PROCEEDINGS

55. IWK

Internationales Wissenschaftliches Kolloquium
International Scientific Colloquium

13 - 17 September 2010

Crossing Borders within the ABC

Automation,

Biomedical Engineering and

Computer Science

Faculty of
Computer Science and Automation

www.tu-ilmenau.de

TECHNISCHE UNIVERSITÄT
ILMENAU

Impressum
Published by

Publisher: Rector of the Ilmenau University of Technology
Univ.-Prof. Dr. rer. nat. habil. Dr. h. c. Prof. h. c. Peter Scharff

Editor: Marketing Department (Phone: +49 3677 69-2520)
Andrea Schneider (conferences@tu-ilmenau.de)

Faculty of Computer Science and Automation
(Phone: +49 3677 69-2860)
Univ.-Prof. Dr.-Ing. habil. Jens Haueisen

Editorial Deadline: 20. August 2010

Implementation: Ilmenau University of Technology
Felix Böckelmann
Philipp Schmidt

USB-Flash-Version.

Publishing House: Verlag ISLE, Betriebsstätte des ISLE e.V.
Werner-von-Siemens-Str. 16
98693 Ilmenau

Production: CDA Datenträger Albrechts GmbH, 98529 Suhl/Albrechts

Order trough: Marketing Department (+49 3677 69-2520)
Andrea Schneider (conferences@tu-ilmenau.de)

ISBN: 978-3-938843-53-6 (USB-Flash Version)

Online-Version:

Publisher: Universitätsbibliothek Ilmenau
ilmedia
Postfach 10 05 65
98684 Ilmenau

Home / Index:
http://www.db-thueringen.de/servlets/DocumentServlet?id=16739

ANALYTICAL FORMING AND COMPUTER SIMULATION OF PROGRAM MOTIONS

I. Dainiak

Laboratory of Mathematical Modelling of Technical Systems,
Belarusian State University of Informatics and Radioelectronics, Minsk, Belarus

ABSTRACT

The analytical method of the forming of program motion is proposed which is built on the solving of inverse problems of dynamics. The method of synthesis of program motion which was built on the basis of Lagrange principle is presented. The forming of program motion on parabolic trajectory with constant speed for four-coordinate spatial motion system *XYZ-phi*, using the presented method, was considered.

Index Terms - Program motion, analytical forming, mathematical model, computer simulation, holonomic automation system

1. INTRODUCTION

There are different approaches for the realizing of program motions in precision coordinate systems [1, 2]. The most used among them is the approach which is built on the changing of motion parameters in according to the necessary quality of motion. For example, by accuracy – it is a position error, deviation from trajectory, amplitude or frequency of forced oscillation. Such approach is realized by follow algorithm:

a) motion program is formed preliminarily by control computer;

b) when motion is realizing, the signals of position and velocity are derived by sensors of drive measurement system;

c) obtained signals are processed by special regulation algorithms of drive control system, and the changes are included into the control action.

Regulation algorithms are based on differential equations, which need to be solved. Additionally, they are needed to carry out of the stability analysis, to find out the range of work values and so on.

Another approach can be built on analytical forming of program motion on the base of the solving of inverse problems of dynamics.

2. ANALYTICAL METHOD OF THE FORMING OF PROGRAM MOTION

Analytical method of the forming of program motion is based on supplementing of drive system dynamic model in accordance with analytical motion program, without solving of differential equation system. To realize this approach, model of coordinate system is needed in the form of differential equations which describe dynamic state of system in phase coordinates.

In general case equation system of one-coordinate stepping drive with $m-$phase winding can be written in form:

$$\begin{cases} M_\Sigma \dfrac{d^2x}{dt^2} + \beta \dfrac{dx}{dt} + F_r = F, \\ i_k r_k + \dfrac{d\psi_k}{dt} = U_k, \end{cases} \qquad (2.1)$$

where $k = 1, 2, \ldots, m$ – numbers of electric contours which formed by phase windings of drive; $i_k, \psi_k, U_k -$ momentary values of current, interlinkage and voltage of *k*-th electric contour; $r_k -$ electric resistance of *k*-th contour; $M_\Sigma -$ total mass of moving part of drive system; $x-$ displacement of inductor with respect to stator; $F_r -$ total resistance force of load and idling losses; $F-$ electromagnetic force of drive, which defined by type and construction of stepping motor.

Similar differential equation systems can be written for any multi-coordinate system on the base of linear stepping motors [3]. Equation system (2.1) describes physical processes in drive system; it is full mathematical model of precision motion system.

For the aims of analytical forming of program motion, these equations of system can be resolved relatively the highest derivative and structured as

$$\ddot{x} = X_i(t, x_1, ..., x_n; \dot{x}_1, ..., \dot{x}_n)$$

where $X = (X_1, X_2, \ldots, X_m)$ is vector of right parts of motion differential equations; $x = (x_1, ..., x_n)$ is vector of generalized coordinates; $\dot{x} = (\dot{x}_1, ..., \dot{x}_n)$ is vector of generalized velocities.

Then suppose, that motion features of mechanical system which is described by system (2.1) is determined by vector of generalized coordinates $x[x_1, \ldots, x_n]$ and vector of generalized velocities $\dot{x}[\dot{x}_1, \ldots, \dot{x}_n]$ which are defined in the form of integral multiformity:

$$\Omega : \omega_\mu(x, \dot{x}, t) = 0; \;\; \mu = 1, \ldots, m \le n. \qquad (2.2)$$

This multiformity Ω of features of motion is essentially integral multiformity of corresponding motion equations of drive system. Thus, to solve the inverse problem of dynamics we should to build motion equation by given integral multiformity Ω so that the expressions $\omega_\mu(x,\dot{x},t)=0$ must be integrals of these equations. Then, we should to determine required generalized control functions (forces, parameters and correlations) from built equations, which permit the motion with given features (2.2).

In cases when the structure of motion equations is known but additional forces or parameters to get the motion with necessary features are unknown, we should to supplement of motion equations using the given integral multiformity and then find the control functions from supplemented equation system.

If we know only part of motion equations of considered mechanical system, to solve the inverse problems of dynamics we need to build missing motion equations using the given integral multiformity.

As result, the solving of the inverse problems of dynamics in the general mathematical statement is coming to the building of motion equations of mechanical system using the integral multiformity of motion features.

The problem of dynamics as a rule doesn't have unique solution. This fact allows solving the inverse problems of dynamics in combination with the problem of stability and optimality of motion; generally, any additional conditions and limitations to dynamic characteristics of motion can be taken into account.

3. SYNTHESIS OF PROGRAM MOTIONS

It is known that the motion of multicoordinate device can be described by a system of differential equations of second order, which was built on the basis of Lagrange principle. In general view this system can be written in the form:

$$\dot{X}_i = f_i(t, x_1,...,x_n, u_1,...,u_r),\ i=1,2,\ldots,n, \tag{3.1}$$

where $X=(x_1,...,x_n)$ – vector of phase coordinates of device;

$u=(u_1,\ldots,u_r)$ – vector of control functions.

The problem of program motion synthesis lies in the building of control function $u=u(t,x)$, which is satisfied to technical limitations $u\in\mathbf{U}$, where $\mathbf{U}-$ is defined set in $\mathbf{R}^n$, for which the respective solution $x=x(t)$ of system (3.1) satisfied to additional condition:

$$\omega_k(t,x_1,...,x_n)=0,\ k=1,2,\ldots,m,\ m\le n \tag{3.2}$$

(or $\omega_k(t,x)=0$, $k=1,2,\ldots,m$). This fact means that motion is realized on the curve or surface defined by equations (3.2).

Equation system (3.2) is called program system.

The task of program motion synthesis hasn't unique solution in general case, then we can consider the task of optimal program motion synthesis, which consists of the forming of control function $u=u(t,x)$, realizing program motion (3.2) and ensuring the minimum of a some functional.

For example, we can consider the task of program motion synthesis with optimization by operating speed, or by using of minimal quantity of resources etc. These tasks are the optimal control problems with phase limitations on interval.

4. FORMING OF PROGRAM MOTION ON PARABOLIC TRAJECTORY

Let us consider four-coordinate spatial motion system *XYZ-phi*, which move tool or sample stage in three orthogonal and one rotary direction. Now find the control functions to realize a motion on parabolic trajectory, using *X*, *Z* and *phi* drives.

Let's denote S_2, S_3 – parameters of *X* and *Z* coordinate motions and φ_1 – parameter of *phi* coordinate motion. Using Lagrange method, we can write differential equations of spatial motion. Kinetic energy is

$$K=\frac{1}{2}\left(J_1+J_2+J_3+m_2h_2^2\right)\dot{\varphi}_1^2+$$
$$+\frac{1}{2}m_3S_3^2\dot{\varphi}_1^2+\frac{1}{2}(m_3+m_2)\dot{S}_2^2+\frac{1}{2}m_3\dot{S}_3^2$$

From here we can write differential equation system of motion:

$$\begin{aligned} &J_z\ddot{\varphi}_1+m_3S_3\ddot{\varphi}_1+2m_3\dot{\varphi}_1\dot{S}_3=u_1-b_1\dot{\varphi}_1,\\ &(m_2+m_3)\ddot{S}_2=u_2-b_2\dot{S}_2,\\ &m_3\ddot{S}_3-m_3S_3\dot{\varphi}_1^2=u_3-b_3\dot{S}_3, \end{aligned} \tag{4.1}$$

where $J_z=\left(J_1+J_2+J_3+m_2h_2^2\right)$;

J_i $(i=1,2,3)$ – moments of inertia of parts relatively their centroidal axes;

h_2 – distance from axis of revolution to center of gravity of part 2;

φ_1, S_2, S_3 – generalized coordinates.

As additional condition, we try to form the motion with constant speed *V*.

Rewrite differential equation system (4.1) in phase coordinates. Let us assume

$\varphi_1=x_1$, $S_2=x_3$, $S_3=x_5$,

$\dot{\varphi}_1=x_2$, $\dot{S}_2=x_4$, $\dot{S}_3=x_6$,

$\ddot{\varphi}_1=x_2$, $\ddot{S}_2=x_4$, $\ddot{S}_3=x_6$.

Then

$$\dot{x}_1=x_2,\ \dot{x}_2=\frac{u_2-b_1x_2-2m_3x_5x_6}{\Im+m_3x_5^2},$$

$$\dot{x}_3=x_4,\ \dot{x}_4=\frac{u_4-b_2^2x_4}{m_4+m_3},$$

$$\dot{x}_5 = x_6,\ \dot{x}_6 = \frac{u_6 - b_3 x_6 + m_3 x_5 x_2^2}{m_3},$$

(4.2)

where $u_2 = \dot{u}_1$, $u_4 = \dot{u}_2$, $u_6 = \dot{u}_3$.

Now try to realize the motion of parabola $x_3 = h - k(x_5 - b)^2$ with constant speed $v^2 = \dot{S}_2^2 + \dot{S}_3^2$, where $\varphi_1 \equiv 0$.

Therefore, the motion program is

$$\omega_1 = \omega_3 + k(x_5 - b)^2 - h = 0,\quad \omega_2 = x_4^2 + x_6^2 - v^2 = 0. \qquad (4.3)$$

As $\varphi_1 = 0$, motion equations can be simplified:

$$\begin{aligned} &\dot{x}_1 = 0,\\ &\dot{x}_2 = 0,\\ &\dot{x}_3 = x_4,\\ &\dot{x}_4 = \alpha_1 x_4 + \beta_1 u_4,\\ &\dot{x}_5 = x_6,\\ &\dot{x}_6 = \alpha_2 x_6 + \beta_2 u_6. \end{aligned} \qquad (4.4)$$

where $\alpha_1 = \frac{-b_2}{m_2 + m_3}$, $\beta_1 = \frac{1}{m_2 + m_3}$, $\alpha_2 = \frac{b_3}{m_3}$, $\beta_2 = \frac{1}{m_3}$.

Since $x_2 = 0$, then control function U_2 is identically defined as $U_2 = b_1 x_2 + 2 m_3 x_5 x_6$.

According to (4.1), in order to the solution $x = x(t)$ of system (4.4) was satisfied to equation $\omega_1(x) = 0$, it is enough, that

$$\omega_3 = \sum_{i=1}^{6} \frac{\partial \omega_1}{\partial x_i} f_i = 0, \qquad (4.5)$$

where $f_i -$ right parts of (3.1).

To satisfy the condition (4.5) it is necessary and enough, that

$$\sum_{i=1}^{6} \frac{\partial \omega_3}{\partial x_i} f_i = R_1(t, x, \omega_3), \qquad (4.6)$$

where $R_1(t, x, 0) \equiv 0$.

Further, to satisfy the condition $\omega_2(x) = 0$ along the solution $x = x(t)$ it is necessary and enough, that

$$\sum_{i=1}^{6} \frac{\partial \omega_2}{\partial x_i} f_i = R_2(t, x, \omega_2), \qquad (4.7)$$

where $R_2(t, x, 0) \equiv 0$.

As result, to realize motion program (4.3) it is enough to satisfy the conditions (4.6) and (4.7), i.e.

$$\begin{cases} \alpha_1 x_4 + \beta_1 u_4 + 2k x_6^2 + \\ \qquad + 2k(x_5 - b)(\alpha_2 x_6 + \beta_2 u_6) = R_1; \\ x_4(\alpha_1 x_4 + \beta_1 u_4) + x_6(\alpha_2 x_6 + \beta_2 u_6) = R_2, \end{cases}$$

where

$$R_1 = R_1(x, x_4 + 2k(x_5 - b)x_6),$$

$$R_2 = R_2(x, x_4^2 + x_6^2 - v^2).$$

Solving this equation system, we can find the control functions U_4 and U_6:

$$\begin{aligned} U_4 = &\frac{R_1 - \alpha_1 x_4}{\beta_1} - \frac{2k x_6^2 + 2k(x_5 - b)\alpha_2 x_6}{\beta_1} - \\ &- \frac{2k(x_5 - b)(x_4 R_1 - R_2 + \alpha_2 x_6^2)}{\beta_1 \beta_2 [2k x_4 (x_5 - b) - x_6]} + \\ &+ \frac{2k(x_5 - b)(2k x_6^2 + 2k\alpha_2 x_4 x_6 (x_5 - b))}{\beta_1 \beta_2 [2k x_4 (x_5 - b) - x_6]}; \end{aligned}$$

$$U_6 = \frac{x_4 R_1 - 2k x_6^2 x_4 - 2k\alpha_2 x_4 x_6 (x_5 - b) - R_2 + \alpha_2 x_6^2}{\beta_2 [2k x_4 (x_5 - b) - x_6]}.$$

In the first approximation we may accept that $R_1 = R_2 \equiv 0$.

5. COMPUTER MODELLING OF PROGRAM MOTION ON PARABOLIC TRAJECTORY

Now we try to find a calculation structure for modelling the motion on parabolic trajectory in the form

$$\frac{dy}{dx} = -\frac{a_{11} x + a_{12} y + a_{13}}{a_{12} x + a_{22} y + a_{23}} \qquad (5.1)$$

where x and y are controlled coordinates.

Using transfer function of motion system with feedback by velocity, we can write

$$\begin{cases} d\varphi = \alpha\, dt; \\ \frac{dx}{d\varphi} = \alpha(a_{12} x + a_{22} y + a_{23}); \\ \frac{dy}{d\varphi} = -\alpha(a_{11} x + a_{12} y + a_{13}), \end{cases} \qquad (5.2)$$

where α is a number, which defines the direction of motion by sign and the velocity by module.

Since the parabola meets the condition $\begin{vmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{vmatrix} = 0$, or $\frac{a_{11}}{a_{12}} = \frac{a_{12}}{a_{22}}$, then we can write denote $a_{11} = a_{12} \cdot \lambda;\ a_{22} = a_{12}/\lambda$. And now we can write equation system for velocities of motion:

$$\begin{cases} \frac{dx}{dt} = \alpha^2 \left(a_{12} x + \frac{a_{12}}{\lambda} y + a_{23} \right); \\ \frac{dy}{dt} = -\omega^2 (a_{12} \lambda x + a_{12} y + a_{13}). \end{cases} \qquad (5.3)$$

The calculation structure for modelling of motion on parabolic trajectory is presented on Figure 1.

The modelling results for $\alpha = -1$; $a_{12} = 0{,}5$; $a_{13} = -7$; $a_{23} = -5$ are presented on Figure 2.

Initial conditions were $x_0 = 4$; $y_0 = 10$ and $\lambda_1 = 0{,}5$; $\lambda_2 = 0{,}1$; $\lambda_3 = 0{,}05$.

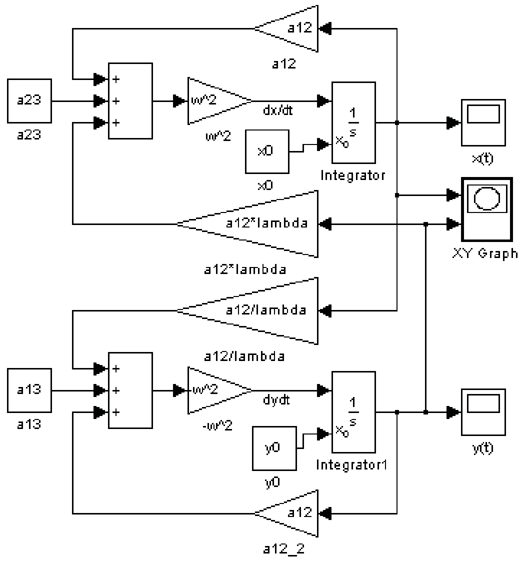

Figure 1 Structure of differential analyzer

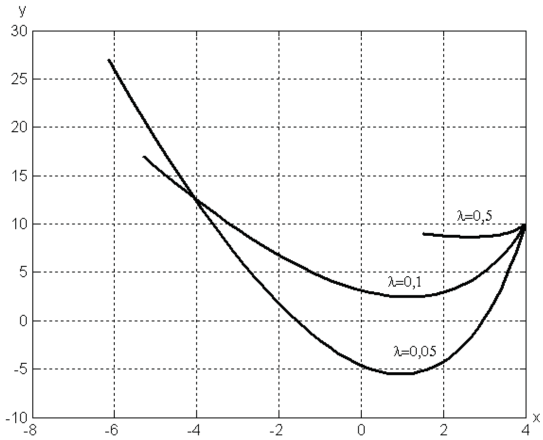

Figure 2 Parabolic trajectories, formed by differential analyzer in MATLAB

6. REFERENCES

[1] I. Dainiak, and S. Karpovich. Building of Electromechanical Robot Systems with Non-Holonomic Constraints // Proceedings of 50th International Scientific Colloquium, Ilmenau (Germany), September 19-23, 2005. – P.141-142.

[2] A. Orlyonok, S. Karpovich, and I. Dainiak. Program Motion Planning And Control For Multicoordinate Mechatronics Systems On The Base Of Linear Stepping Motors // Proceedings of the 14th Power Electronics And Electrical Drives International Conference (EDPE-2001), High Tatras, Slovakia, 3–5 October 2001. – P.247-250.

[3] D. Azentani, I. Dainiak, and A. Ahranovich. The Modeling of Planar Linear Step Motor Functioning on Basis of Multimedia // Scientific Proceedings. – Aachen: Shaker Verlag, 2004. – Vol.2. – P.505-510.