07 - 10 September 2009

PROCEEDINGS

54. IWK

Internationales Wissenschaftliches Kolloquium

International Scientific Colloquium

Information Technology and Electrical Engineering - Devices and Systems, Materials and Technologies for the Future

Faculty of Electrical Engineering and Information Technology

Startseite / Index:
http://www.db-thueringen.de/servlets/DocumentServlet?id=14089

TECHNISCHE UNIVERSITÄT
ILMENAU

Impressum

Herausgeber: Der Rektor der Technischen Universität Ilmenau
Univ.-Prof. Dr. rer. nat. habil. Dr. h. c. Prof. h. c. Peter Scharff

Redaktion: Referat Marketing
Andrea Schneider

Fakultät für Elektrotechnik und Informationstechnik
Univ.-Prof. Dr.-Ing. Frank Berger

Redaktionsschluss: 17. August 2009

Technische Realisierung (USB-Flash-Ausgabe):
Institut für Medientechnik an der TU Ilmenau
Dipl.-Ing. Christian Weigel
Dipl.-Ing. Helge Drumm

Technische Realisierung (Online-Ausgabe):
Universitätsbibliothek Ilmenau
ilmedia
Postfach 10 05 65
98684 Ilmenau

Verlag: isle
Verlag ISLE, Betriebsstätte des ISLE e.V.
Werner-von-Siemens-Str. 16
98693 Ilmenau

ISBN (USB-Flash-Ausgabe): 978-3-938843-45-1
ISBN (Druckausgabe der Kurzfassungen): 978-3-938843-44-4

Startseite / Index:
http://www.db-thueringen.de/servlets/DocumentServlet?id=14089

MEDICAL IMAGE ANALYSIS USING NEURO-FUZZY NETWORK

Ye. Bodyanskiy[1] / *Ye. Gorshkov*[1] / *P. Otto*[2] / *I. Pliss*[1]

[1] Kharkiv National University of Radio Electronics

[2] TU Ilmenau

ABSTRACT

New adaptive fuzzy Kohonen clustering algorithms are proposed to train heterogeneous neuro-fuzzy network. The algorithms perform clustering of the input patterns in on-line mode, do not contain free parameters, are computationally simple and possess the advantages of both the Kohonen's learning rule and fuzzy clustering procedures. Both the probabilistic and possibilistic approaches are considered. The proposed network is applied to the real-life problem of blood flow detection based on the Doppler sonographic images and showed promising results in both classification quality and learning speed.

Index Terms— Neural networks, fuzzy logic, neuro-fuzzy, neo-fuzzy, fuzzy clustering, Kohonen network, Doppler sonography

1. INTRODUCTION

The problem of Doppler sonographic image analysis plays an important role in medical cardiovascular studies. The blood flow areas in a body can be detected using the data from Doppler sonographic scanner by estimating the parameters of received echo signal, reflected from moving blood fractions. The results of the detection are used to visualize the blood flow on the color Doppler digital images in real-time mode.

Hybrid neuro-fuzzy architectures proved themselves as powerful methods for image processing and can be applied to sonographic image analysis as well.

Kohonen's clustering neural network [1] is a widely-used technique which is often applied to data mining, exploratory data analysis, and clustering of large data sets. This network is computationally efficient and possesses the property of on-line learning and operation where the input patterns are processed sequentially.

The learning process of the network weights is based on the principles of "winner-take-all" (WTA) or "winner-takes-most" (WTM) approaches. However, in these cases it is assumed that data clusters are not overlapping, i.e. for each pair of clusters there exists a hyperplane which clearly separates the data points.

However, in real-life applications it is a common situation when an input pattern belongs to several different classes at the same time with some level of membership and the clusters are overlapping. This situation is considered in the methods of fuzzy cluster analysis [2], a great variety of which have been developed in recent years. Thus, it is reasonable to combine the simplicity and computational efficiency of the Kohonen clustering network with the ability to learn on overlapping clusters.

In [3, 4] a modified Kohonen clustering network architecture was introduced in which the neurons were replaced by fuzzy sets and fuzzy rules. Although this network works quite well in many problems of pattern recognition, its learning algorithm is computationally ineffective. In [5] a new Kohonen network with fuzzy inference based on cosine neighborhood-membership functions and combined learning algorithm based on Kohonen and Grossberg rules was proposed. The main disadvantage of this network is the dependence of clustering results on the choice of free parameters of the learning procedure. In [6], the so-called fuzzy Kohonen clustering network (FKCN) was introduced and later improved in [7, 8, 9]. The learning procedure for this network is based on Bezdek's fuzzy c-means clustering algorithm [10], which operates in a batch mode and requires all training data available a priori. Thus, the FKCN cannot be effectively applied to the problems where on-line learning is preferable and the training data must be processed sequentially.

The convergence rate of the learning algorithms plays an important role in this problem because of the large data sets. It can be improved by using special learning procedures, often applied to train heterogeneous networks like RBF-NFN [11].

In this paper we propose using an adaptive modification of the FKCN [2] as a hidden layer of RBF-NFN. A recursive learning algorithm for the hidden layer parameters is proposed, which is a generalization of Kohonen's learning rule and allows to train both centers of the neurons (cluster centroids) and fuzzy membership values of the input samples to each of the clusters.

2. NETWORK ARCHITECTURE

The RBF-NFN network architecture (Fig. 1) is comprised of two layers [11]: the hidden layer of radial-basis functions (RBF), and the output layer of neo-fuzzy neurons [12, 13]. It has D inputs, Q outputs (NFNs), and H neurons in the hidden layer.

The hidden layer can also be represented as a fuzzy Kohonen clustering networks and differs from the RBF layer by the presence of the in-layer latter connections.

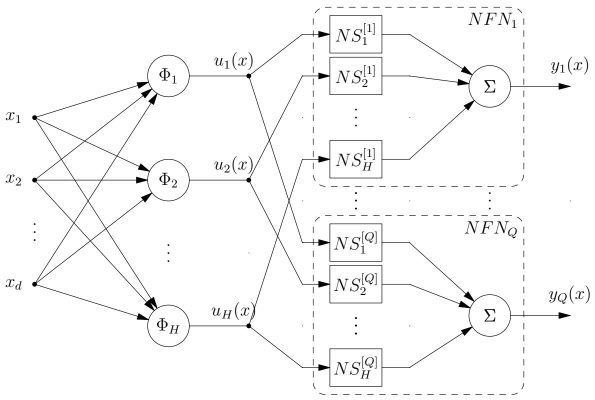

Fig. 1. RBF-NFN network architecture

The output of the hidden layer is a vector $u(x)$ computed as

$$u(x) = (u_1(x), u_2(x), \ldots, u_H(x))^T, \tag{1}$$

$$u_h(x) = \Phi_h(x, c_h), \quad h = 1, 2, \ldots, H, \tag{2}$$

where $x = (x_1, x_2, \ldots, x_D)^T$ is the vector of the input values, $\Phi_h(\cdot)$ are the activation functions of the hidden layer neurons, and $c = (c_1, c_2, \ldots, c_H)^T$ are the centers of the activation functions of receptive fields.

The outputs $u_h(x)$ of the hidden layer neurons are fed to the set of the membership functions (MFs) of neo-fuzzy neurons. The equations for the output layer synapses (see Fig. 2) are

$$f_h^{[q]}(u_h(x)) = \sum_{j=1}^{m_h} \mu_{hj}(u_h(x)) w_{hj}^{[q]}, \tag{3}$$
$$q = 1, 2, \ldots, Q,$$

where m_h is the number of MFs per input h of the synapses, $\mu_{hj}(u_h(x))$ are the output layer MFs, and $w_{hj}^{[q]}$ are the tunable weights.

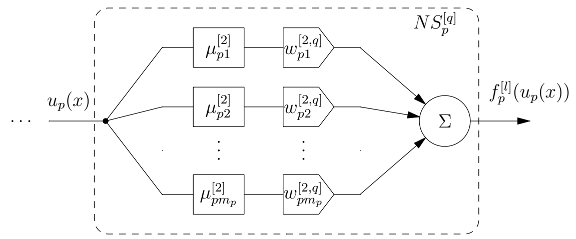

Fig. 2. Nonlinear synapse

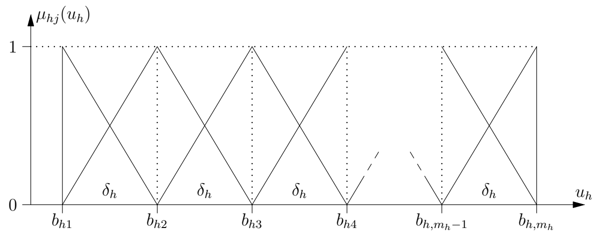

Fig. 3. Array of membership functions of the neo-fuzzy neurons

The MFs usually form a set of functions similar to the function array shown on Fig. 3.

We consider equidistantly distributed triangular MFs:

$$\mu_{hj}(a) = \begin{cases} \dfrac{v_{hj} - a}{v_{hj} - v_{h,j-1}}, & a \in (v_{h,j-1}, v_{hj}], \\ \dfrac{a - v_{hj}}{v_{h,j+1} - v_{hj}}, & a \in (v_{hj}, v_{h,j+1}], \\ 0, \quad \text{otherwise}, \\ j = 1, 2, \ldots, m_h, \end{cases} \tag{4}$$

where v_{hj} are the centers of the MFs with the additional conditions $v_{h0} = v_{h1}$ and $v_{h,m_h+1} = v_{h,m_h}$ for the centers to ensure zero function value when the argument is not in the range $[v_{h1}, v_{h,m_h}]$.

The output of the network is computed as

$$\hat{y}_q(x) = \sum_{h=1}^{H} f_h^{[q]}(u_h(x)) = \sum_{h=1}^{H} \sum_{j=1}^{m_h} \mu_{hj}(u_h(x)) w_{hj}^{[q]}. \tag{5}$$

Introducing a vector of the MFs of the output layer

$$\psi = (\mu_{11}, \mu_{12}, \ldots, \mu_{1m_1}, \mu_{21}, \mu_{22}, \ldots, \mu_{2m_2}, \ldots, \mu_{H1}, \mu_{H2}, \ldots, \mu_{Hm_H})^T, \tag{6}$$

and a $(Q \times m_\Sigma)$-matrix of the output layer weights $w_{hj}^{[q]}$ (here $m_\Sigma = \sum_{h=1}^{H} m_h$):

$$W = (w^{[1]}, w^{[2]}, \ldots, w^{[Q]})^T = \begin{pmatrix} w_{11}^{[1]} & w_{21}^{[1]} & \cdots & w_{Hm_H}^{[1]} \\ w_{11}^{[2]} & w_{21}^{[2]} & \cdots & w_{Hm_H}^{[2]} \\ \cdots & \cdots & \cdots & \cdots \\ w_{11}^{[Q]} & w_{21}^{[Q]} & \cdots & w_{Hm_H}^{[Q]} \end{pmatrix}, \tag{7}$$

we can write the output of the proposed network in a compact form:

$$\hat{y}(x) = W^T \psi(u(x)), \tag{8}$$

where $w^{[q]} = (w_{11}^{[q]}, w_{11}^{[q]}, \ldots, w_{Hm_H}^{[q]})^T$ is the matrix of tunable weights of the q-th NS.

The RBF-NFN network comprises the concepts of radial-basis function networks (RBFNs) and competitive networks (in particular, counter-propagation network (CPN) [14, 15], FKCN), which consists in increasing the dimension of the input space to a sufficiently high value in order to accommodate non-linearities present in the modeled data (e.g. partitioning of the input data into clusters for classification problems) [16].

Thus, the functions $u_h(\cdot) = \Phi_h(\cdot)$ can be considered as both the radial-basis functions and activation functions of the fuzzy Kohonen clustering network.

3. NETWORK LEARNING

To train the RBF-NFN network parameters we propose the use of the following two-stage learning procedure: 1) initialization and training of the hidden layer parameters (e.g. the number of neurons, activation function centers and receptive fields), 2) training of the output layer weights of nonlinear synapses.

The two-stage learning is often used to train different network architectures, like competitive neural networks (e.g. the counter-propagation neural network and soft-competitive basis function networks [17]) and RBFNs.

A training set is assumed to contain N samples $x(k)$, $k = 1, 2, \ldots, N$. We denote by

$$Y = (y(1), y(2), \ldots, y(N))^T$$

the matrix of target values, and by $\hat{y}(k)$ the vector of the network actual outputs at the step k.

3.1. Learning in the Hidden Layer using Adaptive Fuzzy Kohonen Clustering Algorithms

At the first stage of the training procedure any clustering algorithm can be applied, like the unsupervised Kohonen learning rule or k-means clustering used to train Kohonen layer of CPN, or subtractive clustering, fuzzy c-means, or any other clustering algorithm used to initialize the hidden layer of RBFNs and neuro-fuzzy networks.

The output of the hidden layer $u(k)$ defines the levels of membership $u_h(k)$ of the input vector $x(k)$ to each of the H clusters. The cluster centers $c_h(k)$ are fed through the lateral connections and are used to calculate the membership values $u_h(k)$.

The adaptive fuzzy Kohonen clustering algorithms are based on the probabilistic and possibilistic approaches to fuzzy clustering [10, 18].

According to the probabilistic approach the following objective function is optimized [10]:

$$E(u, c) = \sum_{k=1}^{N} \sum_{h=1}^{H} u_h^{\beta}(k) \|x(k) - c_h\|^2 \qquad (9)$$

subject to constraints

$$\sum_{h=1}^{m} u_h(k) = 1,\ 0 \le \sum_{k=1}^{N} u_h(k) \le N,$$

where $u_h(k) \in [0, 1]$, β is a non-negative “fuzzifier” parameter, which determines the fuzzy boundary between clusters and influences the fuzziness level of the final data partitioning.

Using standard non-linear programming techniques based on Lagrange multipliers and solving of Karush-Kuhn-Tucker equations, we can obtain the following well-known procedure:

$$\begin{cases} c_h = \dfrac{\sum_{k=1}^{N} u_j^{\beta}(k) x(k)}{\sum_{k=1}^{N} u_h^{\beta}(k)}, \\ u_h(k) = \dfrac{\left(\|x(k) - c_h\|^2\right)^{\frac{1}{1-\beta}}}{\sum_{l=1}^{H} \left(\|x(k) - c_l\|^2\right)^{\frac{1}{1-\beta}}}, \end{cases} \qquad (10)$$

which coincides with the fuzzy c-means algorithm for $\beta = 2$:

$$\begin{cases} c_h = \dfrac{\sum_{k=1}^{N} u_h^2(k) x(k)}{\sum_{k=1}^{N} u_h^2(k)}, \\ u_h(k) = \dfrac{\|x(k) - c_h\|^{-2}}{\sum_{l=1}^{H} \|x(k) - c_l\|^{-2}}. \end{cases} \qquad (11)$$

The learning procedure of FKCN [19] is based on (11) and assumes that the number of patterns is fixed and equals to N.

In order to overcome this constraint a probabilistic recursive fuzzy clustering algorithm was introduced in [20, 21], which was obtained using the Arrow-Hurwitz-Uzawa procedure [22]:

$$\begin{cases} u_h(k+1) = \dfrac{\|x(k+1) - c_h(k)\|^{\frac{2}{1-\beta}}}{\sum_{l=1}^{H} \|x(k+1) - c_l(k)\|^{\frac{2}{1-\beta}}}, \\ c_h(k+1) = c_h(k) + \eta(k) u_h^{\beta}(k+1) \\ \qquad \cdot (x(k+1) - c_h(k)). \end{cases} \qquad (12)$$

The procedure (12) is a generalization of the gradient based fuzzy c-means (GBFCM) algorithm by Park and Dagher [23].

It can be seen that the multiplier $u_h^{\beta}(k)$ corresponds to the neighborhood function of the WTM learning rule. For $\beta = 1$ the procedure (12) coincides with the hard c-means algorithm and for $\beta = 0$ we obtain the standard WTA rule:

$$c_h(k+1) = c_h(k) + \eta(k)(x(k+1) - c_h(k)). \quad (13)$$

The recursive procedure (13) minimizes the objective function

$$E(c_h) = \sum_{k} \|x(k) - c_h\|^2,$$

with $\eta(k) = \frac{1}{k+1}$, and its direct optimization leads to the conventional approximation of the mean value

$$c_h(k) = \frac{1}{M_h}\sum_{k=1}^{M_h} x(k),$$

where M_h is the number of wins of the h-th neuron, or in a recursive form

$$c_h(k+1) = c_h(k) + \frac{1}{k+1}(x(k+1) - c_h(k)). \quad (14)$$

The final form of the probabilistic adaptive fuzzy Kohonen clustering algorithm can be written as

$$\begin{cases} u_h(k+1) = \dfrac{\|x(k+1) - c_h(k)\|^{\frac{2}{1-\beta}}}{\sum\limits_{l=1}^{m} \|x(k+1) - c_l(k)\|^{\frac{2}{1-\beta}}}, \\ c_h(k+1) = c_h(k) + \dfrac{u_h^\beta(k+1)}{k+1} \\ \qquad \cdot (x(k+1) - c_h(k)). \end{cases} \quad (15)$$

A similar procedure was introduced for unsupervised fuzzy competitive learning in [19].

According to the possibilistic approach, the following objective function is used to find cluster centers and memberships [18]:

$$E^{ps}(u_h^{ps}, c_h^{ps}) = \sum_{h=1}^{m} \mu_j \sum_{k=1}^{N} (1 - u_h^{ps}(k))^\beta + \sum_{k=1}^{N}\sum_{h=1}^{m} (u_h^{ps}(k))^\beta \|x(k) - c_h^{ps}\|^2, \quad (16)$$

where $\mu_h > 0$ determines the distance to the h-th cluster center where the membership u_h^{ps} level equals to 0.5, i.e. $u_h^{ps} = 0.5$ for $\|x(k) - c_h^{ps}\|^2 = \mu_h$.

By applying similar optimization steps to the objective function (16) we can obtain the following recursive procedure:

$$\begin{cases} u_h^{ps}(k+1) = \dfrac{1}{1 + \left(\frac{\|x(k+1) - c_l^{ps}(k)\|^2}{\mu_h(k)}\right)^{\frac{-1}{1-\beta}}}, \\ c_h^{ps}(k+1) = c_h^{ps}(k) + \dfrac{(u_h^{ps}(k+1))^\beta}{k+1} \\ \quad \cdot (x(k+1) - c_h^{ps}(k)), \\ \mu_h(k+1) = \left(\sum\limits_{p=1}^{k+1} (u_h^{ps}(p))^\beta\right)^{-1} \\ \quad \cdot \left(\sum\limits_{p=1}^{k+1} (u_h^{ps}(p))^\beta \|x(p) - c_h^{ps}(k+1)\|^2\right). \end{cases} \quad (17)$$

Procedures (15), (17) combine features of the Kohonen learning rule, such as computational simplicity and the possibility to sequentially process the input patterns, with fuzzy clustering on the basis of the probabilistic and possibilistic approaches provided by FKCN.

These algorithms are very sensitive to the initial values of the cluster prototypes and in some cases can result in merging of the clusters. In order to overcome this difficulty we propose to perform clustering in two stages: at the first stage (for $k \le M$, $M = const$) only the winning neurons are tuned according to the WTA rule, i.e. (15) is computed for

$$k = k* = \arg\max_{l=1,\dots,H} \|x(k+1) - c_l(k)\|,$$

and at the second stage (for $k > M$) the procedure (15) is computed for all h.

Introducing into (15), (17) an additional coefficient

$$\delta_h(k+1) = \begin{cases} 1, \ k > M \ \text{ or} \\ h = \arg\max\limits_{l=1,\dots,H} \|x(k+1) - c_l(k)\|, \\ 0, \ \text{otherwise}, \end{cases} \quad (18)$$

we can write a modified recursive equation for cluster centers as

$$c_h(k+1) = c_h(k) + \delta_h(k+1)\frac{u_h^\beta(k+1)}{k+1} \cdot (x(k+1) - c_h(k)), \quad (19)$$

which is used for both learning stages $k \le M$ and $k > M$.

3.2. Learning in the Output Layer

The output layer of the RBF-NFN contains only the tunable weights $w_{hj}^{[q]}$, and the outputs of the NFNs are, in fact, linear with respect to these weights.

Hence, to find the optimal values of these weights in terms of minimization of the criterion

$$E = \frac{1}{2}\sum_{k=1}^{N} \|y(k) - \hat{y}(k)\|^2, \quad (20)$$

a standard least squares (LS) optimization techniques can be applied instead of derivative-based methods. Introducing matrix

$$\Psi = (\psi(1), \psi(2), \dots, \psi(N))^T, \quad (21)$$

we can write the regularized LS solution as

$$W = (\Psi^T\Psi + \alpha I)^{-1}\Psi^T Y. \quad (22)$$

4. EXPERIMENT

In this section we apply the proposed hybrid neuro-neo-fuzzy network with the adaptive fuzzy Kohonen clustering algorithm to the problem of Doppler sonographic image analysis and detection of the blood flow areas in a body. The results of the detection are used to visualize the blood flow on the color Doppler digital images in real-time mode.

The Doppler sonographic images are derived from the parameters of the echo signal reflected from moving blood fractions, which is received by the Doppler sonographic scanner.

According to the Doppler effect frequency of ultrasonic oscillations reflected from the moving fractions differs from the base frequency f_t of the probe signal. Difference of these signals defines the Doppler frequency shift F_d which is proportional to the radial component of the fractions velocity:

$$F_d = 2 \cdot f_t \cdot \frac{V_r}{C},$$

where C is the speed of ultrasound spread.

By estimating the parameters of the frequency F_d it is possible to detect the motion of the body and blood fractions in each of the measurement volumes. However, the signals reflected from the body and from the moving blood fractions are greatly overlapping and in order to separate areas of blood flow an additional filtering procedure must be applied.

The source data set contains a series of two-dimensional frames. Each pixel of the frames is represented by the complex-valued signal. The length of each signal is 16 samples. The pixels can be classified into two possible classes: volumes with and without the blood flow.

We use 10 low-pass FIR filters with different parameters in order to separate the effective signal for measurement volumes in different conditions. Denote signals after the filtering units by

$$z^{[f]}(k) = (z_1^{[f]}(k), z_2^{[f]}(k), \ldots, z_l^{[f]}(k), \ldots, z_L^{[f]}(k)),$$

where $z^{[0]}(k)$ is an input signal, $f = 1, 2, \ldots, 10$ is a filter number, $l = 1, 2, \ldots, L$ is a number of the sample on the filter output, L is a number of input samples.

We use the following statistical characteristics of the source signal and the filtered signals $z^{[f]}(k)$ as the input data set features for the neural and neuro-fuzzy networks:

1. Logarithm of the signal power:

$$\begin{aligned} P^{[f]}(k) = \ln \Bigg(&\frac{1}{2} \left(\mathrm{Re}^2(z_1^{[f]}(k)) + \mathrm{Im}^2(z_1^{[f]}(k)) \right) \\ &+ \sum_{l=2}^{L-1} \left(\mathrm{Re}^2(z_l^{[f]}(k)) + \mathrm{Im}^2(z_l^{[f]}(k)) \right) \\ &+ \frac{1}{2} \left(\mathrm{Re}^2(z_L^{[f]}(k)) + \mathrm{Im}^2(z_L^{[f]}(k)) \right) \Bigg), \end{aligned}$$

2. Amplitude and phase components of the autocorrelation coefficient of the signal:

$$\begin{aligned} S^{[f]}(k) &= A^{[f]}(k) e^{i\Phi^{[f]}(k)} \\ &= \sum_{l=2}^{L} \left(\mathrm{Re}^2(z_l^{[f]}(k)) + i\mathrm{Im}^2(z_l^{[f]}(k)) \right) \\ &\quad \cdot \left(\mathrm{Re}^2(z_{l-1}^{[f]}(k)) - i\mathrm{Im}^2(z_{l-1}^{[f]}(k)) \right), \end{aligned}$$

where $A^{[f]}(k)$ is an amplitude component and $\Phi^{[f]}(k)$ is a phase component.

Each pixel is encoded as 33-dimensional feature vector:

$$\begin{aligned} x(k) = (&P^{[0]}(k), A^{[0]}(k), \Phi^{[0]}(k), \\ &P^{[1]}(k), A^{[1]}(k), \Phi^{[1]}(k), \ldots \\ &\ldots, P^{[10]}(k), A^{[10]}(k), \Phi^{[10]}(k))^T. \end{aligned}$$

The training data set was constructed by random selection of the samples from the areas with a priori known crisp classification in different frames. It consists of 8458 samples total and 1880 of them representing the areas with blood flow present.

We compare the proposed hybrid neuro-fuzzy network [11] to the conventional neural networks: radial-basis function network and multilayer perceptron (MLP). Classification error rates on the training data set are shown in Table 1. Visual presentation of the classification results are shown on Fig. 4 for the RBFN, Fig. 5 for the MLP, and Fig. 6 for the RBF-NFN. The volumes with blood flow detected are outlined.

Table 1. Classification results of the Doppler sonographic images

Network	**Classification error on the training set**
RBF-NFN	10,18%
RBFN	16,31%
MLP	13,10%

The results show better classification error rates for the RBF-NFN comparatively to the conventional RBFN and MLP. The proposed network show is able to correctly detect smaller areas of blood flow.

5. CONCLUSION

In this paper a novel adaptive fuzzy Kohonen clustering algorithms are proposed to train the hidden Kohonen-like layer of the heterogeneous neuro-fuzzy network RBF-NFN. The RBF-NFN network comprises the concepts of RBFN and competitive networks by combining two layers of the fuzzy Kohonen clustering network and the neo-fuzzy neurons. The learning algorithms are simple and computationally efficient. The proposed network was used to solve the real-life problem of blood flow detection based on the Doppler sonographic images.

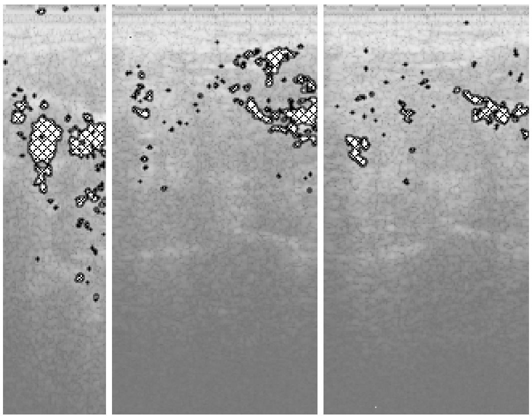

Fig. 4. Sample frames with classification by RBFN

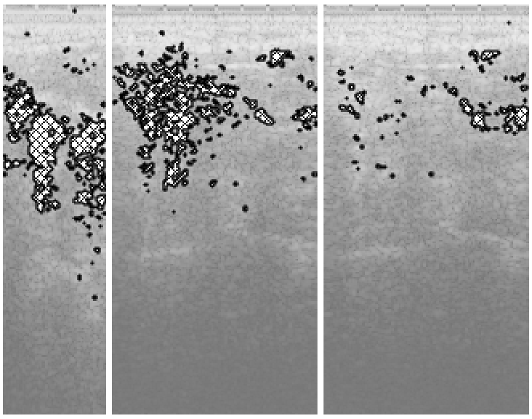

Fig. 5. Sample frames with classification by MLP

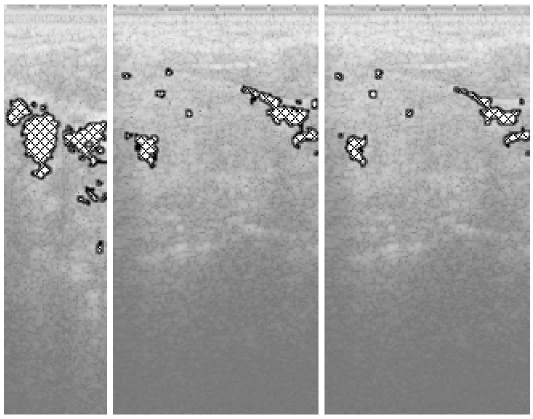

Fig. 6. Sample frames with classification by RBF-NFN

6. REFERENCES

[1] T. Kohonen, *Self-Organizing Maps*, Springer-Verlag, Berlin, 1995.

[2] F. Höppner, F. Klawonn, and R. Kruse, *Fuzzy-Clusteranalyse. Verfahren für die Bilderkennung, Klassifikation und Datenanalyse*, Vieweg, Braunschweig, 1996.

[3] P. Vuorimaa, "Fuzzy self-organizing maps," *Fuzzy Sets and Systems*, vol. 66, pp. 223–231, 1994.

[4] P. Vuorimaa, "Use of the fuzzy self-organizing maps in pattern recognition," in *Proc. 3-rd IEEE Int. Conf. Fuzzy Systems "FUZZ–IEEE'94". Orlando, USA, June 26-29, 1994*, Orlando, 1994, pp. 798–801.

[5] Ye. Bodyanskiy, Ye. Gorshkov, V. Kolodyazhniy, and A. Stephan, "Combined learning algorithm for a self-organizing map with fuzzy inference," in *Computational Intelligence, Theory and Applications. Advances in Soft Computing*, B. Reusch, Ed., pp. 641–650. Springer-Verlag, Berlin; Heidelberg, 2005.

[6] E. C.-K. Tsao, J. C. Bezdek, and N. R. Pal, "Fuzzy Kohonen clustering networks," *Pattern Recognition*, vol. 27, pp. 757–764, 1994.

[7] R. D. Pascual-Marqui, A. D. Pascual-Montano, K. Kochi, and J. M. Caraso, "Smoothly distributed fuzzy c-means: a new self-organizing map," *Pattern Recognition*, vol. 34, pp. 2395–2402, 2001.

[8] A. K. Palit and D. Popovic, *Computational Intelligence in Time-Series Forecasting*, Springer-Verlag, London, 2005.

[9] J. R. Rabunal and J. Dorado, *Artificial Neural Networks in Real-Life Applications*, Idea Group Publ., 2006.

[10] J. C. Bezdek, *Pattern Recognition with Fuzzy Objective Function Algorithms*, Plenum Press, N.Y., 1981.

[11] Ye. Bodyanskiy, Ye. Gorshkov, V. Kolodyazhniy, and T. Rvachova, "Hybrid neuro-fuzzy network with variable shape basis functions," in *Proc. East West Fuzzy Coll. 2006, 13-th Zittau Fuzzy Coll. Zittau, Germany, 13–15 Sep., 2006*, Zittau/Goerlitz, 2006, pp. 322–331, University of Applied Sciences (FH).

[12] T. Yamakawa, E. Uchino, T. Miki, and H. Kusanagi, "A neo fuzzy neuron and its applications to system identification and prediction of the system behavior," in *Proc. 2nd Int. Conf. on Fuzzy Logic and Neural Networks "IIZUKA-92"*, Iizuka, Japan, 1992, pp. 477–483.

[13] T. Miki and T. Yamakawa, "Analog implementation of neo-fuzzy neuron and its on-board learning," in *Computational Intelligence and Applications*, N. E. Mastorakis, Ed., WSES Press, Piraeus, 1999, pp. 144–149.

[14] R. Hecht-Nielsen, "Counterpropagation networks," *Applied Optics*, vol. 26, pp. 4979–4984, 1987.

[15] R. Hecht-Nielsen, "Applications of counterpropagation networks," *Neural Networks*, vol. 1, no. 2, pp. 131–139, 1988.

[16] S. Haykin, *Neural Networks. A Comprehensive Foundation*, Prentice Hall, Inc., Upper Saddle River, N.Y., 1999.

[17] Z. Zhang, N. Zheng, and T. Wang, "Fuzzy generalization of the counter-propagation neural network: a family of soft competitive basis function neural networks," *Soft Computing*, vol. 5, no. 6, pp. 440–450, 2001.

[18] R. Krishnapuram and J. Keller, "A possibilistic approach to clustering," *IEEE Trans. on Fuzzy Systems*, vol. 1, pp. 98–110, 1993.

[19] F.-L. Chung and T. Lee, "Unsupervised fuzzy competitive learning with monotonically decreasing fuzziness," in *Proc. 1993 Int. Joint Conf. on Neural Networks*, 1993, pp. 2929–2932.

[20] Ye. Bodyanskiy, V. Kolodyazhniy, and A. Stephan, "Recursive fuzzy clustering algorithms," in *Proc. East West Fuzzy Coll. 2002, 10-th East-West Fuzzy Coll.*, Zittau/Goerlitz, 2002, pp. 276–283, HS.

[21] Ye. Bodyanskiy, "Computational intelligence techniques for data analysis," in *Lecture Notes in Informatics*, vol. P–72, pp. 15–36. Bonn, Germany, 2005.

[22] K. J. Arrow, L. Hurwitz, and H. Uzawa, *Studies in Linear and Nonlinear Programming*, Stanford University Press, Stanford, CA, 1958.

[23] D. C. Park and I. Dagher, "Gradient based fuzzy c-means (GBFCM) algorithm," in *Proc. IEEE Int. Conf. on Neural Networks*, Orlando, FL, USA, 1994, pp. 1626–1631, IEEE Press.