= l_{\eta} z_{220} \\
z_{634} &= l_{\eta} z_{238} \\
z_{635} &= l_{\eta} z_{256} \\
z_{636} &= -z_{210} z_{633} - z_{228} z_{634} - z_{246} z_{635} \\
z_{637} &= -z_{211} z_{633} - z_{229} z_{634} - z_{247} z_{635} \\
z_{638} &= -z_{212} z_{633} - z_{230} z_{634} - z_{248} z_{635} \\
z_{639} &= -z_{215} z_{633} - z_{233} z_{634} - z_{251} z_{635} \\
z_{640} &= -z_{214} z_{633} - z_{232} z_{634} - z_{250} z_{635} \\
z_{641} &= -z_{213} z_{633} - z_{231} z_{634} - z_{249} z_{635} \\
z_{642} &= z_{462} + z_{220} z_{633} + z_{238} z_{634} + z_{256} z_{635} \\
z_{643} &= z_{463} + z_{221} z_{633} + z_{239} z_{634} + z_{257} z_{635} \\
z_{644} &= z_{216} z_{633} + z_{234} z_{634} + z_{252} z_{635} - z_{460} \\
z_{645} &= z_{464} + z_{219} z_{633} + z_{237} z_{634} + z_{255} z_{635} \\
z_{646} &= z_{461} + z_{218} z_{633} + z_{236} z_{634} + z_{254} z_{635}
\end{aligned}$$

5 Dynamic Equations of the Multi-Lift Model

$$z_{647} = z_{459} + z_{217} z_{633} + z_{235} z_{634} + z_{253} z_{635}$$

$$z_{648} = -z_{222} z_{633} - z_{240} z_{634} - z_{258} z_{635}$$

$$z_{649} = -z_{223} z_{633} - z_{241} z_{634} - z_{259} z_{635}$$

$$z_{650} = -z_{224} z_{633} - z_{242} z_{634} - z_{260} z_{635}$$

$$z_{651} = -z_{227} z_{633} - z_{245} z_{634} - z_{263} z_{635}$$

$$z_{652} = -z_{226} z_{633} - z_{244} z_{634} - z_{262} z_{635}$$

$$z_{653} = -z_{225} z_{633} - z_{243} z_{634} - z_{261} z_{635}$$

$$z_{654} = z_{465} - z_{633} z_{366} - z_{634} z_{367} - z_{635} z_{368}$$

$$z_{655} = l_{\eta} z_{221}$$

$$z_{656} = l_{\eta} z_{239}$$

$$z_{657} = l_{\eta} z_{257}$$

$$z_{658} = -z_{210} z_{655} - z_{228} z_{656} - z_{246} z_{657}$$

$$z_{659} = -z_{211} z_{655} - z_{229} z_{656} - z_{247} z_{657}$$

$$z_{660} = -z_{212} z_{655} - z_{230} z_{656} - z_{248} z_{657}$$

$$z_{661} = -z_{215} z_{655} - z_{233} z_{656} - z_{251} z_{657}$$

$$z_{662} = -z_{214} z_{655} - z_{232} z_{656} - z_{250} z_{657}$$

$$z_{663} = -z_{213} z_{655} - z_{231} z_{656} - z_{249} z_{657}$$

$$z_{664} = z_{463} + z_{220} z_{655} + z_{238} z_{656} + z_{256} z_{657}$$

$$z_{665} = z_{469} + z_{221} z_{655} + z_{239} z_{656} + z_{257} z_{657}$$

$$z_{666} = z_{216} z_{655} + z_{234} z_{656} + z_{252} z_{657} - z_{467}$$

$$z_{667} = z_{470} + z_{219} z_{655} + z_{237} z_{656} + z_{255} z_{657}$$

$$z_{668} = z_{468} + z_{218} z_{655} + z_{236} z_{656} + z_{254} z_{657}$$

$$z_{669} = z_{466} + z_{217} z_{655} + z_{235} z_{656} + z_{253} z_{657}$$

$$z_{670} = -z_{222} z_{655} - z_{240} z_{656} - z_{258} z_{657}$$

$$z_{671} = -z_{223} z_{655} - z_{241} z_{656} - z_{259} z_{657}$$

$$z_{672} = -z_{224} z_{655} - z_{242} z_{656} - z_{260} z_{657}$$

$$z_{673} = -z_{227} z_{655} - z_{245} z_{656} - z_{263} z_{657}$$

$$z_{674} = -z_{226} z_{655} - z_{244} z_{656} - z_{262} z_{657}$$

$$z_{675} = -z_{225} z_{655} - z_{243} z_{656} - z_{261} z_{657}$$

$$z_{676} = z_{471} - z_{655} z_{366} - z_{656} z_{367} - z_{657} z_{368}$$

$$z_{677} = l_{\eta} z_{216}$$

$$z_{678} = l_{\eta} z_{234}$$

5 Dynamic Equations of the Multi-Lift Model

$$\begin{aligned}
 z_{679} &= l_{\eta} z_{252} \\
 z_{680} &= -z_{210} z_{677} - z_{228} z_{678} - z_{246} z_{679} \\
 z_{681} &= -z_{211} z_{677} - z_{229} z_{678} - z_{247} z_{679} \\
 z_{682} &= -z_{212} z_{677} - z_{230} z_{678} - z_{248} z_{679} \\
 z_{683} &= -z_{215} z_{677} - z_{233} z_{678} - z_{251} z_{679} \\
 z_{684} &= -z_{214} z_{677} - z_{232} z_{678} - z_{250} z_{679} \\
 z_{685} &= -z_{213} z_{677} - z_{231} z_{678} - z_{249} z_{679} \\
 z_{686} &= z_{220} z_{677} + z_{238} z_{678} + z_{256} z_{679} - z_{460} \\
 z_{687} &= z_{221} z_{677} + z_{239} z_{678} + z_{257} z_{679} - z_{467} \\
 z_{688} &= z_{473} + z_{216} z_{677} + z_{234} z_{678} + z_{252} z_{679} \\
 z_{689} &= z_{475} + z_{219} z_{677} + z_{237} z_{678} + z_{255} z_{679} \\
 z_{690} &= z_{474} + z_{218} z_{677} + z_{236} z_{678} + z_{254} z_{679} \\
 z_{691} &= z_{472} + z_{217} z_{677} + z_{235} z_{678} + z_{253} z_{679} \\
 z_{692} &= -z_{222} z_{677} - z_{240} z_{678} - z_{258} z_{679} \\
 z_{693} &= -z_{223} z_{677} - z_{241} z_{678} - z_{259} z_{679} \\
 z_{694} &= -z_{224} z_{677} - z_{242} z_{678} - z_{260} z_{679} \\
 z_{695} &= -z_{227} z_{677} - z_{245} z_{678} - z_{263} z_{679} \\
 z_{696} &= -z_{226} z_{677} - z_{244} z_{678} - z_{262} z_{679} \\
 z_{697} &= -z_{225} z_{677} - z_{243} z_{678} - z_{261} z_{679} \\
 z_{698} &= z_{476} - z_{677} z_{366} - z_{678} z_{367} - z_{679} z_{368} \\
 z_{699} &= l_{\eta} z_{219} \\
 z_{700} &= l_{\eta} z_{237} \\
 z_{701} &= l_{\eta} z_{255} \\
 z_{702} &= -z_{210} z_{699} - z_{228} z_{700} - z_{246} z_{701} \\
 z_{703} &= -z_{211} z_{699} - z_{229} z_{700} - z_{247} z_{701} \\
 z_{704} &= -z_{212} z_{699} - z_{230} z_{700} - z_{248} z_{701} \\
 z_{705} &= -z_{215} z_{699} - z_{233} z_{700} - z_{251} z_{701} \\
 z_{706} &= -z_{214} z_{699} - z_{232} z_{700} - z_{250} z_{701} \\
 z_{707} &= -z_{213} z_{699} - z_{231} z_{700} - z_{249} z_{701} \\
 z_{708} &= z_{464} + z_{220} z_{699} + z_{238} z_{700} + z_{256} z_{701} \\
 z_{709} &= z_{470} + z_{221} z_{699} + z_{239} z_{700} + z_{257} z_{701} \\
 z_{710} &= z_{475} + z_{216} z_{699} + z_{234} z_{700} + z_{252} z_{701}
 \end{aligned}$$

5 Dynamic Equations of the Multi-Lift Model

$$\begin{aligned}
z_{711} &= z_{450} + z_{219} z_{699} + z_{237} z_{700} + z_{255} z_{701} \\
z_{712} &= z_{452} + z_{218} z_{699} + z_{236} z_{700} + z_{254} z_{701} \\
z_{713} &= z_{451} + z_{217} z_{699} + z_{235} z_{700} + z_{253} z_{701} \\
z_{714} &= -z_{222} z_{699} - z_{240} z_{700} - z_{258} z_{701} \\
z_{715} &= -z_{223} z_{699} - z_{241} z_{700} - z_{259} z_{701} \\
z_{716} &= -z_{224} z_{699} - z_{242} z_{700} - z_{260} z_{701} \\
z_{717} &= -z_{227} z_{699} - z_{245} z_{700} - z_{263} z_{701} \\
z_{718} &= -z_{226} z_{699} - z_{244} z_{700} - z_{262} z_{701} \\
z_{719} &= -z_{225} z_{699} - z_{243} z_{700} - z_{261} z_{701} \\
z_{720} &= z_{477} - z_{699} z_{366} - z_{700} z_{367} - z_{701} z_{368} \\
z_{721} &= l_{\eta} z_{218} \\
z_{722} &= l_{\eta} z_{236} \\
z_{723} &= l_{\eta} z_{254} \\
z_{724} &= -z_{210} z_{721} - z_{228} z_{722} - z_{246} z_{723} \\
z_{725} &= -z_{211} z_{721} - z_{229} z_{722} - z_{247} z_{723} \\
z_{726} &= -z_{212} z_{721} - z_{230} z_{722} - z_{248} z_{723} \\
z_{727} &= -z_{215} z_{721} - z_{233} z_{722} - z_{251} z_{723} \\
z_{728} &= -z_{214} z_{721} - z_{232} z_{722} - z_{250} z_{723} \\
z_{729} &= -z_{213} z_{721} - z_{231} z_{722} - z_{249} z_{723} \\
z_{730} &= z_{461} + z_{220} z_{721} + z_{238} z_{722} + z_{256} z_{723} \\
z_{731} &= z_{468} + z_{221} z_{721} + z_{239} z_{722} + z_{257} z_{723} \\
z_{732} &= z_{474} + z_{216} z_{721} + z_{234} z_{722} + z_{252} z_{723} \\
z_{733} &= z_{452} + z_{219} z_{721} + z_{237} z_{722} + z_{255} z_{723} \\
z_{734} &= z_{454} + z_{218} z_{721} + z_{236} z_{722} + z_{254} z_{723} \\
z_{735} &= z_{455} + z_{217} z_{721} + z_{235} z_{722} + z_{253} z_{723} \\
z_{736} &= -z_{222} z_{721} - z_{240} z_{722} - z_{258} z_{723} \\
z_{737} &= -z_{223} z_{721} - z_{241} z_{722} - z_{259} z_{723} \\
z_{738} &= -z_{224} z_{721} - z_{242} z_{722} - z_{260} z_{723} \\
z_{739} &= -z_{227} z_{721} - z_{245} z_{722} - z_{263} z_{723} \\
z_{740} &= -z_{226} z_{721} - z_{244} z_{722} - z_{262} z_{723} \\
z_{741} &= -z_{225} z_{721} - z_{243} z_{722} - z_{261} z_{723} \\
z_{742} &= z_{478} - z_{721} z_{366} - z_{722} z_{367} - z_{723} z_{368}
\end{aligned}$$

5 Dynamic Equations of the Multi-Lift Model

$$\begin{aligned}
 z_{743} &= l_{\eta} z_{217} \\
 z_{744} &= l_{\eta} z_{235} \\
 z_{745} &= l_{\eta} z_{253} \\
 z_{746} &= -z_{210} z_{743} - z_{228} z_{744} - z_{246} z_{745} \\
 z_{747} &= -z_{211} z_{743} - z_{229} z_{744} - z_{247} z_{745} \\
 z_{748} &= -z_{212} z_{743} - z_{230} z_{744} - z_{248} z_{745} \\
 z_{749} &= -z_{215} z_{743} - z_{233} z_{744} - z_{251} z_{745} \\
 z_{750} &= -z_{214} z_{743} - z_{232} z_{744} - z_{250} z_{745} \\
 z_{751} &= -z_{213} z_{743} - z_{231} z_{744} - z_{249} z_{745} \\
 z_{752} &= z_{459} + z_{220} z_{743} + z_{238} z_{744} + z_{256} z_{745} \\
 z_{753} &= z_{466} + z_{221} z_{743} + z_{239} z_{744} + z_{257} z_{745} \\
 z_{754} &= z_{472} + z_{216} z_{743} + z_{234} z_{744} + z_{252} z_{745} \\
 z_{755} &= z_{451} + z_{219} z_{743} + z_{237} z_{744} + z_{255} z_{745} \\
 z_{756} &= z_{455} + z_{218} z_{743} + z_{236} z_{744} + z_{254} z_{745} \\
 z_{757} &= z_{457} + z_{217} z_{743} + z_{235} z_{744} + z_{253} z_{745} \\
 z_{758} &= -z_{222} z_{743} - z_{240} z_{744} - z_{258} z_{745} \\
 z_{759} &= -z_{223} z_{743} - z_{241} z_{744} - z_{259} z_{745} \\
 z_{760} &= -z_{224} z_{743} - z_{242} z_{744} - z_{260} z_{745} \\
 z_{761} &= -z_{227} z_{743} - z_{245} z_{744} - z_{263} z_{745} \\
 z_{762} &= -z_{226} z_{743} - z_{244} z_{744} - z_{262} z_{745} \\
 z_{763} &= -z_{225} z_{743} - z_{243} z_{744} - z_{261} z_{745} \\
 z_{764} &= z_{479} - z_{743} z_{366} - z_{744} z_{367} - z_{745} z_{368} \\
 z_{765} &= l_{\eta} z_{222} \\
 z_{766} &= l_{\eta} z_{240} \\
 z_{767} &= l_{\eta} z_{258} \\
 z_{768} &= z_{210} z_{765} + z_{228} z_{766} + z_{246} z_{767} \\
 z_{769} &= z_{211} z_{765} + z_{229} z_{766} + z_{247} z_{767} \\
 z_{770} &= z_{212} z_{765} + z_{230} z_{766} + z_{248} z_{767} \\
 z_{771} &= z_{215} z_{765} + z_{233} z_{766} + z_{251} z_{767} \\
 z_{772} &= z_{214} z_{765} + z_{232} z_{766} + z_{250} z_{767} \\
 z_{773} &= z_{213} z_{765} + z_{231} z_{766} + z_{249} z_{767} \\
 z_{774} &= -z_{220} z_{765} - z_{238} z_{766} - z_{256} z_{767}
 \end{aligned}$$

5 Dynamic Equations of the Multi-Lift Model

$$\begin{aligned}
 z_{775} &= -z_{221} z_{765} - z_{239} z_{766} - z_{257} z_{767} \\
 z_{776} &= -z_{216} z_{765} - z_{234} z_{766} - z_{252} z_{767} \\
 z_{777} &= -z_{219} z_{765} - z_{237} z_{766} - z_{255} z_{767} \\
 z_{778} &= -z_{218} z_{765} - z_{236} z_{766} - z_{254} z_{767} \\
 z_{779} &= -z_{217} z_{765} - z_{235} z_{766} - z_{253} z_{767} \\
 z_{780} &= z_{483} + z_{222} z_{765} + z_{240} z_{766} + z_{258} z_{767} \\
 z_{781} &= z_{484} + z_{223} z_{765} + z_{241} z_{766} + z_{259} z_{767} \\
 z_{782} &= z_{224} z_{765} + z_{242} z_{766} + z_{260} z_{767} - z_{481} \\
 z_{783} &= z_{485} + z_{227} z_{765} + z_{245} z_{766} + z_{263} z_{767} \\
 z_{784} &= z_{482} + z_{226} z_{765} + z_{244} z_{766} + z_{262} z_{767} \\
 z_{785} &= z_{480} + z_{225} z_{765} + z_{243} z_{766} + z_{261} z_{767} \\
 z_{786} &= z_{486} + z_{765} z_{366} + z_{766} z_{367} + z_{767} z_{368} \\
 z_{787} &= l_{\eta} z_{223} \\
 z_{788} &= l_{\eta} z_{241} \\
 z_{789} &= l_{\eta} z_{259} \\
 z_{790} &= z_{210} z_{787} + z_{228} z_{788} + z_{246} z_{789} \\
 z_{791} &= z_{211} z_{787} + z_{229} z_{788} + z_{247} z_{789} \\
 z_{792} &= z_{212} z_{787} + z_{230} z_{788} + z_{248} z_{789} \\
 z_{793} &= z_{215} z_{787} + z_{233} z_{788} + z_{251} z_{789} \\
 z_{794} &= z_{214} z_{787} + z_{232} z_{788} + z_{250} z_{789} \\
 z_{795} &= z_{213} z_{787} + z_{231} z_{788} + z_{249} z_{789} \\
 z_{796} &= -z_{220} z_{787} - z_{238} z_{788} - z_{256} z_{789} \\
 z_{797} &= -z_{221} z_{787} - z_{239} z_{788} - z_{257} z_{789} \\
 z_{798} &= -z_{216} z_{787} - z_{234} z_{788} - z_{252} z_{789} \\
 z_{799} &= -z_{219} z_{787} - z_{237} z_{788} - z_{255} z_{789} \\
 z_{800} &= -z_{218} z_{787} - z_{236} z_{788} - z_{254} z_{789} \\
 z_{801} &= -z_{217} z_{787} - z_{235} z_{788} - z_{253} z_{789} \\
 z_{802} &= z_{484} + z_{222} z_{787} + z_{240} z_{788} + z_{258} z_{789} \\
 z_{803} &= z_{490} + z_{223} z_{787} + z_{241} z_{788} + z_{259} z_{789} \\
 z_{804} &= z_{224} z_{787} + z_{242} z_{788} + z_{260} z_{789} - z_{488} \\
 z_{805} &= z_{491} + z_{227} z_{787} + z_{245} z_{788} + z_{263} z_{789} \\
 z_{806} &= z_{489} + z_{226} z_{787} + z_{244} z_{788} + z_{262} z_{789} \\
 z_{807} &= z_{487} + z_{225} z_{787} + z_{243} z_{788} + z_{261} z_{789}
 \end{aligned}$$

5 Dynamic Equations of the Multi-Lift Model

$$z_{808} = z_{492} + z_{787} z_{366} + z_{788} z_{367} + z_{789} z_{368}$$

$$z_{809} = l_{\eta} z_{224}$$

$$z_{810} = l_{\eta} z_{242}$$

$$z_{811} = l_{\eta} z_{260}$$

$$z_{812} = z_{210} z_{809} + z_{228} z_{810} + z_{246} z_{811}$$

$$z_{813} = z_{211} z_{809} + z_{229} z_{810} + z_{247} z_{811}$$

$$z_{814} = z_{212} z_{809} + z_{230} z_{810} + z_{248} z_{811}$$

$$z_{815} = z_{215} z_{809} + z_{233} z_{810} + z_{251} z_{811}$$

$$z_{816} = z_{214} z_{809} + z_{232} z_{810} + z_{250} z_{811}$$

$$z_{817} = z_{213} z_{809} + z_{231} z_{810} + z_{249} z_{811}$$

$$z_{818} = -z_{220} z_{809} - z_{238} z_{810} - z_{256} z_{811}$$

$$z_{819} = -z_{221} z_{809} - z_{239} z_{810} - z_{257} z_{811}$$

$$z_{820} = -z_{216} z_{809} - z_{234} z_{810} - z_{252} z_{811}$$

$$z_{821} = -z_{219} z_{809} - z_{237} z_{810} - z_{255} z_{811}$$

$$z_{822} = -z_{218} z_{809} - z_{236} z_{810} - z_{254} z_{811}$$

$$z_{823} = -z_{217} z_{809} - z_{235} z_{810} - z_{253} z_{811}$$

$$z_{824} = z_{222} z_{809} + z_{240} z_{810} + z_{258} z_{811} - z_{481}$$

$$z_{825} = z_{223} z_{809} + z_{241} z_{810} + z_{259} z_{811} - z_{488}$$

$$z_{826} = z_{494} + z_{224} z_{809} + z_{242} z_{810} + z_{260} z_{811}$$

$$z_{827} = z_{496} + z_{227} z_{809} + z_{245} z_{810} + z_{263} z_{811}$$

$$z_{828} = z_{495} + z_{226} z_{809} + z_{244} z_{810} + z_{262} z_{811}$$

$$z_{829} = z_{493} + z_{225} z_{809} + z_{243} z_{810} + z_{261} z_{811}$$

$$z_{830} = z_{497} + z_{809} z_{366} + z_{810} z_{367} + z_{811} z_{368}$$

$$z_{831} = l_{\eta} z_{227}$$

$$z_{832} = l_{\eta} z_{245}$$

$$z_{833} = l_{\eta} z_{263}$$

$$z_{834} = z_{210} z_{831} + z_{228} z_{832} + z_{246} z_{833}$$

$$z_{835} = z_{211} z_{831} + z_{229} z_{832} + z_{247} z_{833}$$

$$z_{836} = z_{212} z_{831} + z_{230} z_{832} + z_{248} z_{833}$$

$$z_{837} = z_{215} z_{831} + z_{233} z_{832} + z_{251} z_{833}$$

$$z_{838} = z_{214} z_{831} + z_{232} z_{832} + z_{250} z_{833}$$

$$z_{839} = z_{213} z_{831} + z_{231} z_{832} + z_{249} z_{833}$$

5 Dynamic Equations of the Multi-Lift Model

$$\begin{aligned}
 z_{840} &= -z_{220} z_{831} - z_{238} z_{832} - z_{256} z_{833} \\
 z_{841} &= -z_{221} z_{831} - z_{239} z_{832} - z_{257} z_{833} \\
 z_{842} &= -z_{216} z_{831} - z_{234} z_{832} - z_{252} z_{833} \\
 z_{843} &= -z_{219} z_{831} - z_{237} z_{832} - z_{255} z_{833} \\
 z_{844} &= -z_{218} z_{831} - z_{236} z_{832} - z_{254} z_{833} \\
 z_{845} &= -z_{217} z_{831} - z_{235} z_{832} - z_{253} z_{833} \\
 z_{846} &= z_{485} + z_{222} z_{831} + z_{240} z_{832} + z_{258} z_{833} \\
 z_{847} &= z_{491} + z_{223} z_{831} + z_{241} z_{832} + z_{259} z_{833} \\
 z_{848} &= z_{496} + z_{224} z_{831} + z_{242} z_{832} + z_{260} z_{833} \\
 z_{849} &= z_{450} + z_{227} z_{831} + z_{245} z_{832} + z_{263} z_{833} \\
 z_{850} &= z_{452} + z_{226} z_{831} + z_{244} z_{832} + z_{262} z_{833} \\
 z_{851} &= z_{451} + z_{225} z_{831} + z_{243} z_{832} + z_{261} z_{833} \\
 z_{852} &= z_{498} + z_{831} z_{366} + z_{832} z_{367} + z_{833} z_{368} \\
 z_{853} &= l_{\eta} z_{226} \\
 z_{854} &= l_{\eta} z_{244} \\
 z_{855} &= l_{\eta} z_{262} \\
 z_{856} &= z_{210} z_{853} + z_{228} z_{854} + z_{246} z_{855} \\
 z_{857} &= z_{211} z_{853} + z_{229} z_{854} + z_{247} z_{855} \\
 z_{858} &= z_{212} z_{853} + z_{230} z_{854} + z_{248} z_{855} \\
 z_{859} &= z_{215} z_{853} + z_{233} z_{854} + z_{251} z_{855} \\
 z_{860} &= z_{214} z_{853} + z_{232} z_{854} + z_{250} z_{855} \\
 z_{861} &= z_{213} z_{853} + z_{231} z_{854} + z_{249} z_{855} \\
 z_{862} &= -z_{220} z_{853} - z_{238} z_{854} - z_{256} z_{855} \\
 z_{863} &= -z_{221} z_{853} - z_{239} z_{854} - z_{257} z_{855} \\
 z_{864} &= -z_{216} z_{853} - z_{234} z_{854} - z_{252} z_{855} \\
 z_{865} &= -z_{219} z_{853} - z_{237} z_{854} - z_{255} z_{855} \\
 z_{866} &= -z_{218} z_{853} - z_{236} z_{854} - z_{254} z_{855} \\
 z_{867} &= -z_{217} z_{853} - z_{235} z_{854} - z_{253} z_{855} \\
 z_{868} &= z_{482} + z_{222} z_{853} + z_{240} z_{854} + z_{258} z_{855} \\
 z_{869} &= z_{489} + z_{223} z_{853} + z_{241} z_{854} + z_{259} z_{855} \\
 z_{870} &= z_{495} + z_{224} z_{853} + z_{242} z_{854} + z_{260} z_{855} \\
 z_{871} &= z_{452} + z_{227} z_{853} + z_{245} z_{854} + z_{263} z_{855} \\
 z_{872} &= z_{454} + z_{226} z_{853} + z_{244} z_{854} + z_{262} z_{855}
 \end{aligned}$$

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$$\begin{aligned}
 z_{873} &= z_{455} + z_{225} z_{853} + z_{243} z_{854} + z_{261} z_{855} \\
 z_{874} &= z_{499} + z_{853} z_{366} + z_{854} z_{367} + z_{855} z_{368} \\
 z_{875} &= l_{\eta} z_{225} \\
 z_{876} &= l_{\eta} z_{243} \\
 z_{877} &= l_{\eta} z_{261} \\
 z_{878} &= z_{210} z_{875} + z_{228} z_{876} + z_{246} z_{877} \\
 z_{879} &= z_{211} z_{875} + z_{229} z_{876} + z_{247} z_{877} \\
 z_{880} &= z_{212} z_{875} + z_{230} z_{876} + z_{248} z_{877} \\
 z_{881} &= z_{215} z_{875} + z_{233} z_{876} + z_{251} z_{877} \\
 z_{882} &= z_{214} z_{875} + z_{232} z_{876} + z_{250} z_{877} \\
 z_{883} &= z_{213} z_{875} + z_{231} z_{876} + z_{249} z_{877} \\
 z_{884} &= -z_{220} z_{875} - z_{238} z_{876} - z_{256} z_{877} \\
 z_{885} &= -z_{221} z_{875} - z_{239} z_{876} - z_{257} z_{877} \\
 z_{886} &= -z_{216} z_{875} - z_{234} z_{876} - z_{252} z_{877} \\
 z_{887} &= -z_{219} z_{875} - z_{237} z_{876} - z_{255} z_{877} \\
 z_{888} &= -z_{218} z_{875} - z_{236} z_{876} - z_{254} z_{877} \\
 z_{889} &= -z_{217} z_{875} - z_{235} z_{876} - z_{253} z_{877} \\
 z_{890} &= z_{480} + z_{222} z_{875} + z_{240} z_{876} + z_{258} z_{877} \\
 z_{891} &= z_{487} + z_{223} z_{875} + z_{241} z_{876} + z_{259} z_{877} \\
 z_{892} &= z_{493} + z_{224} z_{875} + z_{242} z_{876} + z_{260} z_{877} \\
 z_{893} &= z_{451} + z_{227} z_{875} + z_{245} z_{876} + z_{263} z_{877} \\
 z_{894} &= z_{455} + z_{226} z_{875} + z_{244} z_{876} + z_{262} z_{877} \\
 z_{895} &= z_{457} + z_{225} z_{875} + z_{243} z_{876} + z_{261} z_{877} \\
 z_{896} &= z_{500} + z_{875} z_{366} + z_{876} z_{367} + z_{877} z_{368} \\
 z_{897} &= z_{369} - z_{522} \\
 z_{898} &= z_{370} - z_{544} \\
 z_{899} &= z_{371} - z_{566} \\
 z_{900} &= z_{374} - z_{588} \\
 z_{901} &= z_{376} - z_{610} \\
 z_{902} &= z_{377} - z_{632} \\
 z_{903} &= z_{378} - z_{654} \\
 z_{904} &= z_{379} - z_{676} \\
 z_{905} &= z_{380} - z_{698} \\
 z_{906} &= z_{381} - z_{720} \\
 z_{907} &= z_{382} - z_{742} \\
 z_{908} &= z_{383} - z_{764} \\
 z_{909} &= z_{384} - z_{786} \\
 z_{910} &= z_{385} - z_{808} \\
 z_{911} &= z_{386} - z_{830} \\
 z_{912} &= z_{387} - z_{852} \\
 z_{913} &= z_{388} - z_{874} \\
 z_{914} &= z_{389} - z_{896}
 \end{aligned}$$

6 Dynamic Equations of the Translational Multi-Lift Model

The following sections present the dynamic equations of the translational model for multi-lift configurations, which is described in the main document [Ber13, pages 100ff.].

6.1 Dynamic Equations

The presented dynamic equations are the solution of

$$\ddot{\mathbf{q}} = \mathbf{a} + \mathbf{M}^{-1} \mathbf{A}^t (\mathbf{A} \mathbf{M}^{-1} \mathbf{A}^t)^* (\mathbf{b} - \mathbf{A} \mathbf{a})$$

The equation itself is described in the thesis main document, see [Ber13, page 102]. The coordinates $(\Delta x_i, \Delta y_i, \Delta z_i)$ describe the position difference between the i 'th helicopter and the load. The length of the modeled ropes, connecting helicopters and load, correspond to $l_{1,2,3}$. The average rope length is described by l . Normally, for simulation $l = l_{1,2,3}$ is assumed, but for initial positions based on real flight data l is only approximately equal to $l_{1,2,3}$. The variable *den* describes a common denominator of the equations. To achieve a compact set of equations the common denominator has been replaced by this variable.

$$\Delta x_{h1} = x_{h1} - x_l$$

$$\Delta y_{h1} = y_{h1} - y_l$$

$$\Delta z_{h1} = z_{h1} - z_l$$

$$\Delta x_{h2} = x_{h2} - x_l$$

$$\Delta y_{h2} = y_{h2} - y_l$$

$$\Delta z_{h2} = z_{h2} - z_l$$

$$\Delta x_{h3} = x_{h3} - x_l$$

$$\Delta y_{h3} = y_{h3} - y_l$$

$$\Delta z_{h3} = z_{h3} - z_l$$

6 Dynamic Equations of the Translational Multi-Lift Model

$$\begin{aligned} den = & \left(((m_{h3} + m_l) (\Delta x_{h3}^2 + \Delta y_{h3}^2 + \Delta z_{h3}^2) ((m_{h1} + m_l)(m_{h2} + m_l) (\Delta x_{h1}^2 + \right. \\ & \Delta y_{h1}^2 + \Delta z_{h1}^2) (\Delta x_{h2}^2 + \Delta y_{h2}^2 + \Delta z_{h2}^2)) / (m_{h1} m_{h2}) - (\Delta x_{h1} \Delta x_{h2} + \\ & \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})^2) / (m_{h3}) - (\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \\ & \Delta z_{h2} \Delta z_{h3}) ((m_{h1} + m_l) (\Delta x_{h1}^2 + \Delta y_{h1}^2 + \Delta z_{h1}^2) (\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \\ & \Delta z_{h2} \Delta z_{h3})) / (m_{h1}) - (\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2}) (\Delta x_{h1} \Delta x_{h3} + \\ & \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})) + (\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) ((\Delta x_{h1} \Delta x_{h2} + \\ & \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2}) (\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - ((m_{h2} + \\ & m_l) (\Delta x_{h2}^2 + \Delta y_{h2}^2 + \Delta z_{h2}^2) (\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})) / (m_{h2})) / (m_l^3) \end{aligned}$$

$$l_{h1} = \sqrt{\Delta x_{h1}^2 + \Delta y_{h1}^2 + \Delta z_{h1}^2}$$

$$l_{h2} = \sqrt{\Delta x_{h2}^2 + \Delta y_{h2}^2 + \Delta z_{h2}^2}$$

$$l_{h3} = \sqrt{\Delta x_{h3}^2 + \Delta y_{h3}^2 + \Delta z_{h3}^2}$$

$$l = (1) / (3) (l_{h1} + l_{h2} + l_{h3})$$

$$\begin{aligned} \ddot{q}_1 = & \left((\Delta x_{h1} ((m_{h1} ((\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) (\Delta x_{h2} \Delta x_{h3} + \right. \\ & \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - (l^2 (m_{h3} + m_l) (\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \\ & \Delta z_{h1} \Delta z_{h2}))) / (m_{h3})) (-\Delta x_{h2} F_{h2,1}^H - F_{h2,2}^H y_{h2} + F_{h2,2}^H y_l - F_{h2,3}^H z_{h2} + \\ & F_{h2,3}^H z_l - m_{h2} ((\dot{x}_{h2} - \dot{x}_l)^2 + (\dot{y}_{h2} - \dot{y}_l)^2 + (\dot{z}_{h2} - \dot{z}_l)^2)) / (m_{h2}) + (m_{h1} ((\Delta x_{h1} \Delta x_{h2} + \\ & \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2}) (\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - (l^2 (m_{h2} + \\ & m_l) (\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})) / (m_{h2})) (-\Delta x_{h3} F_{h3,1}^H - F_{h3,2}^H y_{h3} + \\ & F_{h3,2}^H y_l - F_{h3,3}^H z_{h3} + F_{h3,3}^H z_l - m_{h3} ((\dot{x}_{h3} - \dot{x}_l)^2 + (\dot{y}_{h3} - \dot{y}_l)^2 + \\ & (\dot{z}_{h3} - \dot{z}_l)^2)) / (m_{h3}) + ((l^4 (m_{h2} + m_l) (m_{h3} + m_l)) / (m_{h2} m_{h3}) - (\Delta x_{h2} \Delta x_{h3} + \\ & \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})^2) (-\Delta x_{h1} F_{h1,1}^H - F_{h1,2}^H y_{h1} + \\ & F_{h1,2}^H y_l - F_{h1,3}^H z_{h1} + F_{h1,3}^H z_l - m_{h1} ((\dot{x}_{h1} - \dot{x}_l)^2 + \\ & (\dot{y}_{h1} - \dot{y}_l)^2 + (\dot{z}_{h1} - \dot{z}_l)^2)) / (den m_l^2) + F_{h1,1}^H m_{h1} / (m_{h1}^2) \end{aligned}$$

$$\begin{aligned} \ddot{q}_2 = & \left((\Delta y_{h1} ((m_{h1} ((\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) (\Delta x_{h2} \Delta x_{h3} + \right. \\ & \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - (l^2 (m_{h3} + m_l) (\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \\ & \Delta z_{h1} \Delta z_{h2}))) / (m_{h3})) (-\Delta x_{h2} F_{h2,1}^H - F_{h2,2}^H y_{h2} + F_{h2,2}^H y_l - F_{h2,3}^H z_{h2} + \\ & F_{h2,3}^H z_l - m_{h2} ((\dot{x}_{h2} - \dot{x}_l)^2 + (\dot{y}_{h2} - \dot{y}_l)^2 + (\dot{z}_{h2} - \dot{z}_l)^2)) / (m_{h2}) + (m_{h1} ((\Delta x_{h1} \Delta x_{h2} + \\ & \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2}) (\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - (l^2 (m_{h2} + \\ & m_l) (\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})) / (m_{h2})) (-\Delta x_{h3} F_{h3,1}^H - F_{h3,2}^H y_{h3} + \\ & F_{h3,2}^H y_l - F_{h3,3}^H z_{h3} + F_{h3,3}^H z_l - m_{h3} ((\dot{x}_{h3} - \dot{x}_l)^2 + (\dot{y}_{h3} - \dot{y}_l)^2 + \\ & (\dot{z}_{h3} - \dot{z}_l)^2)) / (m_{h3}) + ((l^4 (m_{h2} + m_l) (m_{h3} + m_l)) / (m_{h2} m_{h3}) - (\Delta x_{h2} \Delta x_{h3} + \\ & \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})^2) (-\Delta x_{h1} F_{h1,1}^H - F_{h1,2}^H y_{h1} + \\ & F_{h1,2}^H y_l - F_{h1,3}^H z_{h1} + F_{h1,3}^H z_l - m_{h1} ((\dot{x}_{h1} - \dot{x}_l)^2 + \\ & (\dot{y}_{h1} - \dot{y}_l)^2 + (\dot{z}_{h1} - \dot{z}_l)^2)) / (den m_l^2) + F_{h1,2}^H m_{h1} / (m_{h1}^2) \end{aligned}$$

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$$\begin{aligned} \ddot{q}_3 = & (\Delta z_{h1} ((m_{h1} ((\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})(\Delta x_{h2} \Delta x_{h3} + \\ & \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - (l^2(m_{h3} + m_l)(\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \\ & \Delta z_{h1} \Delta z_{h2}))) / (m_{h3})) (-\Delta x_{h2} F_{h2,1}^H - F_{h2,2}^H y_{h2} + F_{h2,2}^H y_l - F_{h2,3}^H z_{h2} + \\ & F_{h2,3}^H z_l - m_{h2} ((\dot{x}_{h2} - \dot{x}_l)^2 + (\dot{y}_{h2} - \dot{y}_l)^2 + (\dot{z}_{h2} - \dot{z}_l)^2))) / (m_{h2}) + (m_{h1} ((\Delta x_{h1} \Delta x_{h2} + \\ & \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - (l^2(m_{h2} + \\ & m_l)(\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})) / (m_{h2})) (-\Delta x_{h3} F_{h3,1}^H - F_{h3,2}^H y_{h3} + \\ & F_{h3,2}^H y_l - F_{h3,3}^H z_{h3} + F_{h3,3}^H z_l - m_{h3} ((\dot{x}_{h3} - \dot{x}_l)^2 + (\dot{y}_{h3} - \dot{y}_l)^2 + \\ & (\dot{z}_{h3} - \dot{z}_l)^2))) / (m_{h3}) + ((l^4(m_{h2} + m_l)(m_{h3} + m_l)) / (m_{h2} m_{h3}) - (\Delta x_{h2} \Delta x_{h3} + \\ & \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})^2) (-\Delta x_{h1} F_{h1,1}^H - F_{h1,2}^H y_{h1} + \\ & F_{h1,2}^H y_l - F_{h1,3}^H z_{h1} + F_{h1,3}^H z_l - m_{h1} ((\dot{x}_{h1} - \dot{x}_l)^2 + (\dot{y}_{h1} - \dot{y}_l)^2 + \\ & (\dot{z}_{h1} - \dot{z}_l)^2))) / (den m_{h1}^2 m_l^2) + (F_{h1,3}^H) / (m_{h1}) - G \end{aligned}$$

$$\begin{aligned} \ddot{q}_4 = & ((\Delta x_{h2} ((m_{h2} ((\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})(\Delta x_{h2} \Delta x_{h3} + \\ & \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - (l^2(m_{h3} + m_l)(\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \\ & \Delta z_{h1} \Delta z_{h2}))) / (m_{h3})) (-\Delta x_{h1} F_{h1,1}^H - F_{h1,2}^H y_{h1} + F_{h1,2}^H y_l - F_{h1,3}^H z_{h1} + \\ & F_{h1,3}^H z_l - m_{h1} ((\dot{x}_{h1} - \dot{x}_l)^2 + (\dot{y}_{h1} - \dot{y}_l)^2 + (\dot{z}_{h1} - \dot{z}_l)^2))) / (m_{h1}) + (m_{h2} ((\Delta x_{h1} \Delta x_{h2} + \\ & \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})(\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) - (l^2(m_{h1} + \\ & m_l)(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})) / (m_{h1})) (-\Delta x_{h3} F_{h3,1}^H - F_{h3,2}^H y_{h3} + \\ & F_{h3,2}^H y_l - F_{h3,3}^H z_{h3} + F_{h3,3}^H z_l - m_{h3} ((\dot{x}_{h3} - \dot{x}_l)^2 + (\dot{y}_{h3} - \dot{y}_l)^2 + \\ & (\dot{z}_{h3} - \dot{z}_l)^2))) / (m_{h3}) + ((l^4(m_{h1} + m_l)(m_{h3} + m_l)) / (m_{h1} m_{h3}) - (\Delta x_{h1} \Delta x_{h3} + \\ & \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})^2) (-\Delta x_{h2} F_{h2,1}^H - F_{h2,2}^H y_{h2} + \\ & F_{h2,2}^H y_l - F_{h2,3}^H z_{h2} + F_{h2,3}^H z_l - m_{h2} ((\dot{x}_{h2} - \dot{x}_l)^2 + \\ & (\dot{y}_{h2} - \dot{y}_l)^2 + (\dot{z}_{h2} - \dot{z}_l)^2))) / (den m_l^2) + F_{h2,1}^H m_{h2} / (m_{h2}^2) \end{aligned}$$

$$\begin{aligned} \ddot{q}_5 = & ((\Delta y_{h2} ((m_{h2} ((\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})(\Delta x_{h2} \Delta x_{h3} + \\ & \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - (l^2(m_{h3} + m_l)(\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \\ & \Delta z_{h1} \Delta z_{h2}))) / (m_{h3})) (-\Delta x_{h1} F_{h1,1}^H - F_{h1,2}^H y_{h1} + F_{h1,2}^H y_l - F_{h1,3}^H z_{h1} + \\ & F_{h1,3}^H z_l - m_{h1} ((\dot{x}_{h1} - \dot{x}_l)^2 + (\dot{y}_{h1} - \dot{y}_l)^2 + (\dot{z}_{h1} - \dot{z}_l)^2))) / (m_{h1}) + (m_{h2} ((\Delta x_{h1} \Delta x_{h2} + \\ & \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})(\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) - (l^2(m_{h1} + \\ & m_l)(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})) / (m_{h1})) (-\Delta x_{h3} F_{h3,1}^H - F_{h3,2}^H y_{h3} + \\ & F_{h3,2}^H y_l - F_{h3,3}^H z_{h3} + F_{h3,3}^H z_l - m_{h3} ((\dot{x}_{h3} - \dot{x}_l)^2 + (\dot{y}_{h3} - \dot{y}_l)^2 + \\ & (\dot{z}_{h3} - \dot{z}_l)^2))) / (m_{h3}) + ((l^4(m_{h1} + m_l)(m_{h3} + m_l)) / (m_{h1} m_{h3}) - (\Delta x_{h1} \Delta x_{h3} + \\ & \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})^2) (-\Delta x_{h2} F_{h2,1}^H - F_{h2,2}^H y_{h2} + \\ & F_{h2,2}^H y_l - F_{h2,3}^H z_{h2} + F_{h2,3}^H z_l - m_{h2} ((\dot{x}_{h2} - \dot{x}_l)^2 + \\ & (\dot{y}_{h2} - \dot{y}_l)^2 + (\dot{z}_{h2} - \dot{z}_l)^2))) / (den m_l^2) + F_{h2,2}^H m_{h2} / (m_{h2}^2) \end{aligned}$$

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$$\begin{aligned} \ddot{q}_6 = & (\Delta z_{h2} ((m_{h2} ((\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})(\Delta x_{h2} \Delta x_{h3} + \\ & \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - (l^2(m_{h3} + m_l)(\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \\ & \Delta z_{h1} \Delta z_{h2}))) / (m_{h3})) (-\Delta x_{h1} F_{h1,1}^H - F_{h1,2}^H y_{h1} + F_{h1,2}^H y_l - F_{h1,3}^H z_{h1} + \\ & F_{h1,3}^H z_l - m_{h1} ((\dot{x}_{h1} - \dot{x}_l)^2 + (\dot{y}_{h1} - \dot{y}_l)^2 + (\dot{z}_{h1} - \dot{z}_l)^2))) / (m_{h1}) + (m_{h2} ((\Delta x_{h1} \Delta x_{h2} + \\ & \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})(\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) - (l^2(m_{h1} + \\ & m_l)(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}))) / (m_{h1})) (-\Delta x_{h3} F_{h3,1}^H - F_{h3,2}^H y_{h3} + \\ & F_{h3,2}^H y_l - F_{h3,3}^H z_{h3} + F_{h3,3}^H z_l - m_{h3} ((\dot{x}_{h3} - \dot{x}_l)^2 + (\dot{y}_{h3} - \dot{y}_l)^2 + \\ & (\dot{z}_{h3} - \dot{z}_l)^2))) / (m_{h3}) + ((l^4(m_{h1} + m_l)(m_{h3} + m_l)) / (m_{h1} m_{h3}) - (\Delta x_{h1} \Delta x_{h3} + \\ & \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})^2) (-\Delta x_{h2} F_{h2,1}^H - F_{h2,2}^H y_{h2} + \\ & F_{h2,2}^H y_l - F_{h2,3}^H z_{h2} + F_{h2,3}^H z_l - m_{h2} ((\dot{x}_{h2} - \dot{x}_l)^2 + (\dot{y}_{h2} - \dot{y}_l)^2 + \\ & (\dot{z}_{h2} - \dot{z}_l)^2))) / (den m_{h2}^2 m_l^2) + (F_{h2,3}^H) / (m_{h2}) - G \end{aligned}$$

$$\begin{aligned} \ddot{q}_7 = & ((\Delta x_{h3} ((m_{h3} ((\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})(\Delta x_{h2} \Delta x_{h3} + \\ & \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - (l^2(m_{h2} + m_l)(\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \\ & \Delta z_{h1} \Delta z_{h3}))) / (m_{h2})) (-\Delta x_{h1} F_{h1,1}^H - F_{h1,2}^H y_{h1} + F_{h1,2}^H y_l - F_{h1,3}^H z_{h1} + \\ & F_{h1,3}^H z_l - m_{h1} ((\dot{x}_{h1} - \dot{x}_l)^2 + (\dot{y}_{h1} - \dot{y}_l)^2 + (\dot{z}_{h1} - \dot{z}_l)^2))) / (m_{h1}) + (m_{h3} ((\Delta x_{h1} \Delta x_{h2} + \\ & \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})(\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) - (l^2(m_{h1} + \\ & m_l)(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}))) / (m_{h1})) (-\Delta x_{h2} F_{h2,1}^H - F_{h2,2}^H y_{h2} + \\ & F_{h2,2}^H y_l - F_{h2,3}^H z_{h2} + F_{h2,3}^H z_l - m_{h2} ((\dot{x}_{h2} - \dot{x}_l)^2 + (\dot{y}_{h2} - \dot{y}_l)^2 + \\ & (\dot{z}_{h2} - \dot{z}_l)^2))) / (m_{h2}) + ((l^4(m_{h1} + m_l)(m_{h2} + m_l)) / (m_{h1} m_{h2}) - (\Delta x_{h1} \Delta x_{h2} + \\ & \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})^2) (-\Delta x_{h3} F_{h3,1}^H - F_{h3,2}^H y_{h3} + \\ & F_{h3,2}^H y_l - F_{h3,3}^H z_{h3} + F_{h3,3}^H z_l - m_{h3} ((\dot{x}_{h3} - \dot{x}_l)^2 + \\ & (\dot{y}_{h3} - \dot{y}_l)^2 + (\dot{z}_{h3} - \dot{z}_l)^2))) / (den m_l^2) + F_{h3,1}^H m_{h3} / (m_{h3}^2) \end{aligned}$$

$$\begin{aligned} \ddot{q}_8 = & ((\Delta y_{h3} ((m_{h3} ((\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})(\Delta x_{h2} \Delta x_{h3} + \\ & \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - (l^2(m_{h2} + m_l)(\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \\ & \Delta z_{h1} \Delta z_{h3}))) / (m_{h2})) (-\Delta x_{h1} F_{h1,1}^H - F_{h1,2}^H y_{h1} + F_{h1,2}^H y_l - F_{h1,3}^H z_{h1} + \\ & F_{h1,3}^H z_l - m_{h1} ((\dot{x}_{h1} - \dot{x}_l)^2 + (\dot{y}_{h1} - \dot{y}_l)^2 + (\dot{z}_{h1} - \dot{z}_l)^2))) / (m_{h1}) + (m_{h3} ((\Delta x_{h1} \Delta x_{h2} + \\ & \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})(\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) - (l^2(m_{h1} + \\ & m_l)(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}))) / (m_{h1})) (-\Delta x_{h2} F_{h2,1}^H - F_{h2,2}^H y_{h2} + \\ & F_{h2,2}^H y_l - F_{h2,3}^H z_{h2} + F_{h2,3}^H z_l - m_{h2} ((\dot{x}_{h2} - \dot{x}_l)^2 + (\dot{y}_{h2} - \dot{y}_l)^2 + \\ & (\dot{z}_{h2} - \dot{z}_l)^2))) / (m_{h2}) + ((l^4(m_{h1} + m_l)(m_{h2} + m_l)) / (m_{h1} m_{h2}) - (\Delta x_{h1} \Delta x_{h2} + \\ & \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})^2) (-\Delta x_{h3} F_{h3,1}^H - F_{h3,2}^H y_{h3} + \\ & F_{h3,2}^H y_l - F_{h3,3}^H z_{h3} + F_{h3,3}^H z_l - m_{h3} ((\dot{x}_{h3} - \dot{x}_l)^2 + \\ & (\dot{y}_{h3} - \dot{y}_l)^2 + (\dot{z}_{h3} - \dot{z}_l)^2))) / (den m_l^2) + F_{h3,2}^H m_{h3} / (m_{h3}^2) \end{aligned}$$

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$$\begin{aligned} \ddot{q}_9 = & (\Delta z_{h3} ((m_{h3} ((\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})(\Delta x_{h2} \Delta x_{h3} + \\ & \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - (l^2(m_{h2} + m_l)(\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \\ & \Delta z_{h1} \Delta z_{h3}))) / (m_{h2})) (-\Delta x_{h1} F_{h1,1}^H - F_{h1,2}^H y_{h1} + F_{h1,2}^H y_l - F_{h1,3}^H z_{h1} + \\ & F_{h1,3}^H z_l - m_{h1} ((\dot{x}_{h1} - \dot{x}_l)^2 + (\dot{y}_{h1} - \dot{y}_l)^2 + (\dot{z}_{h1} - \dot{z}_l)^2))) / (m_{h1}) + (m_{h3} ((\Delta x_{h1} \Delta x_{h2} + \\ & \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})(\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) - (l^2(m_{h1} + \\ & m_l)(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}))) / (m_{h1})) (-\Delta x_{h2} F_{h2,1}^H - F_{h2,2}^H y_{h2} + \\ & F_{h2,2}^H y_l - F_{h2,3}^H z_{h2} + F_{h2,3}^H z_l - m_{h2} ((\dot{x}_{h2} - \dot{x}_l)^2 + (\dot{y}_{h2} - \dot{y}_l)^2 + \\ & (\dot{z}_{h2} - \dot{z}_l)^2))) / (m_{h2}) + ((l^4(m_{h1} + m_l)(m_{h2} + m_l)) / (m_{h1} m_{h2})) - (\Delta x_{h1} \Delta x_{h2} + \\ & \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})^2) (-\Delta x_{h3} F_{h3,1}^H - F_{h3,2}^H y_{h3} + \\ & F_{h3,2}^H y_l - F_{h3,3}^H z_{h3} + F_{h3,3}^H z_l - m_{h3} ((\dot{x}_{h3} - \dot{x}_l)^2 + (\dot{y}_{h3} - \dot{y}_l)^2 + \\ & (\dot{z}_{h3} - \dot{z}_l)^2))) / (den m_{h3}^2 m_l^2) + (F_{h3,3}^H) / (m_{h3}) - G \end{aligned}$$

$$\begin{aligned} \ddot{q}_{10} = & - (((\Delta x_{h3} ((\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \\ & \Delta z_{h2} \Delta z_{h3}) - (l^2(m_{h2} + m_l)(\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}))) / (m_{h2})) + \\ & \Delta x_{h2} ((\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \\ & \Delta z_{h2} \Delta z_{h3}) - (l^2(m_{h3} + m_l)(\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2}))) / (m_{h3})) + \\ & \Delta x_{h1} ((l^4(m_{h2} + m_l)(m_{h3} + m_l)) / (m_{h2} m_{h3})) - (\Delta x_{h2} \Delta x_{h3} + \\ & \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})^2) (-\Delta x_{h1} F_{h1,1}^H - F_{h1,2}^H y_{h1} + F_{h1,2}^H y_l - F_{h1,3}^H z_{h1} + \\ & F_{h1,3}^H z_l - m_{h1} ((\dot{x}_{h1} - \dot{x}_l)^2 + (\dot{y}_{h1} - \dot{y}_l)^2 + (\dot{z}_{h1} - \dot{z}_l)^2))) / (m_{h1}) + \\ & ((\Delta x_{h3} ((\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})(\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \\ & \Delta z_{h1} \Delta z_{h3}) - (l^2(m_{h1} + m_l)(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}))) / (m_{h1})) + \\ & \Delta x_{h1} ((\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \\ & \Delta z_{h2} \Delta z_{h3}) - (l^2(m_{h3} + m_l)(\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2}))) / (m_{h3})) + \\ & \Delta x_{h2} ((l^4(m_{h1} + m_l)(m_{h3} + m_l)) / (m_{h1} m_{h3})) - (\Delta x_{h1} \Delta x_{h3} + \\ & \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})^2) (-\Delta x_{h2} F_{h2,1}^H - F_{h2,2}^H y_{h2} + F_{h2,2}^H y_l - F_{h2,3}^H z_{h2} + \\ & F_{h2,3}^H z_l - m_{h2} ((\dot{x}_{h2} - \dot{x}_l)^2 + (\dot{y}_{h2} - \dot{y}_l)^2 + (\dot{z}_{h2} - \dot{z}_l)^2))) / (m_{h2}) + \\ & ((\Delta x_{h2} ((\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})(\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \\ & \Delta z_{h1} \Delta z_{h3}) - (l^2(m_{h1} + m_l)(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}))) / (m_{h1})) + \\ & \Delta x_{h1} ((\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \\ & \Delta z_{h2} \Delta z_{h3}) - (l^2(m_{h2} + m_l)(\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}))) / (m_{h2})) + \\ & \Delta x_{h3} ((l^4(m_{h1} + m_l)(m_{h2} + m_l)) / (m_{h1} m_{h2})) - (\Delta x_{h1} \Delta x_{h2} + \\ & \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})^2) (-\Delta x_{h3} F_{h3,1}^H - F_{h3,2}^H y_{h3} + F_{h3,2}^H y_l - F_{h3,3}^H z_{h3} + \\ & F_{h3,3}^H z_l - m_{h3} ((\dot{x}_{h3} - \dot{x}_l)^2 + (\dot{y}_{h3} - \dot{y}_l)^2 + (\dot{z}_{h3} - \dot{z}_l)^2))) / (m_{h3})) / (den m_l^3) \end{aligned}$$

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$$\begin{aligned}
\ddot{q}_{i1} = & - \left((\Delta y_{h3} ((\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \right. \\
& \Delta z_{h2} \Delta z_{h3}) - (l^2(m_{h2} + m_l)(\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})) / (m_{h2})) + \\
& \Delta y_{h2} ((\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \\
& \Delta z_{h2} \Delta z_{h3}) - (l^2(m_{h3} + m_l)(\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})) / (m_{h3})) + \\
& \Delta y_{h1} ((l^4(m_{h2} + m_l)(m_{h3} + m_l)) / (m_{h2} m_{h3}) - (\Delta x_{h2} \Delta x_{h3} + \\
& \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})^2) \left. \right) (-\Delta x_{h1} F_{h1,1}^H - F_{h1,2}^H y_{h1} + F_{h1,2}^H y_l - F_{h1,3}^H z_{h1} + \\
& F_{h1,3}^H z_l - m_{h1} ((\dot{x}_{h1} - \dot{x}_l)^2 + (\dot{y}_{h1} - \dot{y}_l)^2 + (\dot{z}_{h1} - \dot{z}_l)^2)) / (m_{h1}) + \\
& ((\Delta y_{h3} ((\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})(\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \\
& \Delta z_{h1} \Delta z_{h3}) - (l^2(m_{h1} + m_l)(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})) / (m_{h1})) + \\
& \Delta y_{h1} ((\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \\
& \Delta z_{h2} \Delta z_{h3}) - (l^2(m_{h3} + m_l)(\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})) / (m_{h3})) + \\
& \Delta y_{h2} ((l^4(m_{h1} + m_l)(m_{h3} + m_l)) / (m_{h1} m_{h3}) - (\Delta x_{h1} \Delta x_{h3} + \\
& \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})^2) \left. \right) (-\Delta x_{h2} F_{h2,1}^H - F_{h2,2}^H y_{h2} + F_{h2,2}^H y_l - F_{h2,3}^H z_{h2} + \\
& F_{h2,3}^H z_l - m_{h2} ((\dot{x}_{h2} - \dot{x}_l)^2 + (\dot{y}_{h2} - \dot{y}_l)^2 + (\dot{z}_{h2} - \dot{z}_l)^2)) / (m_{h2}) + \\
& ((\Delta y_{h2} ((\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})(\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \\
& \Delta z_{h1} \Delta z_{h3}) - (l^2(m_{h1} + m_l)(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})) / (m_{h1})) + \\
& \Delta y_{h1} ((\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \\
& \Delta z_{h2} \Delta z_{h3}) - (l^2(m_{h2} + m_l)(\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})) / (m_{h2})) + \\
& \Delta y_{h3} ((l^4(m_{h1} + m_l)(m_{h2} + m_l)) / (m_{h1} m_{h2}) - (\Delta x_{h1} \Delta x_{h2} + \\
& \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})^2) \left. \right) (-\Delta x_{h3} F_{h3,1}^H - F_{h3,2}^H y_{h3} + F_{h3,2}^H y_l - F_{h3,3}^H z_{h3} + \\
& F_{h3,3}^H z_l - m_{h3} ((\dot{x}_{h3} - \dot{x}_l)^2 + (\dot{y}_{h3} - \dot{y}_l)^2 + (\dot{z}_{h3} - \dot{z}_l)^2)) / (m_{h3}) / (den m_l^3)
\end{aligned}$$

$$\begin{aligned}
 \ddot{q}_{12} = & - \left((\Delta z_{h3} ((\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \right. \\
 & \Delta z_{h2} \Delta z_{h3}) - (l^2(m_{h2} + m_l)(\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})) / (m_{h2})) + \\
 & \Delta z_{h2} ((\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \\
 & \Delta z_{h2} \Delta z_{h3}) - (l^2(m_{h3} + m_l)(\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})) / (m_{h3})) + \\
 & \Delta z_{h1} ((l^4(m_{h2} + m_l)(m_{h3} + m_l)) / (m_{h2} m_{h3}) - (\Delta x_{h2} \Delta x_{h3} + \\
 & \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})^2) \left. \right) (-\Delta x_{h1} F_{h1,1}^H - F_{h1,2}^H y_{h1} + F_{h1,2}^H y_l - F_{h1,3}^H z_{h1} + \\
 & F_{h1,3}^H z_l - m_{h1} ((\dot{x}_{h1} - \dot{x}_l)^2 + (\dot{y}_{h1} - \dot{y}_l)^2 + (\dot{z}_{h1} - \dot{z}_l)^2)) / (m_{h1}) + \\
 & ((\Delta z_{h3} ((\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})(\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \\
 & \Delta z_{h1} \Delta z_{h3}) - (l^2(m_{h1} + m_l)(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})) / (m_{h1})) + \\
 & \Delta z_{h1} ((\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \\
 & \Delta z_{h2} \Delta z_{h3}) - (l^2(m_{h3} + m_l)(\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})) / (m_{h3})) + \\
 & \Delta z_{h2} ((l^4(m_{h1} + m_l)(m_{h3} + m_l)) / (m_{h1} m_{h3}) - (\Delta x_{h1} \Delta x_{h3} + \\
 & \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})^2) \left. \right) (-\Delta x_{h2} F_{h2,1}^H - F_{h2,2}^H y_{h2} + F_{h2,2}^H y_l - F_{h2,3}^H z_{h2} + \\
 & F_{h2,3}^H z_l - m_{h2} ((\dot{x}_{h2} - \dot{x}_l)^2 + (\dot{y}_{h2} - \dot{y}_l)^2 + (\dot{z}_{h2} - \dot{z}_l)^2)) / (m_{h2}) + \\
 & ((\Delta z_{h2} ((\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})(\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \\
 & \Delta z_{h1} \Delta z_{h3}) - (l^2(m_{h1} + m_l)(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})) / (m_{h1})) + \\
 & \Delta z_{h1} ((\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \\
 & \Delta z_{h2} \Delta z_{h3}) - (l^2(m_{h2} + m_l)(\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})) / (m_{h2})) + \\
 & \Delta z_{h3} ((l^4(m_{h1} + m_l)(m_{h2} + m_l)) / (m_{h1} m_{h2}) - (\Delta x_{h1} \Delta x_{h2} + \\
 & \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})^2) \left. \right) (-\Delta x_{h3} F_{h3,1}^H - F_{h3,2}^H y_{h3} + F_{h3,2}^H y_l - F_{h3,3}^H z_{h3} + \\
 & F_{h3,3}^H z_l - m_{h3} ((\dot{x}_{h3} - \dot{x}_l)^2 + (\dot{y}_{h3} - \dot{y}_l)^2 + (\dot{z}_{h3} - \dot{z}_l)^2)) / (m_{h3})) / (den m_l^3) - G
 \end{aligned}$$

6.2 Inverse Dynamic Equations

The solution of the inverse dynamic equations is calculated using Singular Value Decomposition. Therefore, the dynamic equations are rewritten into

$$\begin{aligned}
 0 &= \mathbf{a} + \mathbf{M}^{-1} \mathbf{A}^t (\mathbf{A} \mathbf{M}^{-1} \mathbf{A}^t)^* (\mathbf{b} - \mathbf{A} \mathbf{a}) - \ddot{\mathbf{q}} \\
 0 &= \mathbf{C} \mathbf{x} - \mathbf{d}
 \end{aligned}$$

The equation itself is described in the thesis main document, see [Ber13, page 103]. The matrix \mathbf{C} and the vector \mathbf{d} are given by the following equations

$$\begin{aligned}
 \Delta x_{h1} &= x_{h1} - x_l \\
 \Delta y_{h1} &= y_{h1} - y_l \\
 \Delta z_{h1} &= z_{h1} - z_l \\
 \Delta x_{h2} &= x_{h2} - x_l \\
 \Delta y_{h2} &= y_{h2} - y_l
 \end{aligned}$$

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$$\Delta z_{h2} = z_{h2} - z_l$$

$$\Delta x_{h3} = x_{h3} - x_l$$

$$\Delta y_{h3} = y_{h3} - y_l$$

$$\Delta z_{h3} = z_{h3} - z_l$$

$$\begin{aligned} den = & \left((m_{h3} + m_l) (\Delta x_{h3}^2 + \Delta y_{h3}^2 + \Delta z_{h3}^2) \left((m_{h1} + m_l)(m_{h2} + m_l) (\Delta x_{h1}^2 + \right. \right. \\ & \Delta y_{h1}^2 + \Delta z_{h1}^2) (\Delta x_{h2}^2 + \Delta y_{h2}^2 + \Delta z_{h2}^2) \left. \left. / (m_{h1}m_{h2}) - (\Delta x_{h1}\Delta x_{h2} + \right. \right. \\ & \Delta y_{h1}\Delta y_{h2} + \Delta z_{h1}\Delta z_{h2})^2 \left. \left. \right) / (m_{h3}) - (\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \right. \\ & \Delta z_{h2}\Delta z_{h3}) \left((m_{h1} + m_l) (\Delta x_{h1}^2 + \Delta y_{h1}^2 + \Delta z_{h1}^2) (\Delta x_{h2}\Delta x_{h3} + \right. \\ & \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3}) \left. \left. / (m_{h1}) - (\Delta x_{h1}\Delta x_{h2} + \Delta y_{h1}\Delta y_{h2} + \right. \right. \\ & \Delta z_{h1}\Delta z_{h2})(\Delta x_{h1}\Delta x_{h3} + \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3}) \left. \left. \right) + (\Delta x_{h1}\Delta x_{h3} + \right. \\ & \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3}) \left((\Delta x_{h1}\Delta x_{h2} + \Delta y_{h1}\Delta y_{h2} + \Delta z_{h1}\Delta z_{h2})(\Delta x_{h2}\Delta x_{h3} + \right. \\ & \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3}) - (m_{h2} + m_l) (\Delta x_{h2}^2 + \Delta y_{h2}^2 + \Delta z_{h2}^2) \\ & \left. \left. (\Delta x_{h1}\Delta x_{h3} + \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3}) \right) / (m_{h2}) \right) / (m_l^3) \end{aligned}$$

$$l_{h1} = \sqrt{\Delta x_{h1}^2 + \Delta y_{h1}^2 + \Delta z_{h1}^2}$$

$$l_{h2} = \sqrt{\Delta x_{h2}^2 + \Delta y_{h2}^2 + \Delta z_{h2}^2}$$

$$l_{h3} = \sqrt{\Delta x_{h3}^2 + \Delta y_{h3}^2 + \Delta z_{h3}^2}$$

$$l = (1)/(3) (l_{h1} + l_{h2} + l_{h3})$$

$$C(1, 1) = (den m_{h1}m_{h2}m_{h3}m_l^2 + \Delta x_{h1}^2 (m_{h2}m_{h3} ((\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3})^2 - l^4) - l^4 m_l(m_{h2} + m_{h3}) + l^4 (-m_l^2))) / (den m_{h1}^2 m_{h2}m_{h3}m_l^2)$$

$$C(1, 2) = (\Delta x_{h1}(y_{h1} - y_l) (m_{h2}m_{h3} ((\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3})^2 - l^4) - l^4 m_l(m_{h2} + m_{h3}) + l^4 (-m_l^2))) / (den m_{h1}^2 m_{h2}m_{h3}m_l^2)$$

$$C(1, 3) = (\Delta x_{h1}(z_{h1} - z_l) (m_{h2}m_{h3} ((\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3})^2 - l^4) - l^4 m_l(m_{h2} + m_{h3}) + l^4 (-m_l^2))) / (den m_{h1}^2 m_{h2}m_{h3}m_l^2)$$

$$C(1, 4) = (\Delta x_{h1}\Delta x_{h2} (l^2 m_l (\Delta x_{h1}\Delta x_{h2} + \Delta y_{h1}\Delta y_{h2} + \Delta z_{h1}\Delta z_{h2}) - m_{h3} ((\Delta x_{h1}\Delta x_{h3} + \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3})(\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3}) - l^2 (\Delta x_{h1}\Delta x_{h2} + \Delta y_{h1}\Delta y_{h2} + \Delta z_{h1}\Delta z_{h2})))) / (den m_{h1}m_{h2}m_{h3}m_l^2)$$

$$C(1, 5) = - (\Delta x_{h1}(y_{h2} - y_l) (m_{h3} ((\Delta x_{h1}\Delta x_{h3} + \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3})(\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3}) - l^2 (\Delta x_{h1}\Delta x_{h2} + \Delta y_{h1}\Delta y_{h2} + \Delta z_{h1}\Delta z_{h2})) - l^2 m_l (\Delta x_{h1}\Delta x_{h2} + \Delta y_{h1}\Delta y_{h2} + \Delta z_{h1}\Delta z_{h2}))) / (den m_{h1}m_{h2}m_{h3}m_l^2)$$

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$$\begin{aligned}
C(1, 6) &= -(\Delta x_{h1}(z_{h2} - z_l) (m_{h3} ((\Delta x_{h1}\Delta x_{h3} + \Delta y_{h1}\Delta y_{h3} + \\
&\quad \Delta z_{h1}\Delta z_{h3})(\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3}) - l^2(\Delta x_{h1}\Delta x_{h2} + \\
&\quad \Delta y_{h1}\Delta y_{h2} + \Delta z_{h1}\Delta z_{h2})) - l^2 m_l(\Delta x_{h1}\Delta x_{h2} + \\
&\quad \Delta y_{h1}\Delta y_{h2} + \Delta z_{h1}\Delta z_{h2}))) / (den m_{h1}m_{h2}m_{h3}m_l^2) \\
C(1, 7) &= (\Delta x_{h1}\Delta x_{h3} (l^2 m_l(\Delta x_{h1}\Delta x_{h3} + \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3}) - m_{h2} ((\Delta x_{h1}\Delta x_{h2} + \\
&\quad \Delta y_{h1}\Delta y_{h2} + \Delta z_{h1}\Delta z_{h2})(\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \\
&\quad \Delta z_{h2}\Delta z_{h3}) - l^2(\Delta x_{h1}\Delta x_{h3} + \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3})))) / (den m_{h1}m_{h2}m_{h3}m_l^2) \\
C(1, 8) &= -(\Delta x_{h1}(y_{h3} - y_l) (m_{h2} ((\Delta x_{h1}\Delta x_{h2} + \Delta y_{h1}\Delta y_{h2} + \\
&\quad \Delta z_{h1}\Delta z_{h2})(\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3}) - l^2(\Delta x_{h1}\Delta x_{h3} + \\
&\quad \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3})) - l^2 m_l(\Delta x_{h1}\Delta x_{h3} + \\
&\quad \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3}))) / (den m_{h1}m_{h2}m_{h3}m_l^2) \\
C(1, 9) &= -(\Delta x_{h1}(z_{h3} - z_l) (m_{h2} ((\Delta x_{h1}\Delta x_{h2} + \Delta y_{h1}\Delta y_{h2} + \\
&\quad \Delta z_{h1}\Delta z_{h2})(\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3}) - l^2(\Delta x_{h1}\Delta x_{h3} + \\
&\quad \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3})) - l^2 m_l(\Delta x_{h1}\Delta x_{h3} + \\
&\quad \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3}))) / (den m_{h1}m_{h2}m_{h3}m_l^2) \\
C(2, 1) &= (\Delta x_{h1}\Delta y_{h1} (m_{h2}m_{h3} ((\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \\
&\quad \Delta z_{h2}\Delta z_{h3})^2 - l^4) - l^4 m_l(m_{h2} + m_{h3}) + l^4 (-m_l^2))) / (den m_{h1}^2 m_{h2}m_{h3}m_l^2) \\
C(2, 2) &= (den m_{h1}m_{h2}m_{h3}m_l^2 + \Delta y_{h1}(y_{h1} - y_l) (m_{h2}m_{h3} ((\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \\
&\quad \Delta z_{h2}\Delta z_{h3})^2 - l^4) - l^4 m_l(m_{h2} + m_{h3}) + l^4 (-m_l^2))) / (den m_{h1}^2 m_{h2}m_{h3}m_l^2) \\
C(2, 3) &= (\Delta y_{h1}(z_{h1} - z_l) (m_{h2}m_{h3} ((\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \\
&\quad \Delta z_{h2}\Delta z_{h3})^2 - l^4) - l^4 m_l(m_{h2} + m_{h3}) + l^4 (-m_l^2))) / (den m_{h1}^2 m_{h2}m_{h3}m_l^2) \\
C(2, 4) &= (\Delta x_{h2}\Delta y_{h1} (l^2 m_l(\Delta x_{h1}\Delta x_{h2} + \Delta y_{h1}\Delta y_{h2} + \Delta z_{h1}\Delta z_{h2}) - m_{h3} ((\Delta x_{h1}\Delta x_{h3} + \\
&\quad \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3})(\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \\
&\quad \Delta z_{h2}\Delta z_{h3}) - l^2(\Delta x_{h1}\Delta x_{h2} + \Delta y_{h1}\Delta y_{h2} + \Delta z_{h1}\Delta z_{h2})))) / (den m_{h1}m_{h2}m_{h3}m_l^2) \\
C(2, 5) &= -(\Delta y_{h1}(y_{h2} - y_l) (m_{h3} ((\Delta x_{h1}\Delta x_{h3} + \Delta y_{h1}\Delta y_{h3} + \\
&\quad \Delta z_{h1}\Delta z_{h3})(\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3}) - l^2(\Delta x_{h1}\Delta x_{h2} + \\
&\quad \Delta y_{h1}\Delta y_{h2} + \Delta z_{h1}\Delta z_{h2})) - l^2 m_l(\Delta x_{h1}\Delta x_{h2} + \\
&\quad \Delta y_{h1}\Delta y_{h2} + \Delta z_{h1}\Delta z_{h2}))) / (den m_{h1}m_{h2}m_{h3}m_l^2) \\
C(2, 6) &= -(\Delta y_{h1}(z_{h2} - z_l) (m_{h3} ((\Delta x_{h1}\Delta x_{h3} + \Delta y_{h1}\Delta y_{h3} + \\
&\quad \Delta z_{h1}\Delta z_{h3})(\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3}) - l^2(\Delta x_{h1}\Delta x_{h2} + \\
&\quad \Delta y_{h1}\Delta y_{h2} + \Delta z_{h1}\Delta z_{h2})) - l^2 m_l(\Delta x_{h1}\Delta x_{h2} + \\
&\quad \Delta y_{h1}\Delta y_{h2} + \Delta z_{h1}\Delta z_{h2}))) / (den m_{h1}m_{h2}m_{h3}m_l^2)
\end{aligned}$$

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$$\begin{aligned}
C(3, 8) &= -(\Delta z_{h1}(y_{h3} - y_l) (m_{h2} ((\Delta x_{h1}\Delta x_{h2} + \Delta y_{h1}\Delta y_{h2} + \\
&\quad \Delta z_{h1}\Delta z_{h2})(\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3}) - l^2(\Delta x_{h1}\Delta x_{h3} + \\
&\quad \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3})) - l^2 m_l(\Delta x_{h1}\Delta x_{h3} + \\
&\quad \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3}))) / (den m_{h1}m_{h2}m_{h3}m_l^2) \\
C(3, 9) &= -(\Delta z_{h1}(z_{h3} - z_l) (m_{h2} ((\Delta x_{h1}\Delta x_{h2} + \Delta y_{h1}\Delta y_{h2} + \\
&\quad \Delta z_{h1}\Delta z_{h2})(\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3}) - l^2(\Delta x_{h1}\Delta x_{h3} + \\
&\quad \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3})) - l^2 m_l(\Delta x_{h1}\Delta x_{h3} + \\
&\quad \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3}))) / (den m_{h1}m_{h2}m_{h3}m_l^2) \\
C(4, 1) &= (\Delta x_{h1}\Delta x_{h2} (l^2 m_l(\Delta x_{h1}\Delta x_{h2} + \Delta y_{h1}\Delta y_{h2} + \Delta z_{h1}\Delta z_{h2}) - m_{h3} ((\Delta x_{h1}\Delta x_{h3} + \\
&\quad \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3})(\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \\
&\quad \Delta z_{h2}\Delta z_{h3}) - l^2(\Delta x_{h1}\Delta x_{h2} + \Delta y_{h1}\Delta y_{h2} + \Delta z_{h1}\Delta z_{h2})))) / (den m_{h1}m_{h2}m_{h3}m_l^2) \\
C(4, 2) &= -(\Delta x_{h2}(y_{h1} - y_l) (m_{h3} ((\Delta x_{h1}\Delta x_{h3} + \Delta y_{h1}\Delta y_{h3} + \\
&\quad \Delta z_{h1}\Delta z_{h3})(\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3}) - l^2(\Delta x_{h1}\Delta x_{h2} + \\
&\quad \Delta y_{h1}\Delta y_{h2} + \Delta z_{h1}\Delta z_{h2})) - l^2 m_l(\Delta x_{h1}\Delta x_{h2} + \\
&\quad \Delta y_{h1}\Delta y_{h2} + \Delta z_{h1}\Delta z_{h2}))) / (den m_{h1}m_{h2}m_{h3}m_l^2) \\
C(4, 3) &= -(\Delta x_{h2}(z_{h1} - z_l) (m_{h3} ((\Delta x_{h1}\Delta x_{h3} + \Delta y_{h1}\Delta y_{h3} + \\
&\quad \Delta z_{h1}\Delta z_{h3})(\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3}) - l^2(\Delta x_{h1}\Delta x_{h2} + \\
&\quad \Delta y_{h1}\Delta y_{h2} + \Delta z_{h1}\Delta z_{h2})) - l^2 m_l(\Delta x_{h1}\Delta x_{h2} + \\
&\quad \Delta y_{h1}\Delta y_{h2} + \Delta z_{h1}\Delta z_{h2}))) / (den m_{h1}m_{h2}m_{h3}m_l^2) \\
C(4, 4) &= (m_l^2 (den m_{h1}m_{h2}m_{h3} - \Delta x_{h2}^2 l^4) + \Delta x_{h2}^2 m_{h1}m_{h3} ((\Delta x_{h1}\Delta x_{h3} + \Delta y_{h1}\Delta y_{h3} + \\
&\quad \Delta z_{h1}\Delta z_{h3})^2 - l^4) - \Delta x_{h2}^2 l^4 m_l(m_{h1} + m_{h3})) / (den m_{h1}m_{h2}^2 m_{h3}m_l^2) \\
C(4, 5) &= (\Delta x_{h2}(y_{h2} - y_l) (m_{h1}m_{h3} ((\Delta x_{h1}\Delta x_{h3} + \Delta y_{h1}\Delta y_{h3} + \\
&\quad \Delta z_{h1}\Delta z_{h3})^2 - l^4) - l^4 m_l(m_{h1} + m_{h3}) + l^4 (-m_l^2))) / (den m_{h1}m_{h2}^2 m_{h3}m_l^2) \\
C(4, 6) &= (\Delta x_{h2}(z_{h2} - z_l) (m_{h1}m_{h3} ((\Delta x_{h1}\Delta x_{h3} + \Delta y_{h1}\Delta y_{h3} + \\
&\quad \Delta z_{h1}\Delta z_{h3})^2 - l^4) - l^4 m_l(m_{h1} + m_{h3}) + l^4 (-m_l^2))) / (den m_{h1}m_{h2}^2 m_{h3}m_l^2) \\
C(4, 7) &= (\Delta x_{h2}\Delta x_{h3} (l^2 m_l(\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3}) - m_{h1} ((\Delta x_{h1}\Delta x_{h2} + \\
&\quad \Delta y_{h1}\Delta y_{h2} + \Delta z_{h1}\Delta z_{h2})(\Delta x_{h1}\Delta x_{h3} + \Delta y_{h1}\Delta y_{h3} + \\
&\quad \Delta z_{h1}\Delta z_{h3}) - l^2(\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3})))) / (den m_{h1}m_{h2}m_{h3}m_l^2) \\
C(4, 8) &= -(\Delta x_{h2}(y_{h3} - y_l) (m_{h1} ((\Delta x_{h1}\Delta x_{h2} + \Delta y_{h1}\Delta y_{h2} + \\
&\quad \Delta z_{h1}\Delta z_{h2})(\Delta x_{h1}\Delta x_{h3} + \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3}) - l^2(\Delta x_{h2}\Delta x_{h3} + \\
&\quad \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3})) - l^2 m_l(\Delta x_{h2}\Delta x_{h3} + \\
&\quad \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3}))) / (den m_{h1}m_{h2}m_{h3}m_l^2)
\end{aligned}$$

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$$\begin{aligned}
C(4, 9) &= -(\Delta x_{h2}(z_{h3} - z_l) (m_{h1} ((\Delta x_{h1}\Delta x_{h2} + \Delta y_{h1}\Delta y_{h2} + \\
&\quad \Delta z_{h1}\Delta z_{h2})(\Delta x_{h1}\Delta x_{h3} + \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3}) - l^2(\Delta x_{h2}\Delta x_{h3} + \\
&\quad \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3})) - l^2 m_l(\Delta x_{h2}\Delta x_{h3} + \\
&\quad \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3}))) / (den m_{h1} m_{h2} m_{h3} m_l^2) \\
C(5, 1) &= (\Delta x_{h1}\Delta y_{h2} (l^2 m_l(\Delta x_{h1}\Delta x_{h2} + \Delta y_{h1}\Delta y_{h2} + \Delta z_{h1}\Delta z_{h2}) - m_{h3} ((\Delta x_{h1}\Delta x_{h3} + \\
&\quad \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3})(\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \\
&\quad \Delta z_{h2}\Delta z_{h3}) - l^2(\Delta x_{h1}\Delta x_{h2} + \Delta y_{h1}\Delta y_{h2} + \Delta z_{h1}\Delta z_{h2})))) / (den m_{h1} m_{h2} m_{h3} m_l^2) \\
C(5, 2) &= -(\Delta y_{h2}(y_{h1} - y_l) (m_{h3} ((\Delta x_{h1}\Delta x_{h3} + \Delta y_{h1}\Delta y_{h3} + \\
&\quad \Delta z_{h1}\Delta z_{h3})(\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3}) - l^2(\Delta x_{h1}\Delta x_{h2} + \\
&\quad \Delta y_{h1}\Delta y_{h2} + \Delta z_{h1}\Delta z_{h2})) - l^2 m_l(\Delta x_{h1}\Delta x_{h2} + \\
&\quad \Delta y_{h1}\Delta y_{h2} + \Delta z_{h1}\Delta z_{h2}))) / (den m_{h1} m_{h2} m_{h3} m_l^2) \\
C(5, 3) &= -(\Delta y_{h2}(z_{h1} - z_l) (m_{h3} ((\Delta x_{h1}\Delta x_{h3} + \Delta y_{h1}\Delta y_{h3} + \\
&\quad \Delta z_{h1}\Delta z_{h3})(\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3}) - l^2(\Delta x_{h1}\Delta x_{h2} + \\
&\quad \Delta y_{h1}\Delta y_{h2} + \Delta z_{h1}\Delta z_{h2})) - l^2 m_l(\Delta x_{h1}\Delta x_{h2} + \\
&\quad \Delta y_{h1}\Delta y_{h2} + \Delta z_{h1}\Delta z_{h2}))) / (den m_{h1} m_{h2} m_{h3} m_l^2) \\
C(5, 4) &= (\Delta x_{h2}\Delta y_{h2} (m_{h1} m_{h3} ((\Delta x_{h1}\Delta x_{h3} + \Delta y_{h1}\Delta y_{h3} + \\
&\quad \Delta z_{h1}\Delta z_{h3})^2 - l^4) - l^4 m_l(m_{h1} + m_{h3}) + l^4 (-m_l^2))) / (den m_{h1} m_{h2}^2 m_{h3} m_l^2) \\
C(5, 5) &= (den m_{h1} m_{h2} m_{h3} m_l^2 + \Delta y_{h2}(y_{h2} - y_l) (m_{h1} m_{h3} ((\Delta x_{h1}\Delta x_{h3} + \Delta y_{h1}\Delta y_{h3} + \\
&\quad \Delta z_{h1}\Delta z_{h3})^2 - l^4) - l^4 m_l(m_{h1} + m_{h3}) + l^4 (-m_l^2))) / (den m_{h1} m_{h2}^2 m_{h3} m_l^2) \\
C(5, 6) &= (\Delta y_{h2}(z_{h2} - z_l) (m_{h1} m_{h3} ((\Delta x_{h1}\Delta x_{h3} + \Delta y_{h1}\Delta y_{h3} + \\
&\quad \Delta z_{h1}\Delta z_{h3})^2 - l^4) - l^4 m_l(m_{h1} + m_{h3}) + l^4 (-m_l^2))) / (den m_{h1} m_{h2}^2 m_{h3} m_l^2) \\
C(5, 7) &= (\Delta x_{h3}\Delta y_{h2} (l^2 m_l(\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3}) - m_{h1} ((\Delta x_{h1}\Delta x_{h2} + \\
&\quad \Delta y_{h1}\Delta y_{h2} + \Delta z_{h1}\Delta z_{h2})(\Delta x_{h1}\Delta x_{h3} + \Delta y_{h1}\Delta y_{h3} + \\
&\quad \Delta z_{h1}\Delta z_{h3}) - l^2(\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3})))) / (den m_{h1} m_{h2} m_{h3} m_l^2) \\
C(5, 8) &= -(\Delta y_{h2}(y_{h3} - y_l) (m_{h1} ((\Delta x_{h1}\Delta x_{h2} + \Delta y_{h1}\Delta y_{h2} + \\
&\quad \Delta z_{h1}\Delta z_{h2})(\Delta x_{h1}\Delta x_{h3} + \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3}) - l^2(\Delta x_{h2}\Delta x_{h3} + \\
&\quad \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3})) - l^2 m_l(\Delta x_{h2}\Delta x_{h3} + \\
&\quad \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3}))) / (den m_{h1} m_{h2} m_{h3} m_l^2) \\
C(5, 9) &= -(\Delta y_{h2}(z_{h3} - z_l) (m_{h1} ((\Delta x_{h1}\Delta x_{h2} + \Delta y_{h1}\Delta y_{h2} + \\
&\quad \Delta z_{h1}\Delta z_{h2})(\Delta x_{h1}\Delta x_{h3} + \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3}) - l^2(\Delta x_{h2}\Delta x_{h3} + \\
&\quad \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3})) - l^2 m_l(\Delta x_{h2}\Delta x_{h3} + \\
&\quad \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3}))) / (den m_{h1} m_{h2} m_{h3} m_l^2)
\end{aligned}$$

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$$C(6, 1) = (\Delta x_{h1} \Delta z_{h2} (l^2 m_l (\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2}) - m_{h3} ((\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - l^2 (\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})))) / (den m_{h1} m_{h2} m_{h3} m_l^2)$$

$$C(6, 2) = - (\Delta z_{h2} (y_{h1} - y_l) (m_{h3} ((\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - l^2 (\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})) - l^2 m_l (\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2}))) / (den m_{h1} m_{h2} m_{h3} m_l^2)$$

$$C(6, 3) = - (\Delta z_{h2} (z_{h1} - z_l) (m_{h3} ((\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - l^2 (\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})) - l^2 m_l (\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2}))) / (den m_{h1} m_{h2} m_{h3} m_l^2)$$

$$C(6, 4) = (\Delta x_{h2} \Delta z_{h2} (m_{h1} m_{h3} ((\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})^2 - l^4) - l^4 m_l (m_{h1} + m_{h3}) + l^4 (-m_l^2))) / (den m_{h1} m_{h2}^2 m_{h3} m_l^2)$$

$$C(6, 5) = (\Delta z_{h2} (y_{h2} - y_l) (m_{h1} m_{h3} ((\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})^2 - l^4) - l^4 m_l (m_{h1} + m_{h3}) + l^4 (-m_l^2))) / (den m_{h1} m_{h2}^2 m_{h3} m_l^2)$$

$$C(6, 6) = (den m_{h1} m_{h2} m_{h3} m_l^2 + \Delta z_{h2} (z_{h2} - z_l) (m_{h1} m_{h3} ((\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})^2 - l^4) - l^4 m_l (m_{h1} + m_{h3}) + l^4 (-m_l^2))) / (den m_{h1} m_{h2}^2 m_{h3} m_l^2)$$

$$C(6, 7) = (\Delta x_{h3} \Delta z_{h2} (l^2 m_l (\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - m_{h1} ((\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})(\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) - l^2 (\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})))) / (den m_{h1} m_{h2} m_{h3} m_l^2)$$

$$C(6, 8) = - (\Delta z_{h2} (y_{h3} - y_l) (m_{h1} ((\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})(\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) - l^2 (\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})) - l^2 m_l (\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}))) / (den m_{h1} m_{h2} m_{h3} m_l^2)$$

$$C(6, 9) = - (\Delta z_{h2} (z_{h3} - z_l) (m_{h1} ((\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})(\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) - l^2 (\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})) - l^2 m_l (\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}))) / (den m_{h1} m_{h2} m_{h3} m_l^2)$$

$$C(7, 1) = (\Delta x_{h1} \Delta x_{h3} (l^2 m_l (\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) - m_{h2} ((\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - l^2 (\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})))) / (den m_{h1} m_{h2} m_{h3} m_l^2)$$

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$$\begin{aligned}
C(7, 2) &= -(\Delta x_{h3}(y_{h1} - y_l) (m_{h2} ((\Delta x_{h1}\Delta x_{h2} + \Delta y_{h1}\Delta y_{h2} + \\
&\quad \Delta z_{h1}\Delta z_{h2})(\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3}) - l^2(\Delta x_{h1}\Delta x_{h3} + \\
&\quad \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3})) - l^2 m_l(\Delta x_{h1}\Delta x_{h3} + \\
&\quad \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3}))) / (den m_{h1}m_{h2}m_{h3}m_l^2) \\
C(7, 3) &= -(\Delta x_{h3}(z_{h1} - z_l) (m_{h2} ((\Delta x_{h1}\Delta x_{h2} + \Delta y_{h1}\Delta y_{h2} + \\
&\quad \Delta z_{h1}\Delta z_{h2})(\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3}) - l^2(\Delta x_{h1}\Delta x_{h3} + \\
&\quad \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3})) - l^2 m_l(\Delta x_{h1}\Delta x_{h3} + \\
&\quad \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3}))) / (den m_{h1}m_{h2}m_{h3}m_l^2) \\
C(7, 4) &= (\Delta x_{h2}\Delta x_{h3} (l^2 m_l(\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3}) - m_{h1} ((\Delta x_{h1}\Delta x_{h2} + \\
&\quad \Delta y_{h1}\Delta y_{h2} + \Delta z_{h1}\Delta z_{h2})(\Delta x_{h1}\Delta x_{h3} + \Delta y_{h1}\Delta y_{h3} + \\
&\quad \Delta z_{h1}\Delta z_{h3}) - l^2(\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3})))) / (den m_{h1}m_{h2}m_{h3}m_l^2) \\
C(7, 5) &= -(\Delta x_{h3}(y_{h2} - y_l) (m_{h1} ((\Delta x_{h1}\Delta x_{h2} + \Delta y_{h1}\Delta y_{h2} + \\
&\quad \Delta z_{h1}\Delta z_{h2})(\Delta x_{h1}\Delta x_{h3} + \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3}) - l^2(\Delta x_{h2}\Delta x_{h3} + \\
&\quad \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3})) - l^2 m_l(\Delta x_{h2}\Delta x_{h3} + \\
&\quad \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3}))) / (den m_{h1}m_{h2}m_{h3}m_l^2) \\
C(7, 6) &= -(\Delta x_{h3}(z_{h2} - z_l) (m_{h1} ((\Delta x_{h1}\Delta x_{h2} + \Delta y_{h1}\Delta y_{h2} + \\
&\quad \Delta z_{h1}\Delta z_{h2})(\Delta x_{h1}\Delta x_{h3} + \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3}) - l^2(\Delta x_{h2}\Delta x_{h3} + \\
&\quad \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3})) - l^2 m_l(\Delta x_{h2}\Delta x_{h3} + \\
&\quad \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3}))) / (den m_{h1}m_{h2}m_{h3}m_l^2) \\
C(7, 7) &= (m_l^2 (den m_{h1}m_{h2}m_{h3} - \Delta x_{h3}^2 l^4) + \Delta x_{h3}^2 m_{h1}m_{h2} ((\Delta x_{h1}\Delta x_{h2} + \Delta y_{h1}\Delta y_{h2} + \\
&\quad \Delta z_{h1}\Delta z_{h2})^2 - l^4) - \Delta x_{h3}^2 l^4 m_l(m_{h1} + m_{h2})) / (den m_{h1}m_{h2}m_{h3}^2 m_l^2) \\
C(7, 8) &= (\Delta x_{h3}(y_{h3} - y_l) (m_{h1}m_{h2} ((\Delta x_{h1}\Delta x_{h2} + \Delta y_{h1}\Delta y_{h2} + \\
&\quad \Delta z_{h1}\Delta z_{h2})^2 - l^4) - l^4 m_l(m_{h1} + m_{h2}) + l^4 (-m_l^2))) / (den m_{h1}m_{h2}m_{h3}^2 m_l^2) \\
C(7, 9) &= (\Delta x_{h3}(z_{h3} - z_l) (m_{h1}m_{h2} ((\Delta x_{h1}\Delta x_{h2} + \Delta y_{h1}\Delta y_{h2} + \\
&\quad \Delta z_{h1}\Delta z_{h2})^2 - l^4) - l^4 m_l(m_{h1} + m_{h2}) + l^4 (-m_l^2))) / (den m_{h1}m_{h2}m_{h3}^2 m_l^2) \\
C(8, 1) &= (\Delta x_{h1}\Delta y_{h3} (l^2 m_l(\Delta x_{h1}\Delta x_{h3} + \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3}) - m_{h2} ((\Delta x_{h1}\Delta x_{h2} + \\
&\quad \Delta y_{h1}\Delta y_{h2} + \Delta z_{h1}\Delta z_{h2})(\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \\
&\quad \Delta z_{h2}\Delta z_{h3}) - l^2(\Delta x_{h1}\Delta x_{h3} + \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3})))) / (den m_{h1}m_{h2}m_{h3}m_l^2) \\
C(8, 2) &= -(\Delta y_{h3}(y_{h1} - y_l) (m_{h2} ((\Delta x_{h1}\Delta x_{h2} + \Delta y_{h1}\Delta y_{h2} + \\
&\quad \Delta z_{h1}\Delta z_{h2})(\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3}) - l^2(\Delta x_{h1}\Delta x_{h3} + \\
&\quad \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3})) - l^2 m_l(\Delta x_{h1}\Delta x_{h3} + \\
&\quad \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3}))) / (den m_{h1}m_{h2}m_{h3}m_l^2)
\end{aligned}$$

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$$\begin{aligned}
C(8, 3) &= - (\Delta y_{h3}(z_{h1} - z_l) (m_{h2} ((\Delta x_{h1}\Delta x_{h2} + \Delta y_{h1}\Delta y_{h2} + \\
&\quad \Delta z_{h1}\Delta z_{h2})(\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3}) - l^2(\Delta x_{h1}\Delta x_{h3} + \\
&\quad \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3})) - l^2 m_l (\Delta x_{h1}\Delta x_{h3} + \\
&\quad \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3}))) / (den m_{h1} m_{h2} m_{h3} m_l^2) \\
C(8, 4) &= (\Delta x_{h2}\Delta y_{h3} (l^2 m_l (\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3}) - m_{h1} ((\Delta x_{h1}\Delta x_{h2} + \\
&\quad \Delta y_{h1}\Delta y_{h2} + \Delta z_{h1}\Delta z_{h2})(\Delta x_{h1}\Delta x_{h3} + \Delta y_{h1}\Delta y_{h3} + \\
&\quad \Delta z_{h1}\Delta z_{h3}) - l^2(\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3})))) / (den m_{h1} m_{h2} m_{h3} m_l^2) \\
C(8, 5) &= - (\Delta y_{h3}(y_{h2} - y_l) (m_{h1} ((\Delta x_{h1}\Delta x_{h2} + \Delta y_{h1}\Delta y_{h2} + \\
&\quad \Delta z_{h1}\Delta z_{h2})(\Delta x_{h1}\Delta x_{h3} + \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3}) - l^2(\Delta x_{h2}\Delta x_{h3} + \\
&\quad \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3})) - l^2 m_l (\Delta x_{h2}\Delta x_{h3} + \\
&\quad \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3}))) / (den m_{h1} m_{h2} m_{h3} m_l^2) \\
C(8, 6) &= - (\Delta y_{h3}(z_{h2} - z_l) (m_{h1} ((\Delta x_{h1}\Delta x_{h2} + \Delta y_{h1}\Delta y_{h2} + \\
&\quad \Delta z_{h1}\Delta z_{h2})(\Delta x_{h1}\Delta x_{h3} + \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3}) - l^2(\Delta x_{h2}\Delta x_{h3} + \\
&\quad \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3})) - l^2 m_l (\Delta x_{h2}\Delta x_{h3} + \\
&\quad \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3}))) / (den m_{h1} m_{h2} m_{h3} m_l^2) \\
C(8, 7) &= (\Delta x_{h3}\Delta y_{h3} (m_{h1} m_{h2} ((\Delta x_{h1}\Delta x_{h2} + \Delta y_{h1}\Delta y_{h2} + \\
&\quad \Delta z_{h1}\Delta z_{h2})^2 - l^4) - l^4 m_l (m_{h1} + m_{h2}) + l^4 (-m_l^2))) / (den m_{h1} m_{h2} m_{h3}^2 m_l^2) \\
C(8, 8) &= (den m_{h1} m_{h2} m_{h3} m_l^2 + \Delta y_{h3}(y_{h3} - y_l) (m_{h1} m_{h2} ((\Delta x_{h1}\Delta x_{h2} + \Delta y_{h1}\Delta y_{h2} + \\
&\quad \Delta z_{h1}\Delta z_{h2})^2 - l^4) - l^4 m_l (m_{h1} + m_{h2}) + l^4 (-m_l^2))) / (den m_{h1} m_{h2} m_{h3}^2 m_l^2) \\
C(8, 9) &= (\Delta y_{h3}(z_{h3} - z_l) (m_{h1} m_{h2} ((\Delta x_{h1}\Delta x_{h2} + \Delta y_{h1}\Delta y_{h2} + \\
&\quad \Delta z_{h1}\Delta z_{h2})^2 - l^4) - l^4 m_l (m_{h1} + m_{h2}) + l^4 (-m_l^2))) / (den m_{h1} m_{h2} m_{h3}^2 m_l^2) \\
C(9, 1) &= (\Delta x_{h1}\Delta z_{h3} (l^2 m_l (\Delta x_{h1}\Delta x_{h3} + \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3}) - m_{h2} ((\Delta x_{h1}\Delta x_{h2} + \\
&\quad \Delta y_{h1}\Delta y_{h2} + \Delta z_{h1}\Delta z_{h2})(\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \\
&\quad \Delta z_{h2}\Delta z_{h3}) - l^2(\Delta x_{h1}\Delta x_{h3} + \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3})))) / (den m_{h1} m_{h2} m_{h3} m_l^2) \\
C(9, 2) &= - (\Delta z_{h3}(y_{h1} - y_l) (m_{h2} ((\Delta x_{h1}\Delta x_{h2} + \Delta y_{h1}\Delta y_{h2} + \\
&\quad \Delta z_{h1}\Delta z_{h2})(\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3}) - l^2(\Delta x_{h1}\Delta x_{h3} + \\
&\quad \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3})) - l^2 m_l (\Delta x_{h1}\Delta x_{h3} + \\
&\quad \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3}))) / (den m_{h1} m_{h2} m_{h3} m_l^2) \\
C(9, 3) &= - (\Delta z_{h3}(z_{h1} - z_l) (m_{h2} ((\Delta x_{h1}\Delta x_{h2} + \Delta y_{h1}\Delta y_{h2} + \\
&\quad \Delta z_{h1}\Delta z_{h2})(\Delta x_{h2}\Delta x_{h3} + \Delta y_{h2}\Delta y_{h3} + \Delta z_{h2}\Delta z_{h3}) - l^2(\Delta x_{h1}\Delta x_{h3} + \\
&\quad \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3})) - l^2 m_l (\Delta x_{h1}\Delta x_{h3} + \\
&\quad \Delta y_{h1}\Delta y_{h3} + \Delta z_{h1}\Delta z_{h3}))) / (den m_{h1} m_{h2} m_{h3} m_l^2)
\end{aligned}$$

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$$C(9, 4) = (\Delta x_{h2} \Delta z_{h3} (l^2 m_l (\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - m_{h1} ((\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})(\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) - l^2 (\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})))) / (den m_{h1} m_{h2} m_{h3} m_l^2)$$

$$C(9, 5) = -(\Delta z_{h3} (y_{h2} - y_l) (m_{h1} ((\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})(\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) - l^2 (\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})) - l^2 m_l (\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}))) / (den m_{h1} m_{h2} m_{h3} m_l^2)$$

$$C(9, 6) = -(\Delta z_{h3} (z_{h2} - z_l) (m_{h1} ((\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})(\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) - l^2 (\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})) - l^2 m_l (\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}))) / (den m_{h1} m_{h2} m_{h3} m_l^2)$$

$$C(9, 7) = (\Delta x_{h3} \Delta z_{h3} (m_{h1} m_{h2} ((\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})^2 - l^4) - l^4 m_l (m_{h1} + m_{h2}) + l^4 (-m_l^2))) / (den m_{h1} m_{h2} m_{h3}^2 m_l^2)$$

$$C(9, 8) = (\Delta z_{h3} (y_{h3} - y_l) (m_{h1} m_{h2} ((\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})^2 - l^4) - l^4 m_l (m_{h1} + m_{h2}) + l^4 (-m_l^2))) / (den m_{h1} m_{h2} m_{h3}^2 m_l^2)$$

$$C(9, 9) = (den m_{h1} m_{h2} m_{h3} m_l^2 + \Delta z_{h3} (z_{h3} - z_l) (m_{h1} m_{h2} ((\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})^2 - l^4) - l^4 m_l (m_{h1} + m_{h2}) + l^4 (-m_l^2))) / (den m_{h1} m_{h2} m_{h3}^2 m_l^2)$$

$$C(10, 1) = (\Delta x_{h1} (\Delta x_{h1} (l^2 m_l (\Delta x_{h2}^2 (-m_{h2}) - \Delta x_{h3}^2 m_{h3} + l^2 (m_{h2} + m_{h3})) + m_{h2} m_{h3} ((\Delta x_{h2} - l)(\Delta x_{h2} + l)(\Delta x_{h3} - l)(\Delta x_{h3} + l) - (\Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})^2) + l^4 m_l^2) + \Delta x_{h2}^2 \Delta x_{h3} m_{h2} m_{h3} (\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) + \Delta x_{h2} m_{h2} (m_{h3} (\Delta x_{h3}^2 (\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2}) + (\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})(\Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - l^2 (\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})) - l^2 m_l (\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})) + \Delta x_{h3} m_{h3} (m_{h2} ((\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})(\Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - l^2 (\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})) - l^2 m_l (\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})))) / (den m_{h1} m_{h2} m_{h3} m_l^3)$$

$$C(10, 2) = ((y_{h1} - y_l) (\Delta x_{h1} (l^2 m_l (\Delta x_{h2}^2 (-m_{h2}) - \Delta x_{h3}^2 m_{h3} + l^2 (m_{h2} + m_{h3})) + m_{h2} m_{h3} ((\Delta x_{h2} - l)(\Delta x_{h2} + l)(\Delta x_{h3} - l)(\Delta x_{h3} + l) - (\Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})^2) + l^4 m_l^2) + \Delta x_{h2}^2 \Delta x_{h3} m_{h2} m_{h3} (\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) + \Delta x_{h2} m_{h2} (m_{h3} (\Delta x_{h3}^2 (\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2}) + (\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})(\Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - l^2 (\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})) - l^2 m_l (\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})) + \Delta x_{h3} m_{h3} (m_{h2} ((\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})(\Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - l^2 (\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})) - l^2 m_l (\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})))) / (den m_{h1} m_{h2} m_{h3} m_l^3)$$

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$$\begin{aligned}
C(10, 3) &= ((z_{h1} - z_l) (\Delta x_{h1} (l^2 m_l (\Delta x_{h2}^2 (-m_{h2}) - \Delta x_{h3}^2 m_{h3} + l^2 (m_{h2} + m_{h3})) + m_{h2} m_{h3} ((\Delta x_{h2} - l)(\Delta x_{h2} + l)(\Delta x_{h3} - l)(\Delta x_{h3} + l) - (\Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})^2) + l^4 m_l^2) + \Delta x_{h2}^2 \Delta x_{h3} m_{h2} m_{h3} (\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) + \Delta x_{h2} m_{h2} (m_{h3} (\Delta x_{h3}^2 (\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2}) + (\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) (\Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - l^2 (\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})) - l^2 m_l (\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})) + \Delta x_{h3} m_{h3} (m_{h2} ((\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2}) (\Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - l^2 (\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})) - l^2 m_l (\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})))) / (den m_{h1} m_{h2} m_{h3} m_l^3) \\
C(10, 4) &= (\Delta x_{h2} (\Delta x_{h1}^2 m_{h1} (\Delta x_{h2} (\Delta x_{h3}^2 m_{h3} - l^2 (m_{h3} + m_l)) + \Delta x_{h3} m_{h3} (\Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})) + \Delta x_{h1} m_{h1} (m_{h3} (\Delta x_{h3}^2 (\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2}) + (\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) (\Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - l^2 (\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})) - l^2 m_l (\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})) + l^2 m_l (\Delta x_{h2} (l^2 (m_{h1} + m_{h3}) - \Delta x_{h3}^2 m_{h3}) - \Delta x_{h3} m_{h3} (\Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})) + m_{h1} m_{h3} ((\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) (\Delta x_{h3} (\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2}) - \Delta x_{h2} (\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})) - \Delta x_{h3} l^2 (\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) + \Delta x_{h2} l^4) + \Delta x_{h2} l^4 m_l^2)) / (den m_{h1} m_{h2} m_{h3} m_l^3) \\
C(10, 5) &= ((y_{h2} - y_l) (\Delta x_{h1}^2 m_{h1} (\Delta x_{h2} (\Delta x_{h3}^2 m_{h3} - l^2 (m_{h3} + m_l)) + \Delta x_{h3} m_{h3} (\Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})) + \Delta x_{h1} m_{h1} (m_{h3} (\Delta x_{h3}^2 (\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2}) + (\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) (\Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - l^2 (\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})) - l^2 m_l (\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})) + l^2 m_l (\Delta x_{h2} (l^2 (m_{h1} + m_{h3}) - \Delta x_{h3}^2 m_{h3}) - \Delta x_{h3} m_{h3} (\Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})) + m_{h1} m_{h3} ((\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) (\Delta x_{h3} (\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2}) - \Delta x_{h2} (\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})) - \Delta x_{h3} l^2 (\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) + \Delta x_{h2} l^4) + \Delta x_{h2} l^4 m_l^2)) / (den m_{h1} m_{h2} m_{h3} m_l^3) \\
C(10, 6) &= ((z_{h2} - z_l) (\Delta x_{h1}^2 m_{h1} (\Delta x_{h2} (\Delta x_{h3}^2 m_{h3} - l^2 (m_{h3} + m_l)) + \Delta x_{h3} m_{h3} (\Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})) + \Delta x_{h1} m_{h1} (m_{h3} (\Delta x_{h3}^2 (\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2}) + (\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) (\Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - l^2 (\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})) - l^2 m_l (\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})) + l^2 m_l (\Delta x_{h2} (l^2 (m_{h1} + m_{h3}) - \Delta x_{h3}^2 m_{h3}) - \Delta x_{h3} m_{h3} (\Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})) + m_{h1} m_{h3} ((\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) (\Delta x_{h3} (\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2}) - \Delta x_{h2} (\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})) - \Delta x_{h3} l^2 (\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) + \Delta x_{h2} l^4) + \Delta x_{h2} l^4 m_l^2)) / (den m_{h1} m_{h2} m_{h3} m_l^3)
\end{aligned}$$

$$\begin{aligned}
 C(10, 7) = & (\Delta x_{h3} (\Delta x_{h1}^2 m_{h1} (\Delta x_{h2}^2 \Delta x_{h3} m_{h2} + \Delta x_{h2} m_{h2} (\Delta y_{h2} \Delta y_{h3} + \\
 & \Delta z_{h2} \Delta z_{h3}) - \Delta x_{h3} l^2 (m_{h2} + m_l)) + \Delta x_{h1} m_{h1} (m_{h2} (\Delta x_{h2}^2 (\Delta y_{h1} \Delta y_{h3} + \\
 & \Delta z_{h1} \Delta z_{h3}) + (\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2}) (\Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - l^2 (\Delta y_{h1} \Delta y_{h3} + \\
 & \Delta z_{h1} \Delta z_{h3})) - l^2 m_l (\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})) + l^2 m_l (\Delta x_{h3} (l^2 (m_{h1} + \\
 & m_{h2}) - \Delta x_{h2}^2 m_{h2}) - \Delta x_{h2} m_{h2} (\Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})) + \\
 & m_{h1} m_{h2} (-(\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2}) (\Delta x_{h3} (\Delta y_{h1} \Delta y_{h2} + \\
 & \Delta z_{h1} \Delta z_{h2}) - \Delta x_{h2} (\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})) - \Delta x_{h2} l^2 (\Delta x_{h2} \Delta x_{h3} + \\
 & \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) + \Delta x_{h3} l^4) + \Delta x_{h3} l^4 m_l^2)) / (den m_{h1} m_{h2} m_{h3} m_l^3)
 \end{aligned}$$

$$\begin{aligned}
 C(10, 8) = & ((y_{h3} - y_l) (\Delta x_{h1}^2 m_{h1} (\Delta x_{h2}^2 \Delta x_{h3} m_{h2} + \Delta x_{h2} m_{h2} (\Delta y_{h2} \Delta y_{h3} + \\
 & \Delta z_{h2} \Delta z_{h3}) - \Delta x_{h3} l^2 (m_{h2} + m_l)) + \Delta x_{h1} m_{h1} (m_{h2} (\Delta x_{h2}^2 (\Delta y_{h1} \Delta y_{h3} + \\
 & \Delta z_{h1} \Delta z_{h3}) + (\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2}) (\Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - l^2 (\Delta y_{h1} \Delta y_{h3} + \\
 & \Delta z_{h1} \Delta z_{h3})) - l^2 m_l (\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})) + l^2 m_l (\Delta x_{h3} (l^2 (m_{h1} + \\
 & m_{h2}) - \Delta x_{h2}^2 m_{h2}) - \Delta x_{h2} m_{h2} (\Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})) + \\
 & m_{h1} m_{h2} (-(\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2}) (\Delta x_{h3} (\Delta y_{h1} \Delta y_{h2} + \\
 & \Delta z_{h1} \Delta z_{h2}) - \Delta x_{h2} (\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})) - \Delta x_{h2} l^2 (\Delta x_{h2} \Delta x_{h3} + \\
 & \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) + \Delta x_{h3} l^4) + \Delta x_{h3} l^4 m_l^2)) / (den m_{h1} m_{h2} m_{h3} m_l^3)
 \end{aligned}$$

$$\begin{aligned}
 C(10, 9) = & ((z_{h3} - z_l) (\Delta x_{h1}^2 m_{h1} (\Delta x_{h2}^2 \Delta x_{h3} m_{h2} + \Delta x_{h2} m_{h2} (\Delta y_{h2} \Delta y_{h3} + \\
 & \Delta z_{h2} \Delta z_{h3}) - \Delta x_{h3} l^2 (m_{h2} + m_l)) + \Delta x_{h1} m_{h1} (m_{h2} (\Delta x_{h2}^2 (\Delta y_{h1} \Delta y_{h3} + \\
 & \Delta z_{h1} \Delta z_{h3}) + (\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2}) (\Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - l^2 (\Delta y_{h1} \Delta y_{h3} + \\
 & \Delta z_{h1} \Delta z_{h3})) - l^2 m_l (\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})) + l^2 m_l (\Delta x_{h3} (l^2 (m_{h1} + \\
 & m_{h2}) - \Delta x_{h2}^2 m_{h2}) - \Delta x_{h2} m_{h2} (\Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})) + \\
 & m_{h1} m_{h2} (-(\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2}) (\Delta x_{h3} (\Delta y_{h1} \Delta y_{h2} + \\
 & \Delta z_{h1} \Delta z_{h2}) - \Delta x_{h2} (\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})) - \Delta x_{h2} l^2 (\Delta x_{h2} \Delta x_{h3} + \\
 & \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) + \Delta x_{h3} l^4) + \Delta x_{h3} l^4 m_l^2)) / (den m_{h1} m_{h2} m_{h3} m_l^3)
 \end{aligned}$$

$$\begin{aligned}
 C(11, 1) = & (\Delta x_{h1} (\Delta x_{h2}^2 \Delta x_{h3} m_{h2} m_{h3} (\Delta x_{h1} \Delta y_{h3} - \Delta x_{h3} \Delta y_{h1}) + \\
 & \Delta x_{h2} m_{h2} (\Delta x_{h1} m_{h3} (\Delta x_{h3}^2 \Delta y_{h2} + \Delta y_{h2} (\Delta y_{h3} - l) (\Delta y_{h3} + l) + \\
 & \Delta y_{h3} \Delta z_{h2} \Delta z_{h3}) - \Delta x_{h1} \Delta y_{h2} l^2 m_l + \Delta x_{h3} m_{h3} (-2 \Delta y_{h1} \Delta z_{h2} \Delta z_{h3} + \\
 & \Delta y_{h2} \Delta z_{h1} \Delta z_{h3} + \Delta y_{h3} \Delta z_{h1} \Delta z_{h2})) - l^2 m_l (\Delta y_{h3} m_{h3} (\Delta x_{h1} \Delta x_{h3} + \Delta z_{h1} \Delta z_{h3}) + \\
 & \Delta y_{h1} (\Delta y_{h2}^2 m_{h2} + \Delta y_{h3}^2 m_{h3} - l^2 (m_{h2} + m_{h3})) + \Delta y_{h2} \Delta z_{h1} \Delta z_{h2} m_{h2}) + \\
 & m_{h2} m_{h3} (-l^2 (\Delta x_{h1} \Delta x_{h3} \Delta y_{h3} + \Delta y_{h1} (\Delta y_{h2}^2 + \Delta y_{h3}^2) + \Delta y_{h2} \Delta z_{h1} \Delta z_{h2} + \\
 & \Delta y_{h3} \Delta z_{h1} \Delta z_{h3}) + (\Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) (\Delta x_{h1} \Delta x_{h3} \Delta y_{h2} + \\
 & \Delta y_{h1} \Delta y_{h2} \Delta y_{h3} - \Delta y_{h1} \Delta z_{h2} \Delta z_{h3} + \Delta y_{h2} \Delta z_{h1} \Delta z_{h3} + \\
 & \Delta y_{h3} \Delta z_{h1} \Delta z_{h2}) + \Delta y_{h1} l^4) + \Delta y_{h1} l^4 m_l^2)) / (den m_{h1} m_{h2} m_{h3} m_l^3)
 \end{aligned}$$

6 Dynamic Equations of the Translational Multi-Lift Model

$$\begin{aligned}
C(11, 2) = & ((y_{h1} - y_l) (\Delta x_{h2}^2 \Delta x_{h3} m_{h2} m_{h3} (\Delta x_{h1} \Delta y_{h3} - \Delta x_{h3} \Delta y_{h1}) + \\
& \Delta x_{h2} m_{h2} (\Delta x_{h1} m_{h3} (\Delta x_{h3}^2 \Delta y_{h2} + \Delta y_{h2} (\Delta y_{h3} - l) (\Delta y_{h3} + l) + \\
& \Delta y_{h3} \Delta z_{h2} \Delta z_{h3}) - \Delta x_{h1} \Delta y_{h2} l^2 m_l + \Delta x_{h3} m_{h3} (-2 \Delta y_{h1} \Delta z_{h2} \Delta z_{h3} + \\
& \Delta y_{h2} \Delta z_{h1} \Delta z_{h3} + \Delta y_{h3} \Delta z_{h1} \Delta z_{h2})) - l^2 m_l (\Delta y_{h3} m_{h3} (\Delta x_{h1} \Delta x_{h3} + \Delta z_{h1} \Delta z_{h3}) + \\
& \Delta y_{h1} (\Delta y_{h2}^2 m_{h2} + \Delta y_{h3}^2 m_{h3} - l^2 (m_{h2} + m_{h3})) + \Delta y_{h2} \Delta z_{h1} \Delta z_{h2} m_{h2}) + \\
& m_{h2} m_{h3} (-l^2 (\Delta x_{h1} \Delta x_{h3} \Delta y_{h3} + \Delta y_{h1} (\Delta y_{h2}^2 + \Delta y_{h3}^2) + \Delta y_{h2} \Delta z_{h1} \Delta z_{h2} + \\
& \Delta y_{h3} \Delta z_{h1} \Delta z_{h3}) + (\Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) (\Delta x_{h1} \Delta x_{h3} \Delta y_{h2} + \\
& \Delta y_{h1} \Delta y_{h2} \Delta y_{h3} - \Delta y_{h1} \Delta z_{h2} \Delta z_{h3} + \Delta y_{h2} \Delta z_{h1} \Delta z_{h3} + \\
& \Delta y_{h3} \Delta z_{h1} \Delta z_{h2}) + \Delta y_{h1} l^4) + \Delta y_{h1} l^4 m_l^2)) / (den m_{h1} m_{h2} m_{h3} m_l^3)
\end{aligned}$$

$$\begin{aligned}
C(11, 3) = & ((z_{h1} - z_l) (\Delta x_{h2}^2 \Delta x_{h3} m_{h2} m_{h3} (\Delta x_{h1} \Delta y_{h3} - \Delta x_{h3} \Delta y_{h1}) + \\
& \Delta x_{h2} m_{h2} (\Delta x_{h1} m_{h3} (\Delta x_{h3}^2 \Delta y_{h2} + \Delta y_{h2} (\Delta y_{h3} - l) (\Delta y_{h3} + l) + \\
& \Delta y_{h3} \Delta z_{h2} \Delta z_{h3}) - \Delta x_{h1} \Delta y_{h2} l^2 m_l + \Delta x_{h3} m_{h3} (-2 \Delta y_{h1} \Delta z_{h2} \Delta z_{h3} + \\
& \Delta y_{h2} \Delta z_{h1} \Delta z_{h3} + \Delta y_{h3} \Delta z_{h1} \Delta z_{h2})) - l^2 m_l (\Delta y_{h3} m_{h3} (\Delta x_{h1} \Delta x_{h3} + \Delta z_{h1} \Delta z_{h3}) + \\
& \Delta y_{h1} (\Delta y_{h2}^2 m_{h2} + \Delta y_{h3}^2 m_{h3} - l^2 (m_{h2} + m_{h3})) + \Delta y_{h2} \Delta z_{h1} \Delta z_{h2} m_{h2}) + \\
& m_{h2} m_{h3} (-l^2 (\Delta x_{h1} \Delta x_{h3} \Delta y_{h3} + \Delta y_{h1} (\Delta y_{h2}^2 + \Delta y_{h3}^2) + \Delta y_{h2} \Delta z_{h1} \Delta z_{h2} + \\
& \Delta y_{h3} \Delta z_{h1} \Delta z_{h3}) + (\Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) (\Delta x_{h1} \Delta x_{h3} \Delta y_{h2} + \\
& \Delta y_{h1} \Delta y_{h2} \Delta y_{h3} - \Delta y_{h1} \Delta z_{h2} \Delta z_{h3} + \Delta y_{h2} \Delta z_{h1} \Delta z_{h3} + \\
& \Delta y_{h3} \Delta z_{h1} \Delta z_{h2}) + \Delta y_{h1} l^4) + \Delta y_{h1} l^4 m_l^2)) / (den m_{h1} m_{h2} m_{h3} m_l^3)
\end{aligned}$$

$$\begin{aligned}
C(11, 4) = & (\Delta x_{h2} (\Delta x_{h1}^2 \Delta x_{h3} m_{h1} m_{h3} (\Delta x_{h2} \Delta y_{h3} - \Delta x_{h3} \Delta y_{h2}) + \\
& \Delta x_{h1} m_{h1} (\Delta x_{h2} m_{h3} (\Delta x_{h3}^2 \Delta y_{h1} + \Delta y_{h1} (\Delta y_{h3} - l) (\Delta y_{h3} + \\
& l) + \Delta y_{h3} \Delta z_{h1} \Delta z_{h3}) - \Delta x_{h2} \Delta y_{h1} l^2 m_l + \\
& \Delta x_{h3} m_{h3} (\Delta y_{h1} \Delta z_{h2} \Delta z_{h3} - 2 \Delta y_{h2} \Delta z_{h1} \Delta z_{h3} + \\
& \Delta y_{h3} \Delta z_{h1} \Delta z_{h2})) - l^2 m_l (\Delta y_{h3} m_{h3} (\Delta x_{h2} \Delta x_{h3} + \Delta z_{h2} \Delta z_{h3}) + \\
& \Delta y_{h1}^2 \Delta y_{h2} m_{h1} + \Delta y_{h1} \Delta z_{h1} \Delta z_{h2} m_{h1} - \Delta y_{h2} (l^2 (m_{h1} + m_{h3}) - \Delta y_{h3}^2 m_{h3})) + \\
& m_{h1} m_{h3} (-l^2 (\Delta y_{h3} (\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) + \Delta y_{h1}^2 \Delta y_{h2} + \\
& \Delta y_{h1} \Delta z_{h1} \Delta z_{h2}) + (\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) (\Delta x_{h2} \Delta x_{h3} \Delta y_{h1} + \\
& \Delta y_{h1} \Delta y_{h2} \Delta y_{h3} + \Delta y_{h1} \Delta z_{h2} \Delta z_{h3} - \Delta y_{h2} \Delta z_{h1} \Delta z_{h3} + \\
& \Delta y_{h3} \Delta z_{h1} \Delta z_{h2}) + \Delta y_{h2} l^4) + \Delta y_{h2} l^4 m_l^2)) / (den m_{h1} m_{h2} m_{h3} m_l^3)
\end{aligned}$$

6 Dynamic Equations of the Translational Multi-Lift Model

$$\begin{aligned}
C(11, 5) = & ((y_{h2} - y_l) (\Delta x_{h1}^2 \Delta x_{h3} m_{h1} m_{h3} (\Delta x_{h2} \Delta y_{h3} - \Delta x_{h3} \Delta y_{h2}) + \\
& \Delta x_{h1} m_{h1} (\Delta x_{h2} m_{h3} (\Delta x_{h3}^2 \Delta y_{h1} + \Delta y_{h1} (\Delta y_{h3} - l) (\Delta y_{h3} + \\
& l) + \Delta y_{h3} \Delta z_{h1} \Delta z_{h3}) - \Delta x_{h2} \Delta y_{h1} l^2 m_l + \\
& \Delta x_{h3} m_{h3} (\Delta y_{h1} \Delta z_{h2} \Delta z_{h3} - 2 \Delta y_{h2} \Delta z_{h1} \Delta z_{h3} + \\
& \Delta y_{h3} \Delta z_{h1} \Delta z_{h2})) - l^2 m_l (\Delta y_{h3} m_{h3} (\Delta x_{h2} \Delta x_{h3} + \Delta z_{h2} \Delta z_{h3}) + \\
& \Delta y_{h1}^2 \Delta y_{h2} m_{h1} + \Delta y_{h1} \Delta z_{h1} \Delta z_{h2} m_{h1} - \Delta y_{h2} (l^2 (m_{h1} + m_{h3}) - \Delta y_{h3}^2 m_{h3})) + \\
& m_{h1} m_{h3} (-l^2 (\Delta y_{h3} (\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) + \Delta y_{h1}^2 \Delta y_{h2} + \\
& \Delta y_{h1} \Delta z_{h1} \Delta z_{h2}) + (\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) (\Delta x_{h2} \Delta x_{h3} \Delta y_{h1} + \\
& \Delta y_{h1} \Delta y_{h2} \Delta y_{h3} + \Delta y_{h1} \Delta z_{h2} \Delta z_{h3} - \Delta y_{h2} \Delta z_{h1} \Delta z_{h3} + \\
& \Delta y_{h3} \Delta z_{h1} \Delta z_{h2}) + \Delta y_{h2} l^4) + \Delta y_{h2} l^4 m_l^2)) / (den m_{h1} m_{h2} m_{h3} m_l^3) \\
C(11, 6) = & ((z_{h2} - z_l) (\Delta x_{h1}^2 \Delta x_{h3} m_{h1} m_{h3} (\Delta x_{h2} \Delta y_{h3} - \Delta x_{h3} \Delta y_{h2}) + \\
& \Delta x_{h1} m_{h1} (\Delta x_{h2} m_{h3} (\Delta x_{h3}^2 \Delta y_{h1} + \Delta y_{h1} (\Delta y_{h3} - l) (\Delta y_{h3} + \\
& l) + \Delta y_{h3} \Delta z_{h1} \Delta z_{h3}) - \Delta x_{h2} \Delta y_{h1} l^2 m_l + \\
& \Delta x_{h3} m_{h3} (\Delta y_{h1} \Delta z_{h2} \Delta z_{h3} - 2 \Delta y_{h2} \Delta z_{h1} \Delta z_{h3} + \\
& \Delta y_{h3} \Delta z_{h1} \Delta z_{h2})) - l^2 m_l (\Delta y_{h3} m_{h3} (\Delta x_{h2} \Delta x_{h3} + \Delta z_{h2} \Delta z_{h3}) + \\
& \Delta y_{h1}^2 \Delta y_{h2} m_{h1} + \Delta y_{h1} \Delta z_{h1} \Delta z_{h2} m_{h1} - \Delta y_{h2} (l^2 (m_{h1} + m_{h3}) - \Delta y_{h3}^2 m_{h3})) + \\
& m_{h1} m_{h3} (-l^2 (\Delta y_{h3} (\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) + \Delta y_{h1}^2 \Delta y_{h2} + \\
& \Delta y_{h1} \Delta z_{h1} \Delta z_{h2}) + (\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) (\Delta x_{h2} \Delta x_{h3} \Delta y_{h1} + \\
& \Delta y_{h1} \Delta y_{h2} \Delta y_{h3} + \Delta y_{h1} \Delta z_{h2} \Delta z_{h3} - \Delta y_{h2} \Delta z_{h1} \Delta z_{h3} + \\
& \Delta y_{h3} \Delta z_{h1} \Delta z_{h2}) + \Delta y_{h2} l^4) + \Delta y_{h2} l^4 m_l^2)) / (den m_{h1} m_{h2} m_{h3} m_l^3) \\
C(11, 7) = & (\Delta x_{h3} (\Delta x_{h1}^2 \Delta x_{h2} m_{h1} m_{h2} (\Delta x_{h3} \Delta y_{h2} - \Delta x_{h2} \Delta y_{h3}) + \\
& \Delta x_{h1} m_{h1} (m_{h2} (\Delta x_{h2}^2 \Delta x_{h3} \Delta y_{h1} + \Delta x_{h2} (\Delta y_{h1} \Delta z_{h2} \Delta z_{h3} + \\
& \Delta y_{h2} \Delta z_{h1} \Delta z_{h3} - 2 \Delta y_{h3} \Delta z_{h1} \Delta z_{h2}) + \Delta x_{h3} (\Delta y_{h1} (\Delta y_{h2} - l) (\Delta y_{h2} + l) + \\
& \Delta y_{h2} \Delta z_{h1} \Delta z_{h2})) - \Delta x_{h3} \Delta y_{h1} l^2 m_l) - l^2 m_l (\Delta y_{h2} m_{h2} (\Delta x_{h2} \Delta x_{h3} + \\
& \Delta z_{h2} \Delta z_{h3}) + \Delta y_{h1}^2 \Delta y_{h3} m_{h1} + \Delta y_{h1} \Delta z_{h1} \Delta z_{h3} m_{h1} - \Delta y_{h3} (l^2 (m_{h1} + \\
& m_{h2}) - \Delta y_{h2}^2 m_{h2})) + m_{h1} m_{h2} (-l^2 (\Delta y_{h2} (\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) + \\
& \Delta y_{h1} (\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})) + (\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2}) (\Delta x_{h2} \Delta x_{h3} \Delta y_{h1} + \\
& \Delta y_{h1} \Delta y_{h2} \Delta y_{h3} + \Delta y_{h1} \Delta z_{h2} \Delta z_{h3} + \Delta y_{h2} \Delta z_{h1} \Delta z_{h3} - \Delta y_{h3} \Delta z_{h1} \Delta z_{h2}) + \\
& \Delta y_{h3} l^4) + \Delta y_{h3} l^4 m_l^2)) / (den m_{h1} m_{h2} m_{h3} m_l^3)
\end{aligned}$$

$$\begin{aligned}
 C(11, 8) = & ((y_{h3} - y_l) (\Delta x_{h1}^2 \Delta x_{h2} m_{h1} m_{h2} (\Delta x_{h3} \Delta y_{h2} - \Delta x_{h2} \Delta y_{h3}) + \\
 & \Delta x_{h1} m_{h1} (m_{h2} (\Delta x_{h2}^2 \Delta x_{h3} \Delta y_{h1} + \Delta x_{h2} (\Delta y_{h1} \Delta z_{h2} \Delta z_{h3} + \\
 & \Delta y_{h2} \Delta z_{h1} \Delta z_{h3} - 2 \Delta y_{h3} \Delta z_{h1} \Delta z_{h2}) + \Delta x_{h3} (\Delta y_{h1} (\Delta y_{h2} - l) (\Delta y_{h2} + l) + \\
 & \Delta y_{h2} \Delta z_{h1} \Delta z_{h2})) - \Delta x_{h3} \Delta y_{h1} l^2 m_l) - l^2 m_l (\Delta y_{h2} m_{h2} (\Delta x_{h2} \Delta x_{h3} + \\
 & \Delta z_{h2} \Delta z_{h3}) + \Delta y_{h1}^2 \Delta y_{h3} m_{h1} + \Delta y_{h1} \Delta z_{h1} \Delta z_{h3} m_{h1} - \Delta y_{h3} (l^2 (m_{h1} + \\
 & m_{h2}) - \Delta y_{h2}^2 m_{h2})) + m_{h1} m_{h2} (-l^2 (\Delta y_{h2} (\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) + \\
 & \Delta y_{h1} (\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})) + (\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2}) (\Delta x_{h2} \Delta x_{h3} \Delta y_{h1} + \\
 & \Delta y_{h1} \Delta y_{h2} \Delta y_{h3} + \Delta y_{h1} \Delta z_{h2} \Delta z_{h3} + \Delta y_{h2} \Delta z_{h1} \Delta z_{h3} - \Delta y_{h3} \Delta z_{h1} \Delta z_{h2}) + \\
 & \Delta y_{h3} l^4) + \Delta y_{h3} l^4 m_l^2)) / (den m_{h1} m_{h2} m_{h3} m_l^3)
 \end{aligned}$$

$$\begin{aligned}
 C(11, 9) = & ((z_{h3} - z_l) (\Delta x_{h1}^2 \Delta x_{h2} m_{h1} m_{h2} (\Delta x_{h3} \Delta y_{h2} - \Delta x_{h2} \Delta y_{h3}) + \\
 & \Delta x_{h1} m_{h1} (m_{h2} (\Delta x_{h2}^2 \Delta x_{h3} \Delta y_{h1} + \Delta x_{h2} (\Delta y_{h1} \Delta z_{h2} \Delta z_{h3} + \\
 & \Delta y_{h2} \Delta z_{h1} \Delta z_{h3} - 2 \Delta y_{h3} \Delta z_{h1} \Delta z_{h2}) + \Delta x_{h3} (\Delta y_{h1} (\Delta y_{h2} - l) (\Delta y_{h2} + l) + \\
 & \Delta y_{h2} \Delta z_{h1} \Delta z_{h2})) - \Delta x_{h3} \Delta y_{h1} l^2 m_l) - l^2 m_l (\Delta y_{h2} m_{h2} (\Delta x_{h2} \Delta x_{h3} + \\
 & \Delta z_{h2} \Delta z_{h3}) + \Delta y_{h1}^2 \Delta y_{h3} m_{h1} + \Delta y_{h1} \Delta z_{h1} \Delta z_{h3} m_{h1} - \Delta y_{h3} (l^2 (m_{h1} + \\
 & m_{h2}) - \Delta y_{h2}^2 m_{h2})) + m_{h1} m_{h2} (-l^2 (\Delta y_{h2} (\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) + \\
 & \Delta y_{h1} (\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})) + (\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2}) (\Delta x_{h2} \Delta x_{h3} \Delta y_{h1} + \\
 & \Delta y_{h1} \Delta y_{h2} \Delta y_{h3} + \Delta y_{h1} \Delta z_{h2} \Delta z_{h3} + \Delta y_{h2} \Delta z_{h1} \Delta z_{h3} - \Delta y_{h3} \Delta z_{h1} \Delta z_{h2}) + \\
 & \Delta y_{h3} l^4) + \Delta y_{h3} l^4 m_l^2)) / (den m_{h1} m_{h2} m_{h3} m_l^3)
 \end{aligned}$$

$$\begin{aligned}
 C(12, 1) = & (\Delta x_{h1} (\Delta x_{h2}^2 \Delta x_{h3} m_{h2} m_{h3} (\Delta x_{h1} \Delta z_{h3} - \Delta x_{h3} \Delta z_{h1}) + \\
 & \Delta x_{h2} m_{h2} (\Delta x_{h1} \Delta x_{h3}^2 \Delta z_{h2} m_{h3} + \Delta x_{h1} (\Delta y_{h2} \Delta y_{h3} \Delta z_{h3} m_{h3} + \\
 & \Delta z_{h2} (\Delta z_{h3}^2 m_{h3} - l^2 (m_{h3} + m_l))) + \Delta x_{h3} m_{h3} (\Delta y_{h1} \Delta y_{h2} \Delta z_{h3} + \\
 & \Delta y_{h1} \Delta y_{h3} \Delta z_{h2} - 2 \Delta y_{h2} \Delta y_{h3} \Delta z_{h1})) + \Delta y_{h2} \Delta z_{h2} m_{h2} (\Delta x_{h1} \Delta x_{h3} \Delta y_{h3} m_{h3} + \\
 & \Delta y_{h1} (\Delta y_{h3}^2 m_{h3} + \Delta z_{h3}^2 m_{h3} - l^2 (m_{h3} + m_l))) + \\
 & (\Delta z_{h2}^2 m_{h2} - l^2 (m_{h2} + m_l)) (m_{h3} (\Delta x_{h1} \Delta x_{h3} \Delta z_{h3} + \\
 & \Delta y_{h1} \Delta y_{h3} \Delta z_{h3} + \Delta z_{h1} (\Delta z_{h3} - l) (\Delta z_{h3} + l)) - \Delta z_{h1} l^2 m_l) + \\
 & \Delta y_{h2}^2 \Delta y_{h3} m_{h2} m_{h3} (\Delta y_{h1} \Delta z_{h3} - \Delta y_{h3} \Delta z_{h1})) / (den m_{h1} m_{h2} m_{h3} m_l^3)
 \end{aligned}$$

$$\begin{aligned}
 C(12, 2) = & -((y_{h1} - y_l) (\Delta x_{h2}^2 \Delta x_{h3} m_{h2} m_{h3} (\Delta x_{h3} \Delta z_{h1} - \Delta x_{h1} \Delta z_{h3}) + \\
 & \Delta x_{h2} m_{h2} (\Delta x_{h1} \Delta z_{h2} l^2 m_l - m_{h3} (\Delta x_{h1} \Delta x_{h3}^2 \Delta z_{h2} + \\
 & \Delta x_{h1} (\Delta y_{h2} \Delta y_{h3} \Delta z_{h3} + \Delta z_{h2} (\Delta z_{h3} - l) (\Delta z_{h3} + l)) + \\
 & \Delta x_{h3} (\Delta y_{h1} \Delta y_{h2} \Delta z_{h3} + \Delta y_{h1} \Delta y_{h3} \Delta z_{h2} - 2 \Delta y_{h2} \Delta y_{h3} \Delta z_{h1}))) + \\
 & \Delta y_{h2} \Delta z_{h2} m_{h2} (\Delta y_{h1} l^2 m_l - m_{h3} (\Delta x_{h1} \Delta x_{h3} \Delta y_{h3} + \Delta y_{h1} (\Delta y_{h3}^2 + \\
 & \Delta z_{h3}^2 - l^2))) + (\Delta z_{h2}^2 m_{h2} - l^2 (m_{h2} + m_l)) (\Delta z_{h1} l^2 m_l - m_{h3} (\Delta x_{h1} \Delta x_{h3} \Delta z_{h3} + \\
 & \Delta y_{h1} \Delta y_{h3} \Delta z_{h3} + \Delta z_{h1} (\Delta z_{h3} - l) (\Delta z_{h3} + l))) + \\
 & \Delta y_{h2}^2 \Delta y_{h3} m_{h2} m_{h3} (\Delta y_{h3} \Delta z_{h1} - \Delta y_{h1} \Delta z_{h3})) / (den m_{h1} m_{h2} m_{h3} m_l^3)
 \end{aligned}$$

$$\begin{aligned}
 C(12, 3) = & -((z_{h1} - z_l) (\Delta x_{h2}^2 \Delta x_{h3} m_{h2} m_{h3} (\Delta x_{h3} \Delta z_{h1} - \Delta x_{h1} \Delta z_{h3}) + \\
 & \Delta x_{h2} m_{h2} (\Delta x_{h1} \Delta z_{h2} l^2 m_l - m_{h3} (\Delta x_{h1} \Delta x_{h3}^2 \Delta z_{h2} + \\
 & \Delta x_{h1} (\Delta y_{h2} \Delta y_{h3} \Delta z_{h3} + \Delta z_{h2} (\Delta z_{h3} - l) (\Delta z_{h3} + l)) + \\
 & \Delta x_{h3} (\Delta y_{h1} \Delta y_{h2} \Delta z_{h3} + \Delta y_{h1} \Delta y_{h3} \Delta z_{h2} - 2 \Delta y_{h2} \Delta y_{h3} \Delta z_{h1}))) + \\
 & \Delta y_{h2} \Delta z_{h2} m_{h2} (\Delta y_{h1} l^2 m_l - m_{h3} (\Delta x_{h1} \Delta x_{h3} \Delta y_{h3} + \Delta y_{h1} (\Delta y_{h3}^2 + \\
 & \Delta z_{h3}^2 - l^2))) + (\Delta z_{h2}^2 m_{h2} - l^2 (m_{h2} + m_l)) (\Delta z_{h1} l^2 m_l - m_{h3} (\Delta x_{h1} \Delta x_{h3} \Delta z_{h3} + \\
 & \Delta y_{h1} \Delta y_{h3} \Delta z_{h3} + \Delta z_{h1} (\Delta z_{h3} - l) (\Delta z_{h3} + l))) + \\
 & \Delta y_{h2}^2 \Delta y_{h3} m_{h2} m_{h3} (\Delta y_{h3} \Delta z_{h1} - \Delta y_{h1} \Delta z_{h3})) / (den m_{h1} m_{h2} m_{h3} m_l^3)
 \end{aligned}$$

$$\begin{aligned}
 C(12, 4) = & (\Delta x_{h2} (\Delta x_{h1}^2 \Delta x_{h3} m_{h1} m_{h3} (\Delta x_{h2} \Delta z_{h3} - \Delta x_{h3} \Delta z_{h2}) + \\
 & \Delta x_{h1} m_{h1} (\Delta x_{h2} m_{h3} (\Delta z_{h1} (\Delta x_{h3}^2 + \Delta z_{h3}^2 - l^2) + \\
 & \Delta y_{h1} \Delta y_{h3} \Delta z_{h3}) - \Delta x_{h2} \Delta z_{h1} l^2 m_l + \\
 & \Delta x_{h3} m_{h3} (\Delta y_{h1} \Delta y_{h2} \Delta z_{h3} - 2 \Delta y_{h1} \Delta y_{h3} \Delta z_{h2} + \\
 & \Delta y_{h2} \Delta y_{h3} \Delta z_{h1})) - l^2 m_l (\Delta x_{h2} \Delta x_{h3} \Delta z_{h3} m_{h3} + \Delta y_{h1} \Delta y_{h2} \Delta z_{h1} m_{h1} + \\
 & \Delta y_{h2} \Delta y_{h3} \Delta z_{h3} m_{h3} + \Delta z_{h2} (\Delta z_{h1}^2 m_{h1} + \Delta z_{h3}^2 m_{h3} - l^2 (m_{h1} + m_{h3}))) + \\
 & m_{h1} m_{h3} (-l^2 (\Delta x_{h2} \Delta x_{h3} \Delta z_{h3} + \Delta y_{h1} \Delta y_{h2} \Delta z_{h1} + \Delta y_{h2} \Delta y_{h3} \Delta z_{h3} + \\
 & \Delta z_{h2} (\Delta z_{h1}^2 + \Delta z_{h3}^2)) + (\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) (\Delta x_{h2} \Delta x_{h3} \Delta z_{h1} + \\
 & \Delta y_{h1} \Delta y_{h2} \Delta z_{h3} - \Delta y_{h1} \Delta y_{h3} \Delta z_{h2} + \Delta y_{h2} \Delta y_{h3} \Delta z_{h1} + \\
 & \Delta z_{h1} \Delta z_{h2} \Delta z_{h3}) + \Delta z_{h2} l^4) + \Delta z_{h2} l^4 m_l^2) / (den m_{h1} m_{h2} m_{h3} m_l^3)
 \end{aligned}$$

$$\begin{aligned}
 C(12, 5) = & ((y_{h2} - y_l) (\Delta x_{h1}^2 \Delta x_{h3} m_{h1} m_{h3} (\Delta x_{h2} \Delta z_{h3} - \Delta x_{h3} \Delta z_{h2}) + \\
 & \Delta x_{h1} m_{h1} (\Delta x_{h2} m_{h3} (\Delta z_{h1} (\Delta x_{h3}^2 + \Delta z_{h3}^2 - l^2) + \\
 & \Delta y_{h1} \Delta y_{h3} \Delta z_{h3}) - \Delta x_{h2} \Delta z_{h1} l^2 m_l + \\
 & \Delta x_{h3} m_{h3} (\Delta y_{h1} \Delta y_{h2} \Delta z_{h3} - 2 \Delta y_{h1} \Delta y_{h3} \Delta z_{h2} + \\
 & \Delta y_{h2} \Delta y_{h3} \Delta z_{h1})) - l^2 m_l (\Delta x_{h2} \Delta x_{h3} \Delta z_{h3} m_{h3} + \Delta y_{h1} \Delta y_{h2} \Delta z_{h1} m_{h1} + \\
 & \Delta y_{h2} \Delta y_{h3} \Delta z_{h3} m_{h3} + \Delta z_{h2} (\Delta z_{h1}^2 m_{h1} + \Delta z_{h3}^2 m_{h3} - l^2 (m_{h1} + m_{h3}))) + \\
 & m_{h1} m_{h3} (-l^2 (\Delta x_{h2} \Delta x_{h3} \Delta z_{h3} + \Delta y_{h1} \Delta y_{h2} \Delta z_{h1} + \Delta y_{h2} \Delta y_{h3} \Delta z_{h3} + \\
 & \Delta z_{h2} (\Delta z_{h1}^2 + \Delta z_{h3}^2)) + (\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) (\Delta x_{h2} \Delta x_{h3} \Delta z_{h1} + \\
 & \Delta y_{h1} \Delta y_{h2} \Delta z_{h3} - \Delta y_{h1} \Delta y_{h3} \Delta z_{h2} + \Delta y_{h2} \Delta y_{h3} \Delta z_{h1} + \\
 & \Delta z_{h1} \Delta z_{h2} \Delta z_{h3}) + \Delta z_{h2} l^4) + \Delta z_{h2} l^4 m_l^2) / (den m_{h1} m_{h2} m_{h3} m_l^3)
 \end{aligned}$$

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$$\begin{aligned}
C(12, 6) = & ((z_{h2} - z_l) (\Delta x_{h1}^2 \Delta x_{h3} m_{h1} m_{h3} (\Delta x_{h2} \Delta z_{h3} - \Delta x_{h3} \Delta z_{h2}) + \\
& \Delta x_{h1} m_{h1} (\Delta x_{h2} m_{h3} (\Delta z_{h1} (\Delta x_{h3}^2 + \Delta z_{h3}^2 - l^2) + \\
& \Delta y_{h1} \Delta y_{h3} \Delta z_{h3}) - \Delta x_{h2} \Delta z_{h1} l^2 m_l + \\
& \Delta x_{h3} m_{h3} (\Delta y_{h1} \Delta y_{h2} \Delta z_{h3} - 2 \Delta y_{h1} \Delta y_{h3} \Delta z_{h2} + \\
& \Delta y_{h2} \Delta y_{h3} \Delta z_{h1})) - l^2 m_l (\Delta x_{h2} \Delta x_{h3} \Delta z_{h3} m_{h3} + \Delta y_{h1} \Delta y_{h2} \Delta z_{h1} m_{h1} + \\
& \Delta y_{h2} \Delta y_{h3} \Delta z_{h3} m_{h3} + \Delta z_{h2} (\Delta z_{h1}^2 m_{h1} + \Delta z_{h3}^2 m_{h3} - l^2 (m_{h1} + m_{h3}))) + \\
& m_{h1} m_{h3} (-l^2 (\Delta x_{h2} \Delta x_{h3} \Delta z_{h3} + \Delta y_{h1} \Delta y_{h2} \Delta z_{h1} + \Delta y_{h2} \Delta y_{h3} \Delta z_{h3} + \\
& \Delta z_{h2} (\Delta z_{h1}^2 + \Delta z_{h3}^2)) + (\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) (\Delta x_{h2} \Delta x_{h3} \Delta z_{h1} + \\
& \Delta y_{h1} \Delta y_{h2} \Delta z_{h3} - \Delta y_{h1} \Delta y_{h3} \Delta z_{h2} + \Delta y_{h2} \Delta y_{h3} \Delta z_{h1} + \\
& \Delta z_{h1} \Delta z_{h2} \Delta z_{h3}) + \Delta z_{h2} l^4) + \Delta z_{h2} l^4 m_l^2)) / (den m_{h1} m_{h2} m_{h3} m_l^3)
\end{aligned}$$

$$\begin{aligned}
C(12, 7) = & (\Delta x_{h3} (\Delta x_{h1}^2 \Delta x_{h2} m_{h1} m_{h2} (\Delta x_{h3} \Delta z_{h2} - \Delta x_{h2} \Delta z_{h3}) + \\
& \Delta x_{h1} m_{h1} (m_{h2} (\Delta x_{h2}^2 \Delta x_{h3} \Delta z_{h1} + \Delta x_{h2} (-2 \Delta y_{h1} \Delta y_{h2} \Delta z_{h3} + \\
& \Delta y_{h1} \Delta y_{h3} \Delta z_{h2} + \Delta y_{h2} \Delta y_{h3} \Delta z_{h1}) + \Delta x_{h3} (\Delta y_{h1} \Delta y_{h2} \Delta z_{h2} + \\
& \Delta z_{h1} (\Delta z_{h2} - l) (\Delta z_{h2} + l))) - \Delta x_{h3} \Delta z_{h1} l^2 m_l) - l^2 m_l (\Delta z_{h2} m_{h2} (\Delta x_{h2} \Delta x_{h3} + \\
& \Delta y_{h2} \Delta y_{h3}) + \Delta y_{h1} \Delta y_{h3} \Delta z_{h1} m_{h1} + \Delta z_{h3} (\Delta z_{h1}^2 m_{h1} + \\
& \Delta z_{h2}^2 m_{h2} - l^2 (m_{h1} + m_{h2}))) + m_{h1} m_{h2} (-l^2 (\Delta x_{h2} \Delta x_{h3} \Delta z_{h2} + \\
& \Delta y_{h1} \Delta y_{h3} \Delta z_{h1} + \Delta y_{h2} \Delta y_{h3} \Delta z_{h2} + \Delta z_{h3} (\Delta z_{h1}^2 + \Delta z_{h2}^2)) + \\
& (\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2}) (\Delta x_{h2} \Delta x_{h3} \Delta z_{h1} - \Delta y_{h1} \Delta y_{h2} \Delta z_{h3} + \\
& \Delta y_{h1} \Delta y_{h3} \Delta z_{h2} + \Delta y_{h2} \Delta y_{h3} \Delta z_{h1} + \Delta z_{h1} \Delta z_{h2} \Delta z_{h3}) + \\
& \Delta z_{h3} l^4) + \Delta z_{h3} l^4 m_l^2)) / (den m_{h1} m_{h2} m_{h3} m_l^3)
\end{aligned}$$

$$\begin{aligned}
C(12, 8) = & - ((y_{h3} - y_l) (\Delta x_{h1}^2 \Delta x_{h2} m_{h1} m_{h2} (\Delta x_{h2} \Delta z_{h3} - \Delta x_{h3} \Delta z_{h2}) + \\
& \Delta x_{h1} m_{h1} (\Delta x_{h3} \Delta z_{h1} l^2 m_l - m_{h2} (\Delta x_{h2}^2 \Delta x_{h3} \Delta z_{h1} + \Delta x_{h2} (-2 \Delta y_{h1} \Delta y_{h2} \Delta z_{h3} + \\
& \Delta y_{h1} \Delta y_{h3} \Delta z_{h2} + \Delta y_{h2} \Delta y_{h3} \Delta z_{h1}) + \Delta x_{h3} (\Delta y_{h1} \Delta y_{h2} \Delta z_{h2} + \\
& \Delta z_{h1} (\Delta z_{h2} - l) (\Delta z_{h2} + l))) + l^2 m_l (\Delta z_{h2} m_{h2} (\Delta x_{h2} \Delta x_{h3} + \\
& \Delta y_{h2} \Delta y_{h3}) + \Delta y_{h1} \Delta y_{h3} \Delta z_{h1} m_{h1} + \Delta z_{h3} (\Delta z_{h1}^2 m_{h1} + \\
& \Delta z_{h2}^2 m_{h2} - l^2 (m_{h1} + m_{h2}))) - m_{h1} m_{h2} (-l^2 (\Delta x_{h2} \Delta x_{h3} \Delta z_{h2} + \\
& \Delta y_{h1} \Delta y_{h3} \Delta z_{h1} + \Delta y_{h2} \Delta y_{h3} \Delta z_{h2} + \Delta z_{h3} (\Delta z_{h1}^2 + \Delta z_{h2}^2)) + \\
& (\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2}) (\Delta x_{h2} \Delta x_{h3} \Delta z_{h1} - \Delta y_{h1} \Delta y_{h2} \Delta z_{h3} + \\
& \Delta y_{h1} \Delta y_{h3} \Delta z_{h2} + \Delta y_{h2} \Delta y_{h3} \Delta z_{h1} + \Delta z_{h1} \Delta z_{h2} \Delta z_{h3}) + \\
& \Delta z_{h3} l^4) - \Delta z_{h3} l^4 m_l^2)) / (den m_{h1} m_{h2} m_{h3} m_l^3)
\end{aligned}$$

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$$\begin{aligned}
C(12, 9) = & -((z_{h3} - z_l) (\Delta x_{h1}^2 \Delta x_{h2} m_{h1} m_{h2} (\Delta x_{h2} \Delta z_{h3} - \Delta x_{h3} \Delta z_{h2}) + \\
& \Delta x_{h1} m_{h1} (\Delta x_{h3} \Delta z_{h1} l^2 m_l - m_{h2} (\Delta x_{h2}^2 \Delta x_{h3} \Delta z_{h1} + \Delta x_{h2} (-2 \Delta y_{h1} \Delta y_{h2} \Delta z_{h3} + \\
& \Delta y_{h1} \Delta y_{h3} \Delta z_{h2} + \Delta y_{h2} \Delta y_{h3} \Delta z_{h1})) + \Delta x_{h3} (\Delta y_{h1} \Delta y_{h2} \Delta z_{h2} + \\
& \Delta z_{h1} (\Delta z_{h2} - l) (\Delta z_{h2} + l))) + l^2 m_l (\Delta z_{h2} m_{h2} (\Delta x_{h2} \Delta x_{h3} + \\
& \Delta y_{h2} \Delta y_{h3}) + \Delta y_{h1} \Delta y_{h3} \Delta z_{h1} m_{h1} + \Delta z_{h3} (\Delta z_{h1}^2 m_{h1} + \\
& \Delta z_{h2}^2 m_{h2} - l^2 (m_{h1} + m_{h2}))) - m_{h1} m_{h2} (-l^2 (\Delta x_{h2} \Delta x_{h3} \Delta z_{h2} + \\
& \Delta y_{h1} \Delta y_{h3} \Delta z_{h1} + \Delta y_{h2} \Delta y_{h3} \Delta z_{h2} + \Delta z_{h3} (\Delta z_{h1}^2 + \Delta z_{h2}^2)) + \\
& (\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2}) (\Delta x_{h2} \Delta x_{h3} \Delta z_{h1} - \Delta y_{h1} \Delta y_{h2} \Delta z_{h3} + \\
& \Delta y_{h1} \Delta y_{h3} \Delta z_{h2} + \Delta y_{h2} \Delta y_{h3} \Delta z_{h1} + \Delta z_{h1} \Delta z_{h2} \Delta z_{h3}) + \\
& \Delta z_{h3} l^4) - \Delta z_{h3} l^4 m_l^2)) / (den m_{h1} m_{h2} m_{h3} m_l^3)
\end{aligned}$$

$$\begin{aligned}
d(1) = & \ddot{x}_{h1} - (\Delta x_{h1} (-((\dot{x}_{h3} - \dot{x}_l)^2 + (\dot{y}_{h3} - \dot{y}_l)^2 + (\dot{z}_{h3} - \dot{z}_l)^2) ((\Delta x_{h1} \Delta x_{h2} + \\
& \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2}) (\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - (l^2 (m_{h2} + \\
& m_l) (\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})) / (m_{h2})) - ((\dot{x}_{h2} - \dot{x}_l)^2 + (\dot{y}_{h2} - \dot{y}_l)^2 + \\
& (\dot{z}_{h2} - \dot{z}_l)^2) ((\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) (\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \\
& \Delta z_{h2} \Delta z_{h3}) - (l^2 (m_{h3} + m_l) (\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})) / (m_{h3})) + \\
& ((\dot{x}_{h1} - \dot{x}_l)^2 + (\dot{y}_{h1} - \dot{y}_l)^2 + (\dot{z}_{h1} - \dot{z}_l)^2) (- (l^4 (m_{h2} + m_l) (m_{h3} + \\
& m_l)) / (m_{h2} m_{h3}) - (\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})^2))) / (den m_{h1} m_l^2)
\end{aligned}$$

$$\begin{aligned}
d(2) = & \ddot{y}_{h1} - (\Delta y_{h1} (-((\dot{x}_{h3} - \dot{x}_l)^2 + (\dot{y}_{h3} - \dot{y}_l)^2 + (\dot{z}_{h3} - \dot{z}_l)^2) ((\Delta x_{h1} \Delta x_{h2} + \\
& \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2}) (\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - (l^2 (m_{h2} + \\
& m_l) (\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})) / (m_{h2})) - ((\dot{x}_{h2} - \dot{x}_l)^2 + (\dot{y}_{h2} - \dot{y}_l)^2 + \\
& (\dot{z}_{h2} - \dot{z}_l)^2) ((\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) (\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \\
& \Delta z_{h2} \Delta z_{h3}) - (l^2 (m_{h3} + m_l) (\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})) / (m_{h3})) + \\
& ((\dot{x}_{h1} - \dot{x}_l)^2 + (\dot{y}_{h1} - \dot{y}_l)^2 + (\dot{z}_{h1} - \dot{z}_l)^2) (- (l^4 (m_{h2} + m_l) (m_{h3} + \\
& m_l)) / (m_{h2} m_{h3}) - (\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})^2))) / (den m_{h1} m_l^2)
\end{aligned}$$

$$\begin{aligned}
d(3) = & -(\Delta z_{h1} (-((\dot{x}_{h3} - \dot{x}_l)^2 + (\dot{y}_{h3} - \dot{y}_l)^2 + (\dot{z}_{h3} - \dot{z}_l)^2) ((\Delta x_{h1} \Delta x_{h2} + \\
& \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2}) (\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - (l^2 (m_{h2} + \\
& m_l) (\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})) / (m_{h2})) - ((\dot{x}_{h2} - \dot{x}_l)^2 + \\
& (\dot{y}_{h2} - \dot{y}_l)^2 + (\dot{z}_{h2} - \dot{z}_l)^2) ((\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) (\Delta x_{h2} \Delta x_{h3} + \\
& \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - (l^2 (m_{h3} + m_l) (\Delta x_{h1} \Delta x_{h2} + \\
& \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})) / (m_{h3})) + ((\dot{x}_{h1} - \dot{x}_l)^2 + (\dot{y}_{h1} - \dot{y}_l)^2 + \\
& (\dot{z}_{h1} - \dot{z}_l)^2) (- (l^4 (m_{h2} + m_l) (m_{h3} + m_l)) / (m_{h2} m_{h3}) - (\Delta x_{h2} \Delta x_{h3} + \\
& \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})^2))) / (den m_{h1} m_l^2) + G + \ddot{z}_{h1}
\end{aligned}$$

6 Dynamic Equations of the Translational Multi-Lift Model

$$\begin{aligned}
d(4) &= \ddot{x}_{h2} - (\Delta x_{h2} (-((\dot{x}_{h3} - \dot{x}_l)^2 + (\dot{y}_{h3} - \dot{y}_l)^2 + (\dot{z}_{h3} - \dot{z}_l)^2) ((\Delta x_{h1} \Delta x_{h2} + \\
&\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})(\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) - (l^2(m_{h1} + \\
&m_l)(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})) / (m_{h1})) + ((\dot{x}_{h1} - \dot{x}_l)^2 + \\
&(\dot{y}_{h1} - \dot{y}_l)^2 + (\dot{z}_{h1} - \dot{z}_l)^2) (-((\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \\
&\Delta z_{h1} \Delta z_{h3})(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - (l^2(m_{h3} + \\
&m_l)(\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})) / (m_{h3}))) - ((\dot{x}_{h2} - \dot{x}_l)^2 + \\
&(\dot{y}_{h2} - \dot{y}_l)^2 + (\dot{z}_{h2} - \dot{z}_l)^2) ((l^4(m_{h1} + m_l)(m_{h3} + \\
&m_l)) / (m_{h1} m_{h3}) - (\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})^2)) / (den m_{h2} m_l^2) \\
d(5) &= \ddot{y}_{h2} - (\Delta y_{h2} (-((\dot{x}_{h3} - \dot{x}_l)^2 + (\dot{y}_{h3} - \dot{y}_l)^2 + (\dot{z}_{h3} - \dot{z}_l)^2) ((\Delta x_{h1} \Delta x_{h2} + \\
&\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})(\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) - (l^2(m_{h1} + \\
&m_l)(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})) / (m_{h1})) + ((\dot{x}_{h1} - \dot{x}_l)^2 + \\
&(\dot{y}_{h1} - \dot{y}_l)^2 + (\dot{z}_{h1} - \dot{z}_l)^2) (-((\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \\
&\Delta z_{h1} \Delta z_{h3})(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - (l^2(m_{h3} + \\
&m_l)(\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})) / (m_{h3}))) - ((\dot{x}_{h2} - \dot{x}_l)^2 + \\
&(\dot{y}_{h2} - \dot{y}_l)^2 + (\dot{z}_{h2} - \dot{z}_l)^2) ((l^4(m_{h1} + m_l)(m_{h3} + \\
&m_l)) / (m_{h1} m_{h3}) - (\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})^2)) / (den m_{h2} m_l^2) \\
d(6) &= -(\Delta z_{h2} (-((\dot{x}_{h3} - \dot{x}_l)^2 + (\dot{y}_{h3} - \dot{y}_l)^2 + (\dot{z}_{h3} - \dot{z}_l)^2) ((\Delta x_{h1} \Delta x_{h2} + \\
&\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})(\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) - (l^2(m_{h1} + \\
&m_l)(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})) / (m_{h1})) + ((\dot{x}_{h1} - \dot{x}_l)^2 + \\
&(\dot{y}_{h1} - \dot{y}_l)^2 + (\dot{z}_{h1} - \dot{z}_l)^2) (-((\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \\
&\Delta z_{h1} \Delta z_{h3})(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - (l^2(m_{h3} + m_l)(\Delta x_{h1} \Delta x_{h2} + \\
&\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})) / (m_{h3}))) - ((\dot{x}_{h2} - \dot{x}_l)^2 + (\dot{y}_{h2} - \dot{y}_l)^2 + \\
&(\dot{z}_{h2} - \dot{z}_l)^2) ((l^4(m_{h1} + m_l)(m_{h3} + m_l)) / (m_{h1} m_{h3}) - (\Delta x_{h1} \Delta x_{h3} + \\
&\Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})^2)) / (den m_{h2} m_l^2) + G + \ddot{z}_{h2} \\
d(7) &= \ddot{x}_{h3} - (\Delta x_{h3} (-((\dot{x}_{h2} - \dot{x}_l)^2 + (\dot{y}_{h2} - \dot{y}_l)^2 + (\dot{z}_{h2} - \dot{z}_l)^2) ((\Delta x_{h1} \Delta x_{h2} + \\
&\Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})(\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3}) - (l^2(m_{h1} + \\
&m_l)(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3})) / (m_{h1})) + ((\dot{x}_{h1} - \dot{x}_l)^2 + \\
&(\dot{y}_{h1} - \dot{y}_l)^2 + (\dot{z}_{h1} - \dot{z}_l)^2) (-((\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \\
&\Delta z_{h1} \Delta z_{h2})(\Delta x_{h2} \Delta x_{h3} + \Delta y_{h2} \Delta y_{h3} + \Delta z_{h2} \Delta z_{h3}) - (l^2(m_{h2} + \\
&m_l)(\Delta x_{h1} \Delta x_{h3} + \Delta y_{h1} \Delta y_{h3} + \Delta z_{h1} \Delta z_{h3})) / (m_{h2}))) - ((\dot{x}_{h3} - \dot{x}_l)^2 + \\
&(\dot{y}_{h3} - \dot{y}_l)^2 + (\dot{z}_{h3} - \dot{z}_l)^2) ((l^4(m_{h1} + m_l)(m_{h2} + \\
&m_l)) / (m_{h1} m_{h2}) - (\Delta x_{h1} \Delta x_{h2} + \Delta y_{h1} \Delta y_{h2} + \Delta z_{h1} \Delta z_{h2})^2)) / (den m_{h3} m_l^2)
\end{aligned}$$

7 Dynamic Equations of the Flexible Rope Model

This chapter presents the dynamic equations of the flexible rope model. The model has been designed using the Kane method and the equations have been generated using the tool Autolev. The equations for three particular configurations $n \in \{1, 2, 5\}$, composed of one, two and five mass points, are provided.

7.1 Composed of One Mass Point

The equation of this configuration is only provided for the validation of the modeling approach. At least five mass points are required for the model, to reflect the motion of LTD, rope and load correctly, see Sec. 7.3.

For $n = 1$ the following equation is derived:

$$\ddot{\alpha}_1 = -(-\cos(\alpha_1)F_1 + G\sin(\alpha_1)m_1 + k_1l_1\dot{\alpha}_1)/(l_1m_1)$$

Where α_1 is the pendulum angle, with $\dot{\alpha}_1 = d\alpha_1/dt$ and $\ddot{\alpha}_1 = d^2\alpha_1/dt^2$. The constants l_1 , m_1 denote pendulum length and pendulum mass respectively. The coefficient k_1 describes the friction in the pendulum joint and the force F_1 can be used to stimulate the pendulum. It is easy to see, that for $k_1 = 0$ and $F_1 = 0$ the equation become equivalent to the dynamic equation of an idealized pendulum.

7.2 Composed of Two Mass Points

Similar to Sec. 7.1, the equations of this configuration are only provided for the validation of the modeling approach. At least five mass point are required for the model, to reflect the motion of LTD, rope and load correctly, see Sec. 7.3.

For $n = 2$ the following equations are derived:

$$\begin{aligned} \ddot{\alpha}_1 = & (k_2l_1\dot{\alpha}_1\cos^2(\alpha_2) + (\sin(\alpha_2)l_1m_2\dot{\alpha}_1^2 - \cos(\alpha_1 + \alpha_2)F_2 + G\sin(\alpha_1 + \\ & \alpha_2)m_2)\cos(\alpha_2) + \sin(\alpha_2)l_2m_2(\dot{\alpha}_1 + \dot{\alpha}_2)^2 + \cos(\alpha_1)(F_1 + \\ & F_2) - G\sin(\alpha_1)(m_1 + m_2) - (k_1 + k_2)l_1\dot{\alpha}_1)/(l_1(-m_2\cos^2(\alpha_2) + m_1 + m_2)) \\ \ddot{\alpha}_2 = & -(2(((m_1 + m_2)l_1^2 + 2\cos(\alpha_2)l_2m_2l_1 + l_2^2m_2)(-\cos(\alpha_1 + \alpha_2)F_2 + \\ & G\sin(\alpha_1 + \alpha_2)m_2 + l_1\dot{\alpha}_1(\cos(\alpha_2)k_2 + \sin(\alpha_2)m_2\dot{\alpha}_1) + k_2l_2(\dot{\alpha}_1 + \\ & \dot{\alpha}_2))))/(m_2) - (\cos(\alpha_2)l_1 + l_2)(k_1\dot{\alpha}_1l_1^2 - \cos(\alpha_1)(F_1 + F_2)l_1 + G\sin(\alpha_1)(m_1 + \\ & m_2)l_1 + \cos(\alpha_2)k_2l_2(2\dot{\alpha}_1 + \dot{\alpha}_2)l_1 + k_2(\dot{\alpha}_1l_1^2 + l_2^2(\dot{\alpha}_1 + \dot{\alpha}_2)) - l_2(\cos(\alpha_1 + \alpha_2)F_2 + \\ & m_2(\sin(\alpha_2)l_1\dot{\alpha}_2(2\dot{\alpha}_1 + \dot{\alpha}_2) - G\sin(\alpha_1 + \alpha_2)))))/(l_1^2l_2(2m_1 - \cos(2\alpha_2)m_2 + m_2)) \end{aligned}$$

7 Dynamic Equations of the Flexible Rope Model

Where α_i is the angle of the i 'th pendulum, with $\dot{\alpha}_i = \alpha_i \text{ d}/\text{d}t$ and $\ddot{\alpha}_i = \alpha_i \text{ d}^2/\text{d}t^2$. The constants l_i , m_i denote length and mass of the pendulum respectively. The coefficient k_i describes the friction in the i 'th pendulum joint and the force F_i can be used to stimulate the i 'th pendulum. For $k_{1,2} = 0$ and $F_{1,2} = 0$ the equations become equivalent to the dynamic equations of an idealized double pendulum:

$$\begin{aligned}\ddot{\alpha}_1 &= (m_2 (G \cos(\alpha_2) \sin(\alpha_1 + \alpha_2) + \sin(\alpha_2) (\cos(\alpha_2) l_1 \dot{\alpha}_1^2 + l_2 (\dot{\alpha}_1 + \dot{\alpha}_2)^2)) - G \sin(\alpha_1) (m_1 + m_2)) / (l_1 (-m_2 \cos^2(\alpha_2) + m_1 + m_2)) \\ \ddot{\alpha}_2 &= (G \sin(\alpha_1) l_2 (2m_1 - \cos(2\alpha_2) m_2 + m_2) - 2 \sin(\alpha_2) (l_1^2 (m_1 + m_2) \dot{\alpha}_1^2 + l_2^2 m_2 (\dot{\alpha}_1 + \dot{\alpha}_2)^2 + G \cos(\alpha_1) (\cos(\alpha_2) l_2 m_2 + l_1 (m_1 + m_2)) + \cos(\alpha_2) l_1 l_2 m_2 (2\dot{\alpha}_1^2 + 2\dot{\alpha}_2 \dot{\alpha}_1 + \dot{\alpha}_2^2))) / (2l_1 l_2 (-m_2 \cos^2(\alpha_2) + m_1 + m_2))\end{aligned}$$

7.3 Composed of Five Mass Points

The configuration presented in this section is composed of five consecutive pendulums. With a growing number of consecutive pendulums the generated equations quickly become larger. Therefore, $n = 5$ has been chosen as tradeoff between model complexity and size of the generated equations. The author considers this to be the smallest feasible configuration for the approximation of LTD, flexible rope and load. The topmost and lowermost mass points model the motion of LTD and load respectively. The three intermediate mass points model the motion of the flexible rope.

The variable α_i represents the angle of the i 'th pendulum, with $\dot{\alpha}_i = \alpha_i \text{ d}/\text{d}t$ and $\ddot{\alpha}_i = \alpha_i \text{ d}^2/\text{d}t^2$. The constants l_i , m_i denote length and mass of the pendulum respectively. The coefficient k_i describes the friction in the i 'th pendulum joint and the force F_i can be used to stimulate the i 'th pendulum. The variables z_{xyz} represent intermediate results, which are created automatically during the generation of the equations.

The following equations have been generated for the model:

$$\begin{aligned}z_6 &= \cos(\alpha_1) \\ z_7 &= \sin(\alpha_1) \\ z_{16} &= z_6^2 + z_7^2 \\ z_{21} &= l_1 * z_{16} \\ z_8 &= \cos(\alpha_1 + \alpha_2) \\ z_9 &= \sin(\alpha_1 + \alpha_2) \\ z_{17} &= z_8^2 + z_9^2 \\ z_{22} &= l_2 * z_{17} \\ z_{10} &= \cos(\alpha_1 + \alpha_2 + \alpha_3) \\ z_{11} &= \sin(\alpha_1 + \alpha_2 + \alpha_3)\end{aligned}$$

7 Dynamic Equations of the Flexible Rope Model

$$z_{18} = z_{10}^2 + z_{11}^2$$

$$z_{23} = l_3 * z_{18}$$

$$z_{12} = \cos(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4)$$

$$z_{13} = \sin(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4)$$

$$z_{19} = z_{12}^2 + z_{13}^2$$

$$z_{24} = l_4 * z_{19}$$

$$z_{14} = \cos(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5)$$

$$z_{15} = \sin(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5)$$

$$z_{20} = z_{14}^2 + z_{15}^2$$

$$z_{25} = l_5 * z_{20}$$

$$z_{72} = G * m_1$$

$$z_{57} = k_1 * z_{21} * \dot{\alpha}_1$$

$$z_{73} = G * m_2$$

$$z_{74} = G * m_3$$

$$z_{75} = G * m_4$$

$$z_{76} = G * m_5$$

$$z_{58} = k_2 * z_{21} * \dot{\alpha}_1$$

$$z_{60} = k_3 * z_{21} * \dot{\alpha}_1$$

$$z_{63} = k_4 * z_{21} * \dot{\alpha}_1$$

$$z_{67} = k_5 * z_{21} * \dot{\alpha}_1$$

$$z_{59} = k_2 * z_{22} * (\dot{\alpha}_1 + \dot{\alpha}_2)$$

$$z_{61} = k_3 * z_{22} * (\dot{\alpha}_1 + \dot{\alpha}_2)$$

$$z_{64} = k_4 * z_{22} * (\dot{\alpha}_1 + \dot{\alpha}_2)$$

$$z_{68} = k_5 * z_{22} * (\dot{\alpha}_1 + \dot{\alpha}_2)$$

$$z_{62} = k_3 * z_{23} * (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3)$$

$$z_{65} = k_4 * z_{23} * (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3)$$

$$z_{69} = k_5 * z_{23} * (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3)$$

$$z_{66} = k_4 * z_{24} * (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3 + \dot{\alpha}_4)$$

$$z_{70} = k_5 * z_{24} * (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3 + \dot{\alpha}_4)$$

7 Dynamic Equations of the Flexible Rope Model

$$\begin{aligned}
z_{71} &= k_5 * z_{25} * (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3 + \dot{\alpha}_4 + \dot{\alpha}_5) \\
z_{77} &= z_6 * z_8 + z_7 * z_9 \\
z_{80} &= z_6 * z_{10} + z_7 * z_{11} \\
z_{86} &= z_6 * z_{12} + z_7 * z_{13} \\
z_{95} &= z_6 * z_{14} + z_7 * z_{15} \\
z_{83} &= z_8 * z_{10} + z_9 * z_{11} \\
z_{89} &= z_8 * z_{12} + z_9 * z_{13} \\
z_{98} &= z_8 * z_{14} + z_9 * z_{15} \\
z_{92} &= z_{10} * z_{12} + z_{11} * z_{13} \\
z_{101} &= z_{10} * z_{14} + z_{11} * z_{15} \\
z_{104} &= z_{12} * z_{14} + z_{13} * z_{15} \\
z_{107} &= F_2 * z_6 * z_{21} + F_2 * z_8 * z_{22} + F_3 * z_6 * z_{21} + F_3 * z_8 * z_{22} + F_3 * z_{10} * z_{23} + \\
&F_4 * z_6 * z_{21} + F_4 * z_8 * z_{22} + F_4 * z_{10} * z_{23} + F_4 * z_{12} * z_{24} + F_5 * z_6 * z_{21} + \\
&F_5 * z_8 * z_{22} + F_5 * z_{10} * z_{23} + F_5 * z_{12} * z_{24} + F_5 * z_{14} * z_{25} + \\
&z_{21} * (F_1 * z_6 - z_{72} * z_7 - z_{57}) - z_{73} * z_7 * z_{21} - z_{73} * z_9 * z_{22} - z_{74} * z_7 * z_{21} - z_{74} * z_9 \\
&* z_{22} - z_{74} * z_{11} * z_{23} - z_{75} * z_7 * z_{21} - z_{75} * z_9 * z_{22} - z_{75} * z_{11} * z_{23} - z_{75} * z_{13} \\
&* z_{24} - z_{76} * z_7 * z_{21} - z_{76} * z_9 * z_{22} - z_{76} * z_{11} * z_{23} - z_{76} * z_{13} * z_{24} - z_{76} * z_{15} \\
&* z_{25} - z_{21} * z_{58} - z_{21} * z_{60} - z_{21} * z_{63} - z_{21} * z_{67} - z_{22} * z_{59} - z_{22} * z_{61} - z_{22} * z_{64} \\
&- z_{22} * z_{68} - z_{23} * z_{62} - z_{23} * z_{65} - z_{23} * z_{69} - z_{24} * z_{66} - z_{24} * z_{70} - z_{25} * z_{71} - z_{21} \\
&* z_{77} * z_{59} - z_{21} * z_{77} * z_{61} - z_{21} * z_{77} * z_{64} - z_{21} * z_{77} * z_{68} - z_{21} * z_{80} * z_{62} - z_{21} \\
&* z_{80} * z_{65} - z_{21} * z_{80} * z_{69} - z_{21} * z_{86} * z_{66} - z_{21} * z_{86} * z_{70} - z_{21} * z_{95} * z_{71} - z_{22} \\
&* z_{77} * z_{58} - z_{22} * z_{77} * z_{60} - z_{22} * z_{77} * z_{63} - z_{22} * z_{77} * z_{67} - z_{22} * z_{83} * z_{62} - z_{22} \\
&* z_{83} * z_{65} - z_{22} * z_{83} * z_{69} - z_{22} * z_{89} * z_{66} - z_{22} * z_{89} * z_{70} - z_{22} * z_{98} * z_{71} - z_{23} \\
&* z_{80} * z_{60} - z_{23} * z_{80} * z_{63} - z_{23} * z_{80} * z_{67} - z_{23} * z_{83} * z_{61} - z_{23} * z_{83} * z_{64} - z_{23} \\
&* z_{83} * z_{68} - z_{23} * z_{92} * z_{66} - z_{23} * z_{92} * z_{70} - z_{23} * z_{101} * z_{71} - z_{24} * z_{86} * z_{63} - z_{24} \\
&* z_{86} * z_{67} - z_{24} * z_{89} * z_{64} - z_{24} * z_{89} * z_{68} - z_{24} * z_{92} * z_{65} - z_{24} * z_{92} * z_{69} - z_{24} \\
&* z_{104} * z_{71} - z_{25} * z_{95} * z_{67} - z_{25} * z_{98} * z_{68} - z_{25} * z_{101} * z_{69} - z_{25} * z_{104} * z_{70} \\
z_{31} &= z_8 * (\dot{\alpha}_1 + \dot{\alpha}_2) \\
z_{30} &= z_9 * (\dot{\alpha}_1 + \dot{\alpha}_2) \\
z_{32} &= 2 * z_9 * z_{31} - 2 * z_8 * z_{30} \\
z_{33} &= l_2 * (\dot{\alpha}_1 + \dot{\alpha}_2) * z_{32} \\
z_{78} &= z_7 * z_8 - z_6 * z_9 \\
z_{29} &= z_{22} * (\dot{\alpha}_1 + \dot{\alpha}_2) \\
z_{34} &= z_{17} * (\dot{\alpha}_1 + \dot{\alpha}_2) \\
z_{35} &= z_{29} * z_{34}
\end{aligned}$$

7 Dynamic Equations of the Flexible Rope Model

$$\begin{aligned}
 z_{79} &= z_6 * z_9 - z_7 * z_8 \\
 z_{26} &= z_{21} * \dot{\alpha}_1 \\
 z_{27} &= z_{16} * \dot{\alpha}_1 \\
 z_{28} &= z_{26} * z_{27} \\
 z_{38} &= z_{10} * (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3) \\
 z_{37} &= z_{11} * (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3) \\
 z_{39} &= 2 * z_{11} * z_{38} - 2 * z_{10} * z_{37} \\
 z_{40} &= l_3 * (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3) * z_{39} \\
 z_{81} &= z_7 * z_{10} - z_6 * z_{11} \\
 z_{36} &= z_{23} * (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3) \\
 z_{41} &= z_{18} * (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3) \\
 z_{42} &= z_{36} * z_{41} \\
 z_{84} &= z_9 * z_{10} - z_8 * z_{11} \\
 z_{82} &= z_6 * z_{11} - z_7 * z_{10} \\
 z_{85} &= z_8 * z_{11} - z_9 * z_{10} \\
 z_{45} &= z_{12} * (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3 + \dot{\alpha}_4) \\
 z_{44} &= z_{13} * (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3 + \dot{\alpha}_4) \\
 z_{46} &= 2 * z_{13} * z_{45} - 2 * z_{12} * z_{44} \\
 z_{47} &= l_4 * (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3 + \dot{\alpha}_4) * z_{46} \\
 z_{87} &= z_7 * z_{12} - z_6 * z_{13} \\
 z_{43} &= z_{24} * (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3 + \dot{\alpha}_4) \\
 z_{48} &= z_{19} * (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3 + \dot{\alpha}_4) \\
 z_{49} &= z_{43} * z_{48} \\
 z_{90} &= z_9 * z_{12} - z_8 * z_{13} \\
 z_{93} &= z_{11} * z_{12} - z_{10} * z_{13} \\
 z_{88} &= z_6 * z_{13} - z_7 * z_{12} \\
 z_{91} &= z_8 * z_{13} - z_9 * z_{12} \\
 z_{94} &= z_{10} * z_{13} - z_{11} * z_{12} \\
 z_{52} &= z_{14} * (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3 + \dot{\alpha}_4 + \dot{\alpha}_5) \\
 z_{51} &= z_{15} * (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3 + \dot{\alpha}_4 + \dot{\alpha}_5) \\
 z_{53} &= 2 * z_{15} * z_{52} - 2 * z_{14} * z_{51}
 \end{aligned}$$

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$$z_{54} = l_5 * (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3 + \dot{\alpha}_4 + \dot{\alpha}_5) * z_{53}$$

$$z_{96} = z_7 * z_{14} - z_6 * z_{15}$$

$$z_{50} = z_{25} * (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3 + \dot{\alpha}_4 + \dot{\alpha}_5)$$

$$z_{55} = z_{20} * (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3 + \dot{\alpha}_4 + \dot{\alpha}_5)$$

$$z_{56} = z_{50} * z_{55}$$

$$z_{99} = z_9 * z_{14} - z_8 * z_{15}$$

$$z_{102} = z_{11} * z_{14} - z_{10} * z_{15}$$

$$z_{105} = z_{13} * z_{14} - z_{12} * z_{15}$$

$$z_{97} = z_6 * z_{15} - z_7 * z_{14}$$

$$z_{100} = z_8 * z_{15} - z_9 * z_{14}$$

$$z_{103} = z_{10} * z_{15} - z_{11} * z_{14}$$

$$z_{106} = z_{12} * z_{15} - z_{13} * z_{14}$$

$$\begin{aligned} z_{117} = & m_2 * (z_{22} * z_{33} + z_{21} * z_{77} * z_{33} + z_{21} * z_{78} * z_{35} + z_{22} * z_{79} * z_{28}) + \\ & m_3 * (z_{22} * z_{33} + z_{23} * z_{40} + z_{21} * z_{77} * z_{33} + z_{21} * z_{78} * z_{35} + z_{21} * z_{80} * z_{40} + \\ & z_{21} * z_{81} * z_{42} + z_{22} * z_{79} * z_{28} + z_{22} * z_{83} * z_{40} + z_{22} * z_{84} * z_{42} + z_{23} * z_{82} * z_{28} + \\ & z_{23} * z_{83} * z_{33} + z_{23} * z_{85} * z_{35}) + m_4 * (z_{22} * z_{33} + z_{23} * z_{40} + z_{24} * z_{47} + \\ & z_{21} * z_{77} * z_{33} + z_{21} * z_{78} * z_{35} + z_{21} * z_{80} * z_{40} + z_{21} * z_{81} * z_{42} + z_{21} * z_{86} * z_{47} + \\ & z_{21} * z_{87} * z_{49} + z_{22} * z_{79} * z_{28} + z_{22} * z_{83} * z_{40} + z_{22} * z_{84} * z_{42} + z_{22} * z_{89} * z_{47} + \\ & z_{22} * z_{90} * z_{49} + z_{23} * z_{82} * z_{28} + z_{23} * z_{83} * z_{33} + z_{23} * z_{85} * z_{35} + z_{23} * z_{92} * z_{47} + \\ & z_{23} * z_{93} * z_{49} + z_{24} * z_{88} * z_{28} + z_{24} * z_{89} * z_{33} + z_{24} * z_{91} * z_{35} + z_{24} * z_{92} * z_{40} + \\ & z_{24} * z_{94} * z_{42}) + m_5 * (z_{22} * z_{33} + z_{23} * z_{40} + z_{24} * z_{47} + z_{25} * z_{54} + z_{21} * z_{77} * z_{33} + \\ & z_{21} * z_{78} * z_{35} + z_{21} * z_{80} * z_{40} + z_{21} * z_{81} * z_{42} + z_{21} * z_{86} * z_{47} + z_{21} * z_{87} * z_{49} + \\ & z_{21} * z_{95} * z_{54} + z_{21} * z_{96} * z_{56} + z_{22} * z_{79} * z_{28} + z_{22} * z_{83} * z_{40} + z_{22} * z_{84} * z_{42} + \\ & z_{22} * z_{89} * z_{47} + z_{22} * z_{90} * z_{49} + z_{22} * z_{98} * z_{54} + z_{22} * z_{99} * z_{56} + z_{23} * z_{82} * z_{28} + \\ & z_{23} * z_{83} * z_{33} + z_{23} * z_{85} * z_{35} + z_{23} * z_{92} * z_{47} + z_{23} * z_{93} * z_{49} + z_{23} * z_{101} * z_{54} + \\ & z_{23} * z_{102} * z_{56} + z_{24} * z_{88} * z_{28} + z_{24} * z_{89} * z_{33} + z_{24} * z_{91} * z_{35} + z_{24} * z_{92} * z_{40} + \\ & z_{24} * z_{94} * z_{42} + z_{24} * z_{104} * z_{54} + z_{24} * z_{105} * z_{56} + z_{25} * z_{97} * z_{28} + z_{25} * z_{98} * z_{33} + \\ & z_{25} * z_{100} * z_{35} + z_{25} * z_{101} * z_{40} + z_{25} * z_{103} * z_{42} + z_{25} * z_{104} * z_{47} + z_{25} * z_{106} * z_{49}) \end{aligned}$$

$$z_{132} = z_{107} - z_{117}$$

$$\begin{aligned} z_{113} = & m_2 * z_{22} * (z_{22} + z_{21} * z_{77}) + m_3 * (z_{22}^2 + z_{23}^2 + z_{21} * z_{22} * z_{77} + \\ & z_{21} * z_{23} * z_{80} + 2 * z_{22} * z_{23} * z_{83}) + m_4 * (z_{22}^2 + z_{23}^2 + z_{24}^2 + z_{21} * z_{22} * z_{77} + \\ & z_{21} * z_{23} * z_{80} + z_{21} * z_{24} * z_{86} + 2 * z_{22} * z_{23} * z_{83} + 2 * z_{22} * z_{24} * z_{89} + \\ & 2 * z_{23} * z_{24} * z_{92}) + m_5 * (z_{22}^2 + z_{23}^2 + z_{24}^2 + z_{25}^2 + z_{21} * z_{22} * z_{77} + z_{21} * z_{23} * z_{80} + \\ & z_{21} * z_{24} * z_{86} + z_{21} * z_{25} * z_{95} + 2 * z_{22} * z_{23} * z_{83} + 2 * z_{22} * z_{24} * z_{89} + \\ & 2 * z_{22} * z_{25} * z_{98} + 2 * z_{23} * z_{24} * z_{92} + 2 * z_{23} * z_{25} * z_{101} + 2 * z_{24} * z_{25} * z_{104}) \end{aligned}$$

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$$\begin{aligned}
 z_{112} = & m_1 * z_{21}^2 + m_2 * (z_{21}^2 + z_{22}^2 + 2 * z_{21} * z_{22} * z_{77}) + m_3 * (z_{21}^2 + z_{22}^2 + \\
 & z_{23}^2 + 2 * z_{21} * z_{22} * z_{77} + 2 * z_{21} * z_{23} * z_{80} + 2 * z_{22} * z_{23} * z_{83}) + \\
 & m_4 * (z_{21}^2 + z_{22}^2 + z_{23}^2 + z_{24}^2 + 2 * z_{21} * z_{22} * z_{77} + 2 * z_{21} * z_{23} * z_{80} + \\
 & 2 * z_{21} * z_{24} * z_{86} + 2 * z_{22} * z_{23} * z_{83} + 2 * z_{22} * z_{24} * z_{89} + 2 * z_{23} * z_{24} * z_{92}) + \\
 & m_5 * (z_{21}^2 + z_{22}^2 + z_{23}^2 + z_{24}^2 + z_{25}^2 + 2 * z_{21} * z_{22} * z_{77} + 2 * z_{21} * z_{23} * z_{80} + \\
 & 2 * z_{21} * z_{24} * z_{86} + 2 * z_{21} * z_{25} * z_{95} + 2 * z_{22} * z_{23} * z_{83} + 2 * z_{22} * z_{24} * z_{89} + \\
 & 2 * z_{22} * z_{25} * z_{98} + 2 * z_{23} * z_{24} * z_{92} + 2 * z_{23} * z_{25} * z_{101} + 2 * z_{24} * z_{25} * z_{104})
 \end{aligned}$$

$$z_{137} = z_{113} / z_{112}$$

$$\begin{aligned}
 z_{108} = & F_3 * z_8 * z_{22} + F_3 * z_{10} * z_{23} + F_4 * z_8 * z_{22} + F_4 * z_{10} * z_{23} + \\
 & F_4 * z_{12} * z_{24} + F_5 * z_8 * z_{22} + F_5 * z_{10} * z_{23} + F_5 * z_{12} * z_{24} + F_5 * z_{14} * z_{25} + \\
 & z_{22} * (F_2 * z_8 - z_{73} * z_9 - z_{59} - z_{77} * z_{58}) - z_{74} * z_9 * z_{22} - z_{74} * z_{11} * z_{23} - z_{75} * z_9 * z_{22} \\
 & - z_{75} * z_{11} * z_{23} - z_{75} * z_{13} * z_{24} - z_{76} * z_9 * z_{22} - z_{76} * z_{11} * z_{23} - z_{76} * z_{13} * z_{24} - z_{76} \\
 & * z_{15} * z_{25} - z_{22} * z_{61} - z_{22} * z_{64} - z_{22} * z_{68} - z_{23} * z_{62} - z_{23} * z_{65} - z_{23} * z_{69} - z_{24} \\
 & * z_{66} - z_{24} * z_{70} - z_{25} * z_{71} - z_{22} * z_{77} * z_{60} - z_{22} * z_{77} * z_{63} - z_{22} * z_{77} * z_{67} - z_{22} \\
 & * z_{83} * z_{62} - z_{22} * z_{83} * z_{65} - z_{22} * z_{83} * z_{69} - z_{22} * z_{89} * z_{66} - z_{22} * z_{89} * z_{70} - z_{22} \\
 & * z_{98} * z_{71} - z_{23} * z_{80} * z_{60} - z_{23} * z_{80} * z_{63} - z_{23} * z_{80} * z_{67} - z_{23} * z_{83} * z_{61} - z_{23} * z_{83} \\
 & * z_{64} - z_{23} * z_{83} * z_{68} - z_{23} * z_{92} * z_{66} - z_{23} * z_{92} * z_{70} - z_{23} * z_{101} * z_{71} - z_{24} * z_{86} \\
 & * z_{63} - z_{24} * z_{86} * z_{67} - z_{24} * z_{89} * z_{64} - z_{24} * z_{89} * z_{68} - z_{24} * z_{92} * z_{65} - z_{24} * z_{92} * z_{69} \\
 & - z_{24} * z_{104} * z_{71} - z_{25} * z_{95} * z_{67} - z_{25} * z_{98} * z_{68} - z_{25} * z_{101} * z_{69} - z_{25} * z_{104} * z_{70}
 \end{aligned}$$

$$\begin{aligned}
 z_{122} = & m_2 * z_{22} * (z_{33} + z_{79} * z_{28}) + m_3 * (z_{22} * z_{33} + z_{23} * z_{40} + z_{22} * z_{79} * z_{28} + \\
 & z_{22} * z_{83} * z_{40} + z_{22} * z_{84} * z_{42} + z_{23} * z_{82} * z_{28} + z_{23} * z_{83} * z_{33} + z_{23} * z_{85} * z_{35}) + \\
 & m_4 * (z_{22} * z_{33} + z_{23} * z_{40} + z_{24} * z_{47} + z_{22} * z_{79} * z_{28} + z_{22} * z_{83} * z_{40} + \\
 & z_{22} * z_{84} * z_{42} + z_{22} * z_{89} * z_{47} + z_{22} * z_{90} * z_{49} + z_{23} * z_{82} * z_{28} + z_{23} * z_{83} * z_{33} + \\
 & z_{23} * z_{85} * z_{35} + z_{23} * z_{92} * z_{47} + z_{23} * z_{93} * z_{49} + z_{24} * z_{88} * z_{28} + z_{24} * z_{89} * z_{33} + \\
 & z_{24} * z_{91} * z_{35} + z_{24} * z_{92} * z_{40} + z_{24} * z_{94} * z_{42}) + m_5 * (z_{22} * z_{33} + z_{23} * z_{40} + \\
 & z_{24} * z_{47} + z_{25} * z_{54} + z_{22} * z_{79} * z_{28} + z_{22} * z_{83} * z_{40} + z_{22} * z_{84} * z_{42} + \\
 & z_{22} * z_{89} * z_{47} + z_{22} * z_{90} * z_{49} + z_{22} * z_{98} * z_{54} + z_{22} * z_{99} * z_{56} + z_{23} * z_{82} * z_{28} + \\
 & z_{23} * z_{83} * z_{33} + z_{23} * z_{85} * z_{35} + z_{23} * z_{92} * z_{47} + z_{23} * z_{93} * z_{49} + z_{23} * z_{101} * z_{54} + \\
 & z_{23} * z_{102} * z_{56} + z_{24} * z_{88} * z_{28} + z_{24} * z_{89} * z_{33} + z_{24} * z_{91} * z_{35} + z_{24} * z_{92} * z_{40} + \\
 & z_{24} * z_{94} * z_{42} + z_{24} * z_{104} * z_{54} + z_{24} * z_{105} * z_{56} + z_{25} * z_{97} * z_{28} + z_{25} * z_{98} * z_{33} + \\
 & z_{25} * z_{100} * z_{35} + z_{25} * z_{101} * z_{40} + z_{25} * z_{103} * z_{42} + z_{25} * z_{104} * z_{47} + z_{25} * z_{106} * z_{49})
 \end{aligned}$$

$$z_{133} = z_{108} - z_{122}$$

$$z_{142} = z_{137} * z_{132} - z_{133}$$

$$\begin{aligned}
 z_{114} = & m_3 * z_{23} * (z_{23} + z_{21} * z_{80} + z_{22} * z_{83}) + m_4 * (z_{23}^2 + z_{24}^2 + z_{21} * z_{23} * z_{80} + \\
 & z_{21} * z_{24} * z_{86} + z_{22} * z_{23} * z_{83} + z_{22} * z_{24} * z_{89} + 2 * z_{23} * z_{24} * z_{92}) + m_5 * (z_{23}^2 + \\
 & z_{24}^2 + z_{25}^2 + z_{21} * z_{23} * z_{80} + z_{21} * z_{24} * z_{86} + z_{21} * z_{25} * z_{95} + z_{22} * z_{23} * z_{83} + \\
 & z_{22} * z_{24} * z_{89} + z_{22} * z_{25} * z_{98} + 2 * z_{23} * z_{24} * z_{92} + 2 * z_{23} * z_{25} * z_{101} + 2 * z_{24} * z_{25} * z_{104})
 \end{aligned}$$

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$$z_{119} = m_3 * z_{23} * (z_{23} + z_{22} * z_{83}) + m_4 * (z_{23}^2 + z_{24}^2 + z_{22} * z_{23} * z_{83} + z_{22} * z_{24} * z_{89} + 2 * z_{23} * z_{24} * z_{92}) + m_5 * (z_{23}^2 + z_{24}^2 + z_{25}^2 + z_{22} * z_{23} * z_{83} + z_{22} * z_{24} * z_{89} + z_{22} * z_{25} * z_{98} + 2 * z_{23} * z_{24} * z_{92} + 2 * z_{23} * z_{25} * z_{101} + 2 * z_{24} * z_{25} * z_{104})$$

$$z_{139} = z_{114} * z_{137} - z_{119}$$

$$z_{143} = z_{114} / z_{112}$$

$$z_{109} = F_4 * z_{10} * z_{23} + F_4 * z_{12} * z_{24} + F_5 * z_{10} * z_{23} + F_5 * z_{12} * z_{24} + F_5 * z_{14} * z_{25} + z_{23} * (F_3 * z_{10} - z_{74} * z_{11} - z_{62} - z_{80} * z_{60} - z_{83} * z_{61}) - z_{75} * z_{11} * z_{23} - z_{75} * z_{13} * z_{24} - z_{76} * z_{11} * z_{23} - z_{76} * z_{13} * z_{24} - z_{76} * z_{15} * z_{25} - z_{23} * z_{65} - z_{23} * z_{69} - z_{24} * z_{66} - z_{24} * z_{70} - z_{25} * z_{71} - z_{23} * z_{80} * z_{63} - z_{23} * z_{80} * z_{67} - z_{23} * z_{83} * z_{64} - z_{23} * z_{83} * z_{68} - z_{23} * z_{92} * z_{66} - z_{23} * z_{92} * z_{70} - z_{23} * z_{101} * z_{71} - z_{24} * z_{86} * z_{63} - z_{24} * z_{86} * z_{67} - z_{24} * z_{89} * z_{64} - z_{24} * z_{89} * z_{68} - z_{24} * z_{92} * z_{65} - z_{24} * z_{92} * z_{69} - z_{24} * z_{104} * z_{71} - z_{25} * z_{95} * z_{67} - z_{25} * z_{98} * z_{68} - z_{25} * z_{101} * z_{69} - z_{25} * z_{104} * z_{70}$$

$$z_{126} = m_3 * z_{23} * (z_{40} + z_{82} * z_{28} + z_{83} * z_{33} + z_{85} * z_{35}) + m_4 * (z_{23} * z_{40} + z_{24} * z_{47} + z_{23} * z_{82} * z_{28} + z_{23} * z_{83} * z_{33} + z_{23} * z_{85} * z_{35} + z_{23} * z_{92} * z_{47} + z_{23} * z_{93} * z_{49} + z_{24} * z_{88} * z_{28} + z_{24} * z_{89} * z_{33} + z_{24} * z_{91} * z_{35} + z_{24} * z_{92} * z_{40} + z_{24} * z_{94} * z_{42}) + m_5 * (z_{23} * z_{40} + z_{24} * z_{47} + z_{25} * z_{54} + z_{23} * z_{82} * z_{28} + z_{23} * z_{83} * z_{33} + z_{23} * z_{85} * z_{35} + z_{23} * z_{92} * z_{47} + z_{23} * z_{93} * z_{49} + z_{23} * z_{101} * z_{54} + z_{23} * z_{102} * z_{56} + z_{24} * z_{88} * z_{28} + z_{24} * z_{89} * z_{33} + z_{24} * z_{91} * z_{35} + z_{24} * z_{92} * z_{40} + z_{24} * z_{94} * z_{42} + z_{24} * z_{104} * z_{54} + z_{24} * z_{105} * z_{56} + z_{25} * z_{97} * z_{28} + z_{25} * z_{98} * z_{33} + z_{25} * z_{100} * z_{35} + z_{25} * z_{101} * z_{40} + z_{25} * z_{103} * z_{42} + z_{25} * z_{104} * z_{47} + z_{25} * z_{106} * z_{49})$$

$$z_{134} = z_{109} - z_{126}$$

$$z_{148} = z_{143} * z_{132} - z_{134}$$

$$z_{144} = z_{113} * z_{143} - z_{119}$$

$$z_{118} = m_2 * z_{22}^2 + m_3 * (z_{22}^2 + z_{23}^2 + 2 * z_{22} * z_{23} * z_{83}) + m_4 * (z_{22}^2 + z_{23}^2 + z_{24}^2 + 2 * z_{22} * z_{23} * z_{83} + 2 * z_{22} * z_{24} * z_{89} + 2 * z_{23} * z_{24} * z_{92}) + m_5 * (z_{22}^2 + z_{23}^2 + z_{24}^2 + z_{25}^2 + 2 * z_{22} * z_{23} * z_{83} + 2 * z_{22} * z_{24} * z_{89} + 2 * z_{22} * z_{25} * z_{98} + 2 * z_{23} * z_{24} * z_{92} + 2 * z_{23} * z_{25} * z_{101} + 2 * z_{24} * z_{25} * z_{104})$$

$$z_{138} = z_{113} * z_{137} - z_{118}$$

$$z_{161} = z_{144} / z_{138}$$

$$z_{165} = z_{148} - z_{161} * z_{142}$$

$$z_{115} = m_4 * z_{24} * (z_{24} + z_{21} * z_{86} + z_{22} * z_{89} + z_{23} * z_{92}) + m_5 * (z_{24}^2 + z_{25}^2 + z_{21} * z_{24} * z_{86} + z_{21} * z_{25} * z_{95} + z_{22} * z_{24} * z_{89} + z_{22} * z_{25} * z_{98} + z_{23} * z_{24} * z_{92} + z_{23} * z_{25} * z_{101} + 2 * z_{24} * z_{25} * z_{104})$$

$$z_{124} = m_4 * z_{24} * (z_{24} + z_{23} * z_{92}) + m_5 * (z_{24}^2 + z_{25}^2 + z_{23} * z_{24} * z_{92} + z_{23} * z_{25} * z_{101} + 2 * z_{24} * z_{25} * z_{104})$$

$$z_{146} = z_{115} * z_{143} - z_{124}$$

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$$\begin{aligned}
 z_{120} &= m_4 * z_{24} * (z_{24} + z_{22} * z_{89} + z_{23} * z_{92}) + m_5 * (z_{24}^2 + z_{25}^2 + z_{22} * z_{24} * z_{89} + \\
 &\quad z_{22} * z_{25} * z_{98} + z_{23} * z_{24} * z_{92} + z_{23} * z_{25} * z_{101} + 2 * z_{24} * z_{25} * z_{104}) \\
 z_{140} &= z_{115} * z_{137} - z_{120} \\
 z_{163} &= z_{146} - z_{140} * z_{161} \\
 z_{149} &= z_{115} / z_{112} \\
 z_{110} &= F_5 * z_{12} * z_{24} + F_5 * z_{14} * z_{25} + \\
 &\quad z_{24} * (F_4 * z_{12} - z_{75} * z_{13} - z_{66} - z_{86} * z_{63} - z_{89} * z_{64} - z_{92} * z_{65}) - z_{76} * z_{13} * z_{24} - z_{76} \\
 &\quad * z_{15} * z_{25} - z_{24} * z_{70} - z_{25} * z_{71} - z_{24} * z_{86} * z_{67} - z_{24} * z_{89} * z_{68} - z_{24} * z_{92} * z_{69} \\
 &\quad - z_{24} * z_{104} * z_{71} - z_{25} * z_{95} * z_{67} - z_{25} * z_{98} * z_{68} - z_{25} * z_{101} * z_{69} - z_{25} * z_{104} * z_{70} \\
 z_{129} &= m_4 * z_{24} * (z_{47} + z_{88} * z_{28} + z_{89} * z_{33} + z_{91} * z_{35} + z_{92} * z_{40} + \\
 &\quad z_{94} * z_{42}) + m_5 * (z_{24} * z_{47} + z_{25} * z_{54} + z_{24} * z_{88} * z_{28} + z_{24} * z_{89} * z_{33} + \\
 &\quad z_{24} * z_{91} * z_{35} + z_{24} * z_{92} * z_{40} + z_{24} * z_{94} * z_{42} + z_{24} * z_{104} * z_{54} + \\
 &\quad z_{24} * z_{105} * z_{56} + z_{25} * z_{97} * z_{28} + z_{25} * z_{98} * z_{33} + z_{25} * z_{100} * z_{35} + \\
 &\quad z_{25} * z_{101} * z_{40} + z_{25} * z_{103} * z_{42} + z_{25} * z_{104} * z_{47} + z_{25} * z_{106} * z_{49}) \\
 z_{135} &= z_{110} - z_{129} \\
 z_{154} &= z_{149} * z_{132} - z_{135} \\
 z_{150} &= z_{113} * z_{149} - z_{120} \\
 z_{166} &= z_{150} / z_{138} \\
 z_{170} &= z_{154} - z_{166} * z_{142} \\
 z_{151} &= z_{114} * z_{149} - z_{124} \\
 z_{167} &= z_{151} - z_{139} * z_{166} \\
 z_{123} &= m_3 * z_{23}^2 + m_4 * (z_{23}^2 + z_{24}^2 + 2 * z_{23} * z_{24} * z_{92}) + m_5 * (z_{23}^2 + z_{24}^2 + \\
 &\quad z_{25}^2 + 2 * z_{23} * z_{24} * z_{92} + 2 * z_{23} * z_{25} * z_{101} + 2 * z_{24} * z_{25} * z_{104}) \\
 z_{145} &= z_{114} * z_{143} - z_{123} \\
 z_{162} &= z_{145} - z_{139} * z_{161} \\
 z_{176} &= z_{167} / z_{162} \\
 z_{179} &= z_{170} - z_{176} * z_{165} \\
 z_{116} &= m_5 * z_{25} * (z_{25} + z_{21} * z_{95} + z_{22} * z_{98} + z_{23} * z_{101} + z_{24} * z_{104}) \\
 z_{128} &= m_5 * z_{25} * (z_{25} + z_{24} * z_{104}) \\
 z_{153} &= z_{116} * z_{149} - z_{128} \\
 z_{121} &= m_5 * z_{25} * (z_{25} + z_{22} * z_{98} + z_{23} * z_{101} + z_{24} * z_{104}) \\
 z_{141} &= z_{116} * z_{137} - z_{121} \\
 z_{169} &= z_{153} - z_{141} * z_{166}
 \end{aligned}$$

7 Dynamic Equations of the Flexible Rope Model

$$\begin{aligned}
z_{125} &= m_5 * z_{25} * (z_{25} + z_{23} * z_{101} + z_{24} * z_{104}) \\
z_{147} &= z_{116} * z_{143} - z_{125} \\
z_{164} &= z_{147} - z_{141} * z_{161} \\
z_{178} &= z_{169} - z_{164} * z_{176} \\
z_{155} &= z_{116} / z_{112} \\
z_{111} &= z_{25} * (F_5 * z_{14} - z_{76} * z_{15} - z_{71} - z_{95} * z_{67} - z_{98} * z_{68} - z_{101} * z_{69} - z_{104} * z_{70}) \\
z_{131} &= m_5 * z_{25} * (z_{54} + z_{97} * z_{28} + z_{98} * z_{33} + z_{100} * z_{35} + \\
&\quad z_{101} * z_{40} + z_{103} * z_{42} + z_{104} * z_{47} + z_{106} * z_{49}) \\
z_{136} &= z_{111} - z_{131} \\
z_{160} &= z_{155} * z_{132} - z_{136} \\
z_{156} &= z_{113} * z_{155} - z_{121} \\
z_{171} &= z_{156} / z_{138} \\
z_{175} &= z_{160} - z_{171} * z_{142} \\
z_{157} &= z_{114} * z_{155} - z_{125} \\
z_{172} &= z_{157} - z_{139} * z_{171} \\
z_{180} &= z_{172} / z_{162} \\
z_{183} &= z_{175} - z_{180} * z_{165} \\
z_{158} &= z_{115} * z_{155} - z_{128} \\
z_{173} &= z_{158} - z_{140} * z_{171} \\
z_{181} &= z_{173} - z_{163} * z_{180} \\
z_{127} &= m_4 * z_{24}^2 + m_5 * (z_{24}^2 + z_{25}^2 + 2 * z_{24} * z_{25} * z_{104}) \\
z_{152} &= z_{115} * z_{149} - z_{127} \\
z_{168} &= z_{152} - z_{140} * z_{166} \\
z_{177} &= z_{168} - z_{163} * z_{176} \\
z_{184} &= z_{181} / z_{177} \\
z_{186} &= z_{183} - z_{184} * z_{179} \\
z_{130} &= m_5 * z_{25}^2 \\
z_{159} &= z_{116} * z_{155} - z_{130} \\
z_{174} &= z_{159} - z_{141} * z_{171} \\
z_{182} &= z_{174} - z_{164} * z_{180} \\
z_{185} &= z_{182} - z_{178} * z_{184} \\
z_{187} &= z_{186} / z_{185} \\
z_{188} &= (z_{179} - z_{178} * z_{187}) / z_{177} \\
z_{189} &= (z_{165} - z_{163} * z_{188} - z_{164} * z_{187}) / z_{162} \\
z_{190} &= (z_{142} - z_{139} * z_{189} - z_{140} * z_{188} - z_{141} * z_{187}) / z_{138} \\
z_{191} &= (z_{132} - z_{113} * z_{190} - z_{114} * z_{189} - z_{115} * z_{188} - z_{116} * z_{187}) / z_{112} \\
\ddot{\alpha}_1 &= z_{191} \\
\ddot{\alpha}_2 &= z_{190} \\
\ddot{\alpha}_3 &= z_{189} \\
\ddot{\alpha}_4 &= z_{188} \\
\ddot{\alpha}_5 &= z_{187}
\end{aligned}$$

Bibliography

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