ASPECTS OF MEASUREMENT STRATEGY WITH DYNAMIC PROGRAMMING

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Abstract – Measurements are an integral part of process and quality management. Measurements are uncertain, thus there always is uncertainty about the quality and decisions are to be made under uncertainty. In this paper optimal decision policy and action cost structure is researched through quality pipe problem. Two examples with three and four action options is presented and discussed.

Keywords Dynamic programming, SPC, Measurement strategy

1. INTRODUCTION

Measurements are an integral part of process and quality management. From measurement data the information about the state of the process can be derived. System state should always be known with acceptable uncertainty.

This paper concerns quality pipe problem introduced in [1, 2] with the SPC (statistical process control) –type approach. Optimization problem is formed and dynamic programming is used to find optimal solutions. Examples of solving optimal decision policy are presented with three or four decision options – do nothing, control action, and one or two measurements.

This paper is organized as follows – Section 2 illustrates the measurement strategy and quality pipe problem with SPC approach. Section 3 handles basics about the dynamic programming and Section 4 gives quality pipe examples with three or four different decision options. And finally section 5 present conclusions.

2. QUALITY PIPE AND MEASUREMENT STRATEGY

This section illustrates the quality pipe and its relevancy to measurement strategy. The process industries make use of hundreds of on-line and laboratory measurements to monitor and control the process [3]. Information systems are designed with the aim of supporting the daily decision making about the process and product quality by operators and engineers so that the best practice of operation can be achieved continuously. Measurements, soft sensors and process simulators form the basis for such decision support by reducing the uncertainty about the present state of the process and about its future evolution.

Measurement strategy should be such that enough information for controlling the process is acquired by doing measurements. On the other hand the costs of making these measurements should be acceptable level. This sets an optimization problem that is very process dependent; the costs of measurements vary from process to process as well as the difficult level of decisions. But altogether we must have well enough overall picture of our process in order to control it effectively.

In this paper we research an optimal action schedule and cost structure of one quality parameter having one or two measurements with additional options to do nothing or do control action.

2.1. Quality pipe

It is common industrial practice that every quality variable has its own quality specifications, acceptance limits – “quality pipe”. This can also be understood as a tool for monitoring and controlling the process. Different quality pipes can be formed by forming information channels in which measurements, *a priori* information and information based on covariance matrix can be combined [4].

This kind of quality pipe can serve as basis for process monitoring where statistical process control (SPC) tool can be applied or this can be turned around - regular statistical process control problem can be understood as a solution to control problem, in which there is an action that returns the system back to its origin, but has a cost associated to it. In this paper this framework is adopted for quality pipe problem.

3. DYNAMIC PROGRAMMING

This section discusses the basics about dynamic programming – what it is and why it is used here.

Dynamic programming is a powerful algorithmic method of solving problems exhibiting the properties of overlapping sub problems and optimal substructure that takes much less time than naive methods [5]. In case of quality pipe we have different action possibilities and certain time steps (horizon) where dynamic programming can help overcome the difficulties.

Dynamic programming is usually applied to an optimization problem, for example, maximizing or minimizing something. These problems may have many different solutions, but one of them is an optimal solution. The steps to creating a dynamic programming algorithm are:

1. Define the structure of an optimal solution,
2. Generate the value of the optimal solution recursively,
3. Create an optimal solution in a bottom up manner.

4. OPTIMAL ACTION SCHEDULE AND COST STRUCTURE

In this section this framework is illustrated through examples, so let us now formulate the regular SPC problem for scalar state *x* which is fully observable at any time instant as follows. System has two actions - “no action” leading to linear dynamics:

$$x_{n+1} = ax_n + \varepsilon_{n+1} \qquad (1)$$
$$\varepsilon_{n+1} \sim N(0, \sigma_\varepsilon^2)$$

And another one - “make an action” that returns the system to zero

$$x_{n+1} = \varepsilon_{n+1} \qquad (2)$$
$$\varepsilon_{n+1} \sim N(0, \sigma_\varepsilon^2)$$

There is cost $c(x)=\alpha x^2$ associated with state, see fig. 1. and cost c_A associated with action. Actually the form of *c(x)* is rather irrelevant, but this form allows some numerical simplification as it can be averaged. State cost *c(x)* is 4 where as cost of control action is 1 so it is better to make a control action when getting outside the quality pipe.

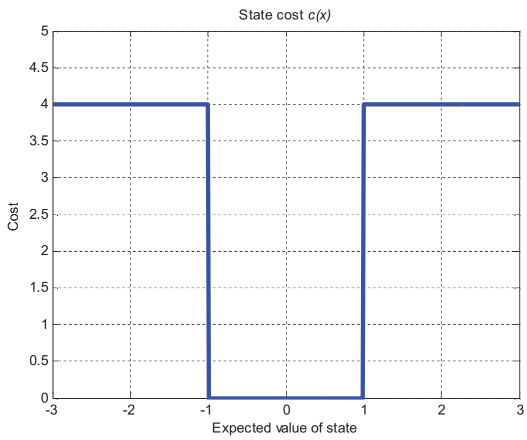

Fig. 1. Form of state cost *c(x)*.

Then the dynamic programming [5, 6] problem reads as

$$V_N(x_0) = \min E\left\{\sum_{i=1}^{N} \gamma^{i-1}\left(c(x_i) + \delta_{A_{i-1},1} c_A\right)\right\} = \qquad (3)$$
$$\min_{A_0}\left(E_{X_1|x_0,A_0}\left\{\alpha x^2\right\} + \delta_{A_0,1} c_A + \gamma \cdot E_{X_1|x_0,A_0}\left\{V_{N-1}(x_1)\right\}\right)$$

where c_A is a cost of action, *c(x)* is a state cost and γ is a discount factor.

This is straight forward to solve numerically – first solve the simple one-step ahead and then iterate. Fig. 2. shows optimal cost for this system. In the flat region the action is chosen whereas at the parabolic region no action is made. So this can be understood as SPC with alarm limits at the points in which the action region starts.

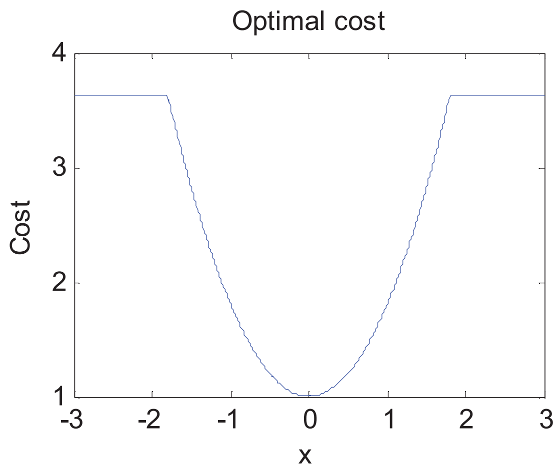

Fig. 2. Optimal cost with parameters $\sigma^2_\varepsilon = 1$, $\alpha = 1$, $a = 0.9$, $c_A = 1$, $\gamma = 0.95$ and $N = 10$.

Now let us add an imperfect (uncertain) measurement with the cost of measurement - c_M and uncertainty - σ^2_m.

$$y_{n+1} = x_{n+1} + v_{n+1}$$
$$v_{n+1} \sim N(0,\sigma_m^2) \tag{4}$$

We allow three decision alternatives:

1. Do nothing - "no measurement, no action" leading to linear dynamics, see Eq. 1.
2. Act - "no measurement, action", returning the system to zero, see Eq. 2.
3. Measure - "measurement, no action", giving us more information about the state of the system, see Eq. 4.

The dynamic programming problem is now:

$$V_N\left(\mu_0,\sigma_0^2\right) =$$
$$\min E\left\{\sum_{i=1}^{N}\gamma^{i-1}\left(c(x_i)+\delta_{A_{i-1},1}c_A+\delta_{M_{i-1},1}c_M\right)\right\} \tag{5}$$
$$= \min_{A_0,M_0}\left(\begin{matrix} E_{X_1|x_0,A_0}\left\{\alpha x^2\right\}+\delta_{A_0,1}c_A+\delta_{M_0,1}c_M + \\ \gamma\cdot E_{Y_1|x_0,A_0,M_0}\left\{V_{N-1}\left(\mu_1(y_1),\sigma_1^2(y_1)\right)\right\}\end{matrix}\right)$$

Again one step ahead is simple and then the solution is iterated. Fig. 3. shows one step ahead cost for state cost *c(x)* shown in Fig. 1.

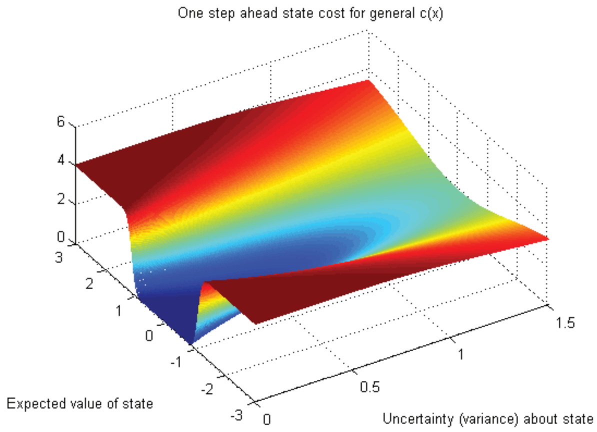

Fig. 3. One step ahead cost for *c(x)* shown in Fig. 1.

The expressions for the decision options are now:

1. Do nothing – "no measurement, no action". This option is shown in blue in the following figures.

$$\alpha\left(a^2\mu_0^2+a^2\sigma_0^2+\sigma_\varepsilon^2\right)+\gamma V_{N-1}(a\mu_0,a^2\sigma_0^2) \tag{6}$$

2. Act – "no measurement, action". This option is shown in green in the following figures.

$$\alpha\sigma_\varepsilon^2+c_A+\gamma\cdot V_{N-1}(0,\sigma_\varepsilon^2) \tag{7}$$

3. Measure – "measurement, no action". This option is shown in red in the following figures.

$$\alpha\left(a^2\mu_0^2+a^2\sigma_0^2+\sigma_\varepsilon^2\right)+c_M$$
$$+\gamma\int_{-\infty}^{\infty}N\left(y;a\mu_0,a^2\sigma_0^2+\sigma_\varepsilon^2+\sigma_m^2\right)\cdot$$
$$V_{N-1}\left(\frac{a\mu_0\sigma_m^2+y\left(a^2\sigma_0^2+\sigma_\varepsilon^2\right)}{a^2\sigma_0^2+\sigma_\varepsilon^2+\sigma_m^2},\frac{\sigma_m^2\left(a^2\sigma_0^2+\sigma_\varepsilon^2\right)}{a^2\sigma_0^2+\sigma_\varepsilon^2+\sigma_m^2}\right) \tag{8}$$

Fig. 4. shows optimal decision policy calculated with following system parameters - system variance σ^2_ε = 0.1, measurement uncertainty σ^2_m = 0.03 and system dynamics a = 0.9. The cost parameters were – state cost factor α = 1, cost of action c_A = 1, cost of measurement c_M = 0.05, discount factor γ = 0.95 and horizon N = 10.

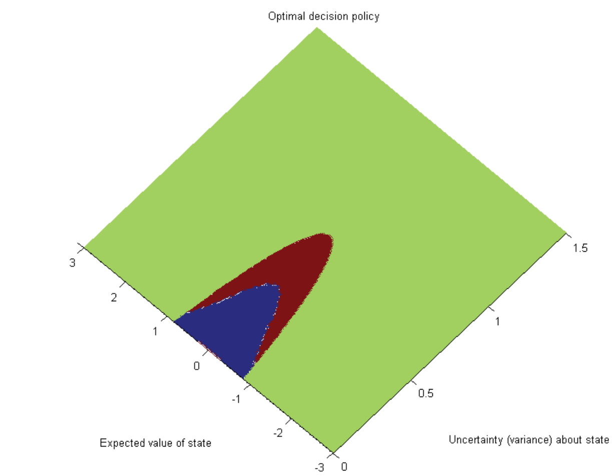

Fig. 4. Horizon (N) is 2. Blue denotes the no measurement/no action, red denotes measurement/no action and green no measurement/action.

As to be expected, the importance of measurement increases when the horizon increases. Fig. 5. show the results with same parameters than previous figure but horizon (N) is 10.

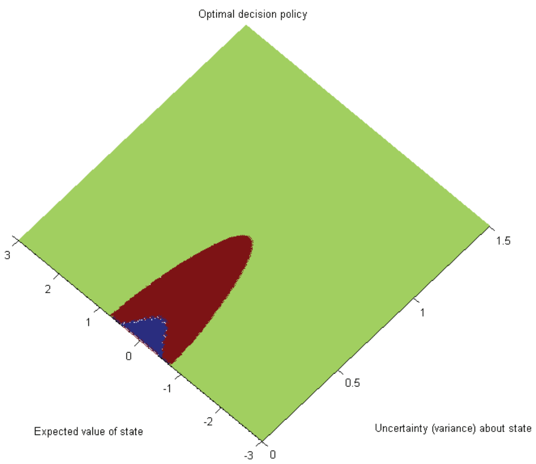

Fig. 5. Horizon (N) is 10. Blue denotes the no measurement/no action, red denotes measurement/no action and green no measurement/action.

It is to be noted that with these parameters, the action takes the state information to (μ = 0, σ^2_ε = 0.1). Next decision is not to make any measurement or action and the state information is ((μ = 0, σ^2_ε = 0.181) and the movement continues until the red region is reached, that is, if point ((μ = 0, σ^2_ε = 0.5263, see Eq. 9) is outside blue region as it is for both horizons.

$$E = \sigma_\varepsilon^2 \frac{1}{1-a^2} = 0.1\frac{1}{1-0.9^2} = 0.5623 \qquad (9)$$

From that point on the further decisions (system state information) will depend on which results are obtained. The frequency of measurements and actions can be analysed with simulation, which is left to future study.

4.1. Two measurements

Now let us add another imperfect measurement with different cost and uncertainty. With that we have another option:

4. Measure 2 –"measurement 2, no action". This option is shown in yellow in the following figures.

$$\alpha\left(a^2\mu_0^2 + a^2\sigma_0^2 + \sigma_\varepsilon^2\right) + c_{M2}$$
$$+\gamma \int_{-\infty}^{\infty} N\left(y; a\mu_0, a^2\sigma_0^2 + \sigma_\varepsilon^2 + \sigma_{m2}^2\right)\cdot$$
$$V_{N-1}\left(\frac{a\mu_0\sigma_{m2}^2 + y\left(a^2\sigma_0^2 + \sigma_\varepsilon^2\right)}{a^2\sigma_0^2 + \sigma_\varepsilon^2 + \sigma_{m2}^2}, \frac{\sigma_{m2}^2\left(a^2\sigma_0^2 + \sigma_\varepsilon^2\right)}{a^2\sigma_0^2 + \sigma_\varepsilon^2 + \sigma_{m2}^2}\right) \qquad (10)$$

System and cost parameters are same as in previous example. Parameters for another measurement is - measurement uncertainty σ^2_{m22} = 0.06 and cost of measurement c_{M2}= 0.04. Fig. 6. show optimal decision policy with these parameters calculation horizon being 2.

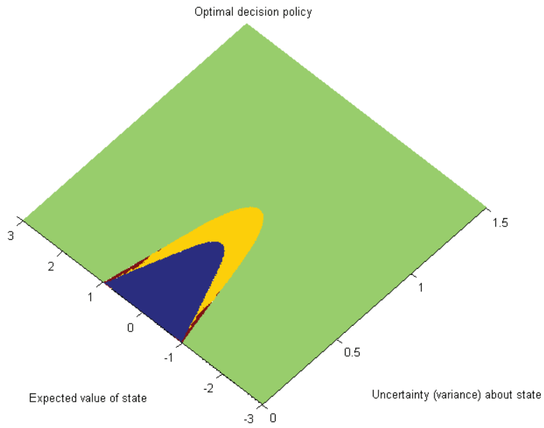

Fig. 6. Horizon (N) is 2. Blue denotes the no measurement/no action, red denotes measurement 1/no action, yellow denotes measurement 2/no action and green no measurement/action.

Again as to be expected, the importance of measurement increases when the horizon increases. The interesting part is how measurement options behave compared to each other. Fig. 7. shows optimal decision policy with same parameters as previous figure but calculation horizon being 5. Now the area of option measure 2, as it is cheaper but more uncertain, is decreased in the area where the state information in more certain.

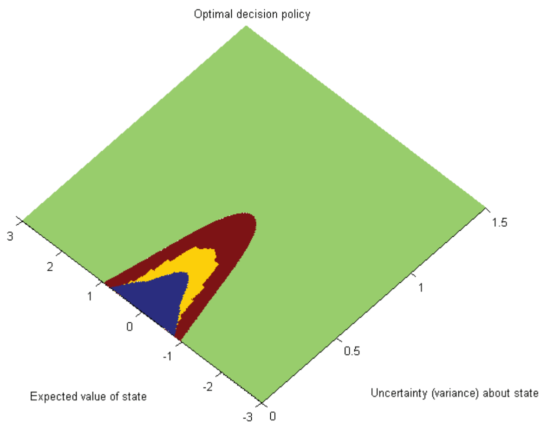

Fig. 7. Horizon (N) is 5. Blue denotes the no measurement/no action, red denotes measurement 1/no action, yellow denotes measurement 2/no action and green no measurement/action.

5. CONCLUSION

This paper addressed quality pipe problem with SPC approach using dynamic programming as tool to solve the problem. Also the measurement strategy was under discussion and examples with three or four decision options were presented to show the importance of measurement.

Main results showed how the importance of measurement increases when our horizon increases. Future study includes simulation and analysis of the frequency of measurements and actions. By this we can address more information to quality pipe problem and thus make more efficient measurement schedules.

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