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## Electric Field Calculation at Cable Terminations Using Conformal Mapping and Equivalent Electrodes Method

### INTRODUCTION

It is possible to use analogies between plan-parallel and axi-symmetrical electric systems for analytical determining of electric potential and field distributions at the cable terminations. Conformal mapping is applied for those calculations. For modeled cable terminations this method can not be used, but equivalent electrodes method (EEM) is generally applicable.

At large distances from the termination, inside the cable, it may be considered that the field is approximately homogenous and charge distribution is continuous on its conductors. It is possible to determine the potential at the cable end as superposition of two components: the first one originates from continuous distribution of the electric charge, and the second one from equivalent electrodes. Equivalent electrodes (EE) are appointed at the end of coaxial cable, where the edge effect exists. Equivalent electrodes are non limited cylindrical electrodes, having radii equivalent to a square of segments wight, which replace these parts of cable termination.

### THEORETICAL APPROACH

Using the Schwartz-Cristoffel's transform and function of complex variable:

$$z = x + jy = \frac{D}{\pi} \left( e^{-\frac{\pi \bar{w}}{U}} + \frac{\pi}{U} \bar{w} - 1 + j\pi \right) \quad (1)$$

for plan-parallel system (Fig.1), where  $w = u + jv$  and  $\bar{w} = u - jv$ , coordinates with electric potential  $\varphi = u$ , can be obtained.  $U$  is voltage the coaxial cable is supplied by. It is possible to choose  $D = \pi$  and  $U = \pi$ . For this case coordinates of equipotential curves are:

$$x = e^{-u} \cos v + u - 1; \quad y = e^{-u} \sin v + \pi - v. \quad (2)$$

If  $E_0 = \frac{U}{D}$ , the intensity of the electric field is

$$\frac{E_{kp}}{E_0} = \frac{1}{\sqrt{1 + e^{-\frac{2\pi u}{U}} - 2e^{-\frac{\pi u}{U}} \cos \frac{\pi v}{U}}} = \frac{e^{\frac{\pi u}{2U}}}{\sqrt{2} \sqrt{\cosh \frac{\pi u}{U} - \cos \frac{\pi v}{U}}}, \quad (3)$$

but electric field for normalized values  $D$  and  $U$  is

$$E_\pi = \frac{1}{\sqrt{e^{-2u} - 2e^{-u} \cos v + 1}} = \frac{1}{\sqrt{2} e^{-u} \sqrt{\cosh u - \cos v}}. \quad (4)$$

Equipotential contours ( $\varphi = const.$ ) and electric field lines ( $v = const.$ ) are shown in Fig.2.

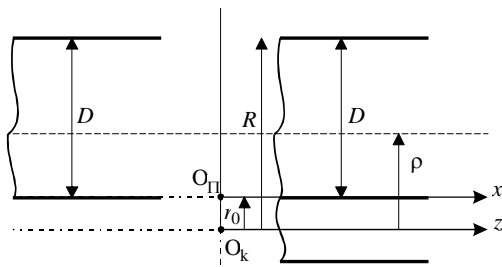


Fig. 1. Plan-parallel and cylindrical electrode systems.

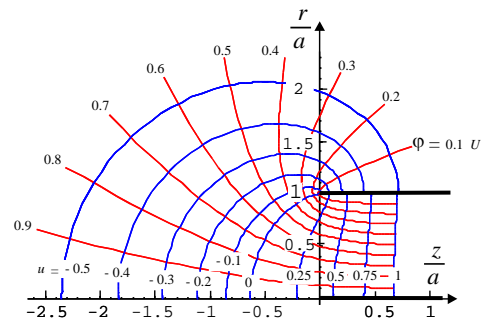


Fig. 2. Equipotential curves and electric field lines.

Using adequate analogies between plan-parallel and axi-symmetrical electric systems, methods, experiences and results, which are very serious and detailed considered at plan-parallel systems, can be successfully extended for solving similar problems at axi-symmetrical electric systems.

It is possible to recalculate values for electric field and potential at cable terminations. This procedure is valid for cable joints determining, too, but at modeled cable terminations and cable joints, any other numerical method is recommended, for example EEM.

Based on previous results, electric field intensity at the end of coaxial line can be determined:

$$E_k = E_\pi \frac{\rho}{r} = E_\pi \frac{r_0 + y_\pi}{r_0 + y}, \quad (5)$$

where  $r_0$  is radius of inner conductor,  $R = r_0 + D$  is radius of outer conductor,  $r = r_0 + y$  is the distance between center lines, and

$$\rho = \frac{D}{\ln \frac{R}{r_0}}. \quad (6)$$

## APPLICATION OF THE EQUIVALENT ELECTRODES METHOD

EEM is applied on non-modeled (Fig.3) and modeled (Fig.4) cable termination. Firstly, electric potential and electric field components are calculated for plan-parallel "equivalent" of terminations (ends of strip lines). The next step is recalculation of their values for real cases. The distance between strip electrodes is  $2d$ .

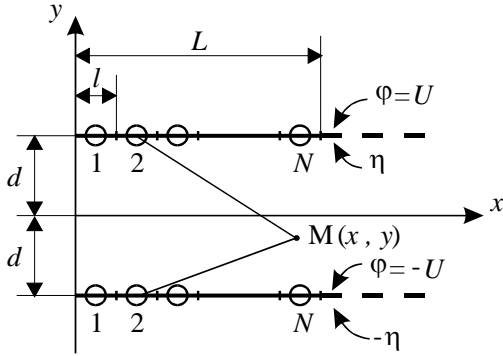


Fig. 3. EE at the non-modeled cable terminations.

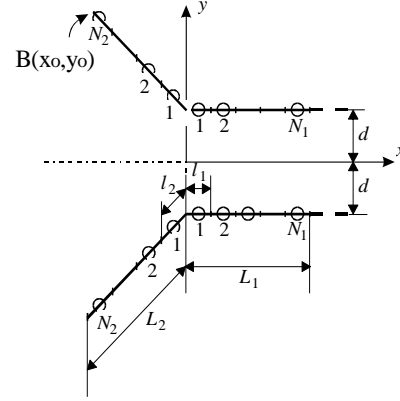


Fig. 4. EE at the modeled cable terminations.

Far away from the strip line ends (Fig.3 and 4) charge distribution is continuous. There is distributed positive charge on upper and negative charge on the lower conductor. Charge density per unit surface is constant in the distant regions from breaks, which are on the upper, and on the lower conductor, respectively:

$$\eta = \varepsilon E_0 \quad \text{and} \quad \eta = -\varepsilon E_0. \quad (7)$$

where  $E_0 = \frac{U}{d}$ .

If it is presumed that such charge distribution is also in the surroundings of the line end, the approximate expression for potential will be:

$$\frac{\varphi_{\text{apr}}(x, y)}{U} = \frac{1}{4\pi d} \left( \begin{aligned} &\pi(a - |b|) - (L - x) \ln \frac{(L - x)^2 + a^2}{(L - x)^2 + b^2} - \\ &- 2 \left( a \arctan \left( \frac{L - x}{a} \right) - b \arctan \left( \frac{L - x}{b} \right) \right) \end{aligned} \right). \quad (8)$$

On the basis of the expression for the potential, approximate expressions for electric field's components are determined:

$$\frac{E_{x \text{ apr}}(x, y)}{E_0} = -\frac{1}{4\pi} \ln \frac{(L - x)^2 + a^2}{(L - x)^2 + b^2}; \quad (9)$$

$$\frac{E_{y\text{apr}}(x, y)}{E_0} = -\frac{1}{2\pi} \begin{cases} \frac{\pi}{2}(1 - \text{sgn}(y - d)) - \arctan\left(\frac{L-x}{a}\right) + \arctan\left(\frac{L-x}{b}\right), & y \neq d \\ \frac{\pi}{2}(1 + \text{sgn}(L-x)) - \arctan\left(\frac{L-x}{2d}\right), & y = d \end{cases}, \quad (10)$$

where

$$\text{sgn}(f(x)) = \begin{cases} 1, & f(x) > 0 \\ 0, & f(x) = 0; \\ -1, & f(x) < 0 \end{cases} \quad (11)$$

Charge distributions mentioned above, do not coincide with the real ones, because the boundary conditions are not satisfied, so consequently the conductors are not of constant potential. Due to this, additional expressions are superposed to the previous ones.

If equivalent electrodes are used as additional elements, excellent results are obtained. Infinite linear electrodes are employed as equivalent electrodes, having cross section's

radius  $a_{e1} = \frac{l_1}{4} = \frac{l}{4} = \frac{L_1}{4N} = \frac{L}{4N} = a_e$ , whose central lines are located at the places:

$$x_n = (2n-1)\frac{l}{2}; \quad y_n = d, \quad (12)$$

where  $n = 1, 2, \dots, N$ .

Using the equivalent electrodes method, it is possible to determine electric potential,

$$\frac{\varphi(x, y)}{U} = \frac{\Phi_{\text{apr}}(x, y)}{U} + Z(x, y, x_n), \quad (13)$$

and electric field components:

$$\frac{E_x(x, y)}{E_0} = -\frac{1}{4\pi} \left( \ln \frac{(L-x)^2 + a^2}{(L-x)^2 + b^2} - 16\pi d^2 y \sum_{n=1}^N \frac{x-x_n}{((x-x_n)^2 + a^2) \cdot ((x-x_n)^2 + b^2)} Q_n \right); \quad (14)$$

$$\frac{E_y(x, y)}{E_0} = -\frac{1}{2\pi} \left( \frac{\pi}{2}(1 - \text{sgn}(y - d)) - \arctan\left(\frac{L-x}{a}\right) + \arctan\left(\frac{L-x}{b}\right) - 4\pi d^2 \times \right. \\ \left. \times \sum_{n=1}^N \frac{(x-x_n)^2 + d^2 - y^2}{((x-x_n)^2 + a^2) \cdot ((x-x_n)^2 + b^2)} Q_n \right), \quad (15)$$

at arbitrary chosen point of the strip line end region.

Relative charge is

$$Q_n = \frac{q'_n}{2\pi\epsilon U}, \quad (16)$$

where  $q'_n$  denotes the total uniform line charge density of the  $n$ -th equivalent electrode,

$\varepsilon$  is permittivity of the medium,

$$W(x, y, x_n) = \ln \sqrt{\frac{(x - x_n)^2 + (y + d)^2}{(x - x_n)^2 + (y - d)^2 + \delta(x, x_n) \delta(y, d) a_e^2}}, \quad (17)$$

and

$$\delta(x, x_n) = \begin{cases} 1, & x = x_n \\ 0 & x \neq x_n \end{cases}, \quad (18)$$

is Dirac's delta function.

Table 1 shows obtained convergence of the results when the number of equivalent electrodes and the length of a strip line termination, which is being modeled by equivalent electrodes, are used as parameters. Electric potential and strength of electric field components are compared with values, obtained by using conformal map. For example, at the point  $x = 0.108d$ ,  $y = 0.33d$ , potential is  $\varphi_t = 0.3U$ . The proper electric field components are  $E_x = 0.155001 U/d$  and  $E_y = -1.029112 U/d$ .

Better results are achieved with a large number of EE (Tab. 2). In this case  $l = 1.4d$ .

**Table 1:** Results obtained by using EEM with different EE number and different  $l/d$ .

$l/d$	$N$	$\varphi/U$	Relative error (%)	$E_x d/U$	$E_y d/U$
1	10	0.30144	0.4798	0.1744	-1.0424
2	20	0.30149	0.4991	0.1714	-1.0422
3	30	0.30151	0.5037	0.1708	-1.0421
4	40	0.30151	0.5058	0.1705	-1.0421

**Table 2:** Results obtained by using EEM with different number of EE and conformal mapping

$N$	$\varphi/U$	Relative error (%)	$E_x d/U$	$E_y d/U$
2	0.27248	9.17115	-0.068	-0.841
5	0.30219	0.73303	0.208	-1.068
10	0.30205	0.68349	0.186	-1.053
15	0.30138	0.46134	0.170	-1.040
20	0.30107	0.35679	0.163	-1.035
30	0.30077	0.25710	0.157	-1.031
40	0.30061	0.20601	0.155	-1.029
60	0.30057	0.15051	0.155	-1.029

In the case when  $n_1$  equivalent electrodes are placed at the cable ends, an accurate result at one decimal digit is obtained. When  $n_1 n_2$  equivalent electrodes are placed at the cable ends, an accurate result at two digits is obtained, and similarly when  $n_1 n_2 \dots n_l$  are used the accuracy of the results are at  $l$  digits.

Equipotential curves, near the cable end, are plotted in Fig.5.

EEM can be successfully applied on modeled (Fig.4) strip line termination, and obtained potential distribution is presented in Fig. 6. Terminated point of added plate is  $B(-0.5d, d)$ .

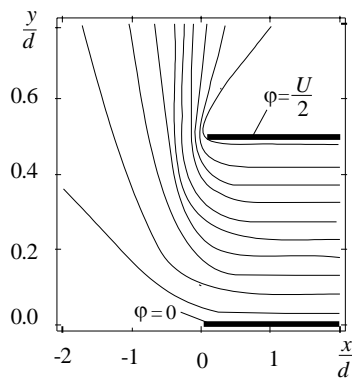


Fig. 5. Equipotential curves at non modeled terminations.

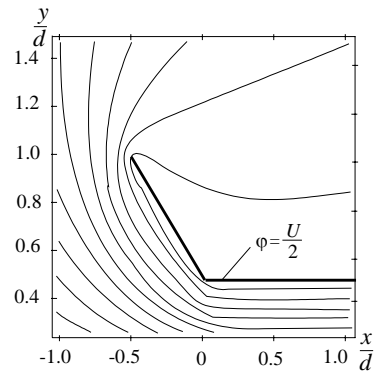


Fig. 6. Equipotential curves at modeled terminations.

## CONCLUSION

There is an analytical solution for electric field in the region near to strip line end. This analysis is carried out by EEM using. It is possible to recalculated those values into cable terminations ones.

But, conformal mapping method is very complex for modeled cable terminations determining. Because of that, EEM is much applicable and gives better accuracy.

Main advantages of the equivalent electrodes methods in comparison to all existing methods lie in the very high precision even in cases when relatively small number of equivalent electrodes is used. In a limit case when the number of the equivalent electrodes is very large (leads to infinity), results are absolutely accurate.

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