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PETKO KIRIAZOV

EFFICIENT APPROACH FOR INTEGRATED STRUCTURE-CONTROL DESIGN OF LEGGED ROBOTS

ABSTRACT

This study is motivated by the need of dynamics-based methodologies for overall design of legged robots (LR). Along with the basic design requirement for strength/load capacity, additional design criteria for LR are needed to meet the continuously increasing demands for faster motion, higher position accuracy and reduced energy consumption. A conceptual framework for their integrated structure-control design is proposed that can be used to create LR with maximum capability to achieve the required dynamic performance. To verify our design optimisation concepts, several interesting examples related with two- and four-LR are considered.

INTRODUCTION

Difficulties to study LR and optimise their dynamic performance are mainly due to complex system dynamics and variable external disturbances. At first, the structure of the input-output relations between their subsystems has to be defined, i.e., numbers and locations of their inputs, outputs and interconnections, [2]. For analysis, design and control purposes, adequate dynamic models are needed and they have to be easily and accurately identifiable.

The dynamic model structure of a robotic system depends on the type of its control subsystem: centralised or decentralised. In practice, the decentralised manner of control has been adopted for its main advantages to the centralised one: simplicity, reliability and faster response. A common feature of the existing control design methods for linear systems with decentralised control structure is that stability in the face of parameter uncertainties can be ensured if the control transfer matrix (TM) is generalised diagonally dominant (GDD), [5]. We extend this property to robotic systems and prove that the GDD-condition is necessary and sufficient for them to be robust against arbitrary, but bounded disturbances, [1]. By using the nonnegative matrix theory [4], relevant design criteria are derived and they are not in conflict with the strength requirement. Moreover, these criteria enable a decomposition of the overall design task into a sequence of design solutions for the LR's components: mechanical structure, actuators, and controls. To verify our concepts and the efficiency of the proposed design approach, several interesting examples related with the optimisation of two- and four-LR are considered.
INTEGRATED STRUCTURE-CONTROL DESIGN APPROACH

LR are highly non-linear and difficult to model, identify, and control dynamical systems mainly due to gravitation, inertia couplings, friction, elasticity, and actuator limits. In addition, the structure of their dynamics may change during locomotion. The dynamic performance of a LR in a specific locomotion phase can be described, in general, by the following system of differential equations

\[ \dot{q} = M(q)^{-1} (Bu - C(q, \dot{q}) + g(q)) \]  

(1)

where, \( q \) is the vector of the links' rotation angles, \( M(q) \) is the inertia matrix, \( C(q, \dot{q}) \) is the vector of velocity forces, \( g(q) \) stands for friction and gravitation forces, matrix \( B \) represents the actuator location, and \( u \) is the vector of actuator torques. To identify the coefficients describing these dynamic terms we can apply the approach proposed in [3].

After compensating to some extent for the inertia, velocity, friction and gravitation forces by feedforward control, the following reduced model for the error dynamics can be used for the feedback stabilisation of the programmed motion.

\[ \dot{e} = A(q)u + d \]  

(2)

where \( e = q - q^{\text{ref}} \), \( A = M(q)^{-1}B \) is the control TM and vector \( d \) stands for all uncompensated terms, as well as for measurement and environment noises. With \( A \) being GDD, the non-negative matrix theory [4] states that there always exists a positive vector \( \overline{u} \) of control magnitudes solving the following system of equations

\[ A_i \overline{u}_i - \sum_{j \neq i} A_{ij} \overline{u}_j = \overline{d}_i \quad i = 1, \ldots, n \]  

(3)

where \( \overline{d}_i \) are some upper bounds.

Eqs. (3) present optimal trade-off relations between the bounds of model uncertainties and the control force limits. The greater the determinant \( \Delta \) of this system of linear equations, the less control forces are required to overcome the disturbances. In other words, \( \Delta \) quantifies the capability of LR to be robustly controlled in a decentralised manner. For these reasons, \( \Delta \) can be taken as a relevant integrated design index for the subsystems whose parameters enter the control TM.

DESIGN OPTIMISATION SCHEME

The linearity of Eqs. (3) makes it possible a decomposition of the overall design problem into much simpler design problems for LR’s components: (1) mechanics, (2) actuators (sensors), and (3)
controls. This order will correspond to the hierarchy in a multi-level optimisation procedure in which a series of design problems for these subsystems are to be solved.

A. Mechanics

*design parameters:* all inertial/geometrical data of the bodies;
*design constraints:* strength and GDD conditions;
*design objective:* maximise $\Delta$;

B. Actuators

*design parameters:* actuator masses and positions;
*design constraints:* strength and GDD conditions;
*design objective:* maximise $\Delta$;

C. Controls

*design parameters:* control gains;
*design constraints:* optimal trade-off relations (3);
*design objective:* minimise the control effort;

With a specified structure of the control transfer matrix, the proposed optimisation procedure is feasible and convergent, [1].

**CASE STUDIES**

To verify the above concepts for integrated structure-control design, several interesting examples regarding the GDD-condition and the $\Delta$-criterion are considered:

- mass/size optimisation of arms/legs with two or three rotational joints;
- mass distribution in the trunk of biped robots;
- four-legged robots: optimisation of the shape and mass distribution of the torso; finding optimal ratio between magnitudes of its front and rear driving forces;

**On Mass/Size Optimisation of Arms (Legs) with Two and Three Rotational Joints**

Consider first a two-degree-of-freedom manipulator and assume, for simplicity, that the masses $m_1$ and $m_2$ of the links are concentrated at their ends, Fig. 1.
The inertia matrix coefficients read as follows

\[ M_{11} = (m_1 + m_2)l_1^2 + m_2l_2^2 + 2m_2l_1l_2c, \quad M_{12} = m_2l_2^2 + m_2l_1l_2c, \quad M_{22} = m_2l_2^2, \]

where \( l_1 \) and \( l_2 \) are the links’ lengths and \( c = \cos \theta \).

In this case, \( \Delta = (l_1^2 l_2^2 (m_1 m_2 + m_2^2 s^2))^{-1}, \quad s = \sin \theta \). We can see that, with given mass of the first link \( m_1 \), the less the second link’s mass \( m_2 \), the more capable for robust decentralised controllability are such manipulation systems.

Similar considerations can be done in a case a three-link manipulator. However, the GDD-conditions will be satisfied in this case if the links’ masses strongly decrease in progressing from the base to the tip, [1]. Two examples are considered:

If \( l_1 = l_2 = l_3 \) then \( m_1 / m_3 - 16m_3 / m_1 > 16 \), and

If \( l_1 = 2l_2 = 4l_3 \) then \( 4m_1 / m_3 - 8m_3 / m_1 - 9m_2 / m_1 > 14 \)

Such mass distribution conditions are in accordance with the strength requirement for cantilever-like beams.

**On Mass Distribution in the Trunk of Biped Robots**

Consider, for definiteness, a planar biped with seven links: a trunk, two thighs, two shanks and two feet, and with six actuators at the legs’ joints, Fig. 2. The walking task is analysed and the points in the gait where the LR dynamics changes its structure are identified. Thus the stride is presented as a consequence of several phases, and, in each of them, we consider the LR as a multibody system (MBS) with a relevant control structure. The structure of the biped dynamics changes and the
structure of the control system needs to be changed accordingly. From this point of view, four phases in performing steps can be distinguished: double-support, taking-off, single-support (SS) and landing.

![Figure 2: Biped just before SS-phase](image)

![Figure 3: Simplified scheme for SS-phase](image)

In each phase of locomotion, the LR is with specified kinematical, dynamical, and control structures and therefore, the structure of the control transfer matrix is known. We can therefore define the corresponding integrated design criterion and perform design optimisation for any of the controlled MBS representing the LR in the different phases. Design recommendations to the geometry and the mass distribution of the biped as well as to the sizes of its actuators can be given for it to have best controllability and dynamic performance.

In the present case-study, we assume that the design of the legs and the mass of the trunk are already specified and the aim is to optimise the controllability of the biped with respect to the parameter $a_2$ – the position of the mass center of the trunk, Figure 3. It is very important to consider such a design optimisation problem because its solution can help designers of autonomous LR on where to put some massive parts of the biped like batteries, motors, etc.

We consider mainly the SS-phase, which is the most important from dynamics point of view. Some numerical results showing the dependence of the index of controllability $\Delta$ on the parameter $a_2$ are given in Table 1.

<table>
<thead>
<tr>
<th>$a_2$</th>
<th>0.155</th>
<th>0.1</th>
<th>0.07</th>
<th>0.05</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta$</td>
<td>14.0727</td>
<td>28.4991</td>
<td>30.9956</td>
<td>31.1411</td>
</tr>
</tbody>
</table>

Table 1: Dependence of the index of controllability $\Delta$ on parameter $a_2$
The main conclusion from this numerical study is that the closer to the axis of the hip joints the trunk’s mass centre is, the better the biped controllability. When this is the case, less power for the trunk stabilisation (easier gravity compensation) will be required as well. Moreover, our design recommendation for the mass distribution of the trunk can be considered consistent with some related design solutions in the nature like those for the human and some animals (e.g., kangaroo) performing dynamic biped locomotion.

**Four-Legged Robots**

*Optimisation of Torso Shape and Mass Distribution*

Consider a simplified geometrical model of a four-legged robot in the plane of pitch/heave motion (the plane of symmetry), Fig. 4. The two front legs as well as the two rear legs are considered acting virtually as one leg. The vertical components of the driving forces produced by the rear and front legs are $u_r$ and $u_f$, respectively. The body has mass $m$ and inertia $I$, and the angle of its rotation $\theta$ is assumed small.

As with any two-degree-of-freedom system, the GDD condition is here fulfilled and the index of controllability is $\Delta = l^2 / (ml) > 0$. This design optimisation index therefore does not depend on the location of the center of gravity (CoG). To provide the robot with best controllability in the pitch/heave motion, we have to maximise this index. With given $l$ and mass $m$, the inertia moment $I$ can be reduced if the body has an ellipsoidal shape instead of rectangular one; $I$ can be further reduced if the mass of the body is concentrated around CoG. These design recommendations are again in accordance with the strength/load capacity criterion.

*Optimisation of Actuator Sizes*

The minimum control force limits that are needed to cope with the disturbances and provide
the robot with a decentralised controllability have to be determined from the following system of linear equations:

\[
(I + ml_f^2)\ddot{u}_f - (I - ml_f l_r)\ddot{u}_r = gml + \ddot{a}_f
\]

\[
-I - ml_f l_r \ddot{u}_f + (I + ml_r^2)\ddot{u}_r = gml + \ddot{a}_r
\]

This system has always a positive solution because its matrix is GDD and the determinant \(\Delta\) is maximised according to the previous design considerations. In this way, minimum values for the actuator sizes can be achieved.

CONCLUSIONS

We have proposed a generic approach for integrated structure-control design optimisation of LR driven by decentralised joint controllers. The optimisation criteria are defined on the basis of full dynamics, generalised diagonal dominance conditions on the control transfer matrix and optimal trade-off design relations between bounds of model uncertainties and control force limits. The approach makes it possible a decomposition of the overall design problem into design solutions for the robot subsystems: mechanics, actuators, and controls. It can be used in developing new modularity concepts in the design of the mechanical and the drive subsystems of LR. The approach has the necessary mathematical guarantees and its efficiency has been verified considering several important examples. Moreover, its application is found to lead to biologically plausible solutions.

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