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Efficient harmonic balance method in nonlinear eddy current problems

Introduction

In the case of nonlinear dynamic electromagnetic eddy current problems with a sinusoidal excitation, time stepping methods usually need many periods to be stepped through until steady state is reached. Therefore, time stepping methods need a large computational effort. Since, for many applications, the steady state only is of interest, methods have to be developed to avoid calulating transient processes. In [2] a method is presented which uses the fixed point method (see [1]) to linearize the equation system of eddy current problems. This leads to an equation systems where the equations are decoupled for each time instant within a period. But this method suffers by approximating the differential quotient and the solution depends on the number of time steps within a period. Another way to determine the steady state only is to approximate the time dependent quantities by Fourier series. In [3] and [5], Fourier series are used to approximate the introduced potentials and the excitation as well as the reluctivity. The disadvantage here is that the equation for a certain harmonic is coupled with all harmonic components of the previous iteration step.
In this paper the fixed point method is used to linearize the equation, and the time dependent quatities are represented by Fourier series. This approach leads to a number of equations which are decoupled for each frequency. Several iterations have to be made to take the nonlinear behaviour into account. Using this method, only the right hand side of the equation is affected by the nonlinearity and must be updated in each step. The method has been applied to two 2D nonlinear eddy current problems which are compared with a time stepping method.

Method

The equation for nonlinear eddy current problems has the following form if Galerkin's method is applied :

$$\mathbf{S}(\mathbf{x})\cdot\mathbf{x}(t)+\frac{d}{dt}\left(\mathbf{M}(\mathbf{x})\cdot\mathbf{x}(t)\right)=\mathbf{f}(t). \tag{1}$$

The matrices **S** and **M** depend on **x** and so they must be updated at each iteration step. The vector **f***(t)* on the right hand side indicates the sinusoidal excitation and the time dependent vector **x**(*t*) is the solution of the equation. If the formulation leading to (1) is based on the flux density **B**, the nonlinear relationship between **H** and **B** can be separated into a linear and a nonlinear term :

$$\mathbf{H}=\mathbf{H}(\mathbf{B})=\nu_{FP}\mathbf{B}-\mathbf{M}_{FP}(\mathbf{B}). \tag{2}$$

Here $\mathbf{M}_{FP}$ is a magnetization-like quantity, which includes the nonlinear behaviour of the material and ν_{FP} is a fixed value that influences the convergence of the method. If the magnetic vector potential **A** and the time integrated electric scalar potential *v* are introduced and relation (2) is taken into account, Ampere's law and the law of conservation can be written as :

$$\begin{aligned} \nu_{FP}\,\mathrm{curl}\left(\mathrm{curl}\,\mathbf{A}\right)+\sigma\left(\frac{\partial\mathbf{A}}{\partial t}+\mathrm{grad}\,\frac{\partial \mathrm{v}}{\partial t}\right)&=\mathrm{curl}\,\mathbf{M}_{FP} \\ \mathrm{div}\left(\sigma\frac{\partial\mathbf{A}}{\partial t}+\sigma\,\mathrm{grad}\,\frac{\partial \mathrm{v}}{\partial t}\right)&=0\,, \end{aligned} \tag{3}$$

where the potentials **A** and *v* satisfy the equations :

$$\begin{aligned} \mathbf{B}&=\mathrm{curl}\,\mathbf{A} \\ \mathbf{E}&=-\frac{\partial\mathbf{A}}{\partial t}-\mathrm{grad}\,\frac{\partial \mathrm{v}}{\partial t}\;. \end{aligned} \tag{4}$$

The relation between the current density **J** and the electric field intesity **E** is given by **J** = σ **E**. If Galerkin's method is applied to (3), an algebraic equation system can be obtained :

$$\mathbf{S}\cdot\mathbf{x}(t)+\mathbf{M}\cdot\dot{\mathbf{x}}(t)=\mathbf{f}\left(\mathbf{x},t\right). \tag{5}$$

In (5) the matrices **S** and **M** are constant and the vector **x**(*t*) represents the unknown time dependent potentials **A** and *v*. The term **f**(**x**,*t*) consists of the excitation and a term which corresponds to $\mathbf{M}_{FP}$. Since $\mathbf{M}_{FP}$ depends on **B** (see (2)), the vector **f**(**x**,*t*) depends on the solution **x**. The vector **x** at the right hand side can be substituted by the previous solution and so **f**(**x**,*t*) becames a function of time only and (5) can be written as:

$$\mathbf{S}\cdot\mathbf{x}(t)+\mathbf{M}\cdot\dot{\mathbf{x}}(t)=\mathbf{g}(t). \tag{6}$$

The time dependent vectors **x**(*t*) and **g**(*t*) are now represented as complex Fourier series, where only harmonics up to a certain number are taken into account :

$$\begin{aligned} \mathbf{x}(t)&\cong\mathrm{Re}\left\{\sum_{k=0}^{N}\mathbf{X}_k e^{j\,k\omega_0\,t}\right\} \\ \mathbf{g}(t)&\cong\mathrm{Re}\left\{\sum_{k=0}^{N}\mathbf{G}_k e^{j\,k\omega_0\,t}\right\}. \end{aligned} \tag{7}$$

In (7), $\mathbf{X}_k$ and $\mathbf{G}_k$ are the complex amplitudes at the angular frequency $k\omega_0$ and ω_0 is the angular frequency of the excitation. The approximations of **x**(*t*) and **g**(*t*) together with (6) lead to *N*+1 linear equations, which are decoupled from each other :

$$\begin{aligned} &\left(\mathbf{S}+j\,k\omega_0\,\mathbf{M}\right)\cdot\mathbf{X}_k=\mathbf{G}_k \\ &k=0,\ldots,N. \end{aligned} \tag{8}$$

With this harmonic approach, the time derivative in (6) can be replaced by a multiplication with $jk\omega_0$ and so the problems of approximating $\dot{\mathbf{x}}(t)$ can be avoided.

The Algorithm

Starting from an arbitrary value for $\mathbf{M}_{FP}$ (usually zero), the time dependent vector $\mathbf{g}(t)$ in (6) and therefore also $\mathbf{G}_k$ can be determined. From the solution $\mathbf{X}_k$ of (8) the vector $\mathbf{x}$ can be evaluated as a function of time. Since $\mathbf{x}$ corresponds to the flux density $\mathbf{B}$, a new $\mathbf{M}_{FP}$ can be obtained by relation (2). This leads to an equation system (8) with new $\mathbf{G}_k$'s which is solved again. The procedure will be repeated, until a certain error criterion is smaller than a prescribed value. The flow chart of the proposed method is shown in Fig.1.

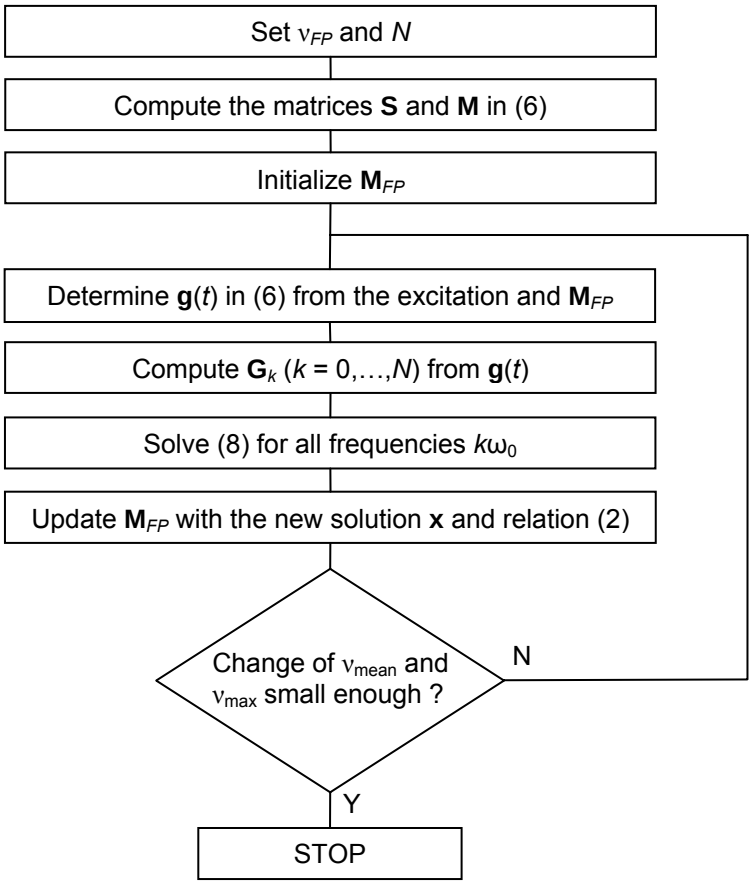

Fig. 1: Flow chart of the proposed method

Numerical Examples

The first example comprises an aluminum conductor with a given sinusoidal voltage of 0.66V per unit length. A conducting ferromagnetic wall shields the conductor (see Fig. 2). The nonlinear B-H curve of the wall is shown in Fig.3. On the symmetry plane, the normal component of the flux density **B** is zero.

The computation with the presented method has been carried out with several harmonics and the value for the relative permeability corresponding to ν_{FP} was chosen to 300 for all computations. The stopping criterion for the nonlinear iterations was chosen to be 0.1% and 1% for the mean and maximum relative variation of the reluctivity.

The present method has been compared with a time stepping method, where each period has been discretized in 40 time steps. For the nonlinear iterations, the same stopping criterions are used as before. For this example the time stepping method needs 6 periods to practically arrive at steady state.

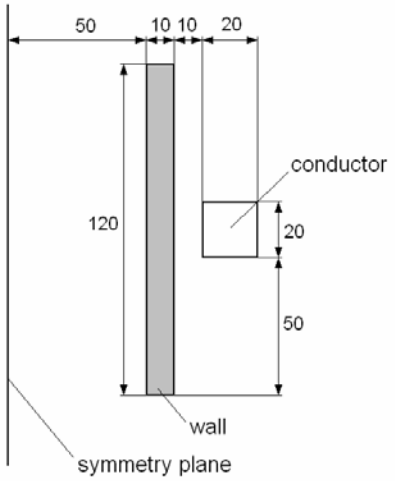

Fig. 2: Aluminum conductor ($\sigma=3.5\cdot10^7$ S/m) with a sinusoidal voltage per unit length of 0.66 V/m peak and 50 Hz. The steel wall has a conductivity of $\sigma=1.0\cdot10^7$ S/m.

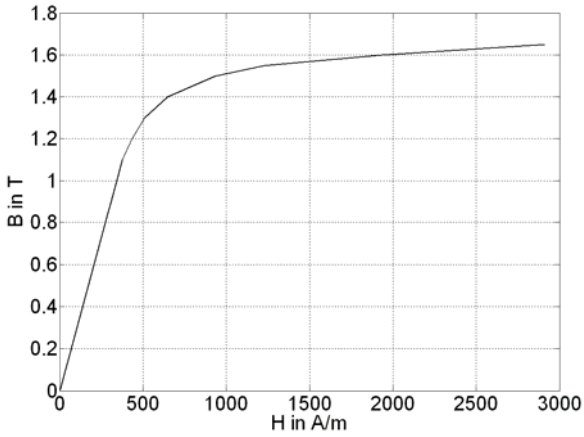

Fig. 3: B-H curve of the ferromagnetic steel wall.

Fig. 4 shows the spectrum of the current in the ferromagnetic wall. The numbers above the bars indicates the number of harmonics taken into account and the asterisk indicates the spectrum of the 6th period obtained by the step-by-step method. Only odd harmonics occur in the spectrum, and it can be seen, that the amplitude for a certain harmonic converge to a fix value as the number of harmonics is increased.

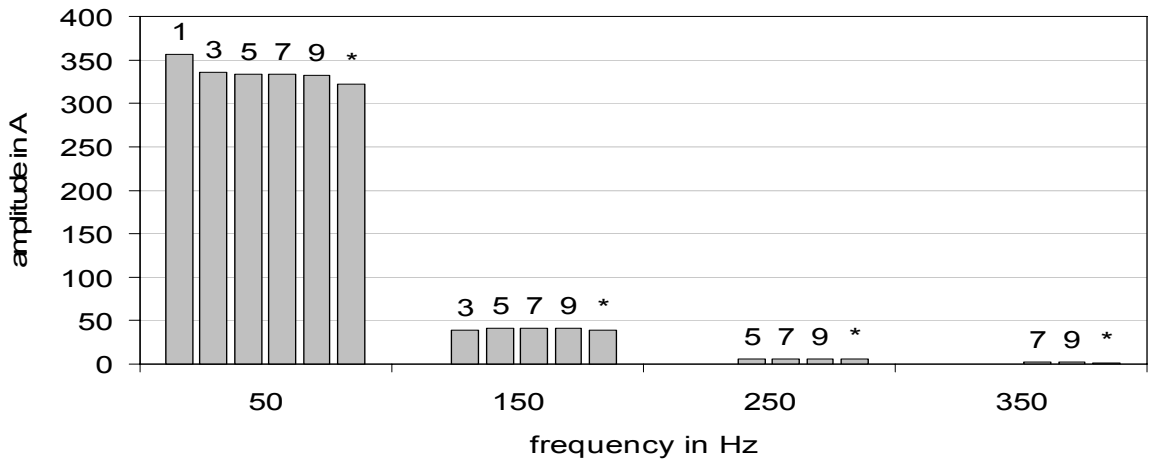

Fig. 4: Spectrum of the current in the ferromagnetic wall.

Table I: Number of equation systems to be solved

method	number of harmonics	number of equations
time stepping		2178
present method	1	32
	3	111
	5	185
	7	245
	9	324

In Table I, the number of equations to be solved as well as the number of harmonics taken into account is shown. It can be seen, that the number of equations are much higher for the time stepping method as for the present method. Also the number of equation systems increase with increasing number of harmonics.

The second 2D example consists of a copper conductor with a given sinusoidal voltage within a conducting ferromagnetic screen (Fig.5). The B-H curve of the screen is the same as in the previous example. With the time stepping method, 60 periods are required to arrive the steady state. One period has been discretized in 40 time steps and the mean and maximum relative variation of the reluctivity has been set to 0.1% and 1.0% respectively. In the present method, the relative permeability corresponding to ν_{FP} is set to 300 and the calculations have been done with several harmonics. The number of equation systems to be solved are given in Table II both for the time stepping method and for the present method.

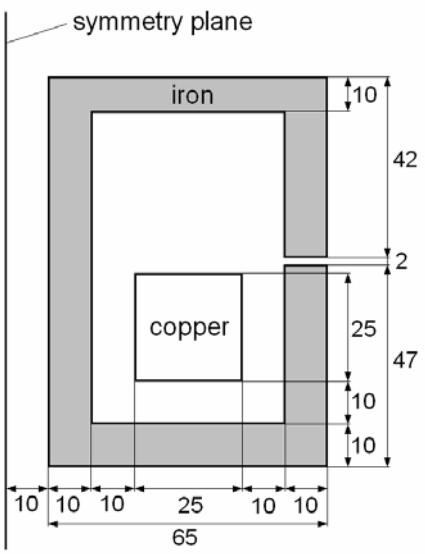

Fig.5: Copper conductor ($\sigma=5.7\cdot10^7$ S/m) surrounded by a ferromagnetic screen ($\sigma=5.7\cdot10^7$ S/m). The conductor is driven by a sinusoidal voltage with 50 Hz and 1.4V per unit length.

Table II: Number of equation systems to be solved

method	number of harmonics	number of equations
time stepping		31967
present method	1	72
	3	210
	5	265
	7	364
	9	459

The spectrum of the current in the copper conductor for the different calculations is given in Fig. 6. Again, the number above the bars gives the number of harmonics taken into account for the present method and the asterisk indicates the spectrum of the 60th period obtained by the step-by-step method. As in the example before, the amplitude of a certain harmonic component converges to a fix value. In the spectrum of the current only odd harmonics occur like in the example before, and only the first and the third harmonics are relevant.

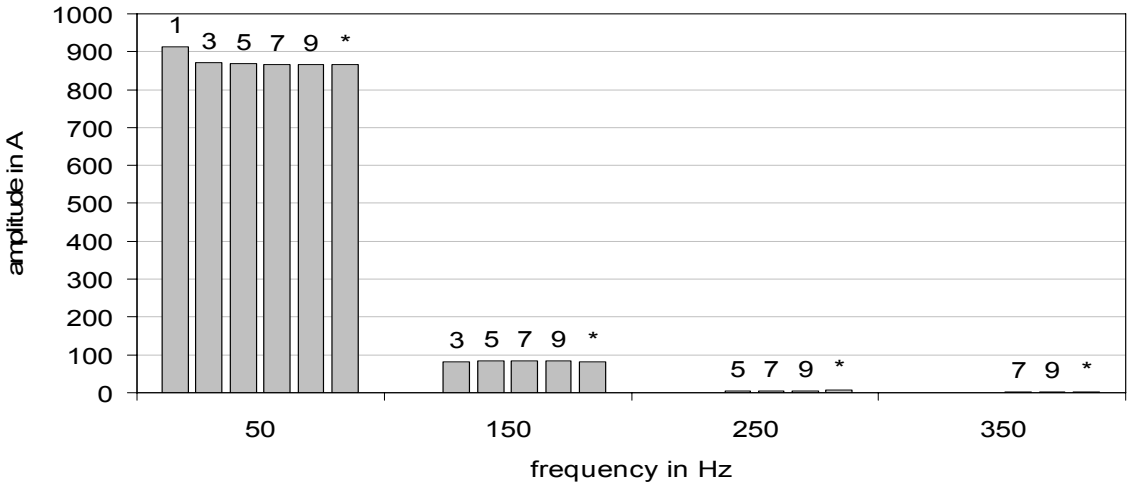

Fig. 6: Spectrum of the current in the copper conductor.

Conclusion

The presented method leads to a number of decoupled equation systems. The matrices **S** and **M** are constant in each iteration step and only the right hand side has to be updated in each step. Since the equations are decoupled, they can be solved separately, which reduces the amount of memory. For weak nonlinearities just a few harmonics are necessary to be taken into account. If it is further known a priori that certain harmonics cannot occur, the number of equation systems to be solved can be reduced.

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