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Impressum
Published by

Publisher:	Rector of the Ilmenau University of Technology Univ.-Prof. Dr. rer. nat. habil. Dr. h. c. Prof. h. c. Peter Scharff
Editor:	Marketing Department (Phone: +49 3677 69-2520) Andrea Schneider (conferences@tu-ilmenau.de) Faculty of Computer Science and Automation (Phone: +49 3677 69-2860) Univ.-Prof. Dr.-Ing. habil. Jens Haueisen
Editorial Deadline:	20. August 2010
Implementation:	Ilmenau University of Technology Felix Böckelmann Philipp Schmidt

USB-Flash-Version.

Publishing House:	Verlag ISLE, Betriebsstätte des ISLE e.V. Werner-von-Siemens-Str. 16 98693 Ilmenau
Production:	CDA Datenträger Albrechts GmbH, 98529 Suhl/Albrechts
Order trough:	Marketing Department (+49 3677 69-2520) Andrea Schneider (conferences@tu-ilmenau.de)

ISBN: 978-3-938843-53-6 (USB-Flash Version)

Online-Version:

Publisher:	Universitätsbibliothek Ilmenau ilmedia Postfach 10 05 65 98684 Ilmenau

Home / Index:
http://www.db-thueringen.de/servlets/DocumentServlet?id=16739

CHANCE CONSTRAINED PROGRAMMING FOR OPTIMAL POWER FLOW TAKING ACCOUNT OF THE LOAD POWER VARIATION

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ABSTRACT

Optimization for power systems under uncertainty is becoming increasingly concerned. In power system operations, the future load power is not known precisely as its value fluctuates from time to time. In this paper the load power variation is taken into account to optimal power flow (OPF) problems and is considered as a random vector associated with normal distribution. First, Monte-Carlo simulation (MCS) is made to investigate the effects of the random inputs to the system operation. Second, a solution strategy with chance constrained programming (CCP) is implemented to deal with the uncertainty with which the inequality constraints of the optimization problem can be transformed into chance constraints for output variables with pre-defined confidence levels. A 5-bus test system is studied to demonstrate the OPF under load uncertainty.

Index Terms – Chance constrained programming, optimal power flow, load power variation, Monte-Carlo simulation

1. INTRODUCTION

The aim of optimal power flow (OPF) is to find the optimal operation strategies of a given power network that optimizes an operational objective function while satisfying its power flow equations, security, and equipment operating limits. It has been widely applied in power system planning and operation for many years. As the power industry is being deregulated, together with the unprecedented development of new technologies during the last few decades, the importance of OPF for the power system planning and operation has increased significantly [1].

Deterministic approaches to OPF have been widely used today in many energy systems. However, many random disturbances or uncertain factors exist during power system operations due to measurement errors, forecast inaccuracies or outages of system elements. The deterministic approaches are usually based on 'worst-case' conditions to deal with uncertainties. Therefore, the deterministic OPF can hardly analyze the variability of important parameters affecting power systems operation and these deterministic approaches provide no idea as to how safe the operation actually is.

Due to the deficiencies of deterministic approaches and the increasing concern about the uncertainties in power systems, different kind of stochastic approaches have been proposed in the last few decades (e.g. probabilistic methods, fuzzy methods, simulation methods, etc.). Prominent among these new approaches are the probabilistic methods which systematically take account of uncertainties. Using a probabilistic method the uncertain input variables are considered as random variables with known probabilistic density functions (PDFs). The results are PDFs of some output variables which describe the resulting quantities and the corresponding probability of each value to occur. Although these probabilistic approaches require more sophisticated analysis, the benefits far outweigh the additional effort required to apply them [2].

Chance constrained programming (CCP) is a competitive method for stochastic optimization problems to address the effects of uncertainties probabilistically. The main feature of CCP is that the inequality constraints are to be satisfied with a user-defined probability level [3]. By using CCP a relationship between the profitability and reliability can be quantitatively established. Recently, CCP based methods have been also applied to power systems. In [4] a probabilistic d.c. power flow calculation was used and a two-step genetic algorithm incorporated to solve the transmission network planning problem. A solution to the stochastic unit commitment problem by using CCP was present in [5] where linear chance constraints were considered. In [6] a two-stage recourse approach and CCP were combined to deal with both discrete and continuous variables for the evaluation of available transfer capability under uncertainty.

In this paper we concentrate on OPF under load uncertainty and implement a CCP based approach to deal with the stochastic optimization problem. The reminder of the paper is organized as follows. In section 2 a general model for OPF under load uncertainty is presented and the effects of the random inputs are analyzed. In section 3, a CCP model is formulated and its solution approach is briefly described. Results of the case study are demonstrated in section 4. The paper is concluded in section 5.

2. OPF UNDER LOAD UNCERTAINTY

2.1. Model of OPF under load uncertainty

During operations the adjustable entities in a power system adapt to the random inputs, so that a necessary equalization between produced and consumed power amount in the system will be achieved, taking into account restrictions of the transmission network. Thus, the optimal power dispatch problem under load uncertainty can be generally described as a nonlinear stochastic optimization problem:

$$\begin{aligned} &\min f(\boldsymbol{x},\boldsymbol{u},\xi) \\ &s.t.\ \boldsymbol{g}(\boldsymbol{x},\boldsymbol{u},\xi)=0 \\ &\boldsymbol{h}(\boldsymbol{x},\boldsymbol{u},\xi)\le \boldsymbol{h}^{\max} \end{aligned} \quad (1)$$

where $\boldsymbol{x}\in\Re^n$, $\boldsymbol{u}\in\Re^m$ and $\xi\in\Re^s$ are the vectors which represent output (dependent), control (decision) variables and random input variables, while $f:\to\Re$, $\boldsymbol{g}:\to\Re^n$ and $\boldsymbol{h}:\to\Re^p$ are the objective function to be minimized, the equality and inequality constraints (with a upper bound $\boldsymbol{h}^{\max}$ as an example) for the system, respectively.

The objective function in (1) can be the fuel cost, the total active power losses or other scalar function defined for the optimization. The equality constraints include network power flow equations of the system. Inequality constraints are incorporated into the OPF problem in order to prevent unacceptable operations according to the restrictive requirements, such as constraints for generator capabilities, line active power flows, line currents and load bus voltage magnitudes.

Control variables are those that can be adjusted for an optimal solution for OPF which usually include active power generations at *PV* buses, transformer tap settings, etc. Output variables are those depending on control variables, such as voltage magnitudes on each *PQ* bus and voltage phase on each bus except the slack. The random vector $\boldsymbol{\xi}$ represents the uncertainty which comes from the load power variation.

2.2. Load uncertainty

The future load is usually not known precisely and the load forecasting error is dependent on how close the forecasted value is to the actual value. Efficient models have been developed with different techniques for load forecasting, such as knowledge based and artificial neural net (ANN) methods [7]. Forecasting errors can be assumed to be random variables associated with certain probability distributions. The load uncertainty is frequently described with normal distributions [8]. Moreover, in power systems complicated correlations may exist between load powers for various reasons [9].

In this study, the load powers are treated as known correlated random parameters which fluctuate around the mean values with some variances. Thus, the *s*-dimensional random input vector $\boldsymbol{\xi}$ representing the load uncertainty is assumed to be normally distributed and its PDF $\rho_s(\xi)$ is given as a multivariate normal distribution:

$$\rho_s(\xi)=\frac{1}{\sqrt{(2\pi)^s|\boldsymbol{\Sigma}|}}\exp\left(-\frac{1}{2}(\xi-\boldsymbol{\mu})^T\boldsymbol{\Sigma}^{-1}(\xi-\boldsymbol{\mu})\right) \quad (2)$$

where $\boldsymbol{\mu}=[\mu_1,\mu_2,...,\mu_s]^T$ is the expected value vector, and $\boldsymbol{\Sigma}$ is the covariance matrix of the random variables given by

$$\boldsymbol{\Sigma}=\begin{bmatrix}\sigma_1^2 & \sigma_1\sigma_2 r_{12}\cdots & \sigma_1\sigma_s r_{1s} \\ \sigma_2\sigma_1 r_{21} & \sigma_2^2\quad\cdots & \sigma_2\sigma_s r_{2s} \\ \cdots & \cdots\quad\cdots & \cdots \\ \sigma_s\sigma_1 r_{s1} & \sigma_s\sigma_2 r_{s2}\cdots & \sigma_s^2\end{bmatrix} \quad (3)$$

where σ_a is the standard deviation of each individual random variable and $r_{ab}\in(-1,1)$ represents the correlation coefficient between ξ_a and ξ_b ($a,b=1,2,...,s$ and $a\ne b$).

2.3. Effects of the Load Uncertainty

Due to the propagation of the input random vector $\boldsymbol{\xi}$ through the model equations $\boldsymbol{g}$ the output variables are also random. The power system considered can be described with Fig. 1. The inputs are the control vector $\boldsymbol{u}$ and the random vector ξ. The output variables $\boldsymbol{x}$ depend on the inputs and are determined by the power flow equations.

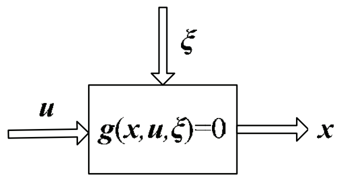

Figure 1 Power systems operations under uncertainty

When we consider the inequality constraints $\boldsymbol{h}$ whose values are dependent on random variables, holding such an inequality constraint for sure is usually impossible. We use Fig. 2 to conceptually explain the relation between input and output variables of optimal power flow under load uncertainty. Suppose the probability distribution of an output (e. g. h_i) is f_1 when the control setting is selected as $\boldsymbol{u}_1$, then the part that distributes on the right side of the limit $h_i^{\max}$ will cause constraint violations. And the area of this part represents the probability of the constraint violation. Moreover, changing the control vector $\boldsymbol{u}$ will shift and change the distribution of the outputs. As an example, another control strategy $\boldsymbol{u}_2$, whose corresponding output distribution is f_2 as shown in Fig. 2 will lead to a higher probability of constraint violation.

Since increasing the reliability level will probably shrink the feasible region of the control variables, in most cases achieving a better objective function value and holding the constraints more reliable are conflicting with each other. It means that a relatively conservative dispatching strategy usually bears lower risks to violate the constraints but achieves inferior objective value comparing with a more aggressive one [10]. Therefore, a rational decision to balance the aspects of profitability and reliability is very important to the system operations.

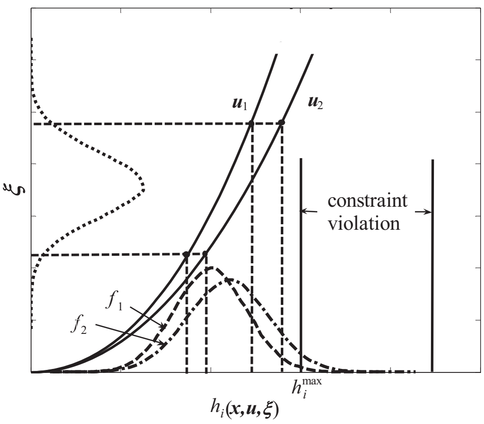

Figure 2 Results for different operation under load uncertainty

3. CHANCE CONSTRAINED OPTIMIZATION

3.1. Formulation with chance constraints

In power system operations the inequality constraints are seldom rigid limits in the strict mathematical sense but are, rather, soft limits. For example, the voltage magnitude $V_i \leq 1.1$ per unit on a *PQ* bus means V_i should not exceed 1.1 by too much, and $V_i = 1.101$ may still be permissible. However, the degree of these constraints violation is usually just marginally permitted and need to be controlled in a certain level. CCP is a useful tool which aims to quantitatively evaluate the probabilities of holding inequality constraints and searches for decisions such that a predefined reliability level will be satisfied.

In chance constrained OPF, each term in the stochastic optimization problem of (1) has to be reformulated due to the random inputs. A probability level of satisfying an inequality h_i with chance constraint is defined as

$$P_r\{h_i(\boldsymbol{u},\boldsymbol{x},\boldsymbol{\xi}) \leq h_i^{\max}\} \geq \alpha_i \quad (i = 1, 2, ..., p) \tag{4}$$

where P_r is the probability measure and $0 < \alpha_i \leq 1$ is a probability level defined based on the operation requirement. In this way, defining different level of confidences can control the reliability degree to the system operation. A larger α_i means a higher reliability for the constraints h_i to be satisfied. However, it is likely that the predefined probability level α_i is higher than reachable. In this case the optimization problem is infeasible, i.e. a feasible solution can not be found for the optimization problem defined. Thus, defining a suitable probability level for the chance constraint is crucial to the solution of the problem. In addition, achieving a higher reliability is at the expense of an increasing cost.

The impact of equality constraints $\boldsymbol{g}$ in (1) is the projection of the uncertain inputs to the output space with a given control vector $\boldsymbol{u}$. Thus, to solve the optimization problem, the outputs $\boldsymbol{x}$ can be eliminated by a simulation scheme since it can be represented in terms of $\boldsymbol{u}$ and ξ. A usual representation of the objective function is the minimization of its expected value to deal with its random feature:

$$E_{\Re^s}(f(\boldsymbol{x},\boldsymbol{u},\boldsymbol{\xi})) = E_{\Re^s}(f(\boldsymbol{u},\boldsymbol{\xi}))$$
$$= \int_{\Re^s} f(\boldsymbol{u},\boldsymbol{\xi})\rho_s(\boldsymbol{\xi})d\boldsymbol{\xi} \tag{5}$$

where E represents the expectation operator for the objective function.

The goal of the chance constrained OPF is to find an optimal strategy $\boldsymbol{u}$ for an optimal power system dispatch so that the expected value of the objective function is minimized, and at the same time, the optimal strategy must satisfy the inequality constraints of the system with a predefined probability [10].

3.2. Solution strategy

It is extremely complicated to solve the formulated chance constrained OPF due to the nonlinearity and its complex propagations. Generally, the chance constrained optimization problem formulated needs first to be relaxed to its equivalent nonlinear programming (NLP) form and then a NLP solver is required to solve the relaxed problem. A direct numerical integration can be applied to evaluate the objective function $E_{\Re^s}(f(\boldsymbol{u},\boldsymbol{\xi}))$ through a cubature technique introduced in [11]. In order to calculate the probability of holding a constraint and its derivatives with respect to the control variables, we use a back-mapping approach to compute these values from the space of the random input proposed by Wendt *et al* [12].

In the study of power systems approximations are widely used to simplify the analysis. It is commonly recognized that linearization of the power flow equations is necessary in probabilistic analysis of power systems. Different methods have been developed for linear approximations so that the output variables can be represented as linear combinations of input variables. In order to implement the back-mapping approach, a monotonic relation between an output and one of the random inputs needs to be found. A linearization method with a.c. power flow analysis around the expected values is used in this paper, with which the coupling between P_i and Q_i can be retained [13]. In this way, the computation for the probabilistic analysis can be significantly simplified.

4. CASE STUDY

A 5-bus system as shown in Fig. 3 is used to illustrate the OPF under load uncertainty and the scope of chance constrained OPF. The load powers at buses 1, 2 and 3 are considered as random parameters. The random inputs are assumed to be normally distributed whose expected values are the nominal values and the standard deviations are set to be 10.0% of the expected values. The correlation coefficients between every pair of the random inputs are all assumed to be 0.3.

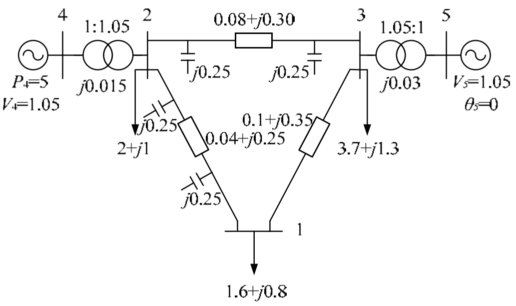

Figure 3 the 5-bus system

We consider an OPF to minimize the generation cost, which is usually expressed in terms of a quadratic function as

$$\min \sum_{i=4,5} (a_{2i}P_{gi}^2 + a_{1i}P_{gi} + a_{0i})$$

where a_{2i}, a_{1i} and a_{0i} are known cost parameters and are listed in Table 1. Bus 5 is chosen as the reference bus. The MVA base value of the system is 100. The control variables are V_4 and P_{g4}. Transformers are assumed to be fixed and absorbed into the transmission line model. Inequality constraints for limiting the voltage magnitudes, line active powers and generation outputs are considered.

Table 1 Cost parameters

Bus	a_{2i}	a_{1i}	a_{0i}
4	0.085	1.2	600
5	0.125	1.0	335

4.1 Analysis with MCS

Monte-Carlo simulation (MCS) techniques are widely used in power system analysis, especially for reliability assessment of electric power systems under uncertainty. Results with MCS were also used as comparison and verification tools for analytical or approximation methods to probabilistic analysis [14] to investigate the effects of the load uncertainty to the problem.

A base-case optimization is first carried out in which the random input variables are treated as their mean values. This deterministic method leads to the optimal solution for the controls $\boldsymbol{u}'$ =[4.281; 1.068]. The base case control will probably lead to constraint violations with different probabilities when considering the load uncertainties. We use Monte-Carlo simulation to estimate the probabilities of constraint violations for line active powers. Table 2 describes the probabilities for these violations, where the upper limits for the line active power are also listed. Moreover, with the results from the MCS the mean values and the standard deviations of the output variables can be statistically estimated.

Table 2 Line active power limits and constraint violations considering load uncertainties

Bus no.		Upper limit (MW)	Violation Probability (%)
From	To		
2	1	1.513	9.4
3	1	0.584	3.2
2	3	1.216	4.1
5	3	0.5950	12.3

4.2 Results of Chance constrained OPF

The maximum reachable probabilities for the inequality constraints are analyzed, and suitable confidence levels for the chance constraints are then defined for the chance constrained OPF. Results show that a 100% of the maximum reachable probability for each single constraint can be obtained. We define the individual chance constraints with a confidence level of 0.95. The base case control $\boldsymbol{u}'$ is used as the initial value of the chance constrained OPF. In the optimization scheme to the chance constrained OPF, linear approximations around the expected values are made in each of the iteration to implement the back-mapping approach. From the linearized model, the monotonic relation between an output variable and one of the input random variable can be easily identified. Table 3 takes some of the variables of the first iteration for example to show the monotonies where the upward arrow represents a positive monotonic relation and a downward arrow indicates a negative monotonic relation.

Provided the monotonic relations are obtained, we can use a sequential quadratic programming (SQP) method to find the optimal solution of the problem. The optimal solution of the control for this chance constrained problem is listed in Table 4, where the base case solution is also compared. The expected values of the objective function for the chance constrained case and the base case are 3.031×10^4 and 3.023×10^4 \$/h, respectively. It can be seen although with the solution from the chance constrained OPF the expected generation cost is inferior to the base case control to an insignificant level, the chance constraints defined to the inequalities are satisfied within the desired confidence level (95%).

Table 3 Monotonic pairs of output and uncertain input variables around the expected values

Input variable	Output variable		
	V_1	V_2	V_3
P_{d1}	↓	↓	↓
P_{d2}	↑	↑	↑
P_{d3}	↓	↑	↑

Table 4 Solution of chance constrained OPF compared with the base case

Variables	Chance constrained case	Base case
V_4 (p.u.)	4.239	4.281
P_{g4} (MW)	1.062	1.068

5. CONCLUSION

In this study, the problem of OPF considering uncertainty is addressed with chance constraints. Based on the concepts and the realization of chance constrained OPF, the aspects of optimality and reliability for power system operations can be quantitatively balanced. The back-mapping strategy is implemented to the chance constrained optimization. Besides bus load uncertainties with correlated normal distribution forms, there are other sources and forms of uncertainty (non-Gaussian distributions) in power system operations. Especially, the appeal of decreasing the CO_2 emission highlights the development of clean, renewable energy such as wind and solar power. Nevertheless, the penetrations of wind or solar power plant will inevitably introduce more uncertain factors to the network because of their uncontrolled primary energy sources. These new challenges will not only make the power system planning and operation much more complicate but also inevitably increase the necessity and potential of the probabilistic analysis for optimal power flow under uncertainty.

6. ACKNOWLEDGMENT

The authors gratefully acknowledge the support of the Siemens/DAAD Post Graduate Program.

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