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NONLINEAR MODEL PREDICTIVE CONTROL OF WATER RESOURCES SYSTEMS IN OPERATIONAL FLOOD FORECASTING

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ABSTRACT

We present a Nonlinear Model Predictive Control (NMPC) algorithm for real-time decision support for water resources systems in operational flood forecasting. The algorithm consists of an iterative, finite horizon optimization of the system over a short-term control horizon. The underlying set of internal process models are embedded in a configurable toolbox for representing relevant physical phenomena such as flood routing in rivers and lakes, and the dynamics of hydraulic structures. Depending on the characteristics of the specific system, the schematized models are solved numerically either by an explicit or implicit time stepping scheme.

Objectives of the control, i.e. the desired damping of flood peaks, and constraints such as the water level dependent capacity of a structure, are mathematically formulated by a set of objective functions and inequality constraints. The resulting optimization problem is solved by Sequential Quadratic Programming (SQP). For enabling the real-time application of the algorithm, we present the derivation of adjoint systems on the discrete level of the process models for computing the gradient of the objective function related to the controlled variables. It is achieved at the computational costs of a single model execution.

The algorithm is applied to i) the control of a generic reservoir system for providing a more systematic demonstration of its characteristics and ii) to the control of six hydraulic structures and two major flood detention basins along the bifurcation points of the Rhine River in The Netherlands.

1. INTRODUCTION

The most common technique for supervisory control of hydraulic structures in water resources is the definition of reactive operating rules. Examples include minimum releases for reservoirs depending on the reservoir level and environmental objectives, the operation of flood detention basins based on water level at reference locations, or the definition of set points for upstream water levels of river weirs. These operating rules typically come along with secondary controllers for controlling the desired variable at site, i.e. a PID-controller for maintaining an upstream water level at a weir.

Whereas this concept works well for smaller water systems, its application gets significantly more complex for larger systems, in particular if these systems have a high degree of interconnectivity such as the Dutch Rhine-Meuse delta. In these cases, the operating rules and the water system may show undesired feedback effects leading to suboptimal control of the total system. Looking for examples related to the control of cascaded hydropower plants, many authors such as Ackermann et al. [1], Glanzmann et al. [4], or Pfuetzenreuter & Rauschenbach [8] report drawbacks of classical feed forward / feedback control methods due to amplification of inflow disturbances. In recent years, the solution to this problem has been found in the application of Model Predictive Control (MPC) for supervisory control of weirs [7] and hydropower plant.

In application to the operation of reservoir systems, Labadie [5] provides a state-of-the-art overview. It includes algorithms for the short-term, real-time control of reservoirs on which we focus in this paper.

MPC is a control concept, which has become an industrial standard in process control over the last two or three decades. It makes use of a process model for predicting future trajectories of the controlled variables over a finite horizon, in order to determine the optimal set of manipulated variables by an optimization algorithm. An integral part of the concept is the explicit consideration of constraints on inputs, states and outputs. Furthermore, the tuning of the control parameters is relatively straightforward even in the presence of contradictory control objectives.

In the next section, we present a general nonlinear MPC scheme for the control of hydraulic structures in complex water resources systems. The underlying set of process models is described in the following, based on an implicit pool routing scheme as well as an explicit kinematic wave model. Finally, we present the application of the NMPC on the control of a generic reservoir system and on several hydraulic structures of two flood detention basins at the bifurcation points of the River Rhine in The Netherlands.

2. NONLINEAR MODEL PREDICTIVE CONTROL SCHEME

Under the assumption that we already applied a spatial schematization to our system of interest, the model of a water resources system can be described by the following generic set of non-linear ordinary differential equation (ODE):

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = f(\mathbf{x},\mathbf{u},\mathbf{d}) \tag{1}$$

where $\mathbf{x} \in \mathbb{R}^l$ is the system state vector, $\mathbf{u} \in \mathbb{R}^m$ the vector of controlled variables, $\mathbf{d} \in \mathbb{R}^n$ the vector of disturbances, l the number of states, m the number of controlled variables, and n the number of disturbances. A formulation of the MPC on-line optimization is given by the solution of the following optimum control problem over a finite prediction horizon $t \in \{0..T\}$ by

$$\min_{\mathbf{u}\in\{0..T\}} J(\mathbf{x},\mathbf{u}) \tag{2}$$

subject to the system dynamics (1) and p additional inequality constraints

$$g_i(\mathbf{x},\mathbf{u}) \le 0 \ , \quad i \in I = \{1,...,p\} \tag{3}$$

We transform the set of ODE (1) into a discrete-time system under the assumption of an explicit or implicit time stepping scheme and get

$$\mathbf{x}^k = f(\mathbf{x}^{k-1},\mathbf{x}^k,\mathbf{u}^{k-1},\mathbf{u}^k,\mathbf{d}^{k-1},\mathbf{d}^k) \tag{4}$$

where k is the time step index. The solution of equation (4), i.e. the computation of new states $\mathbf{x}^k$ based on the data of a previous time step $\mathbf{x}^{k-1}$ and the new state itself in case of an implicit scheme, $\mathbf{u}$ and $\mathbf{d}$, is performed over the finite prediction horizon under consideration of an initial condition $\mathbf{x}^0$.

The resulting optimum control problem (2-4) is solved by the nonlinear programming scheme SNOPT [2] or MINOS [6]. These become available in commercial optimization toolboxes such as TOMLAB (http://tomopt.com/tomlab/) under Matlab.

3. INTERNAL MODELS

We select two out of several other process models for demonstrating the integration of simulation models into the NMPC. These are i) an explicit kinematic wave model and ii) an implicit pool routing model.

The kinematic wave equations can be derived from the complete hydraulic model, i.e. the one-dimensional De Saint-Venant equations, by neglecting the terms for inertia and convection in the momentum equation. The resulting set of equations reads:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q_{lat} \tag{5}$$

$$g\frac{\partial h}{\partial x} = -\frac{gQ|Q|}{C^2A^2m} \tag{6}$$

where A = wetted area, Q = discharge, $qlat$ = lateral discharge per unit length, h = water level, g = acceleration due to gravity, m = hydraulic radius, C = Chezy coefficient.

We now apply a spatial schematisation on a staggered grid, on which the discharge is schematised between an upstream and a downstream storage node including a discrete water level. Defining the distance of these nodes to be Δx, equation (6) can be rearranged to

$$Q = f_{flow}(h_{up},h_{down}) = -\mathrm{sign}(h_{up}-h_{down})CA\sqrt{\left|\frac{h_{up}-h_{down}}{\Delta x}\right|m} \tag{7}$$

where C, A, m can be expressed as functions of the mean water level $(h_{up}+h_{down})/2$. If hydraulic structures exist between two storage nodes, the flow equation (7) can be replaced by a general equation of the hydraulic structure, given by

$$Q = f_{structure}(h_{up},h_{down},dg) \tag{8}$$

where dg = gate or weir setting.

The numerical solution of the continuity equation (5) is done by the Euler Forward Method resulting in:

$$\frac{A(h^k)-A(h^{k-1})}{\Delta t} + \frac{Q_{down}^{k-1}-Q_{up}^{k-1}}{\Delta x} = q_{lat}^{k-1} \tag{9}$$

in which k denotes the time step. By multiplying Δx, substituting $s(h) = A(h)\Delta x$, neglecting the lateral discharge, and introducing equations (7) and (8), we transform equation (9) into a water balance at node level

$$s^k = s^{k-1} + \Delta t \sum_i f(s^{k-1}, s_i^{k-1}, dg_i^{k-1}) \quad (10)$$

where s = storage at a node, i = the index of connected branches to the storage node.

For efficient computation of the gradient of the objective function, we set-up an adjoint system for computing the gradient of J related to the controlled variables dg. The Lagrangian form of the optimum control problem over the prediction horizon reads

$$\begin{aligned} L &= J(\{s^k, s_i^k, dg_i^k\}_{k=1}^T) \\ &+ \sum_{k=1}^{T} \lambda^k \{s^k - s^{k-1} - \Delta t \sum_i f(s^{k-1}, s_i^{k-1}, dg_i^{k-1})\} \end{aligned} \quad (11)$$

in which λ = Lagrange multiplier. In this formulation, we define the system equations at each discrete time step as separate equality constraints of the optimum control problem. We now apply a variational analysis of the Lagrangian form, sort all terms according to spatial derivatives and receive

$$\begin{aligned} \lambda^{k-1} &= \lambda^k - \frac{\partial J}{\partial s^{k-1}} \\ &- \Delta t(\lambda^k - \lambda_i^k) \sum_i \frac{\partial f(s^{k-1}, s_i^{k-1}, dg_i^{k-1})}{\partial s^{k-1}} \end{aligned} \quad (12)$$

$$\begin{aligned} \frac{dJ}{d(dg^{k-1})} &= \frac{\partial J}{\partial dg^{k-1}} \\ &+ \Delta t(\lambda_{up}^k - \lambda_{down}^k) \frac{\partial f(s_{up}^{k-1}, s_{down}^{k-1}, dg^{k-1})}{\partial dg^{k-1}} \end{aligned} \quad (13)$$

The procedure of computing the objective function value and its gradient can be summarised as follows:

1. A model simulation is performed forward in time by applying equation (10). Then, the objective function value J is computed.
2. The Lagrangian multiplier λ^k is computed backwards in time by applying equation (12).
3. The gradient of $dJ / d(dg^k)$ is computed by equation (13).

The procedure above computes a gradient of the cost function J related to a set of m controlled variables by computational costs comparable to a model simulation itself. The numerical alternative would require m executions of the simulation model and therefore requires significantly more computational resources.

The integration of an implicit model is presented using the example of a pool routing scheme of a single reservoir. In particular in the case of water systems with heterogeneous dimensions resulting in stiff equations, the use of implicit internal models is advantageous. The alternative, i.e. the limitation of time steps and the introduction of stabilizing terms [2], seems to be only a second choice in terms of the consistent representation of physical phenomena as well as the stability of the overall MPC algorithm.

The pool routing model in its simplest form can be expressed by the balance equation

$$s^k = s^{k-1} + \Delta t[d^{k-1} - u^{k-1} - f(s^k)] \quad (14)$$

where s = reservoir storage, d = disturbance (inflow), u = controlled release, $f(s^k)$ = release through an uncontrolled spillway as a function of the storage or the water level, respectively.

The derivation of the adjoint model follows the path described above and results in the following two equations for the adjoint model and the cost function derivative:

$$\lambda^t = \frac{\lambda^{t+1} - \partial J / \partial s^t}{1 + f'(s^t)} \quad (15)$$

$$\frac{dJ}{du^t} = \frac{\partial J}{\partial u^t} + \Delta t \lambda^{t+1} \quad (16)$$

It is noteworthy that the implicit simulation model (14) requires an iterative solution, for example by the Newton-Raphson approach used by us, whereas the adjoint model (15) can be solved explicitly. This reduces the costs for the computation of the gradient to a fraction of the execution time of the simulation itself. Keep in mind that this will not be the case for all types of process models and that most of them require an iterative solution of the adjoint model.

4. RESERVOIR SYSTEM TEST CASE

The following academic test case on the optimum control of a generic reservoir system with one up to four sequential reservoirs is proposed in order to i) evaluate the performance and the scalability of the proposed scheme, ii) define a simple benchmark which may be used by other researchers.

The level-storage relation of each reservoir is assumed to be linear, i.e. we assume a constant surface area. The inflow into the most upstream reservoir is a predefined disturbance. The downstream reservoirs are fed by the release from the upstream ones. Each reservoir has a controlled outlet with the capacity

$$0 \le \mathbf{u}^k \le u_{\max} \quad (17)$$

and an uncontrolled spillway for which the release is computed by Poleni's formula according to

$$f(h) = C \max(h - h_{cr}, 0)^{3/2} \tag{18}$$

in which *hcr* = spillway crest level.

The objective function is defined by the terms

$$J_{x1}(\mathbf{x}) = \sum_{k=1}^{T} \left(\mathbf{x}^k - h_{sp}\right)^2 \tag{19}$$

$$J_{x2}(\mathbf{x}) = \sum_{k=1}^{T} \max\left(\mathbf{x}^k - h_{cr}, 0\right)^2 \tag{20}$$

$$J_u(\mathbf{u}) = \sum_{k=1}^{T} \left(\Delta u^k\right)^2 = \sum_{k=1}^{T} \left(u^k - u^{k-1}\right)^2 \tag{21}$$

where *hsp* = set point of the water level.

The three terms represent i) a penalty on the deviation of the water levels from a predefined set point, ii) a penalty on the up-crossing of the spillway crest level, i.e. to spillage, iii) a penalty on release changes of the controlled outlet. The scalar cost function becomes a weighted sum of these terms by

$$J(\mathbf{x}, \mathbf{u}) = w_{x1} J_{x1}(\mathbf{x}) + w_{x2} J_{x2}(\mathbf{x}) + w_u J_u(\mathbf{u}) \tag{22}$$

where w_{x1}, w_{x2}, w_u are weighting coefficients.

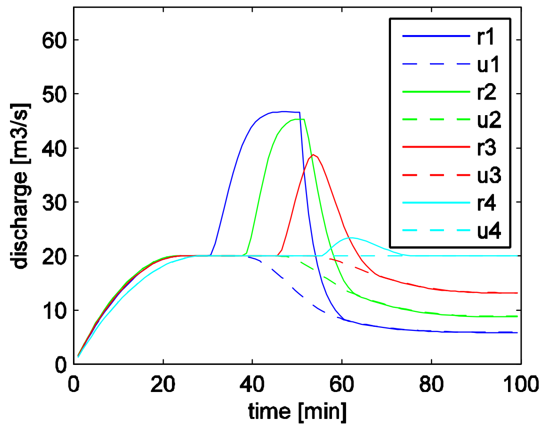
The settings for each reservoir are summarized in Table 1. For simplicity, we assume all reservoirs on the same elevation reference level and apply the same initial condition of a water level of 0m.

Table 1: test case 1 - parameters

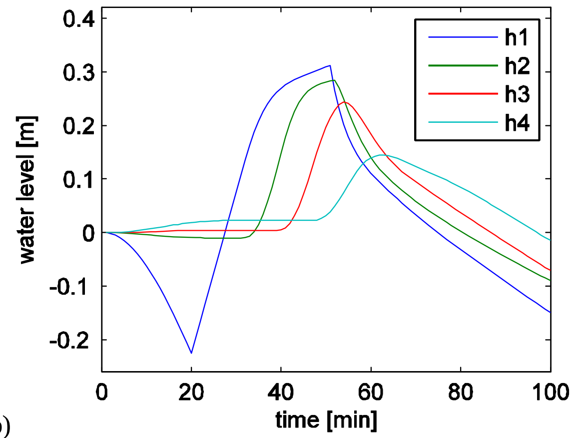
Parameter	Value	Unit
Δt	60	[s]
C	340.979	[$m^{3/2}$/s]
u_{max}	20	[m^3/s]
A1,2,3,4	60, 40, 50, 80	[1000m^2]
hcr1,2,3,4	0.1, 0.1, 0.1, 0.1	[m]
hsp1,2,3,4	0.0, 0.0, 0.0, 0.0	[m]
wx1,2	1.0, 10.0	[-]
wu	0.1	[-]

The control horizon is 100 time steps. The upstream inflow is assumed to be a block function with a value of 50m^3/s in time steps 20-50, and zero otherwise. The stop criteria for the optimization algorithms is set to a tolerance of 10^{-6} for the objective function value.

We apply the NMPC on reservoir systems with the first reservoir only, the first two reservoirs, and all four reservoirs. Furthermore, we run the three cases with both optimization schemes and two time stepping schemes, an explicit Forward Euler scheme as well as an implicit Backwards Euler scheme. This results in a total number of 12 runs which are summarized in Table 2 according to CPU time.



a)



b)

Figure 1: a) total release (r) and release from controlled outlet (u) for 4-reservoir test case, b) reservoir water levels

Table 2: test case 1 - results

	CPU time [s]	
Number reservoirs	Explicit model Snopt / Minos	Implicit model Snopt / Minos
1	0.2807 / 0.1689	0.3658 / 0.2964
2	0.3975 / 0.3547	0.6245 / 0.8644
4	1.1076 / 1.1098	1.1182 / 3.0564

Both optimizers deliver the correct optimum. The Snopt algorithms seem to perform better for larger optimization problems.

The scaling of the controller, which is about linear for the better Snopt solver, is remarkably good with an increasing number of dimensions in the optimization problem. A profiling of the application shows that this strongly depends on the performance of the optimizer itself. The model execution including the evaluation of the objective function value and its gradient does only contribute between 2% (explicit model) and 25% (implicit model) to the total execution time. The reason for the over proportional higher execution time of the Minos solver for the four reservoir test case results from a larger number of iterations for reaching the optimum control.

Figure 1 presents the reservoir releases from the controlled outlet and the uncontrolled spillway and the water levels in the reservoirs. The optimum control is found in such a way that the initial zero releases from all reservoirs are increased gradually to the maximum release. This creates extra storage capacity in the first reservoir which is filled when the flood peak is approaching the reservoir. Although the reservoir system can significantly damp the flood peak, the spillage in the last reservoir can not be completely avoided in the current settings.

5. BIFURCATION TEST CASE

The bifurcation points of the River Rhine are the key to the discharge distribution along the different Dutch river Rhine branches and therefore have a major impact on the water management in The Netherlands. The discharge distribution affects various aspects such as the allocation of drinking water, irrigation, salt intrusion, navigation, and flood protection. The control of the discharge distribution has been the focus of several recent publications such as Schielen et al. [9].

We present the set-up of an NMPC implemented as a pilot in the Dutch flood forecasting system for the rivers Rhine and Meuse. It manages the discharge distribution at the bifurcation points (Figure 2) at low and medium flows by control of a hydraulic structure at Driel (S01). Furthermore, it operates five inlet and outlet structures (S02-S06) of two projected flood detention basins for dampening flood peaks during flood events.

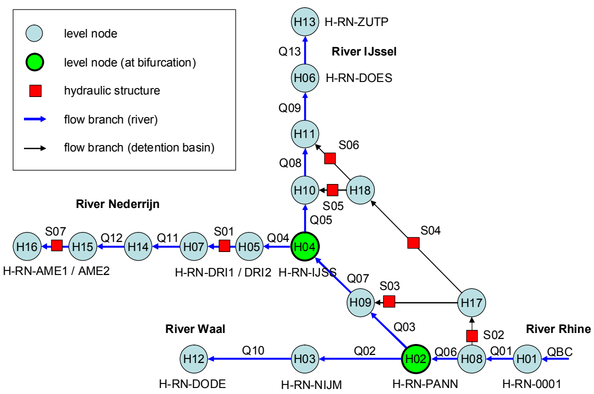
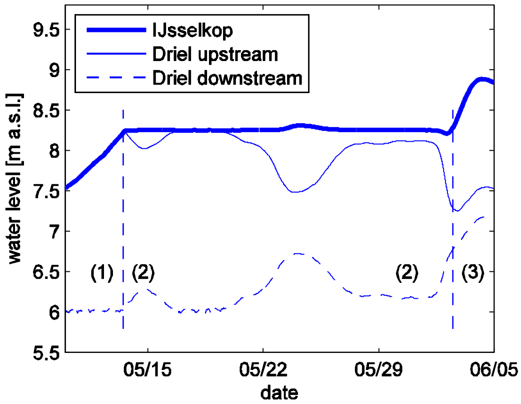
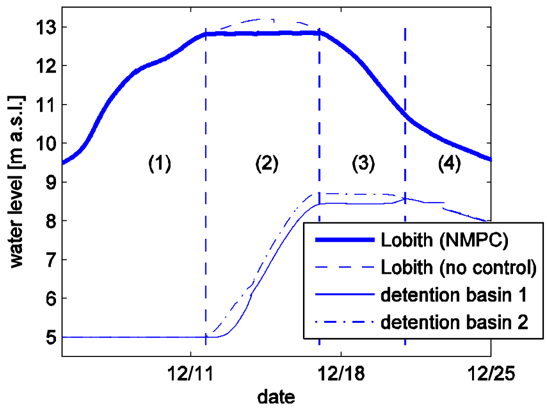


Figure 2: Layout of kinematic wave model: overview about nodes, branches, structures

The model topology of the process model, an explicit kinematic wave model, was set-up as course as possible in order to reduce CPU time. Nodes are only placed at gauges, bifurcation points, upstream and downstream of hydraulic structures in the rivers and inlet / outlet structures of the detention basins. One additional calculation point had to be placed in one of the largest branches to increase the model accuracy.



a)



b)

Figure 3 a) water level control at Driel during low - medium flow regime in May 2007 with water level set point of 8.25 m a.s.l. at gauge IJsselkop, b) damping of small flood peak above 12.75 m a.s.l. in December 2007 at gauge Lobith by control of detention basins

The control horizon of the NMPC is 5 days with model simulation time steps of 2 minutes resulting in a total number of 3600 time steps. We choose an aggregated control time step of one hour, i.e. 120 control time steps for each hydraulic structure.

The objective function is set-up in such a way that we penalize the use of hydraulic structures, the deviation from water level set points and the crossing of maximum allowed water levels. The reader is referred to [10] for additional details.

Figure 3 presents results of the NMPC running in a closed loop setting, using the kinematic wave model also as a replacement of the actual system and for perfect predictions of the disturbance. We intend to repeat the exercise in the near future using a full hydraulic model. In the upper figure, the regime is gradually shifting from low flow (1) for which the set point is not maintained even with fully closed gates, to (2) medium flow for which the set point is well maintained, to (3) a higher flow regime with gates completely opened and balanced water levels upstream and downstream of the gate. The lower figure presents the dampening of a small flood wave. In phase (1) the inlet structures of the detention basins are still inactive. They start discharging the water during phase (2) for keeping the water level at Lobith at a level of 12.75m a.s.l. Inlet gates are closed again in phase (3) till the water is released from the detention basins through the outlet structures in phase (4).

The average execution time of a larger number of optimizations with the NMPC is about 23s on a standard PC. The maximum execution time is 38s enabling the operational use of the scheme in flood forecasting. Profiling of the application shows that the model execution is responsible for about 96% of the total execution time.

6. CONCLUSIONS

This paper presents a Nonlinear Model Predictive Control scheme to support for supporting operational decision-making for hydraulic structures. It is applied to the control of reservoir systems and flood detention basins in a complex river network. Since the proposed framework is generic and allows for the straightforward integration of arbitrary process models and control objectives, it is extendable to various other applications in water resources such as the control of cascades of hydropower plants or irrigation systems.

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