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MATHEMATICAL MODELS OF GEAR TOOTH SPEED SENSORS WITH DUAL OUTPUTS

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Abstract – In this paper mathematical models are proposed for calculating rotational speed, signal period, duty cycle and phase drift of gear tooth speed sensors with dual outputs. The proposed mathematical models are applied to parameter determination/optimization and target wheel design of Hall Effect gear tooth sensors and measuring systems. Experiment results show that the mathematical models are very useful and effective for the design and development of rotational speed sensors and measuring systems.

Keywords: Rotational Speed Sensor, Gear Tooth Sensor, Rotational Speed Measurement.

1. INTRODUCTION

Rotational speed sensors and measurements are widely used in industrial automation, production lines, intelligent robots, wind power stations, and automotive industry for testing, controlling and monitoring engines, motors, generators, spindles of different rotating machines.

There are a lot of rotational speed measuring methods. The most widely used methods are rotational impulse counting methods using proximity switches, encoders and gear tooth sensors [1-10]. Inductive, capacitive, optoelectronic and Hall Effect proximity switches output one impulse per revolution. Therefore they have a high frequency range, but a lower resolution. The rotational direction is not easy to be detected by using the proximity switches.

Inductive, capacitive, optical and magnetic encoders consist of a special target wheel (coding disk, grating disk, multi-pole magnet and magnetic grating etc.) and a detector. These encoders have a relative high resolution. Their disadvantages are expensive, small frequency range. An additional detector built-in the encoders is needed for detecting the rotational direction.

Gear tooth sensors work also according to inductive, capacitive, Hall Effect and magnetoresistive principles. These sensors use a metal gear as target wheel so that they are very easy and cheap for industrial applications. Furthermore, gear tooth sensors have large measuring range, wide frequency bandwidth, simple structure, and adaptability of harsh environments. Therefore they find increasing applications in industries. However, fundamental study of gear tooth speed sensors with dual outputs is still not complete until now. In this paper mathematical models are described for calculating the parameters of gear tooth speed sensors and measuring systems. The sensors have dual outputs for both speed measurement and rotational direction detection. The proposed mathematical models are applied to Hall Effect gear tooth sensor CYGTS104U [3]. Experiment results show that the mathematical models are very useful and effective for the parameter determination/optimization, design and development of rotational speed measuring systems.

2. MATHEMATICAL MODELS

As shown in Fig. 1, a gear tooth rotational speed measuring system consists of a gear tooth sensor (GTS) and a target wheel. Two detectors positioned in distance, *a*, are built in the gear tooth sensor with dual outputs. The detectors sense the addendum of the target wheel according to different physical principles, for instance inductive, capacitive, Hall Effect or magnetoresistive principles.

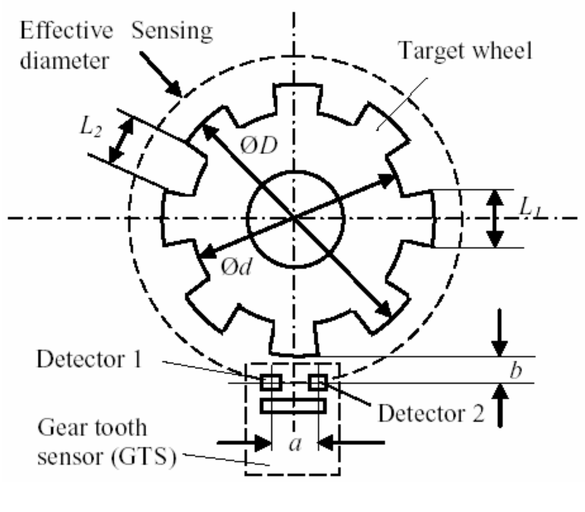

Fig.1. Gear tooth rotational speed measuring system

The gear tooth sensor generates two impulse outputs with a phase drift, $\Delta\Phi$, when the detectors face to the addendum of the target wheel. By counting the number of impulses of one output, n, within a measuring time, t, the rotational speed, w, can be determined by

$$w = \frac{n}{tN} \quad \text{(rps)} \qquad (1)$$

or

$$w = \frac{60n}{tN} \quad \text{(rpm)} \qquad (2)$$

with N as the number of teeth of the target wheel. The time period, T, and frequency, f, of impulse is written by

$$f = \frac{n}{t} = wN \quad \text{(Hz)} \qquad (3)$$

and

$$T = \frac{t}{n} = \frac{1}{wN} \quad \text{(s)} \qquad (4)$$

The rotational speed direction of the target wheel can be detected by the phase drift, $\Delta\Phi$, see Fig. 2.

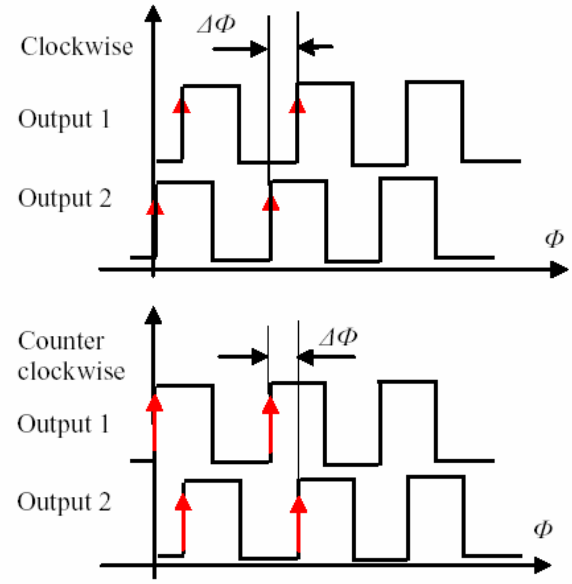

Fig.2. Rotational speed direction detection of target wheel by using phase drift $\Delta\Phi$ for $0 < \Delta\Phi < 180°$.

The phase drift, $\Delta\Phi$, between the two electrical outputs depends on the distance vector of the two detectors, $\underline{a}$, the outer diameter of the target wheel, D, the sensing distance between the detectors and the addendum of the target wheel, b, and the number of teeth, N. It can be calculated by:

$$\Delta\phi = \phi_2(Output_2) - \phi_1(Output_1) = \frac{360° N}{\pi} \sin^{-1}\left(\frac{\underline{a}}{\sqrt{(D+2b)^2 + a^2}}\right) \qquad (5)$$

where the distance vector is defined as follows:

$$\underline{a} = \begin{cases} a & \text{counter clockwise rotation} \\ -a & \text{clockwise rotation} \end{cases} \qquad (6)$$

One can get the phase difference $\Delta\Phi > 0$ for counter clockwise rotation and $\Delta\Phi < 0$ for clockwise rotation of the target wheel under the condition $0 < \Delta\Phi < 180°$.

The duty cycle of the output signals, η, is important for some applications and can be estimated by

$$\eta = \frac{\delta L_1}{L} = \frac{\delta L_1 N}{\pi\sqrt{(D+2b)^2 + a^2}} \qquad (7)$$

with δ as edge effect coefficient, L_1 as addendum arc width and L as effective tooth arc pitch, which is written by

$$L = \frac{\pi}{N}\sqrt{(D+2b)^2 + a^2} \qquad (8)$$

The edge effect coefficient, δ, depends on the material and geometric measures of the target wheel and can be determined by experiment.

In the most applications the condition, $D+2b >> a$, is easy fulfilled. In this case the phase drift and duty cycle can be simplified as

$$\Delta\phi = \frac{360° \underline{a}}{L} = \frac{360° \underline{a} N}{\pi(D+2b)} \qquad (9)$$

$$\eta = \frac{\delta L_1}{L} = \frac{\delta L_1 N}{\pi(D+2b)} \qquad (10)$$

with

$$L = \frac{\pi}{N}(D+2b) \qquad (11)$$

The phase drift, $\Delta\Phi$, increases with the distance vector, $\underline{a}$, and the number of teeth, N, and decreases with the outer diameter of the target wheel, D, and the sensing distance b. The duty cycle of the output signals, η, is proportional to the addendum arc width, L_1, and the number N, and is reverse proportional to the outer diameter, D, and the sensing distance, b.

For target wheel with a relative big outer diameter under the condition $D >> b$, the above parameters are approximated to:

$$\Delta\phi = \frac{360° \underline{a}}{L_g} = \frac{360° \underline{a} N}{\pi D} \qquad (12)$$

$$\eta = \frac{\delta L_1}{L_g} = \delta \eta_g = \frac{\delta L_1 N}{\pi D} \qquad (13)$$

with

$$L_g = \pi \frac{D}{N} = L_1 + L_2 \qquad (14)$$

as geometric tooth arc pitch and

$$\eta_g = \frac{L_1}{L_g} = \frac{L_1 N}{\pi D} = \frac{L_1}{L_1 + L_2} \qquad (15)$$

as geometric duty cycle of the target wheel.

In this case the phase drift and duty cycle depends only on geometric measures L_1, L_2 and the distance vector of the two detectors, $\underline{a}$. One can use these equations to design the target wheel relative easily.

3. EXPERIMENT RESULTS

The mathematical models mentioned above are applied to Hall Effect gear tooth sensors.

3.1 Measuring System

As example Fig. 3 shows an experiment system of Hall Effect gear tooth sensors. The experiment system is composed of an iron gear as target wheel, and a Hall Effect gear tooth sensor, a DC motor (0-6000rpm), an Agilent oscilloscope and power sources. The material of the target wheel is low-carbon steel and has a relative high magnetic permeability. The Hall Effect gear tooth sensor is CYGTS104U with two impulse outputs [3] and detects the rotational speed of the DC motor. The distance between the two Hall detectors is 5.4mm, i.e. a=5.4mm. The oscilloscope is used to sample the impulse outputs of the sensor, in order to measure the phase drift and the duty cycle of the output signals. Experiments are done by using this experiment system.

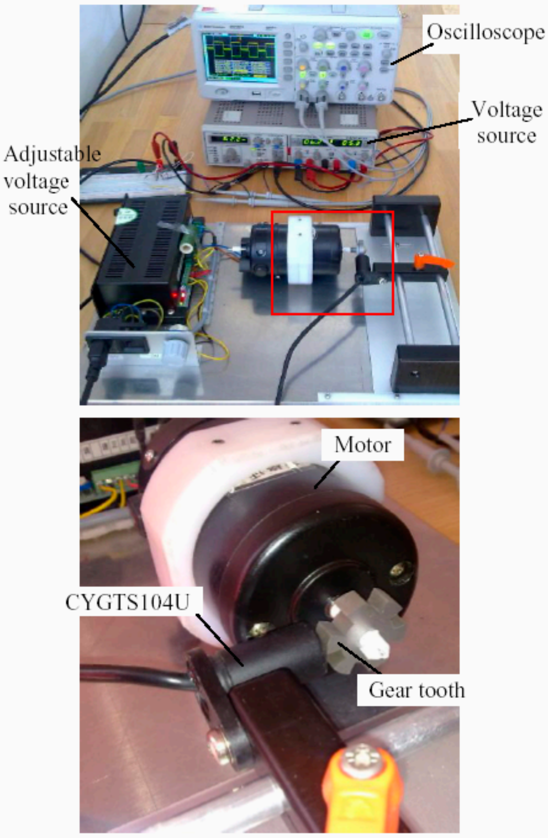

Fig.3. Experiment System of Hall Effect Gear Tooth Sensor

3.2 Phase Drift between the Impulse Outputs

Fig. 4 and Fig. 5 show the measured phase drift between the two impulse outputs of the sensor CYGTS104U in comparison to the value calculated by the models (5) and (9).

The target wheel used for experiments has 13 teeth, the outer diameter of which is 28mm. Both phase drift values decrease with the sensing distance, b. The relative deviation between the measured and calculated phase drifts is a function of the sensing distance. It is less than ±3.0% in sensing distance range from 2.2mm to 4.5mm. Therefore the models (5) and (9) can be used for estimating the phase drift of output signals of rotational measuring systems during design.

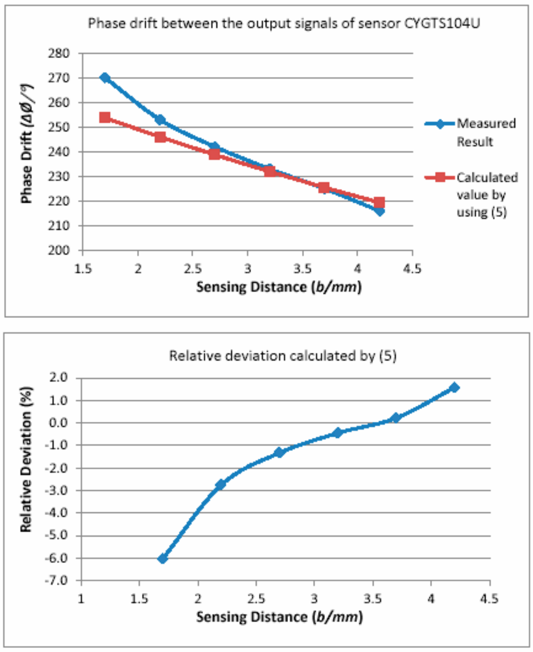

Fig. 4. Phase drift of output signals of sensor CYGTS104U as function of sensing distance, b (under a=5.4mm, N=13, D=28mm, phase drift calculated by (5))

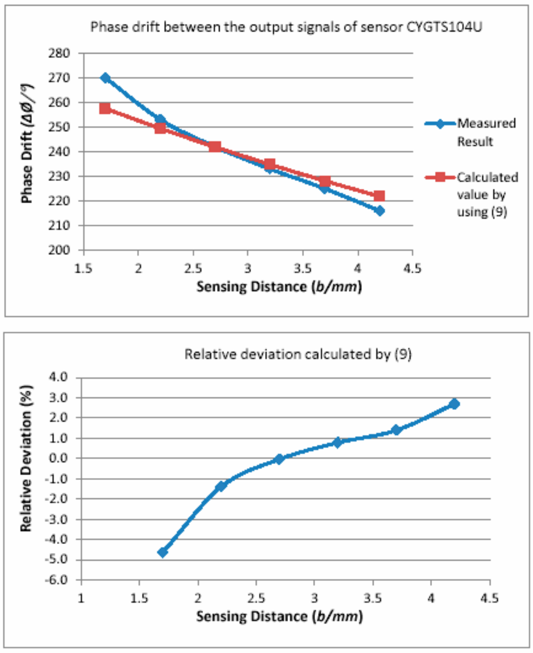

Fig. 5. Phase drift of output signals of sensor CYGTS104U as function of sensing distance, b (under a=5.4mm, N=13, D=28mm, phase drift calculated by (9))

3.2 Duty Cycle of the Impulse Outputs

The measured duty cycles of one output signal of the sensor CYGTS104U are shown in Table 1 and Fig. 6. All used target wheels have 6 teeth and 28mm of outer diameter, and different geometric duty cycle.

TABLE 1. Duty cycle as function of geometric duty cycle (η_g) and sensing distance (b)

Geometric duty cycle (η_g)	Sensing distance (b, mm)				
	2.0	2.5	3.0	3.5	4.0
1/4	0.378	0.375	0.365	0.357	0.355
1/3	0.507	0.500	0.503	0.493	0.500
5/12	0.574	0.551	0.551	0.536	0.535
1/2	0.646	0.615	0.611	0.600	0.591

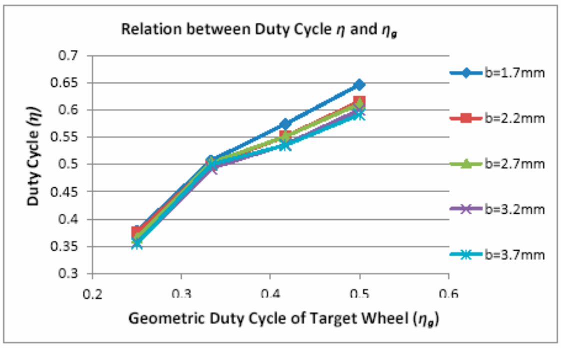

Fig.6. Measured duty cycles at different sensing distance

Using these measured data one can derive a regressive linear function:

$$\eta = 1.3984\frac{L_1}{L} + 0.0287 \qquad (16)$$

Fig. 7 shows the graphic of measured duty cycles as function of the ratio L_1/L in comparison to the regressive line.

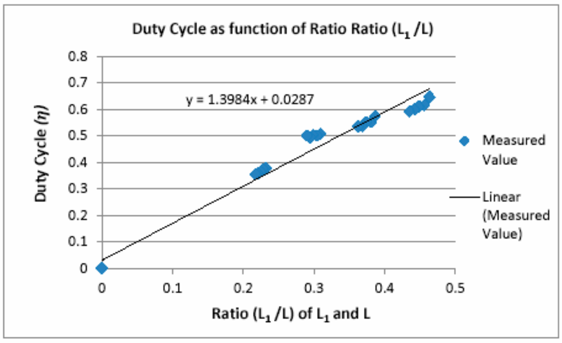

Fig. 7. Graphic of measured duty cycle data and regressive line as function of the ratio L_1/L

The equation (16) can be converted into:

$$\eta = (1.3984 + \frac{0.0287L}{L_1})\frac{L_1}{L} \qquad (17)$$

Comparing the equation (17) to (10) one can get the edge effect coefficient δ approximately:

$$\delta = 1.3984 + \frac{0.0287L}{L_1} \qquad (18)$$

Theoretically the ratio L_1/L changes in the range $0 < L_1/L < 1$. One can use the middle value $L_1/L = 0.5$ for calculation of the edge effect coefficient δ. Therefore the edge effect coefficient δ is written by

$$\delta = 1.3984 + \frac{0.0287}{0.5} = 1.4558$$

By using the parameter δ=1.4558 the duty cycle can be calculated by (7) and (10). Taking the target wheel with η_g=0.5, D=28mm and L_1=7.34mm as example the theoretical duty cycle η is calculated according to (10). Fig. 8 shows the results.

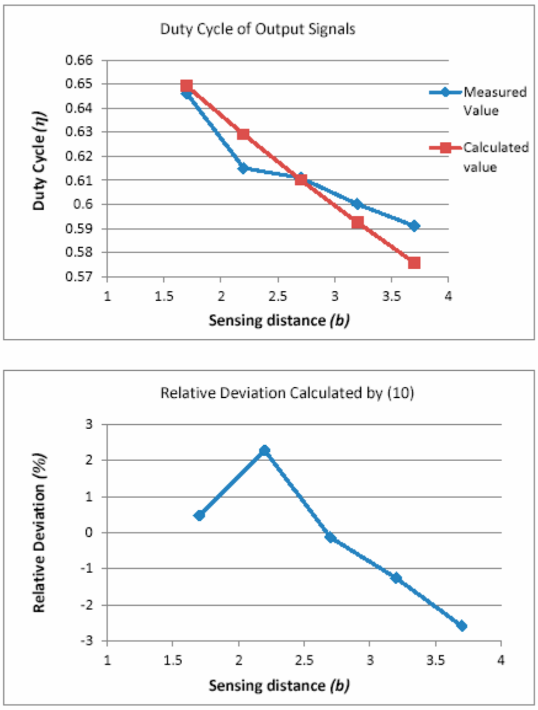

Fig. 8. Duty Cycle of one output signal of sensor CYGTS104U as function of sensing distance, b (Under a=5.4mm, N=6, D=28mm)

The relative deviation between the measured and calculated duty cycles is a function of the sensing distance. It is less than ±3.0% in sensing distance range from 1.7mm to 3.7mm. Therefore the models (7) and (10) can be used for estimating the duty cycle of output signals of rotational measuring systems during design.

4. APPLICATIONS

The mathematical models can be used for design of the target wheel of gear tooth sensors with dual outputs and rotational speed measuring systems. Taking the condition $D+2b>>a$ as example, the phase drift and duty cycle can be described by (9) and (10)

In this case, if the outer diameter, D, the distances, a and b, are known the number of teeth N of the target wheel can be determined by

$$N = \frac{\pi(D+2b)}{360° \underline{a}} \Delta\phi \tag{20}$$

and the addendum arc width L_1 can be calculated by

$$L_1 = \frac{\pi(D+2b)}{\delta N} \eta \tag{21}$$

In order to get a suitable phase drift $\Delta\Phi$ =90° and a duty cycle η=0.5 for a rotational speed measuring system the recommended number of teeth N and the addendum arc width L_1 should be determined by

$$N = \frac{\pi(D+2b)}{4a} \tag{22}$$

and

$$L_1 = \frac{\pi(D+2b)}{2\delta N} \tag{23}$$

In comparison with (11) the effective tooth arc pitch L should be 4 times of distance between the two detectors, i.e. L=4a.

As example one can determine the parameters of target wheel of a rotational speed measuring system under using the Hall Effect gear tooth sensor CYGTS104U [3]. For D=28mm, b=3mm and a=5.4mm one obtains the number of teeth N=5 and the addendum arc width L_1 =7.34mm under using the edge effect coefficient δ=1.4558. The geometric tooth arc pitch L_g is equal to 17.59mm according to (14)

In the same way one can also determine the outer diameter, D, of the target wheel and the sensing distance, b, etc. Further information about this topic will be discussed in another paper.

5. CONCLUSIONS

The proposed mathematical models are tested by Hall Effect gear tooth sensor CYGTS104U and speed measuring systems. From the experiment results one can draw the following conclusions:

- The phase drift between the output signals can be determined by using the models (5) and (9). The relative deviation of the parameter calculation is within ±3.0%.
- The edge effect coefficient δ of target wheel can be estimated by linear regression under using the measuring data of duty cycles of output signals at different geometric duty cycle of the target wheel.
- The duty cycle of the output signals can be calculated by the model (7) and (10). The relative deviation is also within ±3.0%.
- The mathematical models proposed in this paper can be used for determining the parameters of the target wheel and other parameters of the rotational speed measuring systems
- The models are very usable for analysing gear tooth sensors with dual impulse outputs and very helpful and effective for the design and development of rotational speed measuring systems.

The further works should be concentrated on the model-based design of the rotational speed sensors and measuring systems in order to optimize the measuring system and save the design time and development costs.

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