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A Self-Tuning Robust Neo-Fuzzy Controller with Constraints on Control Actions

INTRODUCTION

The control of dynamical plants under structural and parametric uncertainty is one of the important applications of artificial neural networks (ANN) and fuzzy inference systems (FIS), which have been paid much attention in recent years [1 – 4]. While fuzzy controllers demand the expert knowledge from which the rule base of the control law is formulated, neural controllers can learn to generate control signals close to the optimal ones during their operation [5 – 12]. The advantages of neural controllers are the possibility of high-speed processing of large amount of information from sensors due to the parallel organization of computations as well as their ability to cope with substantially nonlinear plants with unknown structure and parameters.

The neural controllers are the result of development of adaptive control systems for linear plants [1, 4, 13, 14] based on the principle of certainty equivalence. The principle states that the uncertainty of knowledge about the controlled plant is reduced through the adaptive identifier connected in parallel to the inputs of the plant and operating in real time adjusting the plant model according to the results of measurements of input and output signals. Such adaptive systems are known as self-tuning regulators (STR) are widely used due to their high performance and noise stability.

Neural controllers based on this principle for a plant model which is an ANN computing the sensitivity functions for subsequent generation of the control actions by means of another neural network – the controller itself. Such a system, in spite of its rich functions, is characterized by slow adaptation that is by far slower than the conventional STR.

This work is an attempt of synthesis of the conventional STR and neural controllers within the framework of indirect adaptive control. The neo-fuzzy neuron (NFN) [15 – 17] is used for the plant emulator. The NFN is characterized by good approximation properties and very high speed of learning. It is proposed to tune the parameters of the NFN by means of a robust adaptive identification procedure. The proposed controller generates the control actions subject to constraints that may exist in the control system.
NEO-FUZZY MODEL

Consider a nonlinear plant described by the following equation:

\[ y(k) = F(u(k-1), \ldots, u(k-n_B), y(k-1), \ldots, y(k-n_A)) + \xi(k) = F(x(k)) + \xi(k), \quad (1) \]

where \( y(k), u(k), \xi(k) \) are the output, control, and disturbance signals respectively at discrete time instance \( k = 0, 1, 2, \ldots; F(\bullet) \) is some function, unknown in the general case; \( n_B \) and \( n_A \) are the number of delayed control and output signals, respectively.

Let the plant be modeled by an NFN whose output is calculated according to

\[ \hat{y}(k) = \sum_{i=1}^{d} f_i(x_i(k)) = \sum_{i=1}^{m} \sum_{h=1}^{n} \mu_{i,h}(x_i(k)) w_{i,h}(k-1), \quad (2) \]

where \( d = n_A + n_B \), \( f_i(x_i(k)) \) is the function of the \( i \)-th nonlinear synapse:

\[ f_i(x_i(k)) = \sum_{h=1}^{m} \mu_{i,h}(x_i(k)) w_{i,h}(k-1), \quad (3) \]

where \( \mu_{i,h} \) is the \( h \)-th membership function of the \( i \)-th input, \( w_{i,h}(k-1) \) is the \( h \)-th tunable weight of the \( i \)-th nonlinear synapse at instant \( k-1 \).

The membership functions in an NFN are triangular

\[ \mu_{i,h}(x_i) = \begin{cases} \frac{x_i - c_{i,h-1}}{c_{i,h} - c_{i,h-1}}, & x_i \in [c_{i,h-1}, c_{i,h}], \\ \frac{c_{i,h+1} - x_i}{c_{i,h+1} - c_{i,h}}, & x_i \in [c_{i,h}, c_{i,h+1}], \\ 0, & x_i \not\in [c_{i,h-1}, c_{i,h+1}], \end{cases} \quad (4) \]

fixed, equidistantly spaced, and chosen such that only two adjacent functions fire at a time with unity sum of degrees of fulfillment. So the equation (3) can be re-written as

\[ f_i(x_i(k)) = \mu_{i,p}(x_i(k)) w_{i,p}(k-1) + \mu_{i,p+1}(x_i(k)) w_{i,p+1}(k-1) \]

\[ = \frac{c_{i,p+1} - x_i(k)}{c_{i,p+1} - c_{i,p}} w_{i,p}(k-1) + \frac{x_i(k) - c_{i,p}}{c_{i,p+1} - c_{i,p}} w_{i,p+1}(k-1), \quad (5) \]

where \( p \) is the active fuzzy interval.

Each nonlinear synapse is a one input-one output FIS with \( m \) rules:

\[ \text{IF } x_i \text{ IS } X_{i,h} \text{ THEN } f_i = w_{i,h}, \quad h = 1, \ldots, m, \quad (6) \]

where \( X_{i,h} \) is the fuzzy set defined by the membership function \( \mu_{i,h} \).
The structure of an NFN is shown in Fig. 1., where NS1,...,NSd denote nonlinear synapses. The only tunable parameters are the weights \( w_{i,h} \).

![Figure 1. Neo-fuzzy neuron](image)

Denoting

\[
a_i(k-1) = \frac{w_{i,p+1}(k-1) - w_{i,p}(k-1)}{c_{i,p+1} - c_{i,p}} , \quad b_i(k-1) = \frac{c_{i,p+1}w_{i,p}(k-1) - c_{i,p}w_{i,p+1}(k-1)}{c_{i,p+1} - c_{i,p}} ,
\]

we obtain the following linear form of the equation (3):

\[
f_i(x_i(k)) = a_i(k-1)x_i(k) + b_i(k-1) ,
\]

which can be used for the synthesis of a control law linear in structure. However, due to the nonlinearity of the neo-fuzzy model, the overall input-output behavior of the NFN model-based controller will be nonlinear, so it can be applied to nonlinear plants.
CONTROL LAW

We will derive a control law that minimizes the following one-step objective function:

\[ J(k) = \rho_1 (y^*(k + 1) - \hat{y}(k + 1))^2 + \rho_2 (u(k) - u(k - 1))^2 \]

\[ = \rho_1 (y^*(k + 1) - \sum_{i=2}^{d} f_i(x_i(k + 1)) - a_i(k)u(k) - b_i(k))^2 + \rho_2 (u(k) - u(k - 1))^2, \]

where \( y^*(k + 1) \) is the reference signal, \( \rho_1 \) and \( \rho_2 \) are the coefficients penalizing the control error and the dynamics of control action, respectively.

Solution of the linear equation

\[ \frac{\partial J(k)}{\partial u(k)} = 0 \]

yields the control action at instant \( k \):

\[ u(k) = \frac{\rho_1 a_1(k)(y^*(k + 1) - \sum_{i=2}^{d} f_i(x_i(k + 1)) - b_1(k)) + \rho_2 u(k - 1)}{\rho_1 a_1^2(k) + \rho_2}. \]

This formula is close in structure to the known regulator of K. Åström [14].

To guarantee zero steady-state error, the reference signal \( y^*(k + 1) \) is computed as

\[ y^*(k + 1) = y^F(k) - e_f(k), \]

\[ y^F(z) = W^F(z)y'(z), \]

where \( y^F(k) \) is the output of the setpoint filter, \( W^F(z) \) is the discrete transfer function of the setpoint filter which defines the desired closed loop transient response, \( y'(z) \) is the external setpoint, and \( z \) is the variable of the discrete Laplace transform.

The structure of the control loop is shown in Fig. 3. The proposed approach is related to the fuzzy model linearization technique for the Takagi-Sugeno models [18, 19], but requires less computing power due to the use of the NFN.

The control law (11) does not take into account any constraints on variables existing in real systems. It is often required to provide such control that the control error should not exceed some pre-specified threshold. Moreover, the control action cannot change instantly. Let us consider a procedure for the adaptation of the control law (11) subject to constraints.

Introducing constraints
\[
\begin{aligned}
\left\{ \begin{array}{l}
e_M^2(k+1) \leq C_e^2, \\
(u(k) - u(k-1))^2 \leq C_u^2,
\end{array} \right.
\end{aligned}
\]  

where \(e_M(k+1) = y^*(k+1) - \hat{y}(k+1)\) is the predicted control error, we can write a Lagrange function

\[
L(k) = J(k) + \lambda_1(e_M^2(k) - C_e^2) + \lambda_2((u(k) - u(k-1))^2 - C_u^2),
\]

\[\lambda_1 \geq 0, \quad \lambda_2 \geq 0.\]

Using the procedure of Arrow-Hurwitz-Uzawa [20]

\[
\begin{aligned}
\tilde{\rho}_1(k) &= \rho_1 + \lambda_1(k), \\
\tilde{\lambda}_1(k+1) &= \lambda_1(k) + \frac{\gamma_1(k+1)\lambda_1(k)}{C_e^2}(e_M^2(k+1) - C_e^2), \\
0 < \gamma_1(k+1) < 1, \\
\tilde{\rho}_2(k) &= \rho_2 + \lambda_2(k), \\
\tilde{\lambda}_2(k+1) &= \lambda_2(k) + \frac{\gamma_2(k+1)\lambda_2(k)}{C_u^2}((u(k) - u(k-1))^2 - C_u^2), \\
0 < \gamma_2(k+1) < 1,
\end{aligned}
\]

we can find the saddle point of the Lagrange function (15). Replacing the constants \(\rho_1\) and \(\rho_2\) in (11) with tunable coefficients \(\tilde{\rho}_1(k)\) and \(\tilde{\rho}_2(k)\) computed in (16), we obtain

\[
u(k) = \frac{\tilde{\rho}_1(k)a_1(k)(y^*(k+1) - \sum_{i=2}^{d} f_i(x_i(k+1)) - b_1(k)) + \tilde{\rho}_2(k)u(k-1)}{\tilde{\rho}_1(k)a_1^2(k) + \tilde{\rho}_2(k)}.
\]

The control law subject to constraint can also improve the closed-loop transient response providing less overshoot and shorter setting times.
ROBUST ADAPTIVE LEARNING ALGORITHM

Since the suitable values of the tunable weights of the neo-fuzzy model (2) are unknown, we have to identify them to be able to implement the control laws (11) and (17).

The tunable weights of the NFN can be adjusted online using the conventional gradient descent algorithm as proposed in [15, 17], or any other recursive identification procedure, such as the improved adaptive learning algorithm previously proposed in [21, 22]:

\[
\begin{align*}
  w(k) &= w(k-1) + r^{-1}(k) e_j(k) \mu(k), \\
  r(k) &= \alpha r(k-1) + \left| \mu(k) \right|^2, \quad 0 \leq \alpha \leq 1,
\end{align*}
\]

where \( w(k) = (w_{1,1}(k), w_{1,2}(k), w_{d,m}(k))^T \), \( \mu(k) = (\mu_{1,1}(k), \mu_{1,2}(k), \mu_{d,m}(k))^T \), and \( e_j(k) = y(k) - \hat{y}(k) \).

However, the disadvantage of most recursive identification procedures and learning algorithms that minimize quadratic error functions lies in the fact that they rely on the assumption of the Gaussian distribution of the disturbance \( \xi(k) \) in (1). If the distribution differs from the Gaussian significantly, e.g., is “heavy-tailed” like the Laplacian distribution, the identification procedure may perform poorly. As the result, the adaptive control system may not be able to provide adequate control quality.

This circumstance has led to the development of a wide range of methods of robust identification [24] based on the minimization of non-quadratic criteria and requiring the solution of nonlinear programming problems, that is difficult in the mode of real-time operation. That is why there is not so many efficient real-time methods of robust identification [25 – 27].

Let us introduce the following modification of (18) based on the robust identification criterion of R. Welsh [28]:

\[
\begin{align*}
  w(k) &= w(k-1) + r^{-1}(k) \psi(e_j(k)) \mu(k), \\
  r(k) &= \alpha r(k-1) + \left| \mu(k) \right|^2, \quad 0 \leq \alpha \leq 1, \\
  \psi(e_j(k)) &= \tanh(e_j(k) / \beta), \quad \beta > 0,
\end{align*}
\]

that will be insensitive to the “heavy tails” of the distribution of the disturbance.

As we expect, the robustness property of the identification procedure should positively impact the robustness of the control laws (11) and (17).

EXPERIMENTS

We evaluated the performance of the developed controller on the well-known benchmark nonlinear plant proposed in [5]:
\[
y(k + 1) = \frac{y(k)y(k - 1)y(k - 2)u(k - 1)(y(k - 2)) + u(k)}{1 + y^2(k - 2) + y^2(k - 1)}.
\]  

(20)

We have carried out four experiments. In the first and the second ones, the control law without constraints was used (equation (11)). In the third and the fourth ones, we introduced constraints (14) and used the control law (17) in conjunction with the procedure (16). In the first and the third experiments we used the non-robust identification procedure (18), and in the second and the fourth ones – the robust procedure (19).

In all the experiments, the setpoint was the same random step-wise signal. The plant output was disturbed with a random sequence with the heavy-tailed Laplacian distribution.

The neo-fuzzy model had 5 inputs for the signals \( y(k), y(k - 1), y(k - 2), u(k), u(k - 1) \), and contained 3 membership functions per input. The total number of tuned weights was 15. These weights were adapted online using the procedure (18) with \( \alpha = 0.95 \). Initial values of all the weights at time \( k=0 \) were set to 0.25 in all of the four runs. The only a priori knowledge of the plant (20) that we used was the number of delayed inputs and outputs (\( n_B=2 \) and \( n_A=3 \)), and the range of input and output variables ([0, 3] and [0, 1] respectively).

For the control law (11) in the first and second experiments, we chose \( \rho_1 = 1 \) and \( \rho_2 = 0.2 \). The parameters of the procedure (16) in the third and fourth experiments were the following: \( C_e = 0.05 \), \( C_u = 0.1 \), \( \rho_1 = 1 \), \( \rho_2 = 0.1 \), \( \gamma_1 = \gamma_2 = 0.001 \).

All the four simulations were carried out for 3000 time steps. The fuzzy model was simultaneously identified and used to compute the control action. The resulting plots are shown in Fig. 3 and Fig. 4.

In Fig. 3, the results of the experiments with the unconstrained control law (11) are shown. As can be seen, the plant output follows the setpoint immediately without steady-state error in both cases (non-robust and robust identification). However, when the robust identification procedure is used, the identification error of the NFN model is smaller, as the difference between the plant output and model output in Fig. 3b becomes practically invisible after about 800 time steps. This results also in somewhat shorter setting times.

Fig. 4 shows the results obtained using the control law (17) in combination with the constraints (14) and adaptation procedure (16) for the penalty coefficients \( \tilde{\rho}_1(k) \) and \( \tilde{\rho}_2(k) \).
Figure 3. Control without constraints: a) non-robust identification; b) robust identification
Figure 4. Control subject to constraints: a) non-robust identification; b) robust identification
As can be seen from Fig. 4a, the use of the non-robust identification procedure (18) does not provide satisfactory results in the case of control with constraints. In contrast, when the robust procedure (19) is used (Fig. 4b), we obtain smooth transient response and up to 50% shorter setting time in comparison with the unconstrained control.

**CONCLUSION**

The use of the neo fuzzy neuron for the modeling of nonlinear plants is proposed. It facilitates the implementation of nonlinear control algorithms in the systems with limited computing power. Experiments confirm the effectiveness of the proposed control scheme and robust identification procedure for nonlinear plants with unknown input-output relation disturbed by random sequence with non-Gaussian heavy-tailed distribution.

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