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On the Existence of Edge Cuts leaving Several Large Components

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Abstract

We characterize graphs of large enough order or large enough minimum degree which contain edge cuts whose deletion results in a graph with a specified number of large components. This generalizes and extends recent results due to Ou (Edge cuts leaving components of order at least m, *Discrete Math.* **305** (2005), 365-371) and Zhang and Yuan (A proof of an inequality concerning k-restricted edge connectivity, *Discrete Math.* **304** (2005), 128-134).

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1 Introduction

Many variants of the edge connectivity of graphs have been proposed and considered [4, 5, 8] to measure the fault-tolerance of interconnection networks with respect to link failure. In this context a connected graph $G = (V, E)$ was called $\lambda_{a,a}$-connected for some $a \in \mathbb{N}$ if it has an edge cut $S \subseteq E$ such that $G - S = (V, E \setminus S)$ has exactly two components of order at least a. While the concept of $\lambda_{a,a}$-connectedness was used to quantify and compare the reliability of special network topologies [2, 3, 15, 14] several authors studied $\lambda_{a,a}$-connected graphs in general [1, 6, 9, 12, 16] focusing on the existence and minimum size of the corresponding edge cuts.

The starting point for the research we present here are recent results due to Ou and Zhang and Yuan who characterized $\lambda_{a,a}$-connected graphs which are either of large order [12] or of large minimum degree [16]. We present a short proof of a generalization of Ou's main result in [12]. As we have noted in [13], one of Zhang and Yuan's main results in [16] can easily be derived from a powerful theorem due to Győri [7] and Lovász [11]. We demonstrate how to extend the result from [16] to edge cuts leaving three or four large components and pose a related conjecture.

For integers $a_1, a_2, ..., a_k \in \mathbb{N}$ we say that a connected graph $G = (V, E)$ is $\lambda_{a_1,a_2,...,a_k}$-connected if it has an edge cut $S \subseteq E$ such that $G - S$ has k components with vertex sets $V_1, V_2, ..., V_k$ such that $|V_i| \geq a_i$ for $1 \leq i \leq k$. In [12] as well as [16] the $\lambda_{a,a}$-connectedness was characterized by the absence of a small set of vertices whose deletion results in a graph all components of which are small. Therefore, for integers $a, k \in \mathbb{N}$ we say that a graph $G = (V, E)$ is (a, k)-stellar, if there is a set $U \subseteq V$ of at most k vertices such that all components of $G - U = G[V \setminus U]$, the subgraph of G induced by $V \setminus U$, are of order at most $a - 1$.

2 Results

Our first result generalizes the main result from [12].

Theorem 2.1 *Let $a, b \in \mathbb{N}$ with $2 \leq a \leq b$ and let $G = (V, E)$ be a connected graph of order $n \geq \max\{2b - 1, 3a - 2\}$.*

Then G is $\lambda_{a,b}$-connected if and only if G is not $(a, 1)$-stellar.

Proof: For the "only if"-part let S be a minimal edge cut such that $G - S$ has a component with vertex set A of cardinality at least a and another component with vertex set B of cardinality at least b. If $u \in A$, then $G - u$ has a component containing B, and, if $u \notin A$, then $G - u$ has a component containing A. This implies that G is not $(a, 1)$-stellar.

For the 'if'-part let $G = (V, E)$ be not $(a, 1)$-stellar.

If G is a tree, then for every vertex $u \in V$ there is a neighbour $p(u) \in V$ such that the component of $G - u$ containing $p(u)$ has order at least a. Since G has more vertices than edges, there is an edge $uv \in E$ such that $p(u) = v$ and $p(v) = u$. This implies that both components of $G - uv$ have order at least a. Since $n \geq 2b - 1$ at least one component of $G - uv$ has order at least b and G is $\lambda_{a,b}$-connected.

If G is not a tree, then we prove the existence of an edge $e \in E$ for which $G - e$ is not $(a, 1)$-stellar which implies the result by an inductive argument. Let $e \in E$ be an edge in a cycle C of G. Clearly, we may assume that $G - e$ is $(a, 1)$-stellar. Let $u \in V$ be such that all components of $(G - e) - u$ have order at most $a - 1$. Clearly, $u \in V(C)$. Since G is not $(a, 1)$-stellar, there are two components of $(G - e) - u$ with vertex sets X and Y such that e joins a vertex in X and a vertex in Y, $|X|, |Y| \leq a - 1$ and $|X| + |Y| \geq a$. Let $Z = V \setminus (X \cup Y)$. Note that $|Z| \geq n - |X| - |Y| \geq 3a - 2 - (a - 1) - (a - 1) = a$. Let $f \in E(C)$ be an edge incident to u. If $v \in Z$, then $(G - f) - v$ has a component containing $X \cup Y$, and, if $v \notin Z$, then $(G - f) - v$ has a component containing Z. Hence, $G - f$ is not $(a, 1)$-stellar and the proof is complete. $\square$

Note that our proof works along the same lines as the proof in [12] but that we present a considerably shorter argument. Choosing $a = b$ in Theorem 2.1 we obtain the main result of [12].

Corollary 2.2 (Ou [12]) *Let $a \in \mathbb{N}$ with $a \geq 2$ and let $G = (V, E)$ be a connected graph of order $n \geq 3a - 2$.*

Then G is $\lambda_{a,a}$-connected if and only if G is not $(a, 1)$-stellar.

Next we consider $\lambda_{a_1,a_2,\dots,a_k}$-connected graphs for $k \geq 3$ and $a_1 = a_2 = \dots = a_k$. Some arguments in the proof of Theorem 2.1 can be extended for trees and lead to the following result.

Theorem 2.3 *Let $a, k \in \mathbb{N}$. A tree is $\lambda_{a_1,a_2,\dots,a_k}$-connected with $a_1 = a_2 = \dots = a_k = a$ if and only if it is not $(a, k-1)$-stellar.*

Proof: For the "only if"-part let S be a minimal edge cut such that $T - S$ has k components of cardinality at least a. Since every set of at most $k-1$ vertices misses at least one component of $T - S$, the "only if"-part follows.

The "if"-part is proved by induction over k. For $k = 1$, the result follows from the fact, that a connected graph is not $(a, 0)$-stellar if and only if its order is at least a. Now let $k \geq 2$ and let $T = (V, E)$ be a tree which is not $(a, k-1)$-stellar. For every edge $uv \in E$, the forest $T - u$ has a component of order at least a. Hence at least one of the two components of $T - uv$ has order at least a.

We direct every edge $uv \in E$ of T from u to v, if the component of $T - uv$ that contains u has less than a vertices.

If all edges of T are directed, then there is a vertex such that all incident edges are directed to this vertex. Deleting this vertex from T results in a forest all components of which are of order less than a. This contradiction implies that there are edges of T which are not directed.

If $u_1u_2...u_l$ is a maximal path in T whose edges are not directed, then all edges incident with u_1 different from u_1u_2 are directed to u_1. Let T' denote the component of $T - u_1$ which contains u_2.

If T' is $(a, k-2)$-stellar, then let U' be a set of at most $k-2$ vertices of T' such that all components of $T' - U'$ are of order at most $a-1$. Clearly, all components of $T - (U' \cup \{u_1\})$ are of order at most $a-1$ which implies the contradiction that T is $(a, k-1)$-stellar. Hence T' is not $(a, k-2)$-stellar and thus, by induction, there is a minimal edge cut S' of T' such that $T' - S'$ has $k-1$ components of order at least a. Clearly, $T - (S' \cup \{u_1u_2\})$ has k components of order at least a and the proof is complete. $\Box$

Note that König's classical theorem [10] relating the cardinalities of a minimum vertex cover and a maximum matching in bipartite graphs can be phrased as follows: *For every $k \in \mathbb{N}$ a bipartite graph is $\lambda_{a_1,a_2,\dots,a_k}$-connected with $a_1 = a_2 = \dots = a_k = 2$ if and only if it is not $(2, k-1)$-stellar.* Thus, in the special case $a = 2$, Theorem 2.3 remains valid for all bipartite graphs.

In general we observe the following.

Proposition 2.4 *Let $a, k \in \mathbb{N}$ and $a_1 = a_2 = \dots = a_k = a$.*

(1) A graph which is $(a, k-1)$-stellar is not $\lambda_{a_1,a_2,\dots,a_k}$-connected.

(2) A graph which is not $(a, a(k-1))$-stellar is $\lambda_{a_1,a_2,\dots,a_k}$-connected.

Proof: Since (1) is obvious we focus on (2). Let $\{V_1, V_2, ..., V_l\}$ be a maximal collection of disjoint sets of vertices each inducing a connected subgraph of G of order exactly a. Clearly, all components of $G - (V_1 \cup V_2 \cup ... \cup V_l)$ are of order at most $a - 1$. Since G is not $(a, a(k-1))$-stellar, we obtain $l \geq k$ which easily implies that G is $\lambda_{a_1,a_2,...,a_k}$-connected. $\square$

The following is one of the main results of [16].

Theorem 2.5 (Z. Zhang and J. Yuan [16]) *Let $a \in \mathbb{N}$ and let $G = (V, E)$ be a connected graph of order $n \geq 2a$ and minimum degree $\delta \geq a - 1$.*

Then G is $\lambda_{a,a}$-connected if and only if G is not $(a, 1)$-stellar.

As we have demonstrated in [13] Theorem 2.5 is a consequence of the following result.

Theorem 2.6 (Győri [7], Lovász [11]) *For $k \in \mathbb{N}$ with $k \geq 2$ let $G = (V, E)$ be a k-connected graph of order n. If $v_1, v_2, ..., v_k \in V$ are k distinct vertices of G and the integers $n_1, n_2, ..., n_k \in \mathbb{N}$ are such that $n_1 + n_2 + ... + n_k = n$, then there exists a partition $V = V_1 \cup V_2 \cup ... \cup V_k$ such that v_i lies in V_i, $|V_i| = n_i$ and $G[V_i]$ is connected for all $1 \leq i \leq k$.*

We conjecture that Theorem 2.5 extends in the following way.

Conjecture 2.7 *Let $a, k \in \mathbb{N}$ with $a, k \geq 2$ and let $G = (V, E)$ be a connected graph of order $n \geq ka$ and minimum degree $\delta \geq a + k - 3$.*

Then G is $\lambda_{a_1,a_2,...,a_k}$-connected with $a_1 = a_2 = ... = a_k = a$ if and only if it is not $(a, k-1)$-stellar.

It is easy to see that the only graphs which satisfy the assumptions of Theorem 2.5 ($k = 2$) or Conjecture 2.7 and are $(a, k-1)$-stellar arise from the union of $l \geq k + 1$ cliques with vertex sets $V_1, V_2, ..., V_l$ of order $a - 1$ by adding $k - 1$ vertices $x_1, x_2, ..., x_{k-1}$ which are adjacent to all vertices in $V_1 \cup V_2 \cup ... \cup V_l$ and possibly to each other.

Our next two results settle the case $k = 3$ of Conjecture 2.7 and establish a slightly weaker result in the case $k = 4$.

Theorem 2.8 *Let $a \in \mathbb{N}$ and let $G = (V, E)$ be a connected graph of order $n \geq 3a$ and minimum degree $\delta \geq a$.*

Then G is $\lambda_{a,a,a}$-connected if and only if G is not $(a, 2)$-stellar.

Proof: If G is 3-connected, then Theorem 2.6 implies that G is $\lambda_{a,a,a}$-connected. Hence we may assume that there is a set $V' \subseteq V$ with $|V'| \leq 1$ such that $G' = G[V \setminus V']$ is a connected graph with at least two endblocks. Let B and C be the vertex sets of two arbitrary endblocks of G' with cutvertices u_B and u_C, respectively.

If $|B| \geq a + 1$, then Theorem 2.6 implies the existence of a partition $B = B_1 \cup B_2$ such that $G[B_1]$ and $G[B_2]$ are connected, $|B_1| = a$ and $u_B \in B_2$. If $V' \neq \emptyset$, then let $v_C \in C \setminus \{u_C\}$ be a neighbour of the unique vertex in V'. Clearly, $|C| \geq \delta \geq a$ and Theorem 2.6 implies the existence of a partition $C = C_1 \cup C_2$ such that $G[C_1]$ and $G[C_2]$

are connected, $|C_1| = a-1$, $v_C \in C_1$ and $u_C \in C_2$. Now $V = V_1 \cup V_2 \cup V_3$ with $V_1 = B_1$, $V_2 = C_1 \cup V'$ and $V_3 = V \setminus (V_1 \cup V_2)$ is a partition of V such that $G[V_i]$ is connected and of order at least a for $1 \leq i \leq 3$, i.e. G is $\lambda_{a,a,a}$-connected. Hence we may assume that all endblocks of G' are of order at most a. By the minimum degree condition, this implies that $V' \neq \emptyset$ and all endblocks of G' are of order exactly a.

Next, we assume that B and C are vertex-disjoint. If G' has at least three endblocks, then we assume without loss of generality the existence of an endblock D of G' with $C \cap D = \emptyset$. If G' has only the two endblocks B and C, then the following argument needs no further assumption. The partition $V = V_1 \cup V_2 \cup V_3$ with $V_1 = (B \cup V') \setminus \{u_B\}$, $V_2 = C$ and $V_3 = V \setminus (V_1 \cup V_2)$ is such that $G[V_i]$ contains a component of order at least a for $1 \leq i \leq 3$. This easily implies that G is $\lambda_{a,a,a}$-connected. Therefore, we may assume that no two endblocks of G' are vertex disjoint. This implies that all blocks of G' are endblocks sharing the same cutvertex. Therefore all components of $G[V \setminus (V' \cup \{u_B\})]$ are of order at most $a-1$, i.e. G is $(a,2)$-stellar and the proof is complete. $\square$

Theorem 2.9 *Let $a \in \mathbb{N}$. If $G = (V, E)$ is a connected graph of order $n \geq 4a+4$ and minimum degree $\delta \geq a+4$, then G is $\lambda_{a,a,a,a}$-connected.*

Proof: If G is 4-connected, then the result follows immediately from Theorem 2.6. Hence we may assume that G is not 4-connected.

Claim 1 *There is a set $V' \subseteq V$ with $|V'| \leq 4$ such that $G' = G[V \setminus V']$*

(1) either has three endblocks,

(2) or has $l \geq 2$ blocks with vertex sets $B_1, B_2, ..., B_l$ such that $|B_i \cap B_{i+1}| = 1$ for $1 \leq i \leq l-1$, $|B_i \cap B_{i+1} \cap B_{i+2}| = 0$ for $1 \leq i \leq l-2$ and

(a) either $|B_1|, |B_l| \leq 2a$,

(b) or $|B_1| > 2a$ and $G[B_1]$ is 3-connected,

(c) or $|B_l| > 2a$ and $G[B_l]$ is 3-connected,

(d) or $|B_1| > 2a$ and $|V \setminus B_1| \geq 2a$,

(e) or $|B_l| > 2a$ and $|V \setminus B_l| \geq 2a$.

Proof of Claim 1: Since G is not 4-connected, there is a set $V_0' \subseteq V$ with $|V_0'| \leq 2$ such that $G_0' = G[V \setminus V_0']$ has at least two endblocks. If G_0' has three endblocks, then (1) holds and we are done. Hence we may assume that G_0' has exactly two endblocks, i.e. G_0' has $l \geq 2$ blocks with vertex sets $B_1, B_2, ..., B_l$ such that $|B_i \cap B_{i+1}| = 1$ for $1 \leq i \leq l-1$ and $|B_i \cap B_{i+1} \cap B_{i+2}| = 0$ for $1 \leq i \leq l-2$. (Note that B_1 and B_l are the endblocks of G_0'.)

In view of (a)-(e), we may assume that $|B_l| > 2a$, $G[B_l]$ is not 3-connected and $|V \setminus B_l| < 2a$. This implies that B_l contains a vertex x different from the cutvertex in B_l such that $G_1' = G[V \setminus (V_0' \cup \{x\})]$ has $l' \geq l+1 \geq 3$ blocks. In view of (1), we may assume that G_1' has exactly two endblocks. Hence G_1' has l' blocks with vertex sets $C_1, C_2, ..., C_{l'}$ such that

$B_i = C_i$ for $1 \leq i \leq l-1$, $|C_i \cap C_{i+1}| = 1$ for $1 \leq i \leq l'-1$ and $|C_i \cap C_{i+1} \cap C_{i+2}| = 0$ for $1 \leq i \leq l'-2$. (Note that the block B_l is replaced by the blocks $C_l, C_{l+1}, ..., C_{l'}$.)

Again, in view of (a)-(e), we may assume that $|C_{l'}| > 2a$, $G[C_{l'}]$ is not 3-connected and $|V \setminus C_{l'}| < 2a$. Repeating the same argument we obtain that $C_{l'}$ contains a vertex y different from the cutvertex in $C_{l'}$ such that $G'_2 = G[V \setminus (V'_0 \cup \{x, y\})]$ has $l'' \geq l'+1 \geq l+2 \geq 4$ blocks with vertex sets $D_1, D_2, ..., D_{l''}$ such that $C_i = D_i$ for $1 \leq i \leq l'-1$, $|D_i \cap D_{i+1}| = 1$ for $1 \leq i \leq l''-1$ and $|D_i \cap D_{i+1} \cap D_{i+2}| = 0$ for $1 \leq i \leq l''-2$.

Since G'_2 has at least 4 blocks and minimum degree at least $\delta - |V'_0 \cup \{x, y\}| \geq \delta - 4 \geq a$, it follows easily that $|V \setminus D_1|, |V \setminus D_l| \geq 2a$ and one of (a), (b), (c), (d) or (e) holds. This completes the proof of the claim. $\square$

We will now prove that the graph G' from Claim 1 is $\lambda_{a,a,a,a}$-connected which clearly implies that also G is $\lambda_{a,a,a,a}$-connected. Note that G' has order at least $4a$ and minimum degree at least a.

Case 1 Condition (1) in Claim 1 holds.

Let G' have three endblocks with vertex sets B, C and D and cutvertices u_B, u_C and u_D, respectively. Theorem 2.6 implies the existence of three sets $V_1 \subseteq B \setminus \{u_B\}$, $V_2 \subseteq C \setminus \{u_C\}$ and $V_3 \subseteq D \setminus \{u_D\}$ such that $|V_1| = |V_2| = |V_3| = a$ and $G[V_1]$, $G[V_2]$, $G[V_3]$ and $G[V \setminus (V_1 \cup V_2 \cup V_3)]$ are connected. Hence G' is $\lambda_{a,a,a,a}$-connected in this case.

Case 2 Condition (2) in Claim 1 holds.

Let G' have $l \geq 2$ blocks with vertex sets $B_1, B_2, ..., B_l$ such that $B_i \cap B_{i+1} = \{u_i\}$ for $1 \leq i \leq l-1$ and $|B_i \cap B_{i+1} \cap B_{i+2}| = 0$ for $1 \leq i \leq l-2$.

Case 2.1 Condition (a) in Claim 1 holds.

Since $|B_1|, |B_l| \leq 2a$, Theorem 2.6 implies the existence of two sets $V_1 \subseteq B_1 \setminus \{u_1\}$ and $V_2 \subseteq B_l \setminus \{u_{l-1}\}$ such that $|V_1| = |V_2| = a$ and $G[V_1]$ and $G[V_2]$ are connected.

There is an index i with $2 \leq i \leq l-1$ such that $|B_1 \cup B_2 \cup ... \cup B_{i-1}| \leq 2a$ and $|B_1 \cup B_2 \cup ... \cup B_i| > 2a$. Applying Theorem 2.6 to the block B_i yields a set $V'_3 \subseteq B_i \setminus \{u_i\}$ such that $u_{i-1} \in V'_3$, $|B_1 \cup B_2 \cup ... \cup B_{i-1} \cup V'_3| = 2a$ and $G[V'_3]$ is connected. For $V_3 = (B_1 \cup B_2 \cup ... \cup B_{i-1} \cup V'_3) \setminus V_1$ we obtain that $|V_3| = a$, $|V \setminus (V_1 \cup V_2 \cup V_3)| \geq a$ and $G[V_3]$ and $G[V \setminus (V_1 \cup V_2 \cup V_3)]$ are connected. Hence G' is $\lambda_{a,a,a,a}$-connected also in this case.

Case 2.2 Condition (b) or (c) in Claim 1 holds.

By symmetry, we may assume that (b) holds. Since $|B_1| > 2a$ and $G[B_1]$ is 3-connected, Theorem 2.6 implies the existence of three disjoint sets $V_1, V_2 \subseteq B_1 \setminus \{u_1\}$ and $V_3 \subseteq B_l \setminus \{u_{l-1}\}$ such that $|V_1| = |V_2| = |V_3| = a$ and $G[V_1]$, $G[V_2]$, $G[V_3]$ and $G[V \setminus (V_1 \cup V_2 \cup V_3)]$ are connected. Hence G' is $\lambda_{a,a,a,a}$-connected also in this case.

Case 2.3 Condition (d) or (e) in Claim 1 holds.

By symmetry, we may assume that (d) holds. Theorem 2.6 implies the existence of a partition $B_1 = V_1 \cup V_2$ and a set $V_3 \subseteq B_l \setminus \{u_{l-1}\}$ such that $|V_1|, |V_2| \geq a$, $|V_3| = a$ and $G[V_1]$, $G[V_2]$, $G[V_3]$ and $G[V \setminus (V_1 \cup V_2 \cup V_3)]$ are connected. Since $|V \setminus (V_1 \cup V_2 \cup V_3)| \geq a$, G' is $\lambda_{a,a,a,a}$-connected also in this case and the proof is complete. $\Box$

It is possible to slightly weaken the assumptions of Theorem 2.9 by using the vertices in the set V' similarly as in the proof of Theorem 2.8. Since we were not able to obtain the full statement of Conjecture 2.7 for $k = 4$, we decided not to further burden the technical proof of Theorem 2.9.

References

[1] P. Bonsma, N. Ueffing and L. Volkmann, Edge-cuts leaving components of order at least three, *Discrete Math.* **256** (2002) 431-439.

[2] Y.-C. Chen and J.J.M. Tan, Restricted connectivity for three families of interconnection networks, to appear in *Appl. Math. Comput.*

[3] Y.-C. Chen, J.J.M. Tan, L.-H. Hsu and S.-S. Kao, Super-connectivity and super-edge-connectivity for some interconnection networks, *Appl. Math. Comput.* **140** (2003), 245-254.

[4] A.H. Esfahanian and S.L. Hakimi, On computing a conditional edge-connectivity of a graph, *Inform. Process. Lett.* **27** (1988), 195-199.

[5] A.H. Esfahanian, Generalized measures of fault tolerance with application to N-cube networks, *IEEE Trans. Comput.* **38** (1989), 1586-1591.

[6] J. Fábrega and M.A. Fiol, On the extraconnectivity of graphs, *Discrete Math.* **155** (1996), 49-57.

[7] E. Győri, On division of graphs to connected subgraphs, Combinatorics (Proc. Fifth Hungarian Colloq., Keszthely, 1976), Vol I, pp. 485-494, Colloq. Math. Soc. János Bolyai, 18, North-Holland, Amsterdam-New York, 1978.

[8] F. Harary, Conditional connectivity, *Networks* **13** (1983), 347-357.

[9] A. Hellwig, D. Rautenbach and L. Volkmann, Cuts leaving components of given minimal order, *Discrete Math.* **292** (2005), 55-65.

[10] D. König, Graphen und Matrizen, *Mat. Fiz. Lapok* **38** (1931), 116-119.

[11] L. Lovász, A homology theory for spanning trees of a graph, *Acta Math. Acad. Sci. Hungar.* **30** (1977), 241-251.

[12] Jianping Ou, Edge cuts leaving components of order at least m, *Discrete Math.* **305** (2005), 365-371.

[13] D. Rautenbach and L. Volkmann, Some Remarks on $\lambda_{p,q}$-Connectedness, manuscript 2007.

[14] M. Wang and Q. Li, Conditional edge connectivity properties, reliability comparisons and transitivity of graphs, *Discrete Math.* **258** (2002), 205-214.

[15] Y.Q. Wang, Super restricted edge-connectivity of vertex-transitive graphs, *Discrete Math.* **289** (2004), 199-205.

[16] Z. Zhang and J. Yuan, A proof of an inequality concerning k-restricted edge connectivity, *Discrete Math.* **304** (2005), 128-134.