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Susanne Jakob
Dipl.-Ing. Helge Drumm

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K. Niayesh, H. Ghorbani, A. Hayati-Soloot

A Maximum – Q inductor design for pulsed power applications

APPLIED ELECTROMAGNETICS AND CIRCUIT THEORY

1 Introduction

Air core inductors find diverse applications in high current pulse technology. Because of very high amplitudes of currents and very fast time variations, it is not efficient to use inductors with ferromagnetic cores. The high current inductors can be used as energy storage components in inductive energy storage systems or as pulse shaping elements in capacitive energy storage systems. In both applications, it is important to decrease the losses of inductors to be able to achieve the highest possible efficiencies or alternatively to realize the necessary inductance using the minimum cable length. High-Q inductors are also very important for other applications like in microwave circuits and in on-chip designs of the RF integrated circuits. These inductors are realized in solenoid or spiral shape.
In this paper, a method to maximize the inductance of an air – core inductor constructed from a given length high voltage cable is proposed. For this purpose, an experimentally verified exact calculation method based on Neumann equation for magnetic vector potential is applied. In this method the inductor is assumed to be a combination of a number of windings located in different positions. This generalized formulation makes it possible to consider solenoid, spiral and any arbitrary combination of solenoid and spiral shapes. The self and mutual inductances between different windings are calculated and a mathematical formulation to find the maximum inductance of the inductor is derived. The results indicate inductance increases of many folds can be achieved using the optimum dimensions for the air – core inductor with the same cable length. The method proposed in this paper can also be applied to other applications like inductors for RF and microwave circuits.

2 Theoretical approach

In general, any air core inductor can be considered as a combination of a number of one-winding units located somewhere in the space. The total inductance (L_{total}) of such a combination can be simply expressed as:

$$L_{total} = \sum_{i=1}^{n} \sum_{j=1}^{n} L_{ij} \qquad (1)$$

Where L_{ii} is the self inductance of the ith unit, L_{ij} the mutual inductance between *ith* and *jth* units and n the number of windings. In this way, the calculation of the inductance of any arbitrary inductor with a number of windings can be reduced to find a general formulation for the self-inductance of a one-winding unit and the mutual inductance between two such units.

2.1 Mutual-inductance between two windings

To formulate the mutual inductance between two windings, the magnetic vector potential ($\vec{A}$) produced by a current source distribution is taken into consideration. Applying the Coloumb's gauge [1], the magnetic vector potential ($\vec{A}$) obtained solving the following equation:

$$\vec{\nabla}^2 \vec{A} = -\mu_0 \vec{j} \tag{2}$$

With the answer as:

$$\vec{A} = \int_{v'} \frac{\mu_0\, \vec{j}(\vec{r}')\cdot dv'}{4\pi|\vec{r}-\vec{r}'|} \tag{3}$$

Under the simplifying assumption that all currents are concentrated at the mid of the cable, the answer can be simplified to:

$$\vec{A} = \int_{c'} \frac{\mu_0\, I(\vec{r}')\cdot d\vec{l}'}{4\pi|\vec{r}-\vec{r}'|} \tag{4}$$

where $\vec{r}'$ is the vector which its end points to current component $I.d\vec{l}'$, $\vec{r}$ a vector that $\vec{A}$ is calculated at its end and c' is the path which current flows in it (see Figure 1).

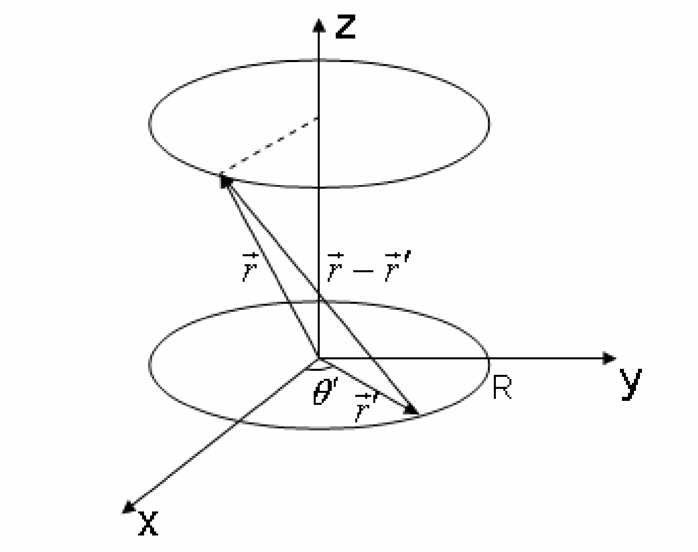

Figure 1: Two windings of an inductor

From the figure 2, the following geometrical relationships can be derived:

$$|\vec{r}-\vec{r}'| = \sqrt{z^2 + 2R^2 - 2R^2\cos(\theta')}$$

$$d\vec{l}' = R\,d\theta'\left(-\sin(\theta')\vec{i} + \cos(\theta')\vec{j}\right) \tag{5}$$

Substituting (5) in (4) results in:

$$\vec{A} = \frac{\mu_0 I}{4\pi}\int_0^{2\pi} \frac{R\left(-\sin(\theta')\vec{i} + \cos(\theta')\vec{j}\right)\cdot d\theta'}{\sqrt{z^2 + 2R^2 - 2R^2\cos(\theta')}} \tag{6}$$

Considering the definition of the magnetic vector potential, the magnetic flux through the second winding generated by the current of first winding can be easily calculated using:

$$\phi = \iint_S \left(\vec{\nabla} \times \vec{A}\right) \cdot \hat{n}\, ds = \oint_c \vec{A} \cdot d\vec{l} \tag{7}$$

So for mutual inductance between two windings, the following expression can be derived:

$$L_{12} = \frac{\psi}{I} = \frac{\phi}{I} = \frac{\mu_0 R^2}{2} \int_0^{2\pi} \frac{\cos(\theta') \cdot d\theta'}{\sqrt{z^2 + 2R^2 - 2R^2 \cos(\theta')}} \tag{8}$$

2.2 Self-inductance of a winding

The magnetic flux through one winding produced by its current can be calculated in a similar way. The only difference is that the magnetic flux through the conductor itself can not be calculated accurately because of the simplifying assumption that the current is concentrated at the middle point of the conductor. Therefore, the formulation used to calculate the mutual inductance between two windings is applied only to calculate a portion of the self inductance related to the magnetic flux, which is not through the conductor. For the other portion (inner inductance), it is assumed that magnetic field of current in ring cable makes the same inductance as that in a long direct cable, because the radius of the inductor R is at least ten times bigger than the radius of conductor a. A long direct cable has 0.05 µH in one meter [2]. This value is simply added to the inductance calculated using equation 10.

According to the figure 2, end of vector $\vec{r}$ points to a ring with radius ($R-a$).

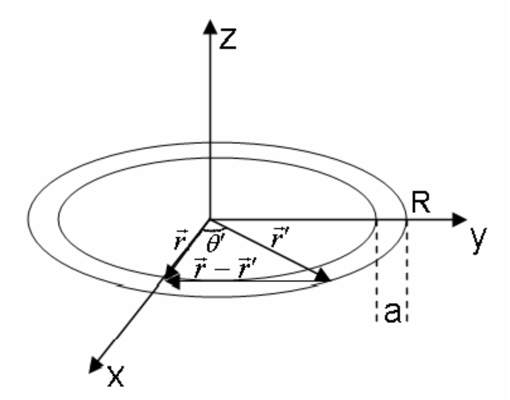

Figure 2: one ring of the inductor

In this case, the following expression for magnetic vector potential and self-inductance are derived:

$$\vec{A} = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{R\left(-\sin(\theta')\vec{i} + \cos(\theta')\vec{j}\right) \cdot d\theta'}{\sqrt{a^2 + 2R^2 - 2Ra - 2R(R-a)\cos(\theta')}} \tag{9}$$

and

$$L_{11} = \frac{\psi}{I} = \frac{\phi}{I} = \frac{\mu_0 R(R-a)}{2} \int_0^{2\pi} \frac{\cos(\theta') \cdot d\theta'}{\sqrt{a^2 + 2R^2 - 2Ra - 2R(R-a)\cos(\theta')}} \tag{10}$$

3 Results

3.1 Experimental verification

To verify the calculation method, simulation and measurement of a number of different inductors constructed using a wire, which has 1 mm conductor diameter, 1.35 mm entire diameter, have been compared. The constructed inductors have been solenoid type with a diameter of 25 cm. As it can be seen in figure 3, there is almost no difference between simulation and measurement results for inductors with lower number of turns. In case of very high number of turns, the differences are still smaller than 2 percent.

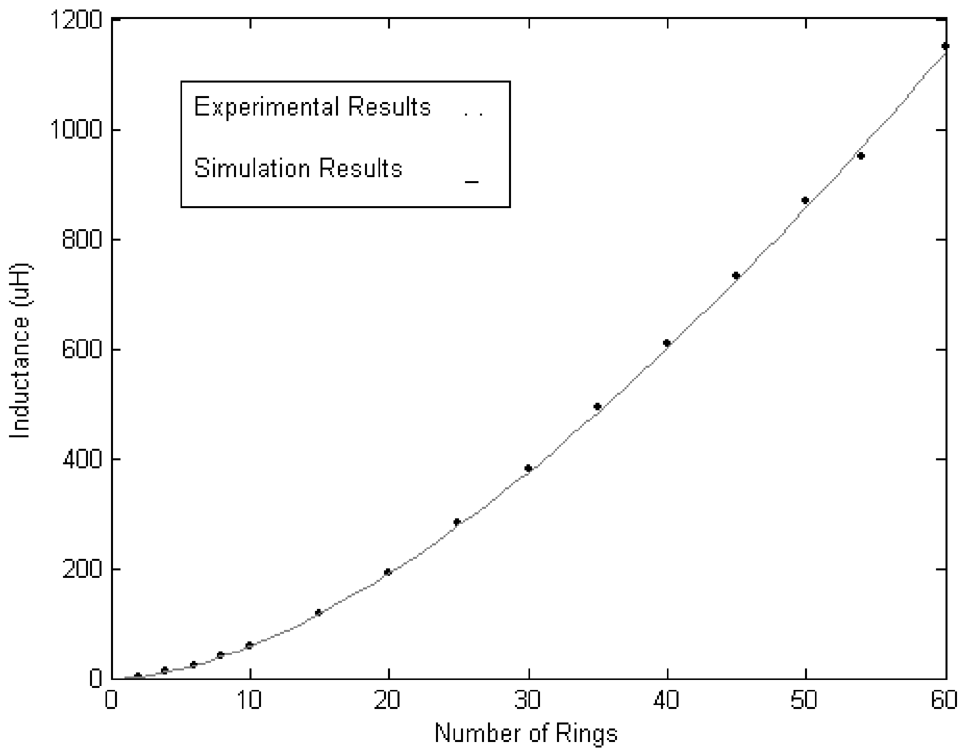

Figure 3: Comparison between experimental results and simulation results for an inductor with *a=0.5 mm*, *b=0.675 mm*, *R=12.5 cm*

3.2 Maximum inductance of one-layer solenoid inductors

In practical applications, it is desired to find out the optimum shape of an inductor constructed from a given cable to maximize its inductance. In those applications the minimum distance between two adjacent windings is given by the characteristics of the cable like the total diameter and the conductor diameter. Figure 4 shows the simulation results for one-layer solenoid type inductors constructed from cables with different length and total diameter of 0.388 cm and conductor diameter of 0.138 cm.

As it can be seen, there is a maximum achievable inductance of 531 µH using the 50 m cable. In this case, the solenoid type inductor has 30 turns and a radius of 26.5 cm. It is also interesting to note that the optimum number of turns for a given inductor shape is dependent on the cable length used to construct it.

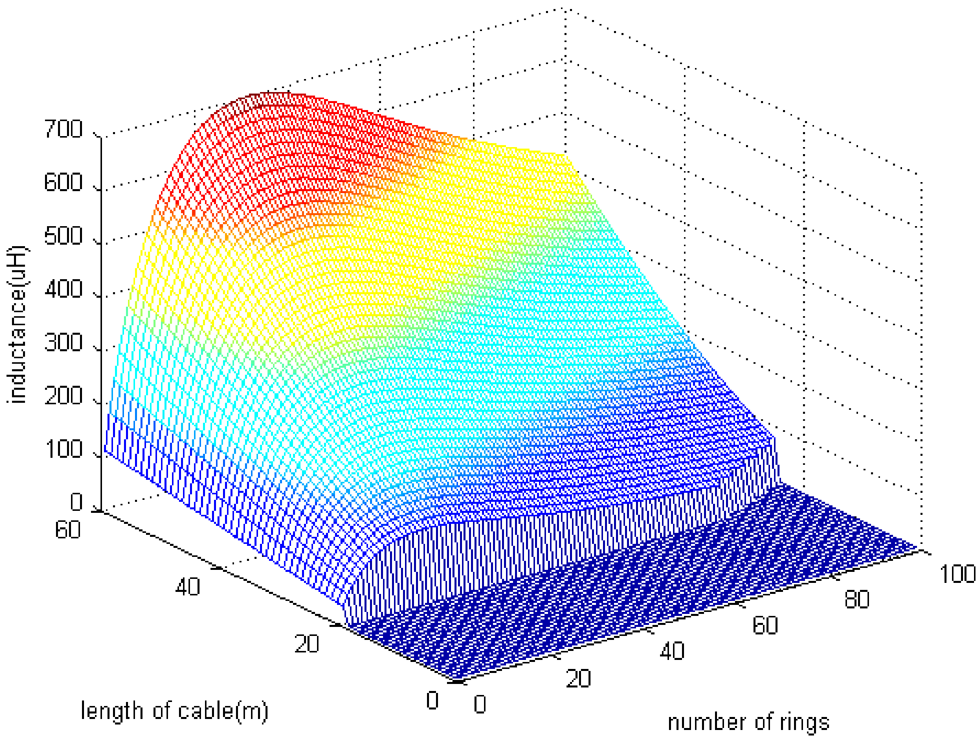

Figure 4: inductance of $1 \times 6 \quad mm^2$ cable in different length and rings

3.3 Maximum inductance of solenoid inductors with several layers

A more general inductor would be a solenoid type inductor with several layers. For such inductors, there are different parameters, i.e. number of layers and number of turns per layer, which could be optimized.

In figure 5, a typical simulation result is shown. In this case, a 10 m cable with total diameter of 0.4 cm and conductor diameter of 0.2 cm is used. The maximum inductance is about L=71 μH for the inductor with 5 layers of winding and 5 turns per layer.

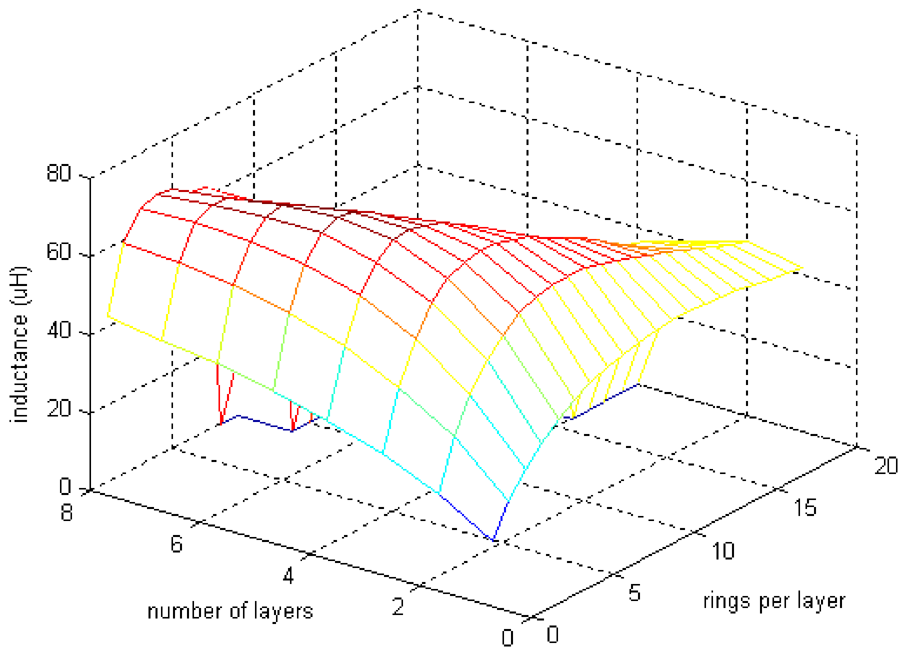

Figure 5: Inductance of inductors with same cable and several layers of winding

So using the proposed formulation for the inductance of an inductor constructed in the form of any arbitrary combination of different round windings, it is possible to find the optimum shape and dimensions of the inductor to achieve the highest inductance and maximum quality factor.

4 Conclusion

In this paper a method has been presented to formulate the inductance of any arbitrary inductor constructed in the form of a combination of a number of windings, based on the self inductance of the windings and the mutual inductances between different windings. The simulation results have been verified by measurements. It has been shown that application of the proposed formulation makes it possible to find out the optimum shape and dimension of the inductor to achieve the maximum inductance level. Especially in cases of solenoid type inductors with one or several layers, the simulation results for inductors constructed from a given cable length are presented. It has been shown that there are maximum values of inductances depending on the number of layers and the number of turns per layer.

References:

[1] Field and Wave Electromagnetic, David k. Cheng
[2] Power System Analysis John Grainger, William Stevenson

Authors:

Dr. Kaveh Niayesh
Hossein Ghorbani
Amir Hayati-Soloot
School of Electrical and Computer Engineerin
Faculty of Engineering, University of Tehran
North Kargar Ave., P.O.B. 14395/515
IR-14395, Tehran, Iran
Phone: +98-21-88011247
Fax: +98-21-88027766
E-mail: kniayesh@ut.ac.ir