TECHNISCHE UNIVERSITÄT
ILMENAU

50. Internationales Wissenschaftliches Kolloquium

September, 19-23, 2005

Maschinenbau von Makro bis Nano / Mechanical Engineering from Macro to Nano

Proceedings

Fakultät für Maschinenbau /
Faculty of Mechanical Engineering

Impressum

Herausgeber:	Der Rektor der Technischen Universität Ilmenau Univ.-Prof. Dr. rer. nat. habil. Peter Scharff
Redaktion:	Referat Marketing und Studentische Angelegenheiten Andrea Schneider Fakultät für Maschinenbau Univ.-Prof. Dr.-Ing. habil. Peter Kurtz, Univ.-Prof. Dipl.-Ing. Dr. med. (habil.) Hartmut Witte, Univ.-Prof. Dr.-Ing. habil. Gerhard Linß, Dr.-Ing. Beate Schlütter, Dipl.-Biol. Danja Voges, Dipl.-Ing. Jörg Mämpel, Dipl.-Ing. Susanne Töpfer, Dipl.-Ing. Silke Stauche
Redaktionsschluss: (CD-Rom-Ausgabe)	31. August 2005
Technische Realisierung: (CD-Rom-Ausgabe)	Institut für Medientechnik an der TU Ilmenau Dipl.-Ing. Christian Weigel Dipl.-Ing. Helge Drumm Dipl.-Ing. Marco Albrecht
Technische Realisierung: (Online-Ausgabe)	Universitätsbibliothek Ilmenau ilmedia Postfach 10 05 65 98684 Ilmenau
Verlag:	isle Verlag ISLE, Betriebsstätte des ISLE e.V. Werner-von-Siemens-Str. 16 98693 Ilmenau

ISBN (Druckausgabe):	3-932633-98-9	(978-3-932633-98-0)
ISBN (CD-Rom-Ausgabe):	3-932633-99-7	(978-3-932633-99-7)

Startseite / Index:
http://www.db-thueringen.de/servlets/DocumentServlet?id=15745

50. Internationales Wissenschaftliches Kolloquium
Technische Universität Ilmenau
19.-23. September 2005

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Robust Fitting of Circles for Optical Dimensional Metrology

ABSTRACT

This paper presents a method for robust recognition and separation of outliers in combination with fitting geometrical primitives into the measurement points with high accuracy.
The known iterative approaches for robust fitting often yield unsatisfactory results if the start conditions are influenced by outliers. Thus, in a first step the outlier detection and filtering in the coordinate space is done. The decision whether the datum belongs to the distortion or to the structure of interest is supported by Hough-transform. The desired geometry is calculated by least square fitting with high and certified accuracy from the outlier free coordinates. The performance of this approach is exemplified on the geometrical primitive circle under practical measuring conditions.
The originality lies with the combination of a recognition approach for robust feature detection with a least squares fitting for highly precise results. This approach has an enormous relevance on applications for non contact dimensional metrology, where the existence of dust leads to a high measurement uncertainty.

Keywords: robust fitting, outlier, dimensional metrology, hough transform

1. INTRODUCTION

Micro- and Nanostructures are in the focus at the beginning of the 20th century. Light microscopy is the key to make them visible. In combination with image processing procedures dimensions of the structures can be measured with submicron resolution.
In the field of microstructure measurement the amount of detection points is very high. The probability for occurrence of outliers arises in existence of dust or badly illuminated images. Moreover the dimension of dust lies in the region of the relevant structures (figure 1). Thus, filtering of statistical and deterministic errors in measurement points is necessary before further calculation of geometric primitives is done.

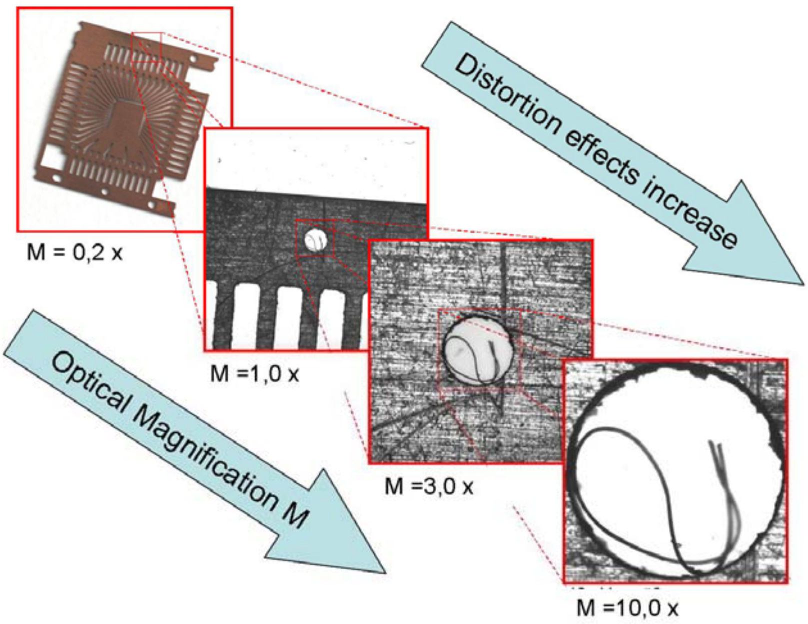

Fig. 1. Micro hole with distortion by dust

2. STATE-OF-THE-ART

The known methods for fitting of geometric primitives can by separated in clustering techniques and least square fitting (see Fig 2).

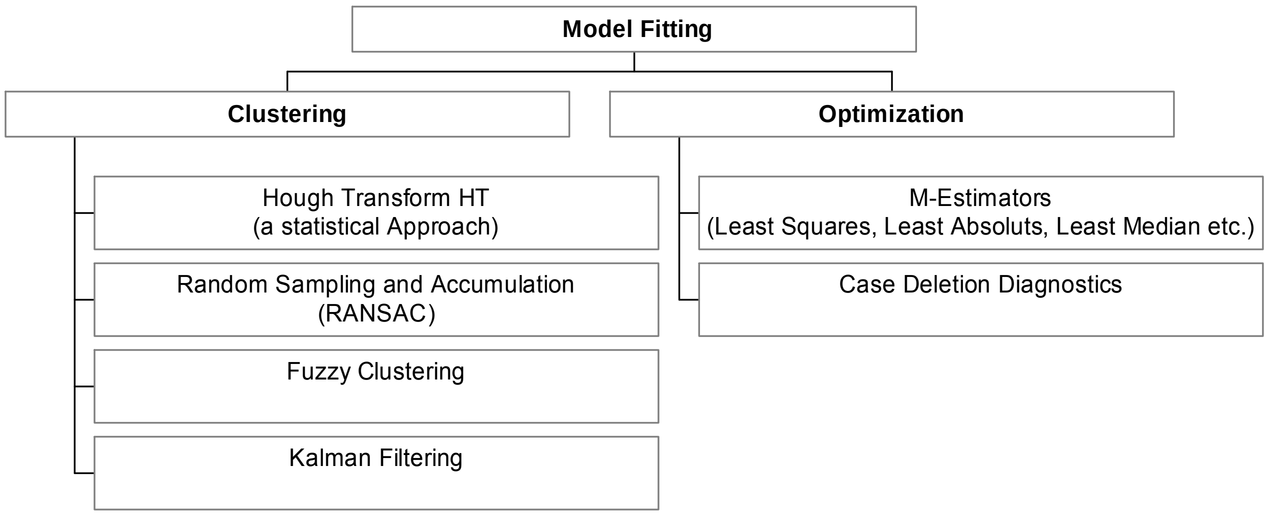

Fig. 2. Classification of model fitting methods [1]

Clustering methods are based on mapping sets of points to the parameter space, such as the Hough transform (HT) [2] and accumulation methods [3]. These methods decide in the parameter space whether data points belong to a defined class. For the assignment of geometric primitives to data points the HT or the "Random Sampling and Accumulation" (RANSAC) can be used.

The HT [4] works on a statistical base. In the parameter space the frequency of occurrence of the geometric primitive which goes through the data points is registered. The parameter tupel with the highest occurrence designates the geometric primitive.

The RANSAC [5] is a closely related procedure. Thereby a minimum number of data points is randomly selected from the whole data. The geometric primitive is calculated and their parameters are accumulated. The expectation values of the parameters describe the geometric primitive.

Both methods are very robust to outliers. But their shortcomings of high computational complexity, non-stable solutions and low accuracy prevent their direct application for metrological tasks.

On the other hand the known optimization methods M-estimation ([6],[7]) and Case Deletion [8] achieve high resolutions. Case Deletion assesses the effect of removing suspect data on the fitting results. M-estimation is based on the Maximum-Likelihood-principle and works optimal under normal distributed deviations. The known Gaussian least squares fitting method (LS) belongs to the M-estimation methods. LS is important in dimensional measurement if the point deviations from the ideal form are normal distributed. This method fits a model function $F(x,y,z;\ \mathbf{a})=0$, with the model parameters $\mathbf{a} = (a_1 .. a_M)^T$ in the two-dimensional coordinate space into the points $P_i=\{y_i(x_i)\}$. The compensation condition for fitting is given by equation (1).

$$\hat{\mathbf{a}} = \underset{\mathbf{a}}{\operatorname{argmin}} \sum_{i=1}^{N} \left(y_i - y(x_i;\mathbf{a})\right)^2 \tag{1}$$

For model fitting of geometric primitives like lines, circles and ellipses orthogonal point distances to the model must be considered. Due to the orthogonal distances the optimization problem in equation (1) becomes nonlinear. It can be solved only in an iterative way. Fitting algorithms of this type are known as orthogonal least squares (OLS).

It can be shown, that equation (1) is influenced by measuring points, with a large distance from the model (outlier). For comparison of optimization methods a influence function was developed in [9]. Moreover the influence increases if the portion of distorted points arises.

The development in the field of fitting geometrical primitives with LS fitting led to algorithms which became more robust due to weighting distances [9]. Thereby a weighting factor γ_i is inserted in equation (1) to reduce the influence of distances d_i which are greater then the standard deviation σ of the distances, see equation (2).

$$\hat{\mathbf{a}} = \underset{\mathbf{a}}{\operatorname{argmin}} \sum_{i=1}^{N} (\gamma_i d_i)^2 \text{ where } d_i = y_i - y(x_i; \mathbf{a}) \tag{2}$$

The following weighting scheme [10] can be used for γ_i:

$$\gamma_i = \begin{cases} 1 & d_i < \sigma \\ \sigma / |d_i| & \sigma \le d_i < 3\sigma \\ 0 & d_i \ge 3\sigma \end{cases} \tag{3}$$

An own approach in this field is decribed in [11] where an OLS method with weighted distances is used for outlier elimination (OLS+OE).

The general disadvantage of optimization methods is that they are strongly influenced by outliers. Furthermore the least squares estimation produces poor starting conditions under the influence of large outliers [12]. Thus, we developed a new method for robust and accurate fitting as a combination of a robust clustering technique to get an ideal start solution, followed by an outlier filter (OE) and a certified orthogonal least squares fit (OLS) for geometric primitives.

3. THE ROBUST PROCESSING METHOD

The basic idea of our approach is to use several methods for data processing in certain stages of incoming data to find the measuring object and determine their geometric parameters (figure 3). It's clear that at first all possibilities must be done to avoid dust in the measurement space. Clean rooms and cleaning of the measuring object are necessary. The illumination of the image scene should be optimized for highly contrast images too [13].

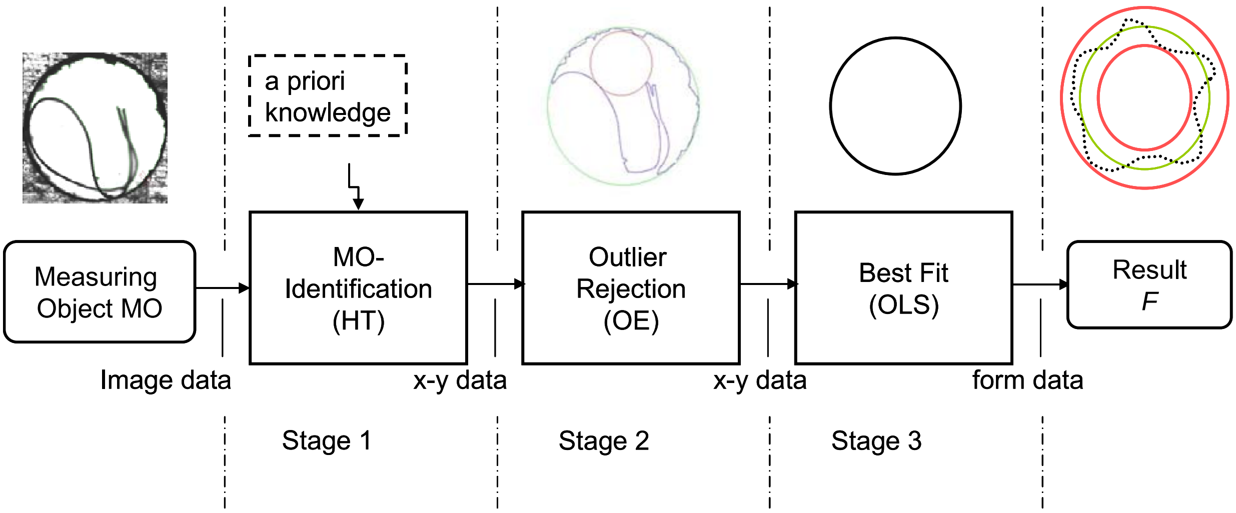

Fig. 3. Stages for processing image data to geometric results

After pre-filtering a robust method for recognition of the object of interest in the image data is necessary [14]. This method does not need the best accuracy but must be tolerant against other objects. Because of its statistical nature and its possibility to deal with geometric primitives the Hough-transform (HT) was selected from the known clustering techniques for our goal. Furthermore geometric primitives like a line or a circle can be described in a two- and three-dimensional parameter space [4],[15]. An other advantageous feature of the HT is that it works well for up to 50% of errors in the data points. The resulting circles by HT (see figure 4) are compared with the a- priori knowledge of the expected circle parameters, i.e the nominal radius. The circle with the smallest difference to the considered parameter is used for subsequent processing.

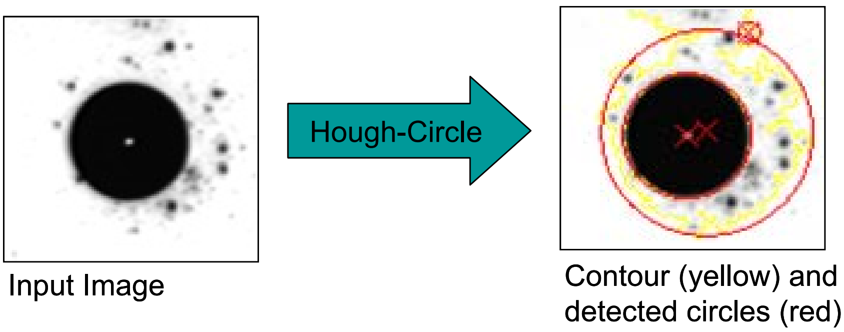

Fig. 4: Hough detection of circles

If the best geometrical primitive $F(x,y,z;\ \mathbf{a})=0$ was found by the parameter comparison the identification and rejection of outliers can be done in the stage of coordinate level data. The discrimination is given by equation (4) and bases on the orthogonal distance d_i between the detected coordinate points $P_i=\{y_i(x_i)\}$ and the geometric primitive F (figure 5).

$$P_i = \begin{cases} Inlier & d_i \leq eps \\ Outlier & eps < d_i \end{cases} \tag{4}$$

A stable start solution results, which is decisive for the subsequent precision fitting with proven procedures like OLS.

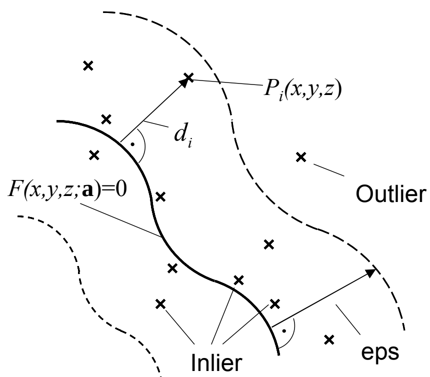

Fig. 5: Outlier Identification related to Model F

4. EXPERIMENTS

4.1 SIMULATION OF DISTORTIONS

The performance of our approach is exemplified on the geometrical primitive circle under practical measuring conditions. Three known methods for robust fitting, namely orthogonal least squares (OLS), OLS with outlier elimination (OLS+OE) and hough-transform (HT), were compared with the new combined approach HT+OLS. They have been proven with test data sets. The test data sets consist of 100 x-y-points which was generated by simulation. They describe a circle with the middle point {*xmN=50* pixel units, *ymN=50* pixel units} and a radius *rN=20* pixel units. A normal distributed orthogonal noise with standard deviation *s1=sN=1.0* pixel units, as shown in figure 6a, was superposed on the ideal circle to get a realistic profile. The response of the four fitting methods is shown in figure 6b.

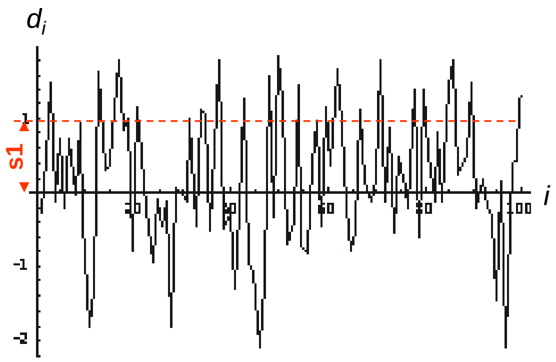

Fig. 6a: Normal distributed profile deviation d_i with standard deviation *s1=1.0* pixel units along the contour point number *i*

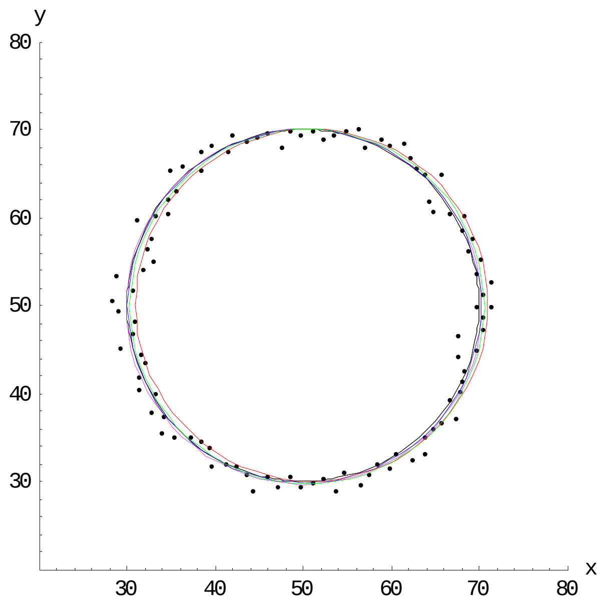

Fig. 6b: Normal distributed circle points and results of circle fitting in x-y-plane, OLS (violet), OLS+OE (blue), HT (red), HT+OLS (green)

4.2 INFLUENCE OF OUTLIER WIDTH

The scope of the first investigation was the influence of a single distortion along the circle contour on the fitting result. Therefore deviations with a defined distance *Ha* orthogonal to the contour and a varying width *Da* along the contour were used (see figure 7). The results are shown in figure 8.

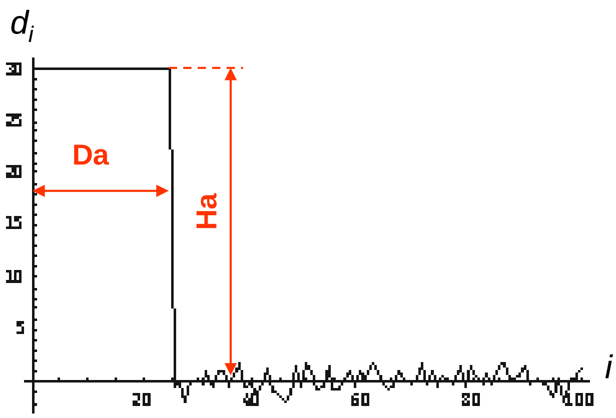

Fig. 7: Profile deviation along the contour point number i with a outlier width $Da=25\%$, a outlier height $Ha=30$ units and a underlying normal distributed noise with $s1=1.0$ pixel units

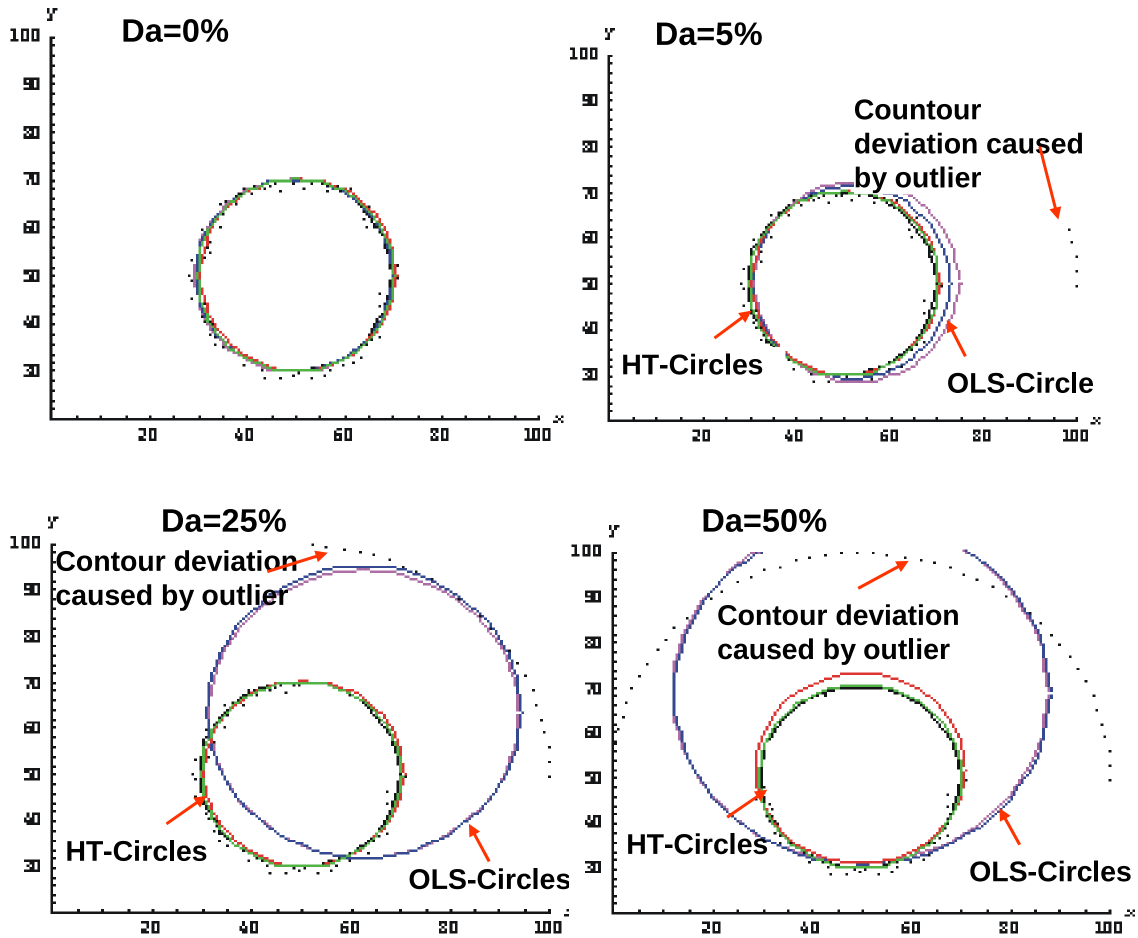

Fig. 8: Results of circle fitting methods, influenced by the outlier width $Da=0\%..50\%$, violet: OLS , blue: OLS+OE , red: HT, green: HT+OLS

The effect of the single distortion with varying parameter *Da* on the radius deviation is quantified in table 1 and figure 9. In contrast to the three known methods the influence on HT+OLS is minimal. Radius deviations smaller than 0.5 pixel units are attainable.

Table 1: Circle radius deviations for tested fitting methods depending on the outlier width *Da*

Fitting Method	r-rN [pixel units]			
	Da=0%	Da=5%	Da=25%	Da=50%
OLS	0,20	1,77	11,13	17,64
OLS+OE	0,08	0,99	11,49	17,73
HT	0,00	0,00	0,00	1,00
HT+OLS	0,15	0,15	0,09	0,28

In case of HT+OLS the deviation of circle radius from the known nominal values is smaller than *0.5* pixel units under influence of outlier width *Da=0%..50%* of circle contour. The deviation of the OLS method increases with *Da.* As to be seen in table 1 the HT suffers from the low resolution of 1 pixel. Higher accuracy can be achieved only by subsequent OLS after outlier rejection.

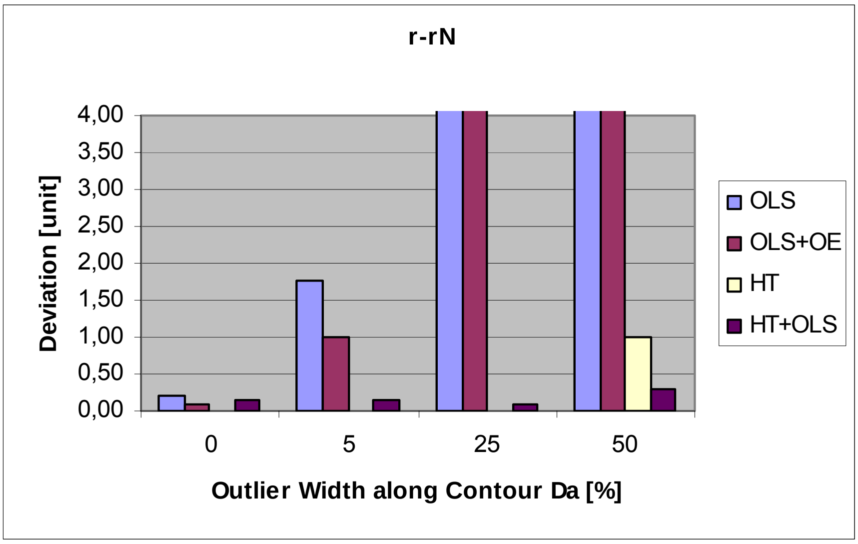

Fig. 9: Influence of *Da* on circle radius deviation *r-rN,* (overshooting bars show large deviations)

4.3 INFLUENCE OF OUTLIER OCCURRENCE PROBABILITY

An other question for the investigation was, how depend the circle results on an outlier population which occurs uniformly distributed along the circle contour. For this test a secondary noise process in addition to the primary one (see figure 6a) was superposed. The primary noise process describes normal distributed point detection deviations and the second noise process describes distortion structures. As the position of the occurrence of distortion structures (outlier population) can not be predicted a uniform distribution along the circle contour is assumed. The probability that outliers occur is described by the outlier occurrence probability *Pa.* The orthogonal deviations from the outlier population to the geometrical primitive are assumed as a normal distribution with the standard deviation *s2=10.0* pixel units. The resulting profile in figure 10 was added orthogonal to the ideal contour of the circle.

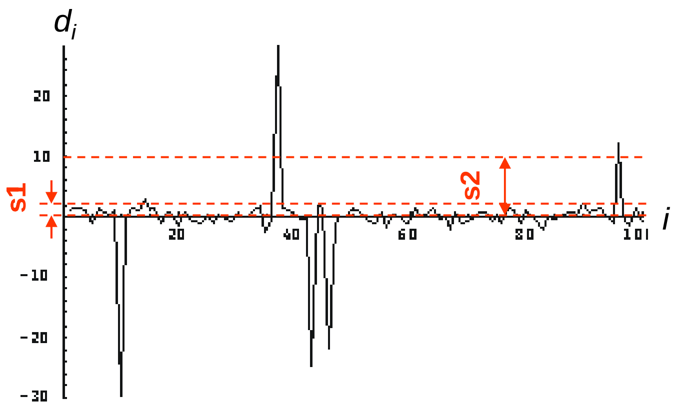

Fig. 10: Normal distributed deviation (*s1=1.0* units) superposed with a outlier population (*s2=10.0* units, *Pa = 5%)* along the circle contour

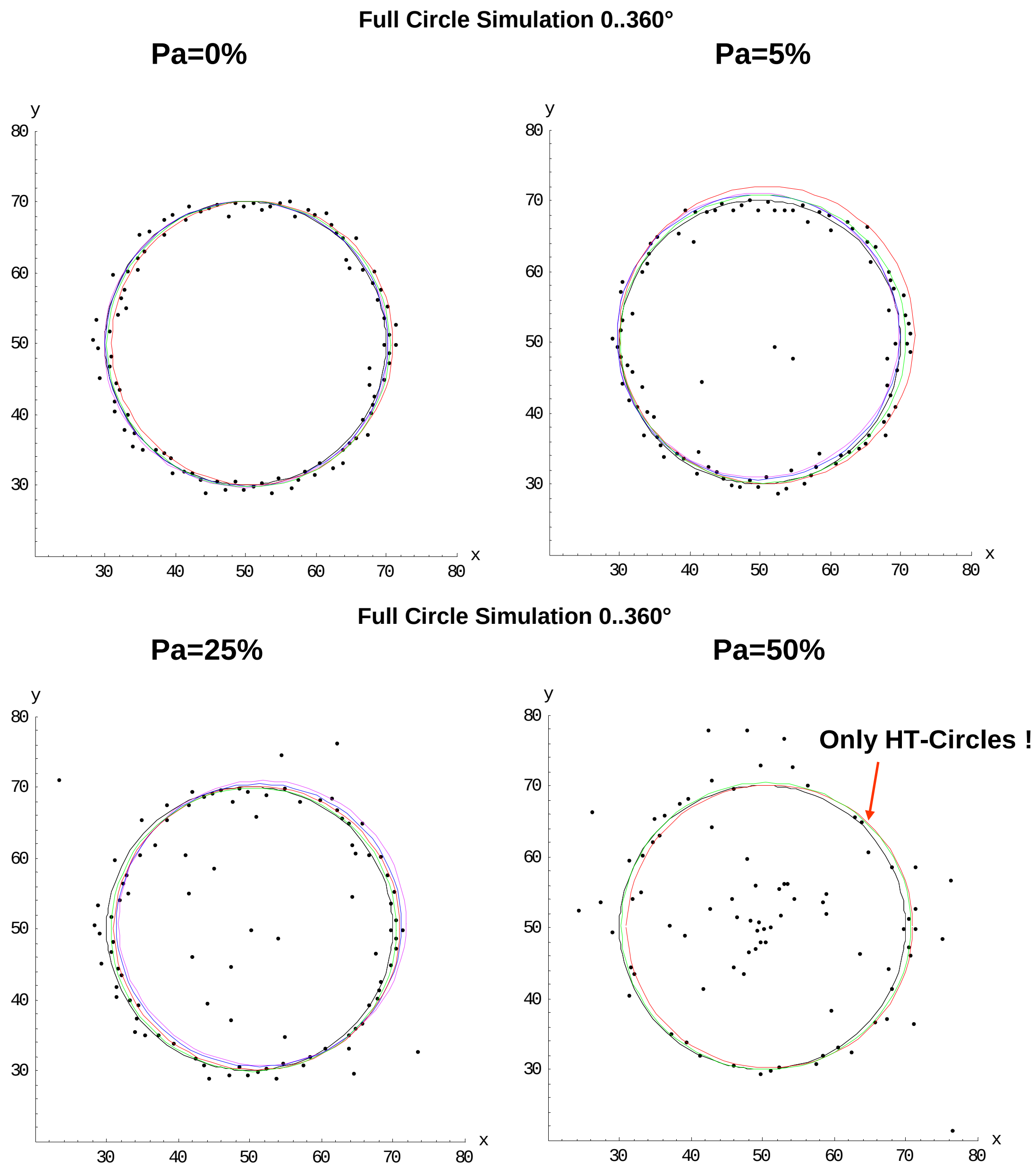

Fig. 11: Results of circle fitting method, influenced by the outlier occurrence probability *Pa=0%..50%*, violet: OLS , blue: OLS+OE , red: HT, green: HT+OLS

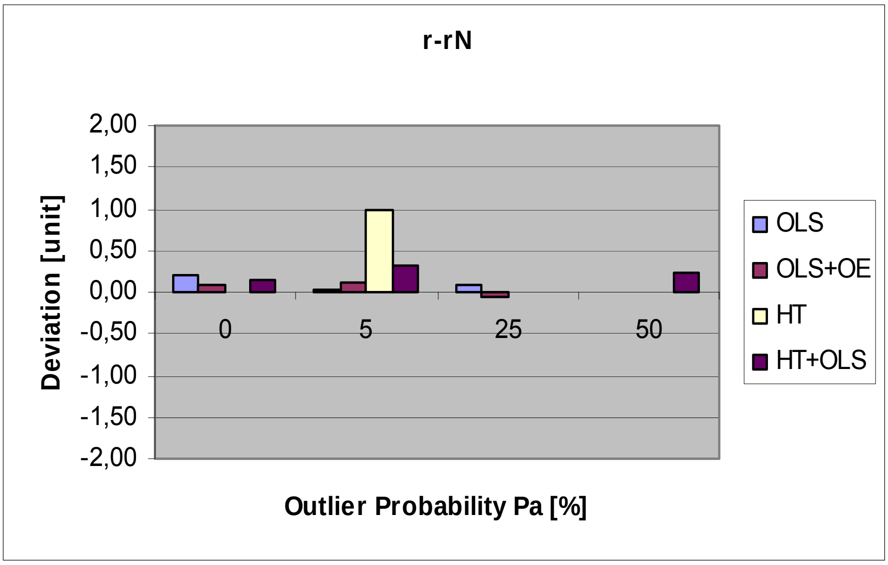

Fig. 12: Influence of *Pa* on circle radius deviation *r-rN,* (overshooting bars show large deviations)

The effect of the varying outer occurrence probability *Pa* on the circle deviations is visualized in figure 11 and quantified in figure 12 and table 2.

Table 2: Radius deviations for tested fitting methods depending on the outlier occurrence probability *Pa,* [1)] no response

Fitting Method	r-rN [pixel units]			
	Pa=0%	Pa=5%	Pa=25%	Pa=50%
OLS	0,20	0,02	0,09	1)
OLS+OE	0,08	0,11	-0,06	1)
HT	0,00	1,00	0,00	1)
HT+OLS	0,15	0,32	-0,01	1)

The HT-based methods get results under all conditions. In combination with OLS the radius deviations from the nominal are less then 0.50 pixel units. In opposite to it the other methods OLS, OLS+OE and HT produce large deviations which increase with *Pa.*

4.4 INFLUENCE OF CIRCLE SEGMENT ANGLE

The effect of varying angle areas of the circle contour from φ={360°, 270°, 180°, 90°}, with *Pa=5%,* was considered in the third investigation. The shortened circle contours and the fitting results are outlined in figure 13.

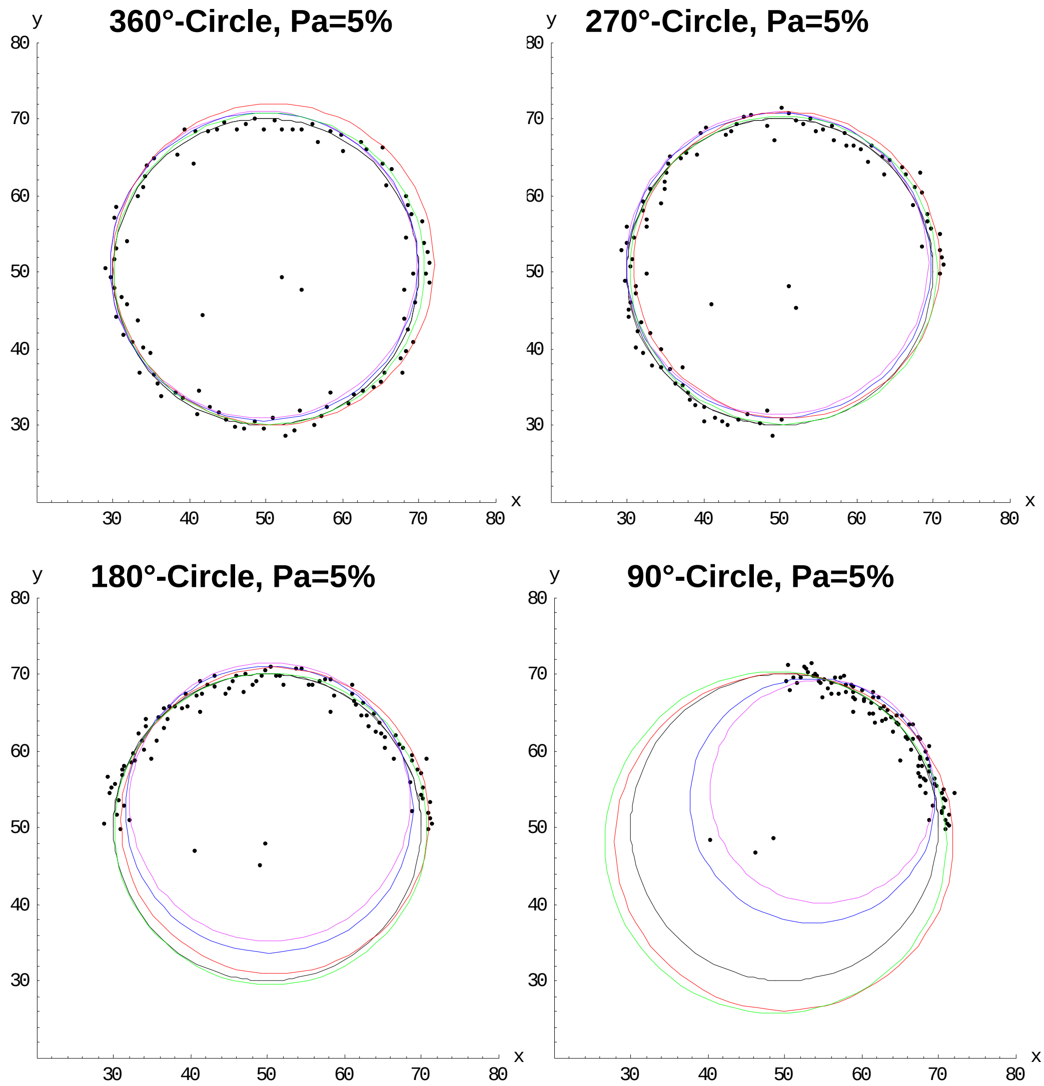

Fig. 13: Results of circle fitting method, influenced by the circle segment angle φ={360°, 270°, 180°, 90°}, *Pa=0%..50%*, violet: OLS , blue: OLS+OE , red: HT, green: HT+OLS

The effect of the varying angel areas on the circle deviations is visualized in figure 13 and quantified in figure 14 and table 3. The deviations of circle results by HT+OLS are smaller than 0.5 pixel units down to $\varphi=180°$.

Table 3: Radius deviations for tested fitting methods depending on the angle area φ, outlier occurrence probability *Pa=5%*

Fitting Method	r-rN [pixel units]			
	φ=360°	φ=270°	φ=180°	φ=90°
OLS	0,02	-0,26	-1,80	-5,46
OLS+OE	0,11	-0,11	-1,29	-4,05
HT	1,00	0,00	0,00	2,00
HT+OLS	0,32	0,10	0,33	2,25

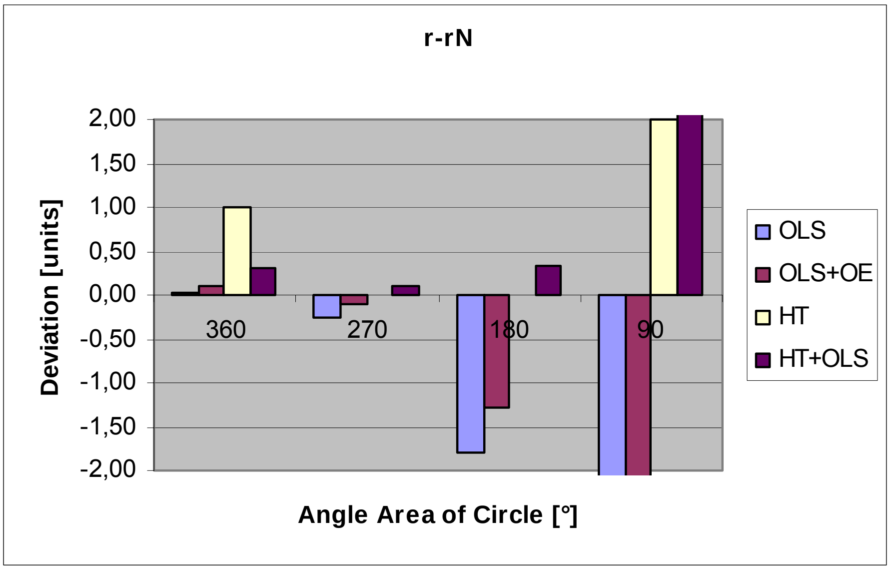

Fig. 14: Influence of varying angle on radius deviation, Pa=5%

As shown in the experimental results the known state-of-the-art fitting method orthogonal least squares (OLS) is deeply influenced by

- a single large outlier
- a population of outliers
- a sortant circle section.

Figures 8, 9, 11, 12, 13 and 14 give information about the capabilities of the new approach. The HT works well and the post processing with OLS is possible. By using the new combination of Hough-Transform for outlier identification and rejection before the orthogonal least squares method is done the critical influences on the circle deviations can be reduced to **0.5** pixel units under the specified test conditions:

- Circle deviation *s1=1.0* pixel units,
- Single outlier width *Da=0..50%* of contour length
- Single outlier deviation *Ra=30* pixel units
- Outlier population deviation *s2=10* pixel units,
- Outlier occurence probability *Pa=0..*50%
- Angle area φ={360°, 270°, 180°, 90°}

Orthogonal outlier deviation can be reduced from 30 pixel units down to < 0.5 pixel units. This is a significant improvement.

5. CONCLUSIONS

A robust fitting technique was described which is based on the combination of a recognition method by hough transform and a precision least squares fit. The performance of this approach is exemplified on the geometrical primitive circle under practical measuring conditions.

The paper on hand proves that even if large distortions occur with a high probability subpixel resolution can be attained. In this way the measurement uncertainty on optical precision measurements at micro parts under existence of dust can be extremely reduced.

It is possible to apply this method also for other geometric primitives like lines and ellipses.

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