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Application of Mathematical Models in Optical Coordinate Metrology

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Abstract – Aim of this paper is to analyse the role of mathematical modelling in the field of optical coordinate metrology. Thus, the definition of an appropriate model for the measurement procedure is outlined. The underlying physical and mathematical models and methods are described. As one example the automatic adjustment of illumination settings is described in detail.

Considering the outlined models in optical coordinate metrology the interconnection between physical and mathematical models as means for defining and performing dimensional measurements becomes apparent.

Keywords: optical coordinate metrology, model based measurement, automatic illumination setting

1. INTRODUCTION

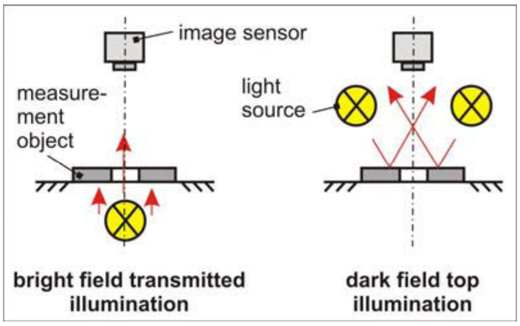


Fig. 1: Measurement setup with two typical illumination types often used in optical coordinate metrology

This paper shall deliver a contribution on the role and importance of mathematical models in the field of measurement science and technology. As application field optical coordinate metrology is considered. The general setup for optical coordinate measurements is depicted in Fig. 1.

In [1] the role of metrology in relation to other sciences is considered, especially the intersection of metrology with mathematics. The author states that metrology acts both as customer and applied branch of theory being one tool for verification of prime knowledge of natural and artificial world. The validity of this thesis can be nicely shown to hold true in the field of optical coordinate metrology.

Section 2 states some basic definitions from the international vocabulary of metrology (VIM) for better understanding of the subsequent sections. Section 3 outlines in detail the underlying theoretic model as well as its physical origin. Thereby the general model is outlined first before a more detailed model for dimensional measurements with image sensors is explained.

Section 4 has been chosen to explain one topic in the field of optical coordinate metrology (OCM) in more detail. Compared with tactile measurements optical measurements are error prone for the operator of the coordinate measurement machine (CMM) has to adjust a large number of measurement parameters. Thus, generally the results of different operators with different knowledge level is not necessarily comparable in OCM since unknown errors due to bad parameter settings may be dominant.

Fig. 2: Typical top light illumination CMM component (image source: courtesy to www.ogpmesstechnik.de)

The selected field comprises the automatic setting of illumination (Fig. 2). This operation is also considered in the context of the measurement models given in section 3. It can be outlined that the automatic parameter setting in optical coordinate metrology does equal the inclusion of a closed loop observer. Thus, in terms of control theory using automatic parameters setting means the utilisation of a Kalman filter. The paper is closing with a brief summary.

2. BASIC DEFINITIONS

The following terms and related explanations in this section are cited from the VIM [1] and have been chosen since they are frequently used subsequently:

- metrology = science of measurement and its application,

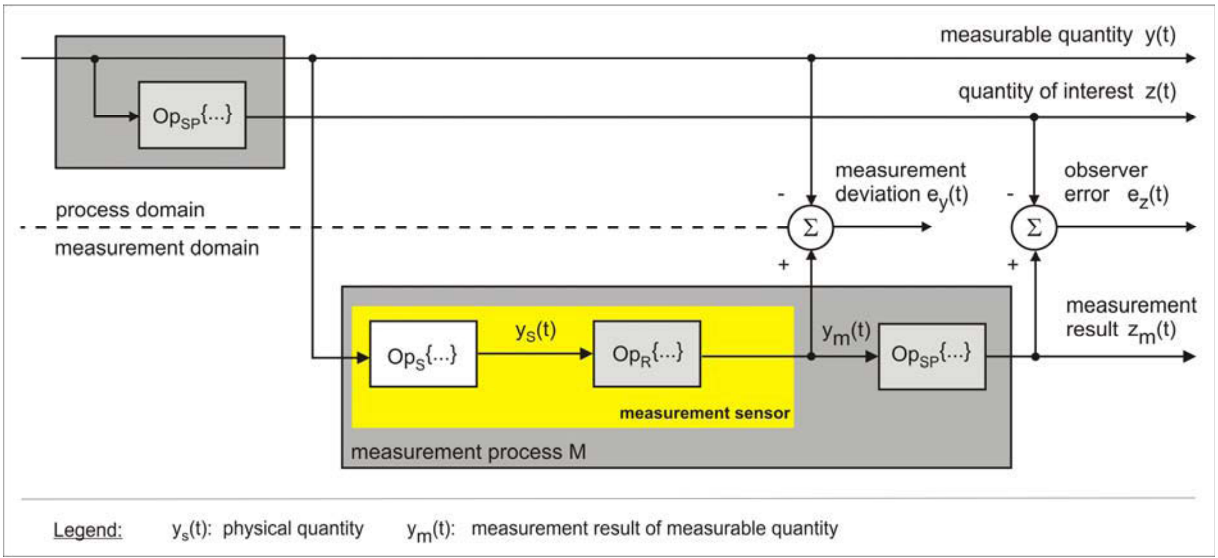


Fig. 3. General model for dimensional measurements with image sensors

- measurand = quantity intended to be measured,
- true value = quantity value consistent with the definition of a quantity [Note 1: In the Error Approach to describing measurement, a true quantity value is considered unique and, in practice, **unknowable**.],
- calibration = operation that, under specified conditions, in a first step, establishes a relation between the quantity values with measurement uncertainties provided by **measurement standards** and corresponding indications with associated measurement uncertainties and, in a second step, uses this information to establish a relation for obtaining a measurement result from an indication,
- measurement uncertainty = non-negative parameter characterising the dispersion of the quantity values being attributed to a measurand, based on the information used.

3. THEORETIC MODEL FOR DIMENSIONAL MEASUREMENTS WITH IMAGE SENSORS

The primary model linked to dimensional measurements with image sensors is the inverse measurement. The measurand of dimensional measurements is typically the length or width of the measurement object. Thereby the grid of the image sensor works as the measurement scale.

The inverse measurement is performed in the image domain and the result is converted back in the physical domain based on calibration data. Calibration data are in the simplest case the imaging scale factor containing the pixel to µm relationship. However, state-of-the-art calibration data are based on far more sophisticated calibration models, comprising up to 30 parameters [3].

This measurement process can be described by a model from the field of system theory as outlined in the following sub chapter.

3.1. General model for dimensional measurements with image sensors

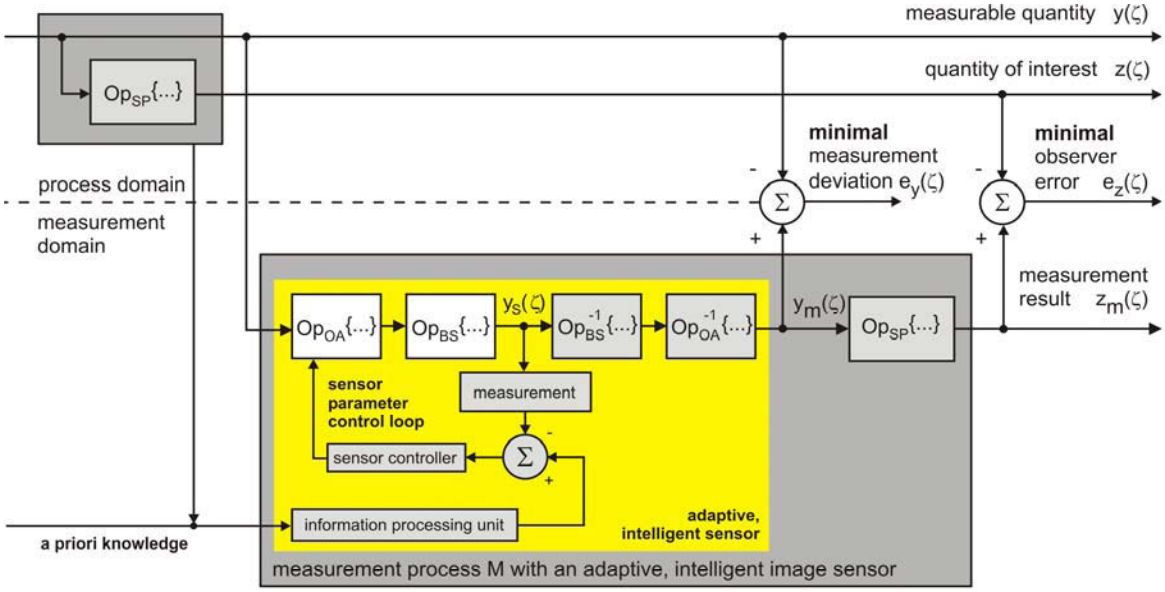
In Fig. 3 the theoretical model for dimensional measurements based on image sensors is depicted. This model is based on a set of general models described by Mr. Ruhm [4]. As can be seen in Fig. 3 there is no feedback path from the measurement domain to the process domain. Thus, the measurement is performed by an open loop observer.

For the wanted measurand z(t) can not be measured directly it is substituted by the measurable quantity y(t). This is applicable only, when an unequivocal causal relationship exists between both variables. This is denoted as sub process $Op_{SP}\{...\}$ which is valid in the process and the measurement domain. Usually the causal relationship is based on physical laws. In the case of optical coordinate metrology this is imaging theory.

Considering the sensor process $Op_S\{...\}$ it usually converts the measurable quantity y(t) into an electrical quantity $y_s(t)$. The value of $y_s(t)$ is determined. Applying the reconstruction process $Op_R\{...\}$ the quantity $y_s(t)$ can be retransformed into the value range of y(t). Now the unknown measurable quantity y(t) can be compared with the measured quantity $y_m(t)$. Thereby the base law of measurement science must be fulfilled. The actually measured quantity must be equivalent to the unknown measurable quantity.

Thus, from system theory it can be derived that the transfer function of the reconstruction process $Op_R\{...\}$ must be the exact inverse transfer function of the sensor process:

$$Op_R\{...\} = Op_S^{-1}\{...\}. \quad (1)$$



Legend:
$Op_{SP}\{...\}$ operator of sub process SP
$Op_{OA}\{...\}$ operator for optical imaging (OA)
$Op_{BS}\{...\}$ operator for the image sensor (BS)
$y_S(\zeta)$: digital 2D image of the measurement object
$e_y(\zeta)$: deviation of the edge position in the pixel grid
$e_z(\zeta)$: deviation of the metric length $\overline{PO}$
$y(\zeta)$: measurable edge position of the structure transition in the pixel grid of the image sensor
$z(\zeta)$: metric length of the distance ($\overline{PO}$) of a point on the measurement object in the sensor coordinate system from the origin of the sensor coordinate system
$y_m(\zeta)$: measured edge position of the structure transition in the pixel grid of the image sensor
$z_m(\zeta)$: indirectly measured position of the point in the sensor coordinate system ($\overline{PO}$)
ζ **replacement variable for the dependence on the spatial coordinates x, y and the time t**

Fig. 4. Detailed model for dimensional measurements based on edge detection with image sensors

The considered measurement is a model based measurement since the inverse model $Op_S^{-1}\{...\}$ of the sensor process is required.

Basically for real systems the actual value of the measurable quantity is unknown. Thus, the measurement deviation $e_y(t)$ is some kind of a virtual quantity. However, applying the process $Op_{SP}\{...\}$ the measured quantity $y_m(t)$ can be converted into the wanted quantity $z_m(t)$. Comparing the unknown wanted z(t) with the measured $z_m(t)$ an observer error $e_z(t)$ can be defined. However, the latter is also a virtual quantity not known.

Causes for the unknown measurement error and observer error is the non ideal measurement process. Specifying detailed causes for the field of optical coordinate metrology delivers the following (non exhaustive) list:

- impact of disturbing quantities (noise, unwanted straylight or unwanted ambient light, influence of temperature of the measurement object and/or the optical coordinate measurement machine),
- inappropriate parameter setting (not suitable edge detection criterion, too low number of search lines, overexposure of image pixel by too large illumination setting, insufficient optical magnification chosen),
- dirt or dust on the measurement object,
- reconstruction process is not accurate enough (calibration of higher order imaging abberations not considered, systematic errors due to extension of measurement object along the optical axis not corrected).

Based on an unbroken chain of comparisons being the necessary but not sufficient condition for traceability [5] one can use a calibrated standard from a national length laboratory in order to get an estimate of the measurement deviation and the observer error for **this! specific** task (only).

3.2. Detailed model for dimensional measurements with image sensors

In this sub section a more detailed description of the previously given general model shall be outlined. In Fig. 4 the theoretical model for dimensional measurements based on image sensors is depicted with greater detail and on the basis of an intelligent, adaptive sensor.

Considering dimensional measurements with an adaptive sensor the following extensions of the model must be introduced [6]:

- internal feedback loops for the sensor and the reconstruction process,
- information processing within the sensor,
- supply of a priori knowledge from external sources and from the process domain
- minimisation of the observer error resp. of the measurement deviation.

Salient feature of an intelligent sensor is its capability to communicate, measure and process other information than sole measurement data [7]. With regard to the minimisation of the observer error it enables:

- adaptation of sensing process on actual measurement object characteristics

- utilisation of a priori knowledge for generation of reference quantities, acceptable value ranges and acceptable signal transitions for the sensing process itself
- evaluation of various quantities in the actual sensing process in order to prevent the occurrence of (unknown) systematic measurement deviations.

The unknown measurable quantity $y(\xi)$ is the edge position of the physical edge of the measurement object. The unknown wanted quantity $z(\xi)$ is the metric value in SI units of the distance of the measured edge position to the origin of the sensor coordinate system. Thus, knowing $z_m(\xi)$ for two different physical edges of the measurement object the length or the width of the related geometric feature of the measurement object e.g. step width is known. The model is valid for in-image measurements where all edges to be measured do fit into the field of view of the image sensor.

If the physical edges to be measured do not fit into the field of view of the used optical coordinate measurement machine (OCMM) the model must be extended. The local coordinate of the measured physical edge must be combined with the global coordinate of the machine axes of the OCMM. Then the machine either moves the measurement object or the imaging system in order to measure the second physical edge of the measurement object. The local and the global position of the second edge are recorded. The overall dimensional measurement result is the distance of both positions in the coordinate system of the OCMM.

The overall sensor process consists of the two sub processes for the optical imaging $Op_{OA}\{...\}$ and the image sensor itself $Op_{BS}\{...\}$. $Op_{OA}\{...\}$ comprises three modules: illumination of the measurement object, interaction with the measurement object and the actual optical imaging.

$Op_{BS}\{...\}$ contains the image capture process and the A/D conversion process. Thus, $y_s(\xi)$ is the digital 2D image of the measurement object (assuming the utilisation of an area image sensor).

3.3. Sub model for optical imaging derived from physics

The core model for imaging is described in physics. The ideal imaging of one point in the object plane always results in an airy ring in the image plane since the intensity function $f_I(x,y)$ in the object plane is convoluted with the point spread function $h(x,y)$ of the imaging system yielding the so called airy ring in the image plane:

$$I(x,y) = f_I(x,y) * h(x,y), \tag{2}$$

where $*$ is the convolution operator and x,y are local coordinates.

This model is the base for the derivation of the actual position of an edge of the object in relation to the signal sensed by the image sensor.

Based on this the algorithms for image analysis have been developed, such as edge detection and subpixel algorithms.

4. AUTOMATIC SETTING OF ILLUMINATION

Regarding automatic setting of illumination various experiments have been performed in order to create a set of reference values resp. conditions in the image domain that if fulfilled provide a measurement setting with low measurement uncertainty. For the establishment of those reference values many experiments have been performed where the measurement result has been compared with the measurable quantity. Thus, a closed loop observer has been used.

The application of the automatic illumination setting is effectively using a Kalman filtering process (see closed loop within measurement domain in the yellow box of the adaptive, intelligent sensor in Fig. 4). The physical law on which the automatic setting of illumination is based on the bidirectional reflectance distribution function (BRDF). For each light source the coefficient of the BRDF is determined. Based on the reference values for the image scene a set of equations can be formulated. This set of equations is solved via nonlinear optimisation yielding the illumination settings that have to be adjusted in order to capture an image scene with the desired reference characteristics.

5. CONCLUSION

As outlined in this paper metrology is a customer for physics and mathematics using a vast variety of different methods and models. On the other hand it also drives the development for even more sophisticated models, if its results are not sufficient for the intended application.

A practical example is the manufacturing of semiconductors where the decreasing size of structures entails stepwise new generations of models for measurement.

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