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M. Ziolkowski, H. Brauer

Comparison of different formulations of 3D magnetostatic problems in finite element method

INTRODUCTION

The goal of presented paper is the comparison of essential features of solutions of 3D linear magnetostatic boundary problems calculated by means of the finite element method [1] using as variables describing the magnetic field the following formulations: scalar potential φ, vector magnetic potential $\mathbf{A}$, and magnetic intensity vector $\mathbf{H}$.To verify the above formulations we solve a simple example and compare the results with the analytical solution. For every method, the boundary conditions and the specific features of implementation are explained in detail.

SIMPLE MAGNETOSTATIC TEST PROBLEM

We define the following simple test example (Fig. 1). We would like to find a distribution of the magnetic field in a magnetically homogeneous cuboid ($\mu=\mu_0$): $x\in(0,a)$, $y\in(0,b)$, $z\in(0,h)$ located in an ideal ferromagnetic recess. On the boundaries we set following boundary conditions: $\mathbf{n}\cdot\mathbf{B}=0$ for $x=0, y\in(0,b), z\in(0,h)$ and $y=0, x\in(0,a), z\in(0,h)$ (boundary type Γ_B) and $\mathbf{H}\times\mathbf{n}=\mathbf{K}_0$ elsewhere (boundary type Γ_H). Vector $\mathbf{K}_0$ is equal to $K_0\mathbf{1}_y$ for $x=a, y\in(0,b), z\in(0,c)$, $-K_0\mathbf{1}_x$ for $y=b, x\in(0,a), z\in(0,c)$, and 0 elsewhere. The above problem can be solved analytically using the separation of variables method [2]. The results of the analytical solution, which are the reference for further comparisons, are shown in Fig. 2. The calculations are performed for the following data: $a=b=1$, $c=0.6$, $K_0=0.6$. Magnetic intensity vector $\mathbf{H}$ is calculated in the element gravity center for every element from the finite element mesh. The results of calculations are presented in a uniform form. For a chosen line, magnetic intensity vector $\mathbf{H}$ is shown in a form of arrows and appriopriate diagrams.

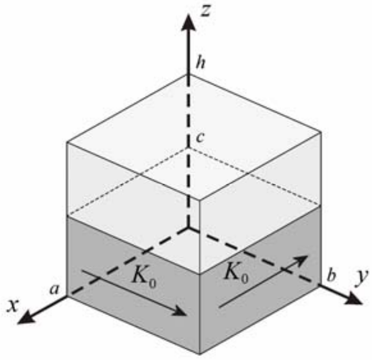

Fig.1 Simple test problem.

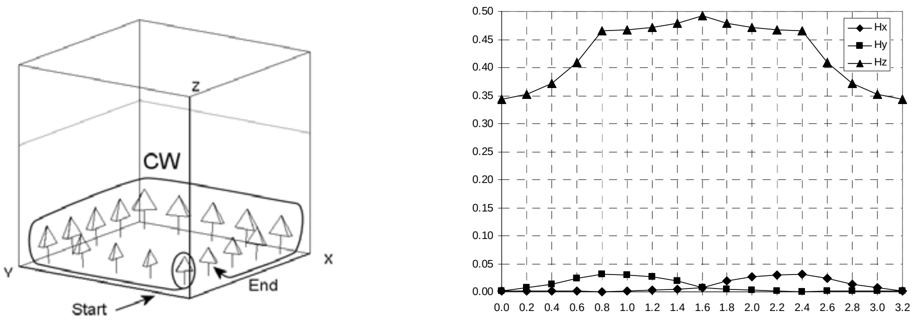

Fig.2 Results of analytical solution at $z/h=0.1$

DIFFERENT FORMULATIONS FOR MAGNETOSTATIC FIELDS

We analyze the following formulations for the description of magnetostatic field:

1) magnetic scalar potential φ defined as $\mathbf{H}=-\mathrm{grad}\ \varphi$,
2) magnetic vector potential $\mathbf{A}$ defined as $\mathbf{H}=\mathrm{rot}\ \mathbf{A}$ together with Coulomb gauge $\mathrm{div}\ \mathbf{A}=0$,
3) direct formulation using magnetic intensity vector $\mathbf{H}$.

In the first formulation, a zero Neumann boundary condition is set on boundary Γ_B ($\partial\varphi/\partial n=0$) and Dirichlet boundary condition on boundary Γ_H ($\varphi=0$ on face $z=0$, potential φ varies linearly versus z in interval $z\in<0,c>$ on faces $x=0$ and $y=b$, and takes the constant value equals $-K_0c$ on the rest part). In the finite element implementation, 8 noded linear brick elements are applied.

For the magnetic vector potential formulation, we consider two cases: 1) potential $\mathbf{A}$ with linear edge elements, and 2) potential $\mathbf{A}$ with separated components (SC) and nodal linear brick elements. In the first case (edge elements), the boundary condition on Γ_H has the form:

$$(\nu\nabla\times\mathbf{A})\times\mathbf{n}=\mathbf{K}_0\,, \tag{1}$$

and can be directly included in the equation used in the finite element method:

$$\int_\Omega \nu(\nabla\times\mathbf{w})\cdot(\nabla\times\mathbf{A})d\Omega=\int_{\Gamma_H}\mathbf{w}\cdot\mathbf{K}_0 d\Gamma\,. \tag{2}$$

On boundary part Γ_B, we set $\mathbf{A}\times\mathbf{n}=0$. If we apply the finite element method based on node vector shape functions the gauge condition has to be inserted into the equation e.g.:

$$\nabla\times(\nu\nabla\times\mathbf{A})-\nabla(\nu\nabla\cdot\mathbf{A})=0, \tag{3}$$

and in order to receive a unique solution we have to set, despite the above boundary conditions, the following boundary conditions:

$$\begin{aligned}\mathbf{n}\cdot\mathbf{A}=0 \quad \text{on} \quad \Gamma_H\\ \nabla\cdot\mathbf{A}=0 \quad \text{on} \quad \Gamma_B\end{aligned} \tag{4}$$

In addition, if magnetic permeability μ is constant in the whole analyzed region then it is possible to split the components of vector $\mathbf{A}$ and the vector equation (3) can be substituted by the following system of scalar equations:

$$\nabla^2 A_x=0,\quad \nabla^2 A_y=0,\quad \nabla^2 A_z=0\,. \tag{5}$$

The table below shows the set of all boundary conditions used for the test problem.

	A_x	$\partial A_x/\partial n$	A_y	$\partial A_y/\partial n$	A_z	$\partial A_z/\partial n$
$x=0$	-	0	0	-	0	-
$x=a,\ z\le c$	0	-	-	$-\mu K_0$	-	0
$x=a,\ z>c$	0	-	-	0	-	0
$y=0$	0	-	-	0	0	-
$y=b,\ z\le c$	-	$-\mu K_0$	0	-	-	0
$y=b,\ z>c$	-	0	0	-	-	0
$z=0$	-	0	-	0	0	-
$z=h$	-	0	-	0	0	-

As the last formulation we have analyzed the direct formulation using magnetic intensity vector $\mathbf{H}$. In that case, the implementation in the finite element method has several restrictions. The primary restriction is that one cannot take into account the non-homogeneity of the environment in such a simple way as it is in the case of scalar potential φ or vector potential $\mathbf{A}$. The second limitation is caused by difficulties in taking into account regions where the currents are flowing (windings). Fortunately, in the formulated test problem we do not have such restrictions. Using the following equation:

$$\nabla \times \nabla \times \mathbf{H} - \lambda \nabla (\nabla \cdot \mathbf{H}) = 0, \qquad (6)$$

we can formulate two kinds of the boundary value problem: with separated components (λ=1) and with edge elements satisfying condition $\nabla \cdot \mathbf{H}=0$ (λ=0).

For λ=1, we can separate $\mathbf{H}$ components as in the case of vector potential $\mathbf{A}$. The appropriate boundary conditions are collected in the table below:

	H_x	$\partial H_x/\partial n$	H_y	$\partial H_y/\partial n$	H_z	$\partial H_z/\partial n$
x=0	0	-	-	0	-	0
$x=a,\ z\le c$	-	$K_0\delta$	0	-	K_0	-
$x=a,\ z>c$	-	0	0	-	0	-
y=0	-	0	0	-	-	0
$y=b,\ z\le c$	0	-	-	$K_0\delta$	K_0	-
$y=b,\ z>c$	0	-	-	0	0	-
z=0	0	-	0	-	-	0
z=h	0	-	0	-	-	0

δ is a Dirac function for $z=c$

The integration of the Dirac function over an element edge at the boundary surface produces an additional term equals to half the length of the element edge located on segment $\{z=c,\ x=a,\ y\in(0,b)\}$ or $\{z=c,\ y=b,\ x\in(0,a)\}$ multiplied by K_0 which should be added to the right hand side vector of the global finite element system in equations corresponding to nodes forming the analyzed element edge.

As we tried to realize the direct $\mathbf{H}$ formulation with the edge elements (λ=0) together with the boundary condition $\mathbf{H}\times\mathbf{n}=\mathbf{K}_0$ on Γ_H and homogeneous natural boundary conditions on walls x=0 and y=0 the results received have a non-physical meaning (Fig.8). The appearance of the non-physical solution in a formally correct formulated boundary value problem is caused by the fact that on Γ_B type boundary instead of the *stronger* principal boundary condition is set the *weaker* natural condition which comes from the used edge approximation.

RESULTS OF CALCULATIONS

In this section, we present the results of calculations received from the finite element method where the above formulations have been implemented. The calculations have been performed in most cases using the finite element mesh consisting of 5×5×5 brick elements (edge or nodal).

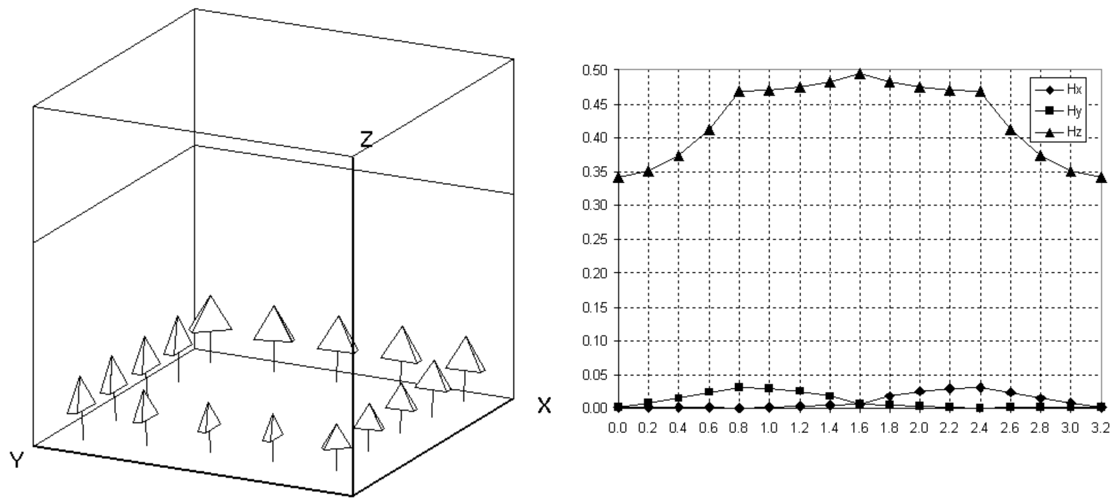

Fig.3 Magnetic intesity field **H** calculated by means of scalar potential φ at z/h=0.1.

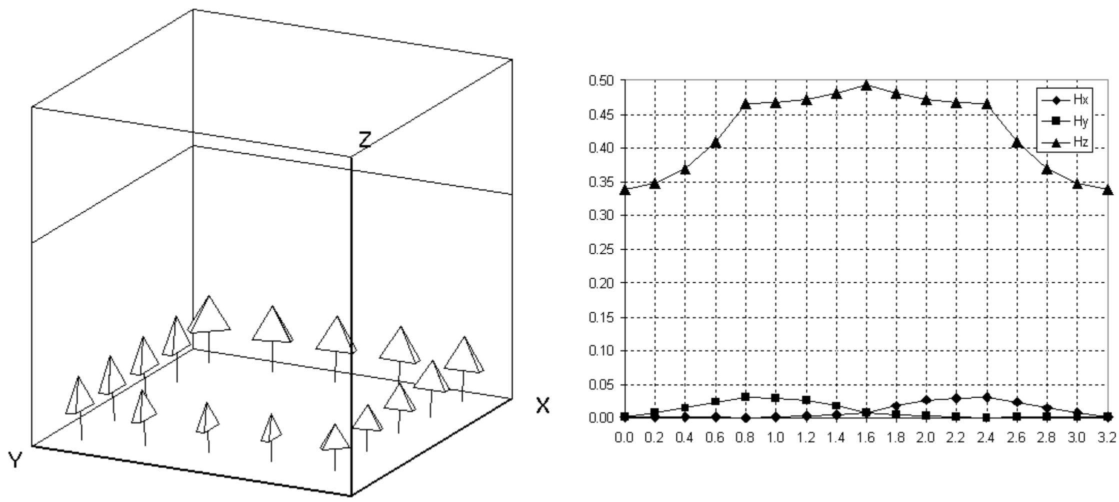

Fig.4 Magnetic intesity field **H** calculated by means of vector potential **A** using edge elements at z/h=0.1.

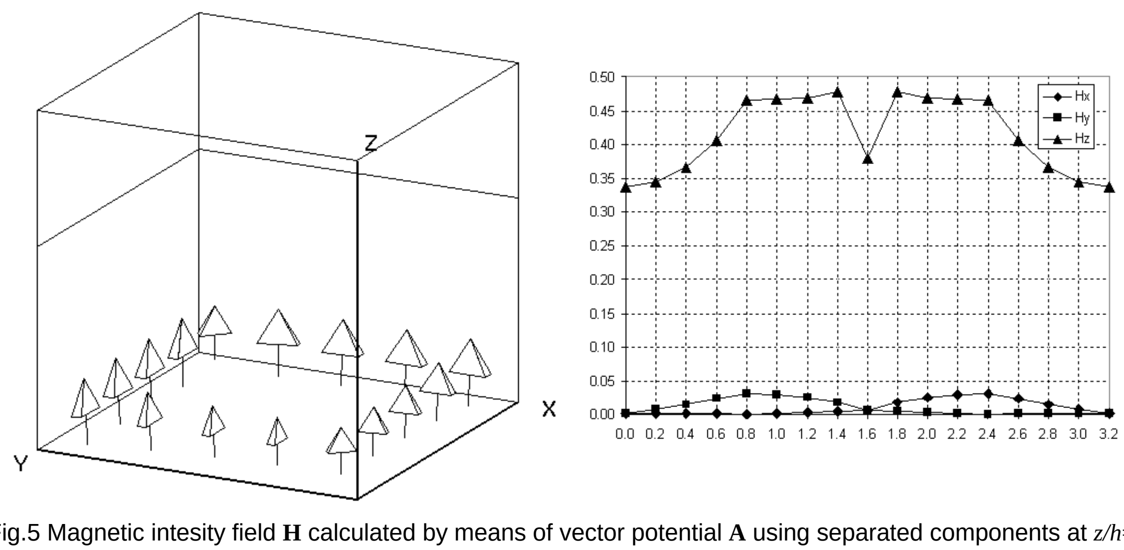

Fig.5 Magnetic intesity field **H** calculated by means of vector potential **A** using separated components at z/h=0.1.

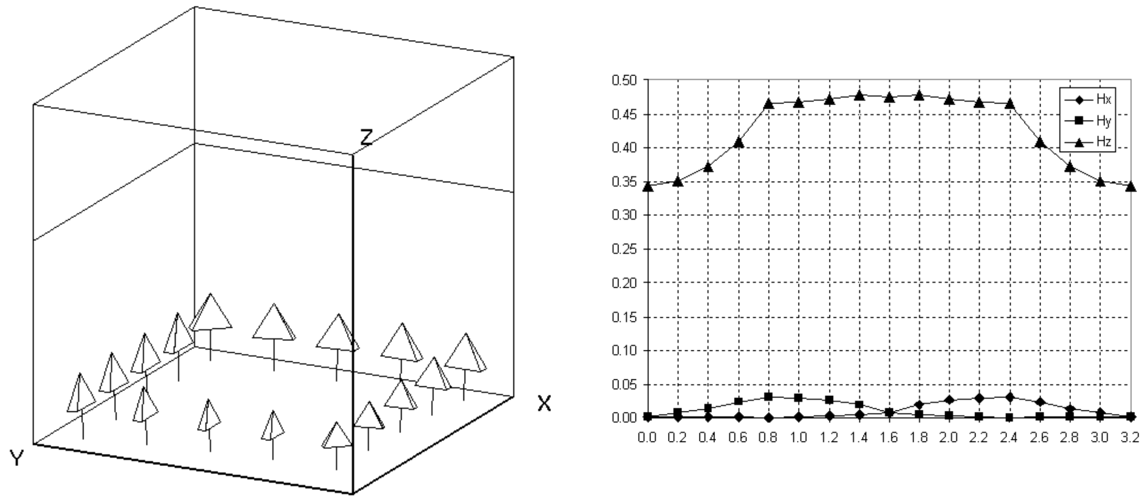

Fig.6 **H** field calculated by means of vector potential **A** with SC applying mesh 15×15×15 (z/h=0.1).

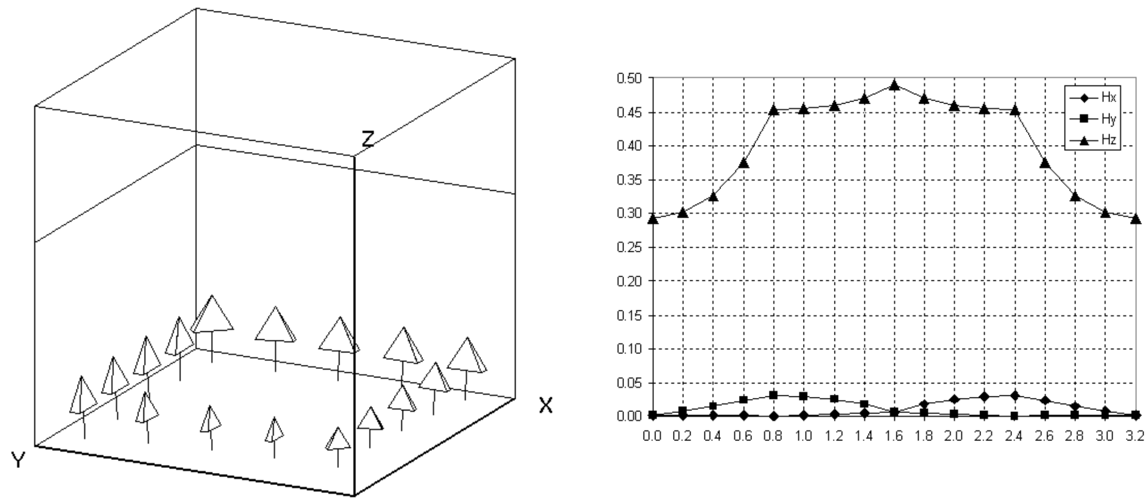

Fig.7 Magnetic intesity field **H** calculated by means of direct **H** formulation using separated components at z/h=0.1.

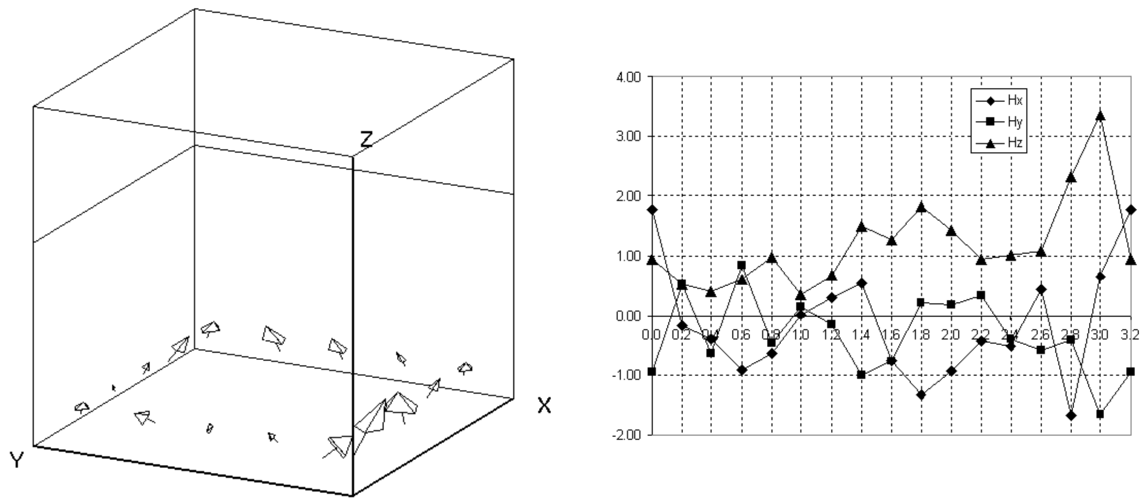

Fig.8 Magnetic intesity field **H** – non-physical solution for direct **H** formulation using edge elements at z/h=0.1.

The comparison of results for scalar potential φ implementation (Fig.3) with the analytical solution (Fig.2) shows that even for relatively poor finite element mesh the accuracy is satisfactory (global relative error 3.81%). Therefore, this formulation can be treated as the basic technique of the analysis of the magnetic field in the non-conducting homogeneous regions. For magnetic vector potential $\mathbf{A}$ formulation with edge elements (Fig.4), the global error is equal to 4.16%. Locally, the greatest error is observed for H_z component in the vicinity where the boundary condition starts to be discontinuous ($z=c$). In vector potential $\mathbf{A}$ formulation with separated components (Fig.5), the greatest error occurs near edge $\{x=a, y=b\}$ (global relative error 6.65%). The error can be reduced by mesh refinement (Fig.6). For direct $\mathbf{H}$ formulation, components H_x and H_y are calculated with high accuracy while errors for component H_z are extremely high at points located in the region of jumping changes of K_0. Mesh refinement improves the quality of the results. We can say that the problem of poor accuracy of H_z component results not from the formulation itself but from the specific features of the test example. The summary of the total relative errors for the magnetic intensity magnitude computed over two meshes consisting of 5×5×5 and 15×15×15 brick elements (nodal or edge) is given in the table below.

	Relative error [%]	
	5×5×5	**15×15×15**
φ	3.81	1.27
A (edge)	4.16	1.32
A (SC, nodal)	6.65	2.18
H (SC, nodal)	17.20	6.37

CONCLUSIONS

Presented comparisons show that the best results we have received with the magnetic scalar potential formulation and this formulation should be treated as a principal for 3D magnetostatic problems. However, when we apply tetrahedral elements of the first order much denser finite element meshes have to be used to receive comparable accuracy in magnetic field intensity calculations. Comparable quality we have received with the magnetic vector potential formulation with edge elements and this formulation should be considered as the alternative one to scalar potential φ.

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[1] O. C. Zienkiewicz,: “The Finite Element Method”, 3rd edition, McGraw-Hill, London, 1977.

[2] M. Gramz, M. Ziolkowski: “Calculations of three dimensional electromagnetic fields”, pt. 1, PN Poltechnika Szczecinska, vol. 444, Szczecin, 1991 (in Polish).

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