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K.M. Gawrylczyk, M. Kugler, P. Putek

Similarity and Differences between Sensitivity Evaluations of Electromagnetic Quantities in the Frequency and the Time Domain

INTRODUCTION

Since a fortieth years of the 20th century, when the famous Bode's work has been published [1], the sensitivity and tolerance analysis constitute a main part of the circuit theory methods. There have been found broad applications in area of circuit design and optimization. The sensitivity analysis could be used in the field theory issues, too. The application to inverse problem using gradient method of optimization is especially interesting. In this case the inverse problem solution proceeds iteratively and the information on the gradient of the goal function is obtained from the sensitivity analysis.

The inverse problem of great technical importance is the recognition of defects or flaws in metallic materials basing on identification of conductivity distribution. As input data the field distribution in a nearby defects is used, which will be obtained from the measurement with eddy current method and from computer simulation. For the measurement either the multi-frequency or impulse current excitations are used. The identification task works in iterative manner and it usually starts with basic configuration, e.g. with the model of analyzed object without cracks. Then, it carries out modification of material parameters basing on the difference between the measured and modeled field distribution. One can assume that similarity of measured and modeled field distribution occurs only for the same conductivity distribution. Depending on the choice of field excitation (multi-frequency or impulse) the algorithm for the sensitivity evaluation in frequency [5,17] or time [6] domain is the necessary tool.

TEST PROBLEM

Let us consider the following model of eddy-current equipment for testing of heat exchanger tubes of steam generator in nuclear plants. The eddy-current sensor consisting of three coils moves inside the long, conducting tube. The coil in the middle is used for magnetic field excitation, other two are differential measurement coils.

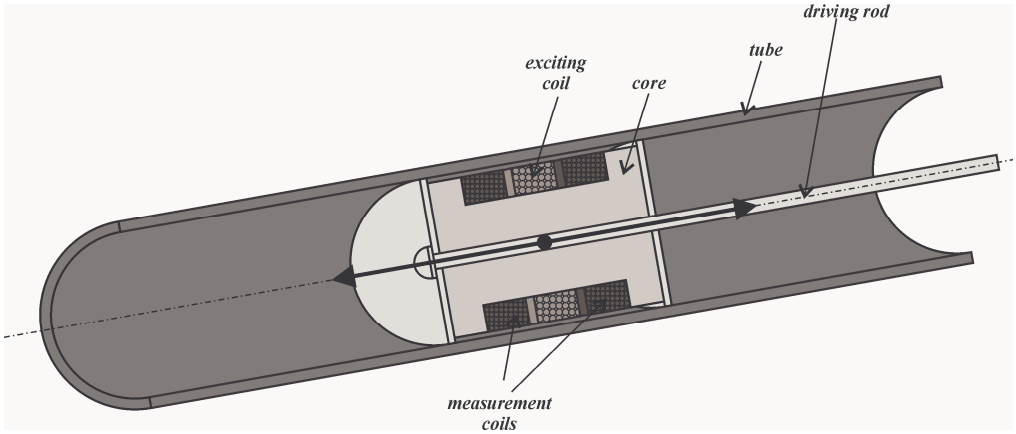

Fig. 1. Differential eddy-current sensor inside tube

In each position the sensor is excited with either exponential current in time domain analysis (Fig. 3) or multi-frequency current in frequency domain (Fig. 2) and the voltage is registered. The model exhibits cylindrical symmetry and can be analyzed using a 2D formulation. The zero Dirichlet boundary condition was assumed along the axis of symmetry and the zero Neumann-conditions on the remaining boundaries. The numerical model with geometrical and material parameters is shown in Fig. 4. Both measurement coils have 500 turns.

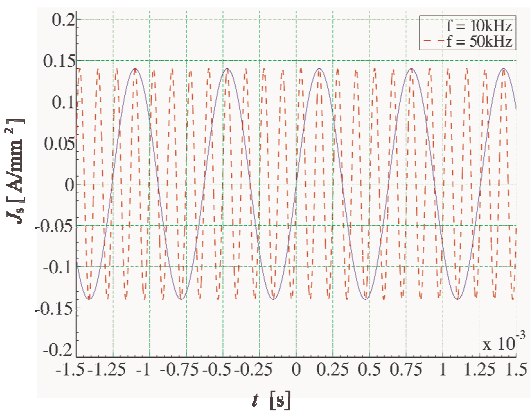

Fig. 2. Exemplary excitation for the frequency analysis

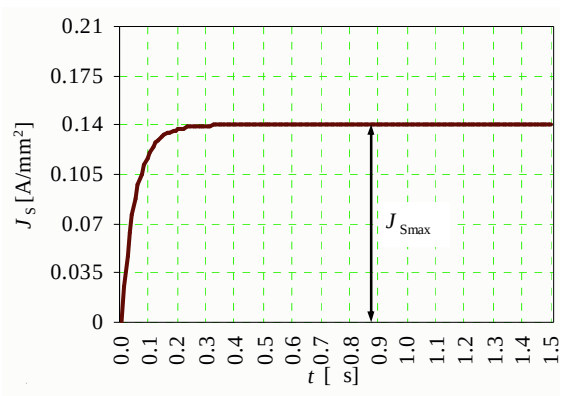

Fig. 3. Excitation shape in the time domain

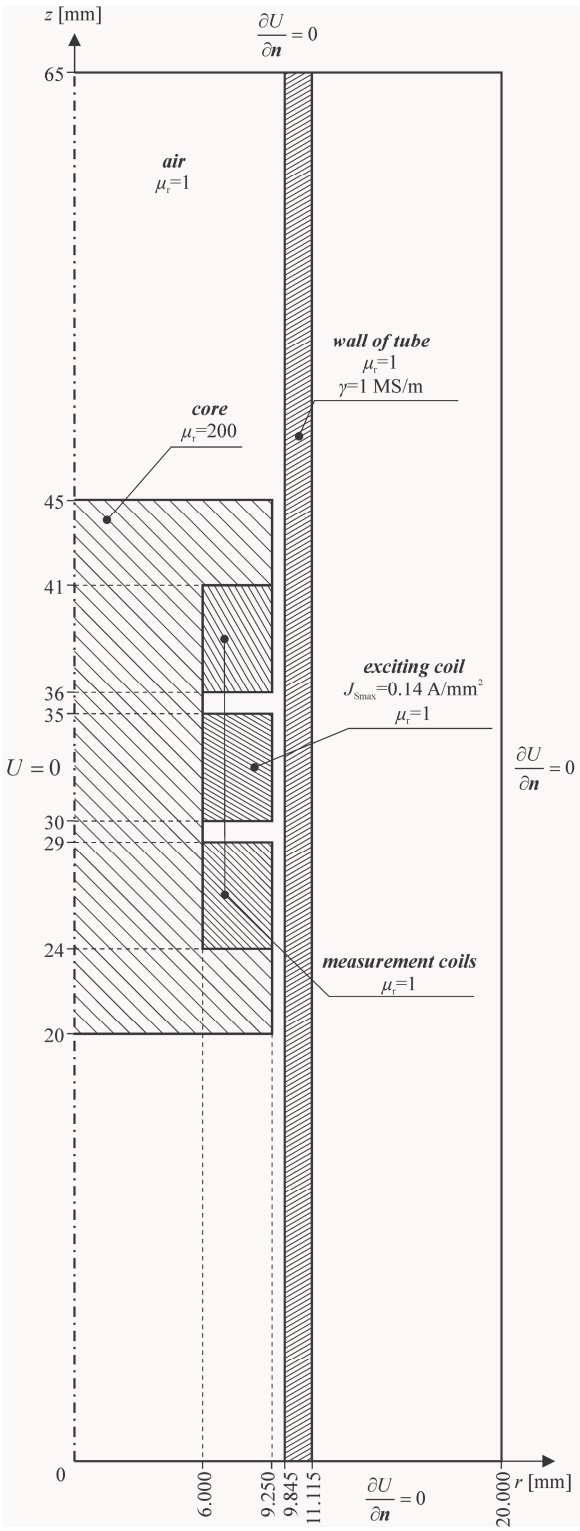

Fig. 4. Model under investigation

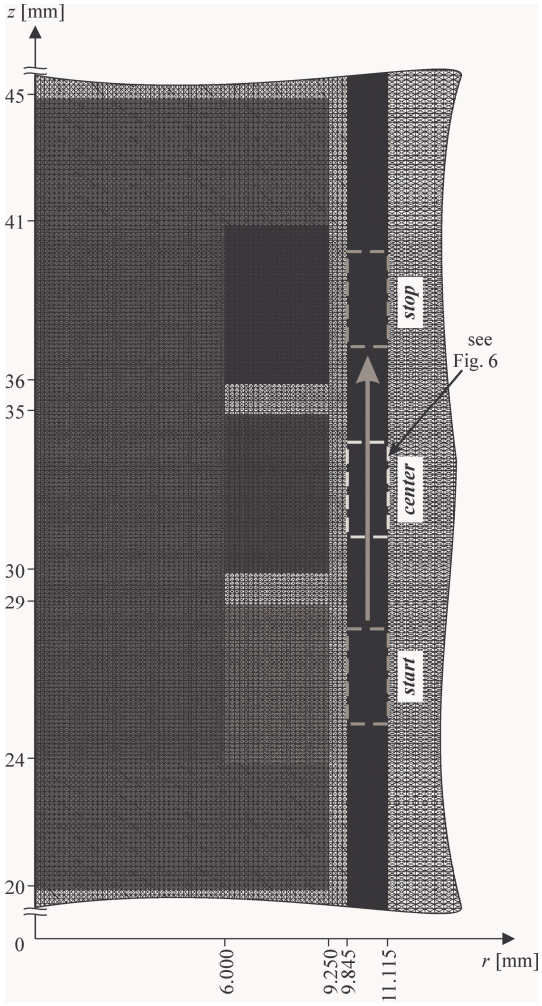

Fig. 5. Fragment of finite element mesh

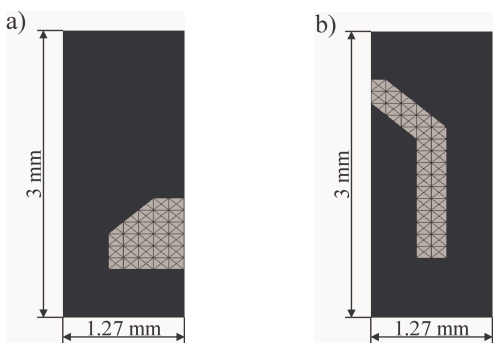

Fig. 6. Shapes of cracks I and II

FEM ALGORITHM

For the transient analysis of magnetic field typical finite element algorithm is used, utilizing generalized time stepping scheme *theta* [9]. Dividing the time range $(0, T)$

into n time steps of the length $\Delta t = T / n$, the differential scheme can be written as follows:

$$\left[\Theta[\boldsymbol{K}]+\frac{1}{\Delta t}[\boldsymbol{M}]\right]\{\boldsymbol{A}_i\}=\left\{\left(\frac{1}{\Delta t}[\boldsymbol{M}]-(1-\Theta)[\boldsymbol{K}]\right)\{\boldsymbol{A}_{i-1}\}+(1-\Theta)\{\boldsymbol{R}_i\}+\Theta\{\boldsymbol{R}_{i-1}\}\right\}, \quad (1)$$

where $[\boldsymbol{K}]$ and $[\boldsymbol{M}]$ are the stiffness and mass matrices of finite elements containing the material parameters and geometric properties of the simulated model, $\{\boldsymbol{A}_i\}$ is the vector of the desired node values and $\{\boldsymbol{R}_i\}$ is the discretized excitation for time steps $i \cdot \Delta t$, with $i = 1...n$ and the parameter Θ determines the time stepping scheme. $\{\boldsymbol{A}_0\}$ is the initial condition vector. In most cases of field penetration into conducting region, this vector is set to zero.

In the case of multi-frequancy analysis ditribution of vector potential $\{\boldsymbol{A}_f\}$ could be decribed using Helmholz equation, what after application to Finite Elements Methods takes the form of:

$$\left[[\boldsymbol{K}]+[\underline{\boldsymbol{M}}]\right]\{\underline{\boldsymbol{A}}_f\}=\{\underline{\boldsymbol{R}}_f\} \quad (2)$$

where $[\boldsymbol{K}]$ and $[\underline{\boldsymbol{M}}]$ constitute the stiffness and mass matrices, respectively, wheras $\{\boldsymbol{R}_f\}$ is the excitation vector corresponding to every frequency $f = 1...n$.

The calculations of the voltage induced in the coil for the examplary eddy-current crack detection is based eiher on [10] in time domain or on [11] for frequency analysis.

The algorithm of finite element analysis in the time domain is shown in Fig. 8. First, the global matrix is formed and decomposed. Next, the analysis time is set to zero and after that the nodal potentials are calculated in the loop for consecutive time steps.

In the frequency domain the FEM algorithm for one frequency was shown in Fig.7. The FEM analysis starts from decomposing stifness and mass matrices and forming excitation vector for every position and frequency. Then, the ditribution of vecor potential $\{\boldsymbol{A}_f\}$ is calculated utilizing Cholesky's decomposition. First, all positions are considered for i-*th* frequency utilizing possibility of calculations only excitation vector corresponding to k-*th* position of probe. Then, the loop of frequency is increased. This treatment improves efficiency of FEM algorithm. In such case the question of modeling boundary condition is very important, because of hard to determine discretization errors. Hence, the size of model should be suitable chosen or infinite elements should be used [12].

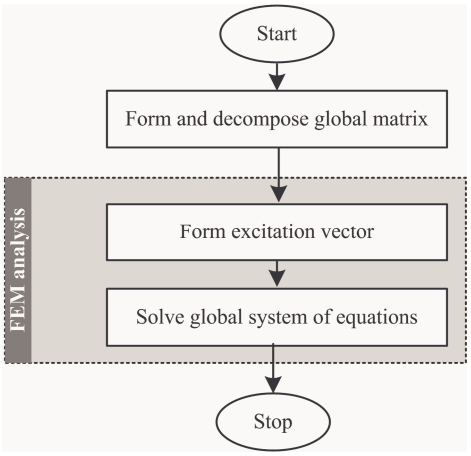

Fig. 7. Algorithm of finite element analysis in the frequency domain

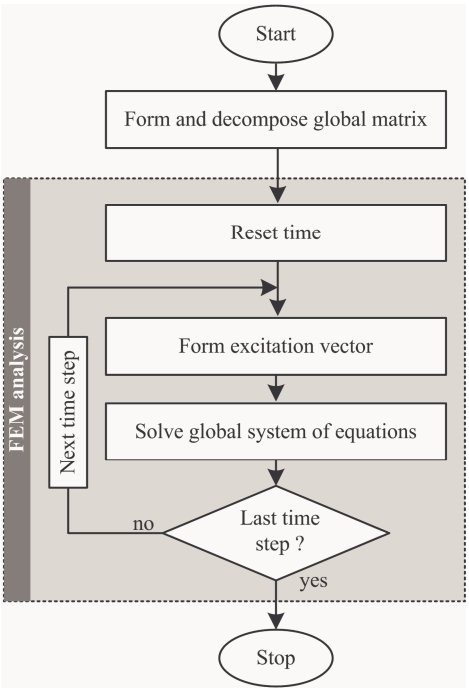

Fig. 8. Algorithm of finite element analysis in the time domain

SENSITIVITY ANALYSES

Voltage sensitivity of coil m versus electric conductivity in single finite element e is defined for time step $i{\cdot}\Delta t$ and for position of sensor b as:

$$\boldsymbol{S}_{(e)}^{b,m,i} = \frac{1}{\Omega_m} \frac{\partial}{\partial \gamma^{(e)}} \int_{\Omega_m} \boldsymbol{A}_i^b \, r \, \mathrm{d}\Omega \,, \tag{3}$$

whereas sensitivity of voltage in frequency domain could be written following:

$$\underline{S}_{(e)}^{b,m,f} = \frac{1}{\Omega_m} \frac{\partial}{\partial \gamma^{(e)}} \int_{\Omega_m} \underline{\boldsymbol{A}}_f^b \, r \mathrm{d}\Omega \tag{4}$$

where: Ω_m is the cross-section of the measurement coil. For the purposes of sensitivity evaluation in time as well as in frequency domain, the two equivalent techniques might be applied – either the incremental or adjoint model method.

Numerical calculation of sensitivity with above methods always requires the field analysis of the original model and the second analysis with either incremental or adjoint model. All models utilize the same stiffness and mass matrices (topology, geometrical and material parameters are the same) but different boundary conditions and excitations.

The first, incremental model's [13] method (see Fig. 9 for algorithm) calls for differentiation of equation (1) versus the electric conductivity $\gamma^{(e)}$ in element e. The stiffness matrix $[\boldsymbol{K}]$ and the excitation vector $\{\boldsymbol{R}_i\}$ do not depend on conductivity. Hence, to obtain the vector of nodal sensitivities defined as

$$\left\{\boldsymbol{S}^{i}_{(e)}\right\} = \frac{\partial}{\partial\gamma^{(e)}}\{\boldsymbol{A}_i\}, \tag{5}$$

the following sensitivity equation has to be solved:

$$\left[\Theta[\boldsymbol{K}] + \frac{1}{\Delta t}[\boldsymbol{M}]\right]\left\{\boldsymbol{S}^{i}_{(e)}\right\} = \left\{\left(\frac{1}{\Delta t}[\boldsymbol{M}] - (1-\Theta)[\boldsymbol{K}]\right)\left\{\boldsymbol{S}^{i-1}_{(e)}\right\} - \frac{1}{\Delta t}\frac{[\boldsymbol{M}]}{\Delta\gamma^{(e)}}\left(\{\boldsymbol{A}_i\} - \{\boldsymbol{A}_{i-1}\}\right)\right\}. \tag{6}$$

The solution of above equation follows step-by-step with zero initial condition. In this manner the vectors of nodal sensitivities for all time steps are obtained – this is incremental model analysis.

The same quantity decribed by eq. (5) could define in frequency domain but in this case index i ought to be replaced by frequancy indicator f. Regarding multi-frequency sensitivity analysis, one could obtain the sensitivity equation diferentating the linear system of equations (2) versus $\gamma^{(e)}$

$$[\underline{\boldsymbol{M}}]\left\{\underline{\boldsymbol{S}}^{f}_{(e)}\right\} = -\frac{\partial}{\partial\gamma^{(e)}}[\underline{\boldsymbol{M}}]\left\{\underline{\boldsymbol{A}}_f\right\} \tag{7}$$

where $\left\{\underline{\boldsymbol{S}}^{f}_{(e)}\right\}$ means the vector of nodal sensitivity versus conductivity in chosen finite element e for f-th frequency, which was defined in the same manner as quantity described by (5).

Using this method, in one cycle of sensitivity loop, the sensitivity of all nodal potentials versus conductivity in only one finite element is obtained. While the terms of matrix $[\boldsymbol{M}]$ or $[\underline{\boldsymbol{M}}]$ are linear functions of electrical conductivity γ, matrix of derivatives contains only constants and zeroes. The coil voltage sensitivity can be derived by integrating nodal sensitivity values over the cross-section of the coil.

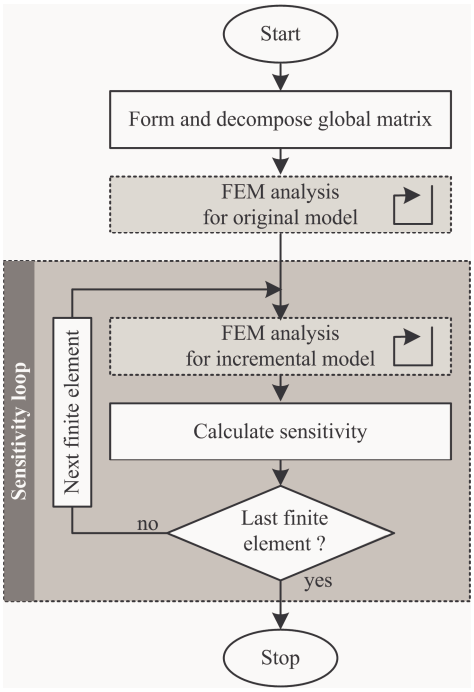

Fig. 9. Algorithm of sensitivity analysis using incremental model

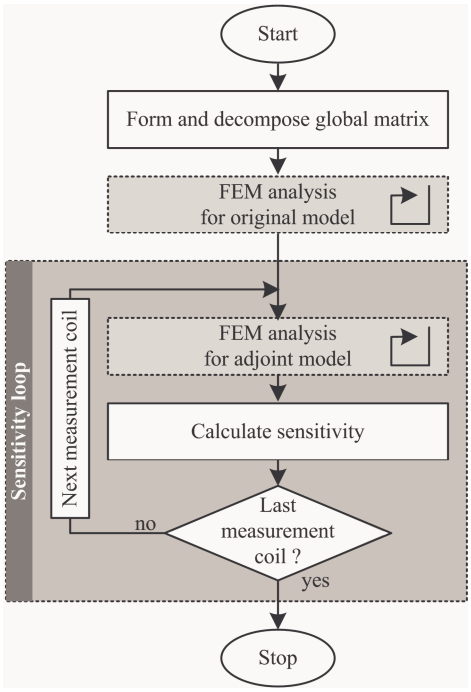

Fig. 10. Algorithm of sensitivity analysis using adjoint model

The second, adjoint model method, is based on the Tellegen's theorem, which is well known in circuit theory [7]. The application of this method in field theory for the frequency domain was shown in [2]. It has been extended by the authors of this paper for application in the time domain [6].

The sensitivity equation for the time domain analysis of electromagnetic fields has a following form of

$$\begin{aligned}&\int_0^T \iint_{\Omega} \left(\Delta \boldsymbol{H} \mu^+ \frac{\partial \boldsymbol{H}^+}{\partial t} - \boldsymbol{H}^+ \mu \frac{\partial}{\partial t} \Delta \boldsymbol{H} + \boldsymbol{E}^+ \gamma \Delta \boldsymbol{E} - \Delta \boldsymbol{E} \gamma^+ \boldsymbol{E}^+ + \boldsymbol{E}^+ \Delta \gamma \boldsymbol{E} - \Delta \boldsymbol{E} \boldsymbol{J}_S^+ + \Delta \boldsymbol{H} \boldsymbol{L}_S^+ \right) \mathrm{d}\Omega \, \mathrm{d}t = \\ &= \int_0^T \oint_{\Gamma(\Omega)} \left(\Delta \boldsymbol{E} \times \boldsymbol{H}^+ - \boldsymbol{E}^+ \times \Delta \boldsymbol{H} \right) \boldsymbol{n} \mathrm{d}\Gamma \, \mathrm{d}t,\end{aligned} \tag{8}$$

while analogous equation for frequency domain is [14]

$$\begin{aligned}&\iint_{\Omega} \left(\mathrm{j}\omega\mu^+ \Delta \underline{\boldsymbol{H}} \, \underline{\boldsymbol{H}}^+ - \underline{\mathrm{j}\omega\mu \boldsymbol{H}}^+ \Delta \underline{\boldsymbol{H}} + \underline{\boldsymbol{E}}^+ \gamma \Delta \underline{\boldsymbol{E}} - \Delta \underline{\boldsymbol{E}} \gamma^+ \underline{\boldsymbol{E}}^+ + \underline{\boldsymbol{E}}^+ \Delta \gamma \underline{\boldsymbol{E}} - \Delta \underline{\boldsymbol{E}} \underline{\boldsymbol{J}}_S^+ + \Delta \underline{\boldsymbol{E}} \underline{\boldsymbol{L}}_S^+ \right) \mathrm{d}\Omega = \\ &= \oint_{\Gamma(\Omega)} \left(\Delta \underline{\boldsymbol{E}} \times \underline{\boldsymbol{H}}^+ - \underline{\boldsymbol{E}}^+ \times \Delta \underline{\boldsymbol{H}} \right) \boldsymbol{n} \mathrm{d}\Gamma \, \mathrm{d}t,\end{aligned} \tag{9}$$

where $\boldsymbol{E}$ is the electric intensity vector, $\boldsymbol{H}$ is the magnetic intensity vector, $\boldsymbol{J}_S$ is the excitation current density, μ is the magnetic permeability and γ is the electric conductivity. Underlined symbols mean complex value and the symbol ($^+$) refers to the adjoint model, the remaining parameters belong to the original model. Both models are

analyzed for the same area Ω with the boundary Γ. The original model is analyzed for time t and the adjoint one for time $\tau = T - t$, where T is the time of the sensitivity evaluation. As distinguished from time sensitivity, the analysis in frequency domain both: the original and adjoint models are calculated for f-th frequency. In the sensitivity equation (8) and (9) one can find the component $\boldsymbol{L}^{+}_{S}$ meaning a magnetic current density vector and being equal to zero for all physical models. The boundary integral in eq. (8) and (9) may be eliminated assuming appropriate boundary conditions in the adjoint model. The excitation of the adjoint model can be chosen in this way, that the sensitivity for desired area, e.g. the cross-section of measurement coil, will be calculated directly. Unlike to first method, while using adjoint model (see Fig. 10), in one cycle of sensitivity loop the sensitivity of one coil voltage versus conductivities in all finite elements are obtained.

CRACKS RECOGNITION – MEASUREMENT RESULTS

For the aim of measurement simulation the area of analysis was divided into 189 696 finite elements with 95 409 nodes (Fig. 5).

The magnitude of (simulated) measurement voltage for frequency domain was shown in Fig.11. The multi-frequency analysis has been calculated for frequency spectrum from f_1 = 10 kHz to f_{61} = 410 kHz for 97-th positions of measurement probe.

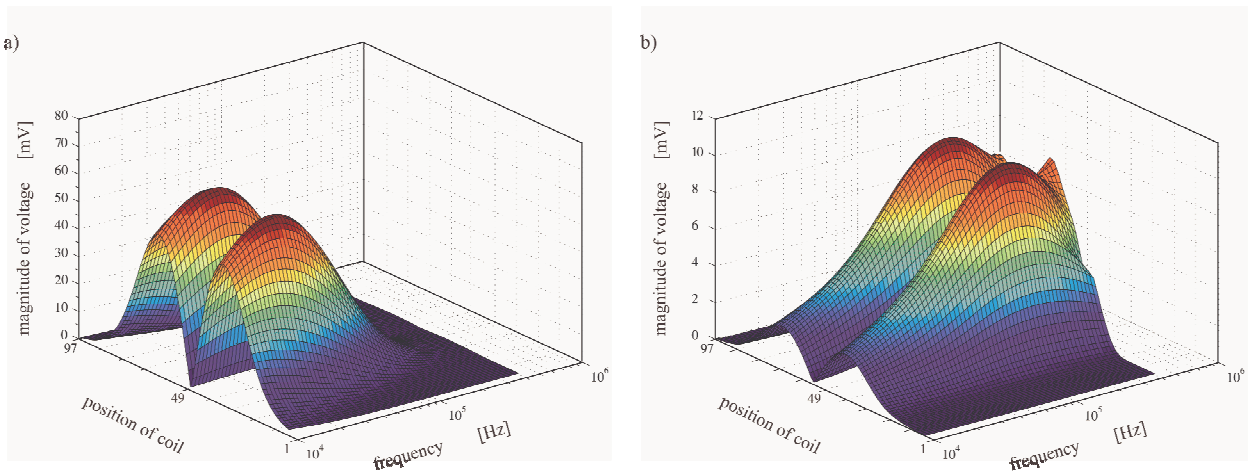

Fig. 11. Results of measurement simulation in the frequency domain for test cracks

The transient field analysis was carried out with the backward Euler scheme ($\Theta = 1$), with n = 150 constant time steps, each of Δt = 10 ns. For d = 97 positions of the sensor the induced voltage shapes were registered.

As measurement data are obtained, the inverse job of conductivity recognition can start. Two exemplary cracks are shown in Fig. 6, their electrical conductivities differing from tube wall.

The registered shapes of induced voltage for all positions of sensor b = 1...d simulated with the help of FEM, for two test cracks are shown in Fig. 12.

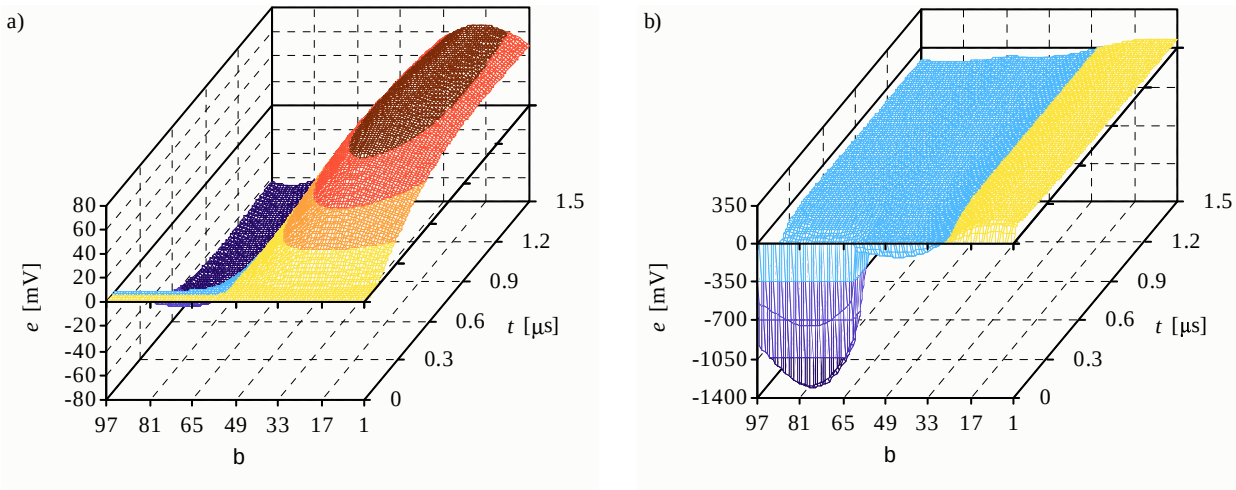

Fig. 12. Results of measurement simulation in the time domain for test cracks

INVERSE JOB

The recognition of material parameters on the basis of (simulated) measurement data using appropriate optimization algorithm constitutes inverse job, which solution, in opposite to field analysis, may be ambiguous. Thus, the goal function and optimization method are essential for efficiency and accuracy of inverse solution. For this aim the regularized Gauss-Newton with TSVD (Truncated Singular Value Decomposition) algorithm was applied [4,5], which was shown on Fig.13.

For recognition process one coarse mesh was used with 128 700 elements and 64 775 nodes. From this reason only 61 sensor positions were used and the measurement data had to be interpolated for some positions. Hence, the sensitivity matrix in time domain consisted of $d \cdot v \cdot n = 61 \cdot 1 \cdot 150 = 9150$ rows (v = 1 means, that two measurement coils were treated as one differential coil system) and k = 360 columns (number of finite elements in search area).

The inverse job in time domain is defined as optimization of the discrete goal function $\boldsymbol{F}$ in the form of the mean square error:

$$F = \frac{1}{2}\sum_{b=1}^{d}\sum_{m=1}^{v}\sum_{i=1}^{n}\left(\frac{1}{\Omega_m}\int_{\Omega_m} \boldsymbol{A}_i^b \,\mathrm{d}\Omega - \frac{1}{\Omega_m}\int_{\Omega_m} \overline{\boldsymbol{A}_i^b}\,\mathrm{d}\Omega\right)^2 , \tag{10}$$

where $\boldsymbol{A}_i^b$ is the simulated magnetic vector potential for the time step *i* and for the sensor position *b* , and $\overline{\boldsymbol{A}}_i^b$ is the referenced value from experimental measurement, *d* is the number of possible positions of sensor, *v* is the number of measurement coils of the sensor, and *n* is the number of time steps.

While for frequency domain the goal function could be described as:

$$F = \frac{1}{2}\sum_{j}(\boldsymbol{U}(\gamma)_i - \overline{\boldsymbol{U}}_i)(\boldsymbol{U}(\gamma)_i - \overline{\boldsymbol{U}}_i)^* \tag{11}$$

where $\mathbf{U}(\gamma)_j$ means the *j*-th component of simulated voltage and $\overline{\mathbf{U}}_j$ is the reference voltage for the *j*-th position of measurement coil, whereas the symbol (*) means conjugate operator. Thus, the sensitivity matrix in this case consists of $d \cdot n = 61 \cdot 61 = 3721$ rows and *k* = 360 columns because of quantity of conductivity for finite elements in search area.

In general case the sensitivity matrix as the component of gradient the goal function (10) or (11) could be written in following form of

$$[\boldsymbol{S}] = \begin{bmatrix} \boldsymbol{S}_{(1)}^1 & \boldsymbol{S}_{(2)}^1 & \dots & \boldsymbol{S}_{(k)}^1 \\ \boldsymbol{S}_{(1)}^2 & \boldsymbol{S}_{(2)}^2 & \dots & \boldsymbol{S}_{(k)}^2 \\ \dots & \dots & \dots & \dots \\ \boldsymbol{S}_{(1)}^m & \boldsymbol{S}_{(2)}^m & \dots & \boldsymbol{S}_{(k)}^m \end{bmatrix}_{m\times k} . \tag{12}$$

The SVD is effective and superior tool for analysis of discrete ill-posed problems, and, in authors opinion, in association with the traditional Gauss-Newton method, may constitute the easy in implementation algorithm of inverse identification problem [15]. Then, SVD of $\mathbf{S} \in \mathbb{R}^{J\times I}$ takes a form of:

$$\mathbf{S} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathrm{T}} = \sum_{i=1}^{I}\mathbf{u}_i\sigma_i\mathbf{v}_i , \tag{13}$$

where $\mathbf{U} = (\mathbf{u}_1,...,\mathbf{u}_n)$ and $\mathbf{V} = (\mathbf{v}_1,...,\mathbf{v}_n)$ are orthogonal columns matrices with $\mathbf{U}\mathbf{U}^{\mathrm{T}} = \mathbf{V}^{\mathrm{T}}\mathbf{V} = \mathbf{I}_n$, and where $\boldsymbol{\Sigma} = \mathrm{diag}(\sigma_1,...,\sigma_n)$ has non-negative diagonal elements appearing in non-increasing order such that:

$$\sigma_1 \geq ... \geq \sigma_n \geq 0 , \tag{14}$$

which are so called the singular values of the matrix $[\boldsymbol{S}]$. If matrix $[\boldsymbol{S}]$ indicates the Jacobian of goal function determined as (10) or (11), the minimization process for each iteration may be defined as

$$\left([\boldsymbol{S}]^{\mathrm{T}<q>}[\boldsymbol{S}]^{<q>}\right)\{\Delta\gamma\}^{<q>} = -[\boldsymbol{S}]^{\mathrm{T}<q>}\{\boldsymbol{p}\}^{<q>} \tag{15}$$

and

$$\{\Delta\gamma\}^{<q+1>} = \{\gamma\}^{<q>} + \{\Delta\gamma\}^{<q>}, \tag{16}$$

where $[\boldsymbol{S}]$ is the Jacobian of goal function (sensitivity matrix), $\{\gamma\}$ is the vector of electric conductivities in finite elements, $\{\Delta\gamma\}$ is the vector of conductivity corrections and q is the iteration number.

Moreover, solving the equation system (15) by means of SVD (for notice clarity in the first iteration step the index q was omitted), one may define such as:

$$\boldsymbol{\Delta}\gamma = \sum_{i=1}^{I} \frac{\mathbf{u}_i^{\mathrm{T}}\boldsymbol{g}}{\sigma_{\mathrm{i}}}\mathbf{v}_i, \tag{17}$$

where $[\boldsymbol{g}] = -[\boldsymbol{S}]^{\mathrm{T}<q>}\{\boldsymbol{p}\}^{<q>}$.

Thus, in the simplest case of regularization, it is enough to define the filter factors for singular values of the form of [4,7]:

$$f_{TSVD}(\sigma) = \begin{cases} 0 & \sigma \le \delta \\ 1 & \sigma > \delta \end{cases}, \tag{18}$$

where δ is the chosen threshold.

In this way one can eliminate component in the numerical null-space of A, spanned by the columns of $\boldsymbol{v}_{\mathrm{i}}$, therefore, the TSVD solution $\boldsymbol{\Delta}\gamma$ assumes the following form of:

$$\boldsymbol{\Delta}\gamma_{TSVD} = \sum_{i=1}^{K} \frac{\mathbf{u}_{\mathrm{i}}^{\mathrm{T}}\boldsymbol{g}}{\sigma_i}\mathbf{v}_i = \sum_{i=1}^{I} f_{TSVD}(\sigma)\frac{\mathbf{u}_{\mathrm{i}}^{\mathrm{T}}\boldsymbol{g}}{\sigma_i}\mathbf{v}_i \tag{19}$$

In the numerical implementation the regularization parameter δ was chosen in indirect way t.i. through assuming the minimal conditioned coefficient κ, which ought to characterize the matrix $[\boldsymbol{S}]$.

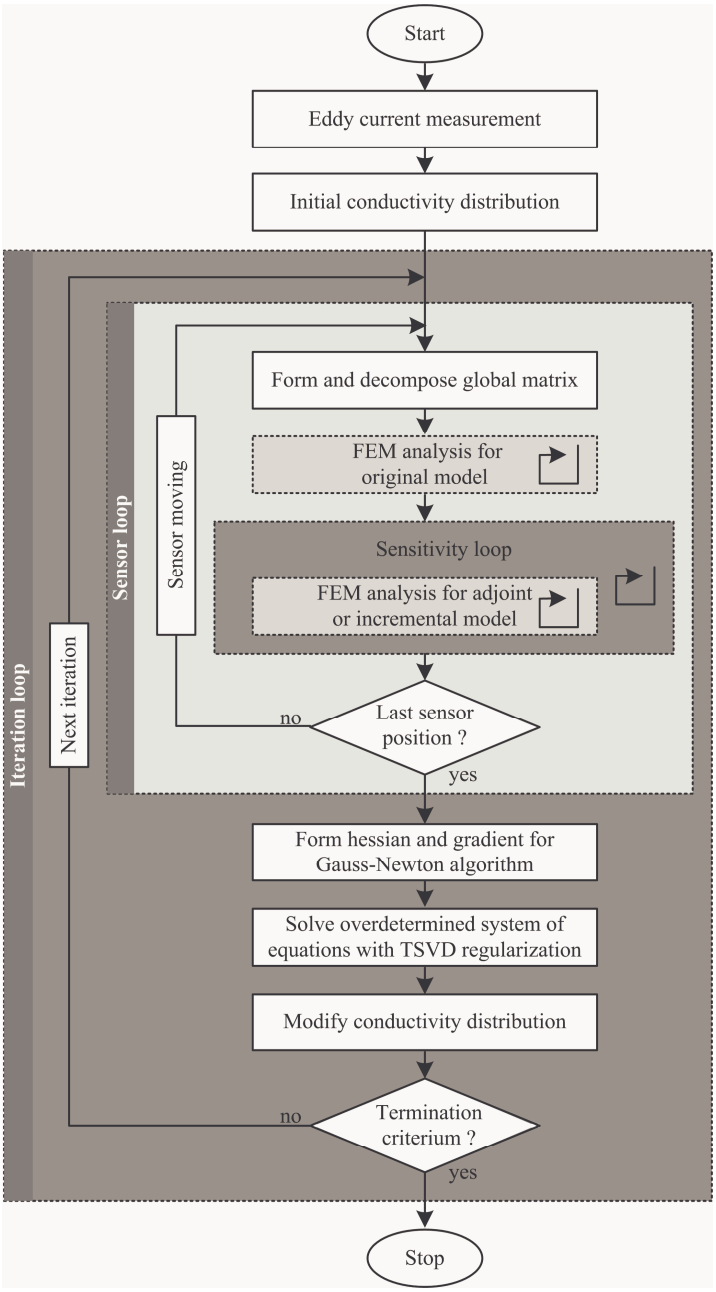

Fig. 13. Algorithm of recognition of conductivity distribution

The conductivity distributions (crack shapes) and crack positions on search area were correctly recognized after 20 iterations.

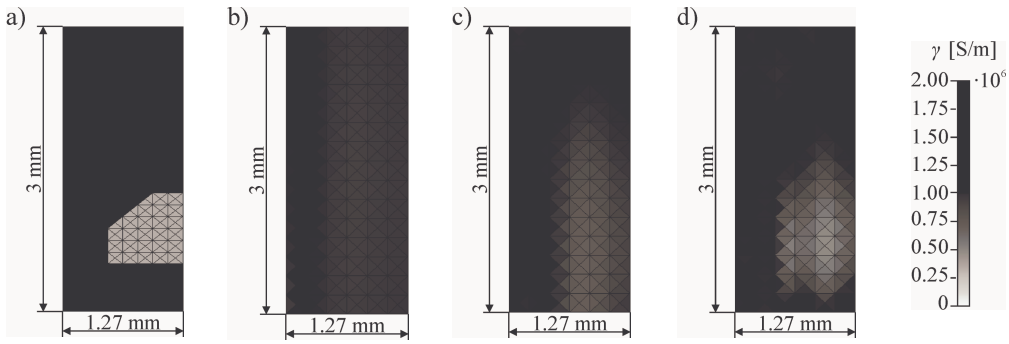

Fig. 14. Conductivity distribution for crack I: a) assumed, b) recognized after 5 iterations,

c) after 10 iterations, d) after 20 iterations

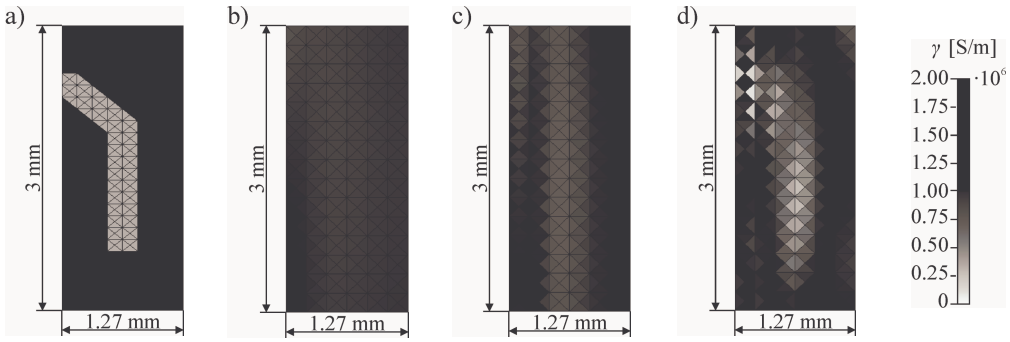

Fig. 15. Conductivity distribution for crack I: a) assumed, b) recognized after 5 iterations, c) after 10 iterations, d) after 20 iterations

CONCLUSIONS

In this paper similarity and differences between sensitivity analysis in time and frequency domain had been presented. The main advantage of time domain sensitivity analysis is the simultaneous penetration of investigated material by the field for whole frequency spectrum. It depends surly on the type of excitation current. This property influences on efficiency of sensitivity algorithm (especially by adjoint model) and also course of identification process. In the second hand, this kind of analysis requires iteration process for solution of diffusion equation (1).

The multi-frequency sensitivity analysis demands a lot of FEM calculations corresponding to the spectrum of chosen frequencies, what causes time extension of identification process, too. For the time sensitivity analysis the more rough regularization method [4,5] to approximate optimal rank of $[\boldsymbol{S}]$ [14] then in multi-frequency identification algorithms could be used.

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