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METHOD OF MAGNETIC FIELD COMPUTATION IN PRESENCE OF THIN FERROMAGNETIC PLATES OR CASES

Kirill Shaposhnikov

Southern Scientific Centre of Russian Academy of Sciences

ABSTRACT

The problem of magnetic field simulation in presence of thin ferromagnetic layers is considered. Its solving is reduced to the projection problem using Orthogonal Projections Method. Theory of the method and its numerical realization are described. Examples of calculation are given.

Index Terms— Thin magnetic layer, stationary magnetic field, orthogonal projections method

1. INTRODUCTION

Thin ferromagnetic layers (plates or cases) are widely used in many engineering devices. They can be parts of sensors for magnetic field, induction motors, magnetic shields, etc.

Use of well known methods (such as Finite Element Method, Integral Equations Method) for simulation of magnetic field in presence of such these layers leads to numerical models of very big dimensions. They are also ill-conditioned because nodes of discretization on opposite sides of the layer are close together.

Use of Orthogonal Projections Method for solving of such problems is considered in this paper. Theory of the method is relatively simple and obvious due to geometrical interpretation. The theory and the numerical realization of the method are described here. The method leads to low cost in numerical model in sense of its dimension. Also the model is well defined.

2. PROBLEM STATEMENT

Consider the problem of stationary magnetic field computation in presence of thin magnetic layer (see fig. 1). Let's introduce the following notation.

- V_∞ is physical three dimensional space;
- V is the volume of the layer;
- S is the median surface of the layer;
- h is the thickness of the layer;
- $\mathbf{n}$ is a outward normal to S;
- $\mathbf{H}^0$ is intensity of an initial field;
- $\mu_0 = const$, $(0 < \mu_0 < \infty)$ is permeability of exterior medium;
- μ is permeability of the magnetic layer.

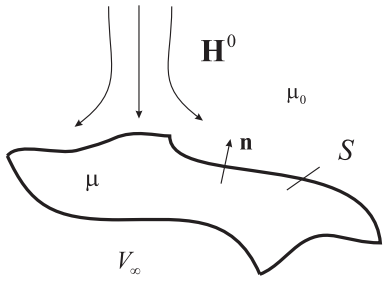

Fig. 1. Problem statement.

Let the thickness of the layer be far smaller than its other dimensions (i. e. $h \ll \sqrt{mes(S)}$). Then we can tend $h \to 0$ and consider the median surface S instead of the layer. But we have to define permeability of the surface in this case.

The case if surface permeability μ tends to ∞ as $h \to 0$ is often used. Such assumption is possible if the layer has high permeability and not saturated yet. Another one assumption is used in the case if properties of the material (such as anisotropy, heterogeneity, etc.) have to be taken into account. For that we replace μ by linear permeability $\bar{\mu} = \mu h$ such that $\bar{\mu}$ – inv as $h \to 0,\ \mu \to \infty$. Further, both cases are considered below. Denote them by a) and b) respectively.

Let S be a piecewise-smooth Lipschitz surface. Suppose that S and the sources of initial field both can be located in the ball V_R of radius R.

Let's the initial field has finite energy, i. e.,

$$\int_{V_\infty} \left|\mathbf{H}^0\right|^2 dV < \infty. \tag{1}$$

Otherwise we consider an equivalent in sense [1] field instead of the initial one. According [1] this means that the initial field coincides with the equivalent field on S but condition (1) is held for the last one.

Represent intensity of magnetic field $\mathbf{H}$ in the form $\mathbf{H} = \mathbf{H}^0 + \mathbf{H}^*$. Here $\mathbf{H}^*$ is intensity of reaction field. Its sources are microcurrents induced on S. Thus $\mathbf{H}^*$

is potential field, i. e. $\mathbf{H}^* = -\nabla\varphi^*$ in V_∞. We obtain the following boundary problem for φ^*

$$\begin{aligned}
&\Delta\varphi^*(M) = 0,\ M \in V_\infty;\\
&\text{a) } \varphi^*(M) = C - \varphi^0(M),\ M \in S;\\
&\text{b) } (\varphi^*(M))^+ = (\varphi^*(M))^-,\ M \in S,\\
&\left(\frac{\partial\varphi^*(M)}{\partial n}\right)^- - \left(\frac{\partial\varphi^*(M)}{\partial n}\right)^+ =\\
&= -div_s(\kappa(\nabla_s\varphi^* - \mathbf{H}_s^0)),\ M \in S;\\
&\varphi^*(M) \to 0,\ M \to \infty,
\end{aligned}$$

where φ^0 is a potential of initial field (suppose it is defined on S at least), $C = const$, $\kappa = \bar{\mu}/\mu_0$, "s" in subscript denote the tangent to S components vectors, div_s is surface divergence.

There was used the following.

- Initial field intensity $\mathbf{H}^0$ and resulting flux density $\mathbf{B}$ are solenoidal in V_∞;
- The medium is homogeneous and linear;
- $\mathbf{H}_s$ and $\mathbf{B}$ are continuous on theg the boundary of the layer;
- Tangent to S component of resulting field intensity is equal to zero in the case a);
- Magnetic flux through the layer is held in the case b).

Obviously, φ^* can be represented by a simple layer potential with density σ, i. e.,

$$\varphi^*(M) = \frac{1}{4\pi}\int_S \frac{\sigma(Q)}{r_{QM}} dS_Q,\ M \in V_\infty.$$

It directly follows from the conditions of the boundary problem. Due to the properties of a simple layer potential we obtain

$$\sigma = \left(\frac{\partial\varphi^*}{\partial n}\right)^- - \left(\frac{\partial\varphi^*}{\partial n}\right)^+,$$

at that in the case b) the following is held

$$\sigma = -div_s(\kappa(\nabla_s\varphi^* - \mathbf{H}_s^0)) = div_s(\kappa\mathbf{H}_s). \quad (2)$$

It is obvious that the density σ must have zero mean value on S, i. e.

$$\int_S \sigma dS = 0. \quad (3)$$

It is proved [2] that the following condition is held for any simple layer potentials ψ_1 and ψ_2 with densities ξ_1 and ξ_2 respectively. These densities are distributed on a closed surface Σ.

$$\int_{V_\infty} \nabla\psi_1\nabla\psi_2 dV = \oint_\Sigma \psi_1\xi_2 dS = \oint_\Sigma \xi_1\psi_2 dS. \quad (4)$$

This equality will be used in the sequel.

The solution that has finite energy is only needed, i. e.,

$$\int_{V_\infty} |\mathbf{H}^*|^2 dV = \int_{V_\infty} |\nabla\varphi^*|^2 dV < \infty. \quad (5)$$

3. ORTHOGONAL PROJECTIONS METHOD

According [3], we choose a function space such that the reaction field and the resulting field are orthogonal in it. Then the reaction field is defined as the projection of the initial field to the subspace which the reaction field belongs to. These is the main idea of Orthogonal Projections Method. Consider the usage of the method in our case.

Due to conditions (1), (5) we can use the space $L_2(V_\infty)$. This space consists of square-summable vector fields in V_∞. The inner product and the norm are defined by following

$$\langle\mathbf{a},\mathbf{b}\rangle_{L_2} = \int_{V_\infty} \mathbf{ab}dV,\ \|\mathbf{a}\|_{L_2} = \langle\mathbf{a},\mathbf{a}\rangle_{L_2}^{1/2}. \quad (6)$$

Here and in the sequel all integral operations are considered in Lebesgue sense [4].

According to Weyl decomposition [5] we have

$$L_2(V_\infty) = L_2^{(p)}(V_\infty) \oplus L_2^{(s)}(V_\infty).$$

Here $L_2^{(p)}(V_\infty)$ and $L_2^{(s)}(V_\infty)$ are subspaces of potential and solenoidal fields respectively. Their elements are orthogonal in sense of (6). Note that vector fields presented in the form $\mathbf{a} = \nabla f$ where f is a simple layer potential form a subspace in $L_2^{(p)}(V_\infty)$. Denote it by $L_{2,\sigma}^{(p)}(V_\infty)$.

Obviously, $\mathbf{H}^* \in L_2^{(p)}(V_\infty)$ and $\mathbf{B} \in L_2^{(s)}(V_\infty)$. The reaction field intensity can be represented by the following

$$\mathbf{H}^* = \hat{\mu}^{-1}\mathbf{B} - \mathbf{H}^0,\ \hat{\mu} = \begin{cases} \mu_0 \text{ in } V_\infty\backslash V,\\ \mu \text{ in } V.\end{cases}$$

Let's introduce an equivalent metric in the space $L_2(V_\infty)$. It has the following form

$$\langle\mathbf{a},\mathbf{b}\rangle_{L_{2,\mu}} = \frac{1}{\mu_0}\int_{V_\infty} \hat{\mu}\mathbf{ab}dV,\ \|\mathbf{a}\|_{L_{2,\mu}} = \langle\mathbf{a},\mathbf{a}\rangle_{L_{2,\mu}}^{1/2}. \quad (7)$$

Thus we have a new Hilbert space $L_{2,\mu}(V_\infty)$. Note that $L_2^{(p)}(V_\infty)$ and $L_2^{(s)}(V_\infty)$ are subspaces in $L_{2,\mu}(V_\infty)$ but their elements are not orthogonal in sense of (7). Each of them will has its own orthogonal complement in $L_{2,\mu}(V_\infty)$. According to [4], there are operators that projected the space $L_{2,\mu}(V_\infty)$ on the subspaces $L_2^{(p)}(V_\infty)$ and $L_2^{(s)}(V_\infty)$.

$\mathbf{H}^*$ and $\hat{\mu}^{-1}\mathbf{B}$ are orthogonal in sense of (7). Hence we can write

$$\mathbf{H}^* = P^{(p)}\mathbf{H}^0,$$

where $P^{(p)}$ is the orthogonal projection of $L_2(V_\infty)$ on $L_2^{(p)}(V_\infty)$ in sense (7).

Let's consider the Orthogonal Projections Method usage in the case b). Consider doubled energy of magnetic field. We obtane

$$\int_{V_\infty} \mathbf{BH}dV = \int_{V_\infty \setminus V} \mathbf{BH}dV + \int_V \mathbf{BH}dV, \quad (8)$$

It is obvious that $\mathbf{B}_s = \mu \mathbf{H}_s \to \infty$ and $H_n = B_n/\mu \to 0$ as $\mu \to \infty$ and $h \to 0$. Assuming that thickness nonuniformity of the magnetic field is unimportant, we obtain

$$\int_V \mathbf{BH}dV \to \int_S \int_{-h/2}^{h/2} \mu \mathbf{H}_s \mathbf{H}_s dn dS =$$
$$= \int_S \bar{\mu} \mathbf{H}_s \mathbf{H}_s dS.$$

Thus

$$\langle \mathbf{B}, \mathbf{H} \rangle_{L_2} = \mu_0 \int_{V_\infty} |\mathbf{H}|^2 dV + \int_S \bar{\mu} |\mathbf{H}_s|^2 dS.$$

Let now prove that $\langle \mathbf{H}^*, \mathbf{H} \rangle_{L_{2,\mu}} = 0$. For that execute

$$\langle \mathbf{H}, \mathbf{H}^* \rangle_{L_{2,\mu}} = \int_{V_\infty} \mathbf{H}\mathbf{H}^* dV + \frac{1}{\mu_0} \int_S \bar{\mu} \mathbf{H}_s \mathbf{H}_s^* dS =$$
$$= \int_{V_\infty} \mathbf{H}^* \mathbf{H}^* dV - \frac{1}{\mu_0} \int_S \bar{\mu} \mathbf{H}_s \nabla_s \varphi^* dS =$$
$$= \int_{V_\infty} \nabla \varphi^* \nabla \varphi^* dV + \frac{1}{\mu_0} \int_S div_s(\bar{\mu} \mathbf{H}_s) \varphi^* dS =$$
$$= \int_S \varphi^* \sigma dS - \int_S \varphi^* \sigma dS = 0.$$

Equalities (2), (4) are used here. Also, the condition $H_\nu = 0$ on ∂S if S is unclosed surface. $\boldsymbol{\nu}$ is normal to the boundary of the surface.

In the case a) doubled magnetic field energy has the following form

$$\langle \mathbf{B}, \mathbf{H} \rangle_{L_2} = \int_{V_\infty \setminus V} \mathbf{BH}dV + \int_V \mathbf{BH}dV \to$$
$$\to \int_{V_\infty} \mathbf{BH}dV, \ h \to 0, \ \mu \to \infty,$$

because $\mathbf{H}(M) \to \mathbf{0}, \ M \in V$ as $\mu \to \infty$.

In this case orthogonality condition is held for fields $\mathbf{H}^0$ and $\mathbf{H}^*$. We have to consider an equivalent in sense [1] potential field $\tilde{\mathbf{H}}^0$ instead of $\mathbf{H}^0$ for use the method . According to [1] $\mathbf{H}^0(M) = \tilde{\mathbf{H}}^0(M), \ M \in S$. Then equivalent resulting field has following form $\tilde{\mathbf{H}} = \tilde{\mathbf{H}}^0 + \mathbf{H}^*$. Let's show that $\tilde{\mathbf{H}}$ and $\mathbf{H}^*$ are orthogonal fields in $L_{2,\mu}(V_\infty)$. First we complement the surface S to a closed surface $\bar{S}$, i. e., $\bar{S} = S + S'$. If S is closed then $\bar{S} = S$. Secondly we suppose σ is equal 0 on S'. We obtain

$$\left\langle \tilde{\mathbf{H}}, \mathbf{H}^* \right\rangle_{L_{2,\mu}} = \int_{V_\infty} \tilde{\mathbf{H}}\mathbf{H}^* dV =$$
$$= \int_{V_\infty} \nabla \tilde{\varphi} \nabla \varphi^* dV = \oint_{\bar{S}} \tilde{\varphi} \sigma dS$$
$$= \int_S \varphi \sigma dS = C \int_S \sigma dS = 0.$$

Thus metric (7) in the cases a) and b) is represented by following

$$\text{a) } \langle \mathbf{a}, \mathbf{b} \rangle_{L_{2,\mu}} = \int_{V_\infty} \mathbf{ab}dV, \quad (9)$$

$$\text{b) } \langle \mathbf{a}, \mathbf{b} \rangle_{L_{2,\mu}} = \int_{V_\infty} \mathbf{ab}dV + \frac{1}{\mu_0} \int_S \bar{\mu} \mathbf{a}_s \mathbf{b_s} dS. \quad (10)$$

After that the problem's solution is reduced to finding of initial field projection on the subspace $L_2^{(p)}(V_\infty)$ in sense of (9) and (10) in the cases a) and b) respectively. We have to calculate of initial field coordinates in some basis for $L_2^{(p)}(V_\infty)$. The coordinates are defined by solving of system of linear algebraic equations with Gram matrix. If

$$\mathbf{H}^* \cong \sum_{i=1}^n c_i \mathbf{g}_i,$$

we obtain

$$\sum_{i=1}^n c_i \langle \mathbf{g}_i, \mathbf{g}_k \rangle_{L_{2,\mu}} = -\left\langle \mathbf{H}^0, \mathbf{g}_k \right\rangle_{L_{2,\mu}}, k = \overline{1,n}. \quad (11)$$

If the coordinate functions are given by following

$$\mathbf{g}_k = -\nabla f_k = -\nabla \frac{1}{4\pi} \int_S \frac{\sigma_k}{r} dS, \ k = \overline{1,n},$$

then using (4) expression (11) has the following form for the cases a) and b)

$$\text{a) } \sum_{i=1}^n c_i \int_S f_i \sigma_k dS = -\int_S \varphi^0 \sigma_k dS, k = \overline{1,n},$$

$$\text{b) } \sum_{i=1}^n c_i \left(\int_S \sigma_i f_k dS + \frac{1}{\mu_0} \int_S \bar{\mu} \nabla_s f_i \nabla_s f_k dS \right) =$$
$$= \frac{1}{\mu_0} \int_S \bar{\mu} \mathbf{H}_s^0 \nabla_s f_k dS, k = \overline{1,n}.$$

Here the condition $\varphi^0(M) = \tilde{\varphi}^0(M), \ M \in S$ is taken into account.

4. NUMERICAL REALIZATION

Obviously, the choose of coordinate functions is important part of simulation process.

In the case a) it is convenient to use simple layer potentials with finite step densities as functions $f_k, \ k =$

$\overline{1,n}$. The densities σ_k are distributed on S_k. They also must have zero mean values on S_k. In this case gradients of f_k form a dense set in terms of approximate convergence in $L^{(p)}_{2,\sigma}$ [2]. Surely the system of coordinate functions has to be linearly independent.

We obtain the following formulas for elements of system (11)

$$\langle \mathbf{g}_i, \mathbf{g}_k \rangle_{L_{2,\mu}} = \frac{1}{4\pi} \int_{S_i} \sigma_i \int_{S_k} \frac{\sigma_k}{r} dS dS, \qquad (12)$$

$$\langle \mathbf{H}^0, \mathbf{g}_k \rangle_{L_{2,\mu}} = \int_{S_k} \varphi^0 \sigma_i dS. \qquad (13)$$

Using system of coordinate functions $\{f_k\}_{k=1}^n$ that is described above we can simplify expression (12). At that the most part of calculations are produced analytically. Radon's cubature formula is used for calculation of (13).

Such system of coordinate functions can't be used in the case b). In this case function $\nabla_s f_i \nabla_s f_k$ has singularities in the nodes that belong to S_i and S_k. Thus we cant't calculate the following integral with acceptable precision.

$$\int_S \bar{\mu} \nabla_s f_i \nabla_s f_k dS \qquad (14)$$

For its calculation we use eigenfunctions of the operator PK as coordinate functions.

$$P\zeta(M) = \zeta(M) - \frac{1}{mes(S)} \int_S \zeta dS,$$
$$K\zeta(M) = \frac{1}{4\pi} \int_S \frac{\zeta(Q)}{r_{QM}} dS_Q.$$

The operator PK is self-adjoint because P is a projection operator and K is an operator with symmetric kernel. Therefore its eigenvalues are real. Its eigenfunctions form a complete basis. Condition (3) is held for the eigenfunctions.

If f_k and α_k are operator's eigenfunction and eigenvalue, we obtaine

$$\frac{1}{4\pi} \int_S \frac{f_k}{r} dS = PKf_k +$$
$$+ \frac{1}{mes(S)} \int_S \frac{1}{4\pi} \int_S \frac{f_k}{r} dS = \alpha_k f_k + C_k,$$

where $C_k - const$.

Elements of system (11) in the case b) are calculated by the following

$$\langle \mathbf{g}_i, \mathbf{g}_k \rangle_{L_{2,\mu}} = \int_S f_i f_k dS + \frac{\alpha_i}{\mu_0} \int_S \bar{\mu} \nabla_s f_i \nabla_s f_k dS, \qquad (15)$$

$$\langle \mathbf{H}^0, \mathbf{g}_k \rangle_{L_{2,\mu}} = \frac{1}{\mu_0} \int_S \bar{\mu} \mathbf{H}^0_s \nabla_s f_k dS. \qquad (16)$$

Obviously, for calculation (15), (16) $\nabla_s f_k, k = \overline{1,n}$ have to exist. At that f_k, $k = \overline{1,n}$ must be a continuous and piecewise continuously differentiable functions. We can get such system of coordinate functions if we know their values in the nodes of the surface discretization. After that calculation of these values is reduced to calculation of eigenvectors of the operator's matrix.

5. EXAMPLES OF CALCULATIONS

The software tool for simulation of magnetic field in presence of thin magnetic layer has been created. This tool allows to compute magnetic field in the cases a) and b) both. Microsoft Visual C# 2008 Express programming language has been used for its development. Three dimensional triangulation of a surface is performed using Gmsh software package [1].

In particular case we can get analytical solution of the problem. This case is computation of plane magnetic field in presence of infinitely long thin flat with width $2a$, thickness h, and permeability μ.

Analytically formula for density of reaction field potential has the following form

$$\sigma(x) = \frac{2\mu h/a}{1 + 2\mu h/a} \frac{x}{\sqrt{a^2 - x^2}}. \qquad (17)$$

Results obtained using developed software tool are compared with analytical result below.

For the numerical calculation rectangular plate with dimensions $2a \times 2b$ ($-a \le x \le a$, $-b \le y \le b$) was used. The densities distributions on the line L : $-a \le x \le a$, $y = 0$ are shown in fig. 2.

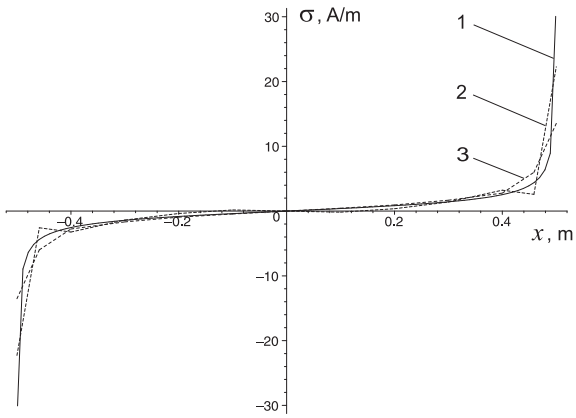

Fig. 2. Comparison of analytical and numerical results.

It is assumed that $a = 0.5$ m, $b = 1$ m, $h = 0.001$ m, $\mu = 4\pi \times 10^{-3}$ H/m, $\mathbf{H}^0 = \{1, 0, 0\}$ A/m.

Here solid line 1 corresponds to the analytical solution (17). Dash lines 2 and 3 are calculated using developed software. They are correspond to the cases b) and a) respectively. One can see that numerical and

[1] See www.geuz.org/gmsh

analytical results are close. At that only 13 nodes of triangulation was situated on the line L in considered example.

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