Light deflection in the post-linear gravitational field of bounded point-like masses

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Summary

Today, technology has achieved a level at which the extremely high precision of current ground-based radio interferometric observations are approaching an accuracy of 1 μarcsec. Moreover the planned space-based interferometers, such as the Global Astrometric Interferometer for Astrophysics (GAIA) and the Space Interferometric Mission (SIM), are going to measure the positions and/or the parallaxes of celestial objects with uncertainties in the range $10^{-5} - 10^{-6}$ arcsec. Furthermore the interferometer for the planned Laser Astrometric Test of Relativity Mission (LATOR) will be able to measure light deflection angles of the order 10^{-8} arcsec.

In order to reach the desired accuracies of $10^{-6}-10^{-8}$ arcsec in the computation of light deflection in gravitational fields, corrections arising from the lack of spherical symmetry of the gravitating system, the motion of the gravitating masses and the relativistic definition of the centre of mass must be taken into account.

In this thesis, the light deflection in the post-linear gravitational field of two bounded point-like masses is treated. Both the light source and the observer are assumed to be located at infinity in an asymptotically flat space. The equations of light propagation are explicitly integrated to the second order in G/c^2. Some of the integrals are evaluated by making use of an expansion in powers of the ratio of the relative separation distance r_{12} to the impact parameter ξ, r_{12}/ξ. A discussion of which orders must be retained to be consistent with the expansion in terms of G/c^2 is given. It is shown that the expression obtained in this thesis for the angle of light deflection is fully equivalent to the expression obtained by Kopeikin and Schäfer up to the order given there. The deflection angle takes a particularly simple form for a light ray originally propagating orthogonal to the orbital plane of a binary with equal masses. Application of the formulae for the deflection angle to the double pulsar PSR J0737-3039 for an impact parameter five times greater than the relative separation distance of the binary's components shows that the corrections to the Epstein-Shapiro light deflection angle of about 10^{-6} arcsec lie between 10^{-7} and 10^{-8} arcsec. The corrections coming from the spins of the components of PSR J0737-3039 lie between 10^{-8} and 10^{-9} arcsec.

Zusammenfassung

Astronomische Beobachtungen mit den heutigen erdgebundenen Interferometern im Radiofrequenzbereich sowie zukünftige astrometrische Beobachtungen mit geplanten Weltrauminterferometern (wie z.B. GAIA und SIM) erreichen eine Genauigkeit von $10^{-5} - 10^{-6}$ arcsec. Außerdem wird das Interferometer der geplanten LATOR-Mission Lichtablenkungen mit einer Genauigkeit von 10^{-8} arcsec messen.

Um bei der Berechnung der Lichtablenkung in Gravitationsfeldern eine entsprechend hohe Genauigkeit zu erreichen, muss man Korrekturen, die durch das Fehlen der sphärischen Symmetrie des gravitierenden Systems, die Bewegung der gravitierenden Massen und der relativistischen Definition des Schwerpunktes verursacht werden, berücksichtigen.

Diese Arbeit behandelt die Lichtablenkung im post-linearen Gravitationsfeld eines Systems zweier gebundenen Punktmassen. Es wird angenommen, dass sowohl die Lichtquelle als auch der Beobachter im Unendlichen in einem asymptotisch flachen Raum lokalisiert sind. Die Gleichungen der Lichtausbreitung werden bis zur Ordnung G^2/c^4 explizit integriert. Um die nicht elementaren Integrale zu berechnen, werden zunächst die Integranden in Potenzreihen vom Verhältnis r_{12}/ξ entwickelt, wobei r_{12} der relative Abstand der Komponenten des Binärsystems und ξ der Stoßparameter ist. In der Arbeit wird gezeigt, wie die Ordnung dieser Potenzreihenentwicklung zu wählen ist, damit sie konsistent mit den Entwicklungstermen in G/c^2 ist. Es wird auch gezeigt, dass der in dieser Arbeit berechnete Ausdruck für die Lichtablenkung mit dem Ergebnis von Kopeikin und Schäfer bis zu deren berechneter Ordnung übereinstimmt. Der Ausdruck für die Lichtablenkung nimmt insbesondere dann eine einfache Form an, wenn sich der ungestörte Lichtstrahl senkrecht zur Bahnebene eines Binärsystems, dessen Komponenten gleiche Masse haben, ausbreitet. Die Anwendung der berechneten Formeln für die Lichtablenkung auf den Doppelpulsar PSR J0737-3039 unter Annahme eines Stoßparameters der fünf Mal größer als der Abstand der Komponenten des Binärsystems ist, zeigt, dass die Korrekturen zu einem Epstein-Shapiro Lichtablenkungswinkel von 10^{-6} arcsec zwischen 10^{-7} und 10^{-8} arcsec liegen. Die Korrekturen, die durch die Eigendrehimpulse der Komponenten von PSR J0737-3039 verursacht werden, liegen zwischen 10^{-8} und 10^{-9} arcsec.

Contents

Notation

Summary of the notation and symbols used in this thesis:

G is the Newtonian constant of gravitation.

c is the velocity of light.

By $\Re$ we denote the real part of a quantity.

The Greek indices α, β, γ..., are space-time indices and run from 0 to 3.

The Latin indices i, j, k,..., are spatial indices and run from 1 to 3.

$g_{\mu\nu}$ is a metric tensor of curved, four-dimensional space-time, depending on spatial coordinates and time.

The signature adopted for $g_{\mu\nu}$ is $(-+++)$.

We suppose that space-time is covered by a harmonic coordinate system $(x^\mu) = (x^0, x^i)$, where $x^0 = c\,t$, t being the time coordinate.

The three-dimensional quantities (3-vectors) are denoted by $\vec{a} = a^i$.

The three-dimensional unit vector in the direction of $\vec{a}$ is denoted by $\vec{e}_a = e^i_a$.

The Latin indices are lowered and raised by means of the unit matrix $\delta_{ij} = \delta^{ij} = \mathrm{diag}(1,1,1)$.

The scalar product of any two 3-vectors $\vec{a}$ and $\vec{b}$ with respect to the Euclidean metric δ_{ij} is denoted by $\vec{a}\cdot\vec{b}$ and can be computed as $\vec{a}\cdot\vec{b} = \delta_{ij}a^i b^j = a^i b^i$.

The Euclidean norm of a 3-vector $\vec{a}$ is denoted by $a = |\vec{a}|$ and can be computed as $a = \Big[\delta_{mn}a^m a^n\Big]^{1/2}$.

By $\vec{l}_{(0)}$ we denote the vector tangent to the unperturbed light ray $\vec{z}(t)$ and the unit vector $\vec{e}_{(0)}$ is defined by $\vec{e}_{(0)} = \vec{l}_{(0)}/|\vec{l}_{(0)}|$.

$\eta^{\alpha\beta\gamma\nu}$ is the Levi-Civita tensor: $\eta^{\alpha\beta\gamma\nu} = -(-g)^{-1/2}\epsilon_{\alpha\beta\gamma\nu}$, where $\epsilon_{0123} = +1$ and g is the determinant of the covariant metric $g_{\mu\nu}$.

Partial derivative: $T_{\alpha,\,\mu} = \partial T_\alpha / \partial x^\mu$.

Christoffel symbols: $\Gamma^\alpha{}_{\mu\nu} = \frac{1}{2} g^{\alpha\rho}(g_{\rho\mu,\,\nu} + g_{\rho\nu,\,\mu} - g_{\mu\nu,\,\rho})$.

Covariant derivative: $$T^\alpha{}_{;\,\mu} = T^\alpha{}_{,\,\mu} + \Gamma^\alpha{}_{\mu\rho} T^\rho,$$
$$T_{\alpha;\,\mu} = T_{\alpha,\,\mu} - \Gamma^\rho{}_{\alpha\mu} T_\rho\,.$$

The symbol $|_{(\rightarrow)}$ denotes the replacement in the integrals of the photon trajectory by its unperturbed approximation before performing the integral and after evaluating the partial derivatives of the metric coefficients with respect to the photon's coordinates (i.e. $(z^0, z^i(t))$).

The symbol $(1 \leftrightarrow 2)$ refers to the preceding term but with the labels 1 and 2 exchanged.

1 Introduction

Light deflection by a gravitational field [1] is one of the observational cornerstones of general relativity. The observational confirmation during a solar eclipse in 1919 of Einstein's prediction that light would be deflected by the gravitational field of the Sun [2] brought general relativity to the attention of the general public in the 1920's. However, these first measurements of the angle of light deflection had only 30 percent accuracy, and succeeding experiments were not better. On account of the substantial improvements that have been made in radio astronomy, the angle of light deflection in the gravitational field of the Sun was measured with an uncertainty of less than one percent in the seventies [3, 4]. Today, technology has reached a level at which the extremely high precision of current ground-based radio interferometric astronomical observations approaches 1 μarcsec and within the next decade the accuracy of space-based astrometric positional observations is also expected to reach this accuracy. At this level of accuracy, we can no longer treat the gravitational field of a system of moving bodies as static and spherically symmetric. This fact is one of the principal reasons for the necessity of a more accurate solution to the problem of the propagation of electromagnetic waves in non-stationary gravitational fields of celestial bodies. To reach the accuracy of 1 μarcsec, many subtle relativistic effects must be taken into account. One of the most intricate problems is the computation of the effects of translational motion of the gravitating bodies on light propagation.

This question was treated for the first time by Hellings in 1986 [5]. In 1989 Klioner [6] solved the problem completely to the first post-Newtonian (1PN) order (i.e. to the order $1/c^2$) for the case of bodies moving with constant velocity. The complete solution of the problem for arbitrarily moving bodies in the first post-Minkowskian approximation (linear in the gravitational constant G) was found by Kopeikin and Schäfer in 1999 [7]. They succeeded in integrating analytically the post-Minkowskian equations of light propagation in the field of arbitrarily moving masses. The effect of the spin of the moving masses on the propagation of light was computed in 2002 by Kopeikin and Mashhoon [8]. In Ref. [9], Le Poncin-Lafitte *et al.* have recently developed an alternative approach to the problem of light deflection and time/frequency transfer in post-Minkowskian gravitational fields based on an expansion of the Synge world function for null geodesics.

In that paper, the world function and time transfer function were computed for a static, spherically symmetric body to the second post-Minkowskian approximation.

In this thesis we treat light deflection in the post-linear gravitational field of two bounded point-like masses (binary system) for the case when the light source as well the observer are located at infinity in an asymptotically flat space and the impact parameter is much larger than the separation distance between the components of the binary. For large impact parameters we assume, on physical grounds that the gravitational field along the light path is weak. To compute the light deflection, we integrate the equations of light propagation explicitly to the second order in G/c^2, i.e. to the order G^2/c^4.

The assumption that the gravitational field is weak along the light path allows us to consider the metric as a perturbation of a flat metric represented by a power series in the gravitational constant G

$$g_{\mu\nu}[x^\sigma, G] \equiv g^{(0)}_{\mu\nu} + \sum_{n=1}^{\infty} G^n g^{(n)}_{\mu\nu}(x^\sigma), \tag{1.1}$$

with

$$g^{(0)}_{\mu\nu} = \eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1).$$

For the same reason, we can consider the light trajectory as a perturbation of its trajectory in flat space (a straight line) represented by a power series in G

$$\vec{z}(t) = \vec{z}_{(0)}(t) + \sum_{n=1}^{\infty} G^n \delta\vec{z}_{(n)}(t). \tag{1.2}$$

It follows from (1.2) that the vector tangent to the light trajectory takes the form

$$\vec{l}(t) \equiv \frac{d\vec{z}(t)}{dt} = \vec{l}_{(0)} + \sum_{n=1}^{\infty} G^n \delta\vec{l}_{(n)}(t), \tag{1.3}$$

where $\vec{l}_{(0)}$ is the constant vector tangent to the unperturbed light trajectory.

In order to obtain the post-linear equations for light propagation, we introduce into the differential equations for the null geodesics the metric as given in (1.1) and the expression for the tangent vector $\vec{l}(t)$ as given in (1.3). As a result we get a set of ordinary coupled differential equations of first order for the perturbation terms $\delta\vec{l}_{(n)}(t)$. Each term $\delta\vec{l}_{(n)}$ is given in the form of a line integral along a straight line in the fictitious metric $g^{(0)}_{\mu\nu}$, i.e. along the original unperturbed light trajectory. We get the post-linear light deflection to the order G^2/c^4 after computing the perturbation terms $\delta\vec{l}_{(1)}(t)$, $\delta\vec{l}_{(2)}(t)$ and the corrections arising from introducing the linear perturbation of the light trajectory

$\delta\vec{z}_{(1)}(t)$, the motion of the masses and the shift of the 1PN-centre of mass with respect to the Newtonian centre of mass in the expression for linear light deflection. The final result is the expression for light deflection in the post-linear gravitational field of two bounded point-like masses. The deflection angle takes a particularly simple form for a light ray originally propagating orthogonal to the orbital plane of a binary with equal masses.

This thesis is organized as follows. In Chapter 2, by means of Maxwell's equations in curved space-time we find the fundamental laws of geometric optics in gravitational fields. Then we derive the post-linear light propagation equations. We introduce an approximation scheme to integrate these equations. The deflection angle as a function of the perturbations of the vector tangent to the light ray is introduced. Chapter 3 starts with a recapitulation of the light deflection in the post-Minkowskian gravitational field of a system of arbitrarily moving and spinning masses. The limit for the part of the light deflection caused by the point-mass piece of the energy-momentum tensor in the event that the speeds of the masses are small with respect to the speed of light and the retarded times are close to the time of closest approach of the unperturbed light ray to the origin of the coordinate system is computed. Furthermore, it is shown that the light deflection is mainly determined by the near zone gravitational field. Chapter 4 is devoted to the computation of the post-linear near zone metric in harmonic coordinates for two bounded point-like masses up to the second order in G/c^2. The coordinate frame is chosen so that the 1PN-centre of mass is at rest at the origin. In Chapter 5 we calculate the light deflection in the post-linear gravitational field of two bounded point-like masses. In Section 5.1 the perturbation of the vector tangent to the unperturbed light ray and the corresponding light deflection in the linear gravitational field are computed. In Section 5.2 we compute the light deflection in the post-linear gravitational field. To facilitate the computations we separate the light deflection terms that are functions of the post-linear metric coefficients from the terms that are functions of the linear metric coefficients and the perturbations of the first order in G of the vector tangent to the unperturbed light ray. The resulting integrals are given in Appendices C and D. In Section 5.3 we calculate the additional linear and post-linear light deflection terms arising from the introduction of the motion of the masses into the expression for the linear perturbation. In Section 5.4 we compute the corrections to the post-linear light deflection arising from the introduction of the linear perturbed light trajectory into the expression for the linear light deflection. The resulting integrals are given in Appendix E. Section 5.5 is devoted to the computation of the corrections to the linear and post-linear light deflection arising from the introduction of the shift of the 1PN-centre of mass with respect to the Newtonian centre of mass into the expression for the linear light deflection. The resulting expressions for the total linear perturbation and the linear and post-linear light deflection by the gravitational field of two bounded point-like masses are given in an explicit form in Sections 5.6 and 5.7. In Section 5.6 it is also shown that the expression for the angle of light deflection computed in Chapter 5 is fully equivalent to the expression obtained by Kopeikin and Schäfer in [7] up to the order given there. In Chapter 6 we present our results and give the light deflection expression for some simple cases in an explicit form. The derived formulae for the angle of light deflection are applied to the double pulsar PSR J0737-3039. In Appendix F we compute the linear light deflection terms arising from the acceleration terms in the metric coefficients $h_{00}^{(1)}$ and $h_{pq}^{(1)}$. Finally, Chapter 7 is devoted to a discussion of the results.

2 Light propagation in the post-linear gravitational field

In this chapter we use Maxwell's equations to derive the laws for light propagation in the post-linear gravitational field. Since in most instances gravitational fields vary even over macroscopic distances very little, we can assume that light propagation is well governed by the laws of geometric optics. In Section 2.1 we shall derive these laws in the presence of gravitational fields from Maxwell's equations. In Section 2.2 we give the equations of light propagation in an explicit form to the second order in G/c^2. An approximation scheme to solve the equations of light propagation is presented in Section 2.3. Finally, in Section 2.4 the angle of light deflection as a function of the perturbations of the vector tangent to the light ray is introduced. Part of this chapter is based on Sec. II of the paper by the author [10].

2.1 Geometric optics in gravitational fields

The general formalism describing the behaviour of electromagnetic radiation in the presence of arbitrary gravitational fields is well known [11, 12, 13]. It is governed by the source-free Maxwell equations in curved space-time:

$$F_{\beta\gamma;\alpha} + F_{\gamma\alpha;\beta} + F_{\alpha\beta;\gamma} = 0, \tag{2.1}$$

$$F^{\alpha\beta}{}_{;\beta} = 0, \tag{2.2}$$

where the electromagnetic field tensor $F_{\mu\nu}$ in terms of the four-vector potential A^μ is given by

$$F_{\mu\nu} = A_{\nu;\mu} - A_{\mu;\nu}. \tag{2.3}$$

In the derivation of the laws of geometric optics the following characteristic lengths are important [11, 13]:

1. the wavelength λ
2. a typical length L over which the amplitude, polarization and wavelength of the wave vary significantly (e.g. the radius of curvature of a wave front)

3. a typical "radius of curvature" for the geometry; more precisely, take

$$R = \left| \begin{array}{l} \text{typical component of the Riemannian tensor} \\ \text{in typical local inertial system} \end{array} \right|^{-1/2}.$$

Geometric optics is valid whenever the wavelength is very short compared to the others characteristic lengths:

$$\lambda \ll L \quad \text{and} \quad \lambda \ll R. \tag{2.4}$$

We consider an electromagnetic wave which is highly monochromatic[1] in regions having a size smaller than L. Now we separate the four-vector potential A_μ into a rapidly varying real phase ψ and a slowly varying complex amplitude $\mathcal{A}_\mu$ (eikonal ansatz)

$$A_\mu = \Re\{\mathcal{A}_\mu \mathrm{e}^{\mathrm{i}\psi}\}. \tag{2.5}$$

In order to obtain the equations of geometric optics, it is useful to introduce the small parameter $\epsilon = \lambda/\min(L, R)$ and expand the amplitude $\mathcal{A}_\mu$ in powers of ϵ:

$$\mathcal{A}_\mu = a_\mu + \epsilon b_\mu + \epsilon^2 c_\mu + ..., \tag{2.6}$$

where $a_\mu, b_\mu, ...$ are independent of λ. After introducing the expression for the amplitude $\mathcal{A}_\mu$ given by (2.6) into (2.5) we obtain

$$A_\mu = \Re\{(a_\mu + \epsilon b_\mu + \epsilon^2 c_\mu + ...)\mathrm{e}^{\frac{i\psi}{\epsilon}}\}, \tag{2.7}$$

where we have replaced ψ by ψ/ϵ since $\psi \propto \lambda^{-1}$.

Here it is important to note that in the expression for the amplitude $\mathcal{A}_\mu$ given by (2.6) the terms in powers of ϵ are small corrections (deviations from geometric optics) to the dominant part, which is independent of λ, due to a finite wavelength. The resulting equations for geometric optics, which we are going to derive below, take their simplest form in terms of the following quantities:

1. the wave four-vector $k_\mu = \partial_\mu \psi$
2. the scalar amplitude $a = \sqrt{(a_\mu \bar{a}^\mu)}$
3. the polarization four-vector $f_\mu = a_\mu/a$, where f_μ is a complex unit vector.

[1] More general cases can be treated through Fourier analysis.

By definition, light rays are integral curves of the vector field k^μ and are thus orthogonal to the surfaces of constant phase ψ, in other words orthogonal to the wave fronts.

From equations (2.2) and (2.3) it follows that

$$A^{\nu;\mu}{}_{;\nu} - A^{\mu;\nu}{}_{;\nu} = 0. \tag{2.8}$$

Upon applying the Ricci identity

$$A^{\nu;\mu}{}_{;\nu} = A^{\nu}{}_{;\nu}{}^{;\mu} + R^{\mu}_{\ \nu} A^{\nu} \tag{2.9}$$

and imposing the Lorenz gauge condition

$$A^{\nu}{}_{;\nu} = 0, \tag{2.10}$$

equation (2.8) becomes

$$A^{\mu;\nu}{}_{;\nu} - R^{\mu}_{\ \nu} A^{\nu} = 0. \tag{2.11}$$

If we now insert (2.7) into the Lorenz condition (2.10), we obtain

$$0 = A^{\nu}{}_{;\nu} = \Re\{(\frac{i}{\epsilon} k_\mu (a^\mu + \epsilon b^\mu + ...) + (a^\mu + \epsilon b^\mu + ...)_{;\mu}) e^{\frac{i\psi}{\epsilon}}\}. \tag{2.12}$$

From the term of order ϵ^{-1}, we deduce that $k_\mu a^\mu = 0$, or equivalently

$$k_\mu f^\mu = 0. \tag{2.13}$$

The preceding equation shows that the polarization vector is orthogonal to the wave vector. Now we introduce (2.7) into (2.11) to get

$$\begin{aligned} 0 &= -A^{\mu;\nu}{}_{;\nu} + R^{\mu}_{\ \nu} A^{\nu} \\ &= \Re\Big\{ \big[\frac{1}{\epsilon^2} k^\nu k_\nu (a^\mu + \epsilon b^\mu + ...) - 2\frac{i}{\epsilon} k^\nu (a^\mu + \epsilon b^\mu + ...)_{;\nu} \\ &- \frac{i}{\epsilon} k^{\nu}{}_{;\nu} (a^\mu + \epsilon b^\mu + ...) - (a^\mu + \epsilon b^\mu + ...)^{;\nu}{}_{;\nu} + R^{\mu}_{\ \nu} (a^\nu + \epsilon b^\nu + ...) \big] e^{\frac{i\psi}{\epsilon}} \Big\}. \end{aligned} \tag{2.14}$$

From the term of order ϵ^{-2} we infer that $k^\nu k_\nu a^\mu = 0$, which is equivalent to

$$k^\nu k_\nu = 0. \tag{2.15}$$

Equation (2.15) proves that the wave vector is null. The terms of order ϵ^{-1} give

$$k^\nu k_\nu b^\mu - 2i\big(k^\nu a^{\mu}{}_{;\nu} + \frac{1}{2} k^{\nu}{}_{;\nu} a^\mu\big) = 0. \tag{2.16}$$

With (2.15) the equation above implies that

$$k^\nu a^\mu{}_{;\nu} = -\frac{1}{2} k^\nu{}_{;\nu} a^\mu. \tag{2.17}$$

As a consequence of these equations, we obtain the geodesic law for the propagation of light rays. From equation (2.15) we have

$$0 = (k^\nu k_\nu)_{;\mu} = 2k^\nu k_{\nu;\mu}. \tag{2.18}$$

Now $k_\nu = \partial_\nu \psi$, and since $\psi_{;\nu;\mu} = \psi_{;\mu;\nu}$ we get, after interchanging indices,

$$k^\nu k_{\mu;\nu} = 0. \tag{2.19}$$

Equations (2.19) and (2.15) show that the paths of light rays are null geodesics.

If we now write the amplitude a^μ as $a^\mu = af^\mu$ and take into account (2.17) we have

$$\begin{aligned} 2ak^\nu a_{,\nu} &= 2ak^\nu a_{;\nu} = k^\nu (a^2)_{;\nu} = k^\nu (a_\mu \bar{a}^\mu)_{;\nu} \\ &= \bar{a}^\mu k^\nu a_{\mu;\nu} + a_\mu k^\nu \bar{a}^\mu_{;\nu} = -k^\nu{}_{;\nu} a^2, \end{aligned}$$

so that

$$k^\nu a_{,\nu} = -\frac{1}{2} k^\nu{}_{;\nu} a. \tag{2.20}$$

This can be regarded as a propagation law for the scalar amplitude. After introducing $a^\mu = af^\mu$ into (2.15) we get

$$\begin{aligned} 0 &= k^\nu (af^\mu)_{;\nu} + \frac{1}{2} k^\nu{}_{;\nu} (af^\mu) \\ &= ak^\nu f^\mu_{;\nu} + k^\nu f^\mu a_{;\nu} + \frac{1}{2} k^\nu{}_{;\nu} (af^\mu) \\ &= ak^\nu f^\mu_{;\nu} + f^\mu (k^\nu a_{;\nu} + \frac{1}{2} k^\nu{}_{;\nu} a) \\ &= ak^\nu f^\mu_{;\nu} \end{aligned}$$

or

$$k^\nu f^\mu_{;\nu} = 0. \tag{2.21}$$

We thus see that the polarization vector f^μ is perpendicular to the light rays and is parallel propagated along them.

After multiplying equation (2.20) by a we find

$$ak^\nu a_{,\nu} + \frac{1}{2} k^\nu{}_{;\nu} a^2 = 0 \tag{2.22}$$

We can rewrite the equation above as

$$k^\nu (a^2)_{;\nu} + a^2 k^\nu{}_{;\nu} = (a^2 k^\nu)_{;\nu} = 0, \tag{2.23}$$

where $(a^2 k^\nu)$ can be regarded as a conserved current.

Quantum mechanically, (2.23) expresses the conservation law for the number of photons. Since the photon number is not in general conserved, here it is an adiabatic invariant, in other words, a quantity which varies very slowly for $R \gg \lambda$ in comparison to the photon frequency.

Finally, we give a summary of the fundamental laws of geometric optics, which we derived in this section:

1. Light rays are null geodesics;
2. The polarization four-vector is perpendicular to the rays and is parallel-propagated along the rays;
3. The amplitude is governed by an adiabatic invariant which, in quantum language, expresses that the number of photons is conserved.

2.2 The light propagation equation

In the present work, we calculate light deflection in the post-linear gravitational field of two bounded masses for the case when the impact parameter $|\vec{\xi}\,|$ is much larger (e.g. 5 times or more) than the coordinate distance r_{12} between the two accelerating masses, so that we can suppose that the gravitational field is weak along the light path.

For weak gravitational fields, as was shown in the preceding section, we can assume that light propagation is very well governed by the laws of geometric optics, whereby light rays (photons) move in curved space-time along null geodesics. The equation for a single null geodesic[2] reads

$$\frac{d^2 z^\mu}{d\lambda^2} + \Gamma^\mu_{\alpha\beta} \frac{dz^\alpha}{d\lambda} \frac{dz^\beta}{d\lambda} = 0, \tag{2.24}$$

where

$$\Gamma^\mu_{\rho\sigma} = \frac{1}{2} g^{\mu\omega} \big(g_{\rho\omega,\sigma} + g_{\sigma\omega,\rho} - g_{\rho\sigma,\omega} \big) \tag{2.25}$$

[2]Contrasted with equation (2.19) which describes a congruence of null geodesics.

are the Christoffel symbols of the second kind and λ is an affine parameter. In this work we shall use the time coordinate t instead of the affine parameter λ to parameterize the null geodesics.

After substituting λ by the time coordinate $z^0 = c\,t$ in the geodesic equation (2.24) by means of

$$\frac{d^2t}{d\lambda^2} + c^{-1}\Gamma^0_{\nu\sigma}\frac{dz^\mu}{d\lambda}\frac{dz^\nu}{d\lambda} = 0, \tag{2.26}$$

we obtain

$$\frac{d^2z^i}{dt^2} + \Gamma^i_{\alpha\beta}\frac{dz^\alpha}{dt}\frac{dz^\beta}{dt} = c^{-1}\Gamma^0_{\nu\sigma}\frac{dz^\nu}{dt}\frac{dz^\sigma}{dt}\frac{dz^i}{dt}. \tag{2.27}$$

The condition for the geodesic to be light-like can be formulated as

$$g_{\mu\nu}[z^0, z^i(t), G]\frac{dz^\mu}{dt}\frac{dz^\nu}{dt} = 0. \tag{2.28}$$

If we substitute in (2.27) dz^μ/dt by $l^\mu = (c, l^i)$ we find

$$\frac{dl^i}{dt} + \Gamma^i_{\alpha\beta}l^\alpha l^\beta = c^{-1}\Gamma^0_{\nu\sigma}l^\nu l^\sigma l^i. \tag{2.29}$$

Notice that l^μ is not exactly a 4-vector because we differentiate with respect to the time coordinate t. So l^μ is a 4-vector up to a factor. Here, $l^i = dz^i/dt$ is the 3-vector tangent to the light ray $z^i(t)$. Now we consider a light ray $z^i(t)$ that is propagating in a curved space-time $g_{\mu\nu}[z^0, z^i(t), G]$ with the signature $(-+++)$. If the gravitational field is weak, we can write the fundamental metric tensor $g_{\mu\nu}[z^0, z^i(t), G]$ as a power series in the gravitational constant G

$$g_{\mu\nu}[z^0, z^i(t), G] \equiv \eta_{\mu\nu} + \sum_{n=1}^{\infty} h^{(n)}_{\mu\nu}[z^0, z^i(t), G], \tag{2.30}$$

where $\eta_{\mu\nu}$ is the Minkowski metric and $h^{(n)}_{\mu\nu}[z^0, z^i(t), G]$ is a perturbation of the order n in the gravitational constant G equivalent to $G^n g^{(n)}_{\mu\nu}(z^0, z^i(t))$ (physically, this means an expansion in the dimensionless number Gm/c^2d which is usually very small, d being the characteristic length of the problem and m a characteristic mass).

In order to obtain from (2.29) the equations of light propagation for the metric (2.30), we substitute the Christoffel symbols into (2.29). To save writing, we denote the metric coefficients $h^{(1)}_{pq}[z^0, z^i(t), G]$, $h^{(2)}_{pq}[z^0, z^i(t), G]$ by $h^{(1)}_{pq}$ and $h^{(2)}_{pq}$. Then the resulting equation of light propagation to the second order in G/c^2 is

$$
\begin{aligned}
\frac{dl^i}{dt} &= \frac{1}{2}c^2 h^{(1)}_{00,i} - c^2 h^{(1)}_{0i,0} - c\,h^{(1)}_{0i,m} l^m + c\,h^{(1)}_{0m,i} l^m - c\,h^{(1)}_{mi,0} l^m - h^{(1)}_{mi,n} l^m l^n \\
&+ \frac{1}{2} h^{(1)}_{mn,i} l^m l^n - \frac{1}{2} c\,h^{(1)}_{00,0} l^i - h^{(1)}_{00,k} l^k l^i + \left(\frac{1}{2}c^{-1} h^{(1)}_{mp,0} - c^{-1} h^{(1)}_{0p,m}\right) l^m l^p l^i \\
&+ \frac{1}{2}c^2 h^{(2)}_{00,i} - \frac{1}{2}c^2 h^{(1)ik} h^{(1)}_{00,k} - h^{(2)}_{00,k} l^k l^i - \left(h^{(2)}_{mi,n} - \frac{1}{2} h^{(2)}_{mn,i}\right) l^m l^n \\
&+ h^{(1)ik}\left(h^{(1)}_{mk,n} - \frac{1}{2} h^{(1)}_{mn,k}\right) l^m l^n - h^{(1)}_{00} h^{(1)}_{00,k} l^k l^i,
\end{aligned} \tag{2.31}
$$

where by $_{,0}$ and $_{,i}$ we denote $\partial/\partial z^0$ and $\partial/\partial z^i$. To calculate the light deflection, we need to solve equation (2.31) for l^i. In order to solve this complicated, nonlinear differential equation we turn to approximation techniques.

2.3 The approximation scheme

The 3-vector $l^i(t)$ can be written as

$$
l^i(t) = l^i_{(0)} + \sum_{n=1}^{\infty} \delta l^i_{(n)}(t), \tag{2.32}
$$

where $l^i_{(0)}$ denotes the constant incoming tangent vector $l^i(-\infty)$ and $\delta l^i_{(n)}(t)$ the perturbation of the constant tangent vector $l^i_{(0)}$ of order n in G equivalent to $G^n \delta \vec{l}_{(n)}(t)$. After introducing the expression for $l^i(t)$ given by (2.32) into (2.31) we obtain differential equations for the perturbations $\delta l^i_{(1)}$ and $\delta l^i_{(2)}$:

$$
\begin{aligned}
\frac{d\delta l^i_{(1)}}{dt} &= \frac{1}{2}c^2 h^{(1)}_{00,i} - c^2 h^{(1)}_{0i,0} - c\,h^{(1)}_{0i,m} l^m_{(0)} + c\,h^{(1)}_{0m,i} l^m_{(0)} - c\,h^{(1)}_{mi,0} l^m_{(0)} - h^{(1)}_{mi,n} l^m_{(0)} l^n_{(0)} \\
&+ \frac{1}{2} h^{(1)}_{mn,i} l^m_{(0)} l^n_{(0)} - \frac{1}{2} c\,h^{(1)}_{00,0} l^i_{(0)} - h^{(1)}_{00,k} l^k_{(0)} l^i_{(0)} + \left(\frac{1}{2}c^{-1} h^{(1)}_{mp,0} - c^{-1} h^{(1)}_{0p,m}\right) l^m_{(0)} l^p_{(0)} l^i_{(0)}
\end{aligned} \tag{2.33}
$$

and

$$
\begin{aligned}
\frac{d\delta l^i_{(2)}}{dt} &= \frac{1}{2}c^2 h^{(2)}_{00,i} - \frac{1}{2}c^2 h^{(1)ik} h^{(1)}_{00,k} - h^{(2)}_{00,k} l^k_{(0)} l^i_{(0)} - \left(h^{(2)}_{mi,n} - \frac{1}{2}h^{(2)}_{mn,i}\right) l^m_{(0)} l^n_{(0)} \\
&+ h^{(1)ik}\left(h^{(1)}_{mk,n} - \frac{1}{2}h^{(1)}_{mn,k}\right) l^m_{(0)} l^n_{(0)} - h^{(1)}_{00} h^{(1)}_{00,k} l^k_{(0)} l^i_{(0)} \\
&- c\, h^{(1)}_{0i,m} \delta l^m_{(1)} + c\, h^{(1)}_{0m,i} \delta l^m_{(1)} - c\, h^{(1)}_{mi,0} \delta l^m_{(1)} \\
&- h^{(1)}_{mi,n} \delta l^m_{(1)} l^n_{(0)} - h^{(1)}_{mi,n} l^m_{(0)} \delta l^n_{(1)} + h^{(1)}_{mn,i} \delta l^m_{(1)} l^n_{(0)} \\
&- \frac{1}{2} c\, h^{(1)}_{00,0} \delta l^i_{(1)} - h^{(1)}_{00,k} \delta l^k_{(1)} l^i_{(0)} - h^{(1)}_{00,k} l^k_{(0)} \delta l^i_{(1)} \\
&+ c^{-1} h^{(1)}_{mp,0} \delta l^m_{(1)} l^p_{(0)} l^i_{(0)} - c^{-1} h^{(1)}_{0p,m} \delta l^m_{(1)} l^p_{(0)} l^i_{(0)} - c^{-1} h^{(1)}_{0p,m} l^m_{(0)} \delta l^p_{(1)} l^i_{(0)} \\
&+ \left(\frac{1}{2} c^{-1} h^{(1)}_{mp,0} - c^{-1} h^{(1)}_{0p,m}\right) l^m_{(0)} l^p_{(0)} \delta l^i_{(1)} .
\end{aligned}
\tag{2.34}
$$

In order to calculate the perturbations $\delta l^i_{(1)}(t)$ and $\delta l^i_{(2)}(t)$, we have to integrate equations (2.33) and (2.34) along the light ray trajectory to the appropriate order.

Before performing the integration it is convenient to introduce a new independent parameter τ along the photon's trajectory as defined by Kopeikin and Schäfer [7]. The relationship between the parameter τ and the time coordinate t is

$$
\tau = t - t^*, \tag{2.35}
$$

where t^* is the time of closest approach of the unperturbed trajectory of the photon to the origin in an asymptotically flat harmonic coordinate system. Then the equation of the unperturbed light ray can be represented as

$$
z^i(\tau)_{\text{unpert.}} = \tau l^i_{(0)} + \xi^i, \tag{2.36}
$$

where ξ^i is the vector directed from the origin of the coordinate system towards the point of closest approach. The vector ξ^i is often called the impact parameter and is orthogonal to the vector $l^i_{(0)}$. The distance $r(\tau) = |\vec{z}(\tau)|$, of the photon from the origin of the coordinate system reads

$$
r(\tau) = \sqrt{c^2\tau^2 + \xi^2}. \tag{2.37}
$$

It follows from equation (2.35) that the differential identity $dt = d\tau$ is valid, so that we can always replace the integration along the unperturbed light ray with respect to t by the integration with respect to the variable τ.

Then the resulting expression for $\delta l^i_{(1)}$ is given by

$$
\begin{aligned}
\delta l^i_{(1)}(\tau) &= \frac{1}{2}\int_{-\infty}^{\tau} d\sigma\, l^\alpha_{(0)} l^\beta_{(0)} h^{(1)}_{\alpha\beta,i}|_{(\rightarrow)} - c\, h^{(1)}_{0i} - h^{(1)}_{mi} l^m_{(0)} - h^{(1)}_{00} l^i_{(0)} \\
&+ \frac{1}{2} c \int_{-\infty}^{\tau} d\sigma\, h^{(1)}_{00,0} l^i_{(0)}|_{(\rightarrow)} + \int_{-\infty}^{\tau} d\sigma\, l^m_{(0)} l^p_{(0)} \left[\frac{1}{2} c^{-1} h^{(1)}_{mp,0} - c^{-1} h^{(1)}_{0p,m}\right] l^i_{(0)}\, |_{(\rightarrow)} \,.
\end{aligned}
\tag{2.38}
$$

On the right-hand side of equation (2.38) after evaluating the partial derivatives of the metric coefficients with respect to the photon's coordinates (i.e. $(z^0, z^i(t))$), we replace in the integrals the photon trajectory by its unperturbed approximation $z^i(\sigma)_{\text{unpert.}} = \sigma l^i_{(0)} + \xi^i$ and the time coordinate z^0 by $\sigma + t^*$. In this work we denote this operation by the symbol $|_{(\rightarrow)}$. Then we perform the integration with respect to σ. After substituting the expression obtained for $\delta l^i_{(1)}$ into equation (2.34), we can integrate it to get $\delta l^i_{(2)}$. To calculate the perturbation $\delta l^i_{(2)}$, we separate the part of $\delta l^i_{(2)}$ which depends on the post-linear metric coefficients from the part which depends on the linear metric coefficients. We denote these parts of $\delta l^i_{(2)}$ by $\delta l^i_{(2)\mathrm{I}}$ and $\delta l^i_{(2)\mathrm{II}}$ respectively. As in the case of equation (2.38) we replace the photon trajectory by its unperturbed approximation and the time coordinate z^0 by $\sigma + t^*$ after evaluating the partial derivatives of the metric coefficients with respect to the photon coordinates. The resulting expressions for $\delta l^i_{(2)\mathrm{I}}$ and $\delta l^i_{(2)\mathrm{II}}$ are

$$\delta l^i_{(2)\mathrm{I}}(\tau) = \int_{-\infty}^{\tau} d\sigma \left[\frac{1}{2}c^2 h^{(2)}_{00,i} - h^{(2)}_{00,k} l^k_{(0)} l^i_{(0)}\right]|_{(\rightarrow)} + \int_{-\infty}^{\tau} d\sigma \left[\frac{1}{2} h^{(2)}_{mn,i} - h^{(2)}_{mi,n}\right] l^m_{(0)} l^n_{(0)}|_{(\rightarrow)} \tag{2.39}$$

and

$$\begin{aligned}
\delta l^i_{(2)\mathrm{II}}(\tau) = &- \int_{-\infty}^{\tau} d\sigma \left[\frac{1}{2}c^2 h^{(1)ik} h^{(1)}_{00,k} + h^{(1)}_{00} h^{(1)}_{00,k} l^k_{(0)} l^i_{(0)}\right]|_{(\rightarrow)} \\
&+ \int_{-\infty}^{\tau} d\sigma \left[h^{(1)ik}\left(h^{(1)}_{mk,n} - \frac{1}{2}h^{(1)}_{mn,k}\right)\right] l^m_{(0)} l^n_{(0)}|_{(\rightarrow)} \\
&+ c\int_{-\infty}^{\tau} d\sigma \left[h^{(1)}_{0m,i} - h^{(1)}_{0i,m} - h^{(1)}_{mi,0}\right] \delta l^m_{(1)}(\sigma)|_{(\rightarrow)} \\
&+ \int_{-\infty}^{\tau} d\sigma \left[h^{(1)}_{mn,i}\delta l^m_{(1)}(\sigma) l^n_{(0)} - h^{(1)}_{mi,n}\delta l^m_{(1)}(\sigma) l^n_{(0)} - h^{(1)}_{mi,n} l^m_{(0)} \delta l^n_{(1)}(\sigma)\right]|_{(\rightarrow)} \\
&- \int_{-\infty}^{\tau} d\sigma \left[\frac{1}{2}c h^{(1)}_{00,0}\delta l^i_{(1)}(\sigma) + h^{(1)}_{00,k}\delta l^k_{(1)}(\sigma) l^i_{(0)} + h^{(1)}_{00,k} l^k_{(0)} \delta l^i_{(1)}(\sigma)\right]|_{(\rightarrow)} \\
&+ c^{-1}\int_{-\infty}^{\tau} d\sigma \left[h^{(1)}_{mp,0}\delta l^m_{(1)}(\sigma) l^p_{(0)} - h^{(1)}_{0p,m}\delta l^m_{(1)}(\sigma) l^p_{(0)} - h^{(1)}_{0p,m} l^m_{(0)} \delta l^p_{(1)}(\sigma)\right] l^i_{(0)}|_{(\rightarrow)} \\
&+ c^{-1}\int_{-\infty}^{\tau} d\sigma \left[\frac{1}{2}h^{(1)}_{mp,0} - h^{(1)}_{0p,m}\right] l^m_{(0)} l^p_{(0)} \delta l^i_{(1)}(\sigma)|_{(\rightarrow)}.
\end{aligned} \tag{2.40}$$

2.4 The light deflection

The dimensionless vector $\alpha^i_{(n)}$ of order n in G, describing the angle of total deflection of the light ray measured at the point of observation and calculated with respect to the vector $l^i_{(0)}$ (see [7]), is given by

$$\alpha^i_{(n)}(t) = P^i_q \frac{\delta l^q_{(n)}(t)}{|\vec{l}_{(0)}|}, \tag{2.41}$$

where $\delta l^i_{(n)}$ is the perturbation of the constant tangent vector of order n in G. Here,

$$P^i_q = \delta^i_q - e^i_{(0)} e_{(0)q} \tag{2.42}$$

is the projection tensor, which projects tensors onto the plane orthogonal to the vector $l^i_{(0)}$. In the case of light rays (photons) $|\vec{l}_{(0)}| = c$.

3 Light deflection in the linear gravitational field of arbitrarily moving and spinning masses

Since the linear metric perturbation $h^{(1)}_{\mu\nu}$ for a system of arbitrarily moving and spinning masses can be split into two pieces, one arising from the point-mass part of the stress-energy tensor and one caused by the spin part of the stress-energy tensor, we can calculate the light deflection corresponding to each part separately.

The angle of light deflection arising from the point-mass part was computed by Kopeikin and Schäfer in 1999 [7] and the spin part by Kopeikin and Mashhoon in 2002 [8].

In this chapter we present their computation and derive the expression for the angle of light deflection resulting from the expression obtained by Kopeikin and Schäfer in the event that the speeds of the masses are small with respect to the speed of light and the retarded times are close to the time of closest approach of the unperturbed light ray to the origin of the coordinate system. We also compute the angle of light deflection caused by the quadrupole moment of the system of arbitrarily moving masses. This chapter is primarily based on the papers by S. M. Kopeikin and G. Schäfer [7], S. M. Kopeikin and B. Mashhoon [8], and the author [10].

3.1 The linear gravitational field generated by arbitrarily moving and spinning masses

In the linear approximation (2.30), reduces to

$$g_{\mu\nu}(t,\vec{x}) = \eta_{\mu\nu} + h^{(1)}_{\mu\nu}(t,\vec{x}). \tag{3.1}$$

The metric perturbation $h^{(1)}_{\mu\nu}(t,\vec{x})$ can be found by solving the Einstein field equations which read in the first post-Minkowskian approximation and in the harmonic gauge (see [14]) as follows:

$$\Box h^{\mu\nu}_{(1)}(t,\vec{x}) = -16\pi\frac{G}{c^4}\mathcal{T}^{\mu\nu}(t,\vec{x}), \tag{3.2}$$

where $\Box = \eta^{\mu\nu}\partial_\mu\partial_\nu$ is the flat d'Alembertian operator and

$$\mathcal{T}^{\mu\nu}(t,\vec{x}) = T^{\mu\nu}(t,\vec{x}) - \frac{1}{2}\eta^{\mu\nu}T^{\lambda}{}_{\lambda}(t,\vec{x}). \tag{3.3}$$

In the present case $T^{\mu\nu}(t,\vec{x})$ is the energy-momentum tensor for a system of spinning bodies, the explicit expression of which will be given in the next subsection.

As is well known, the solution of these equations has the form of a Liénard-Wiechert potential [15], which in terms of the integral of the retarded tensor potential $\mathcal{T}_{\mu\nu}$ is given by

$$h^{\mu\nu}_{(1)}(t,\vec{x}) = 4\frac{G}{c^4}\int d^3x' \frac{\mathcal{T}^{\mu\nu}(t - \frac{|\vec{x}-\vec{x}\,'|}{c}, \vec{x}\,')}{|\vec{x}-\vec{x}\,'|}. \tag{3.4}$$

3.1.1 Energy-momentum tensor of a system of spinning bodies

The energy-momentum tensor $T^{\mu\nu}_a$ of the a^{th} spinning body reads

$$T^{\mu\nu}_a(t,\vec{x}) = T^{\mu\nu}_{aM}(t,\vec{x}) + T^{\mu\nu}_{aS}(t,\vec{x}), \tag{3.5}$$

where $T^{\mu\nu}_{aM}$ and $T^{\mu\nu}_{aS}$ are parts of the tensor generated by the mass and spin of the a^{th} body, and t and $\vec{x}$ are the time coordinate and spatial coordinates in the underlying inertial coordinate system. In the case of a system of spinning bodies the total tensor of energy-momentum is a linear sum of tensors of the form (3.5) corresponding to each body. Since in this chapter we are considering only the linear gravitational field, the total gravitational field of the system of bodies results from the linear superposition of the fields due to individual bodies.

In equation (3.5), $T^{\mu\nu}_{aM}$ and $T^{\mu\nu}_{aS}$ are defined in terms of the Dirac function as follows [8, 16, 17]:

$$T^{\mu\nu}_{aM}(t,\vec{x}) = c\int_{-\infty}^{\infty} d\eta\, p_a^{(\mu}u_a^{\nu)}(-g)^{-1/2}\delta(x^0 - x^0_a(\eta))\delta^{(3)}(\vec{x}-\vec{x}_a(\eta)), \tag{3.6}$$

$$T^{\mu\nu}_{aS}(t,\vec{x}) = -c\nabla_\gamma\int_{-\infty}^{\infty} d\eta\, S_a^{\gamma(\mu}u_a^{\nu)}(-g)^{-\frac{1}{2}}\delta(x^0 - x^0_a(\eta))\delta^{(3)}(\vec{x}-\vec{x}_a(\eta)), \tag{3.7}$$

where η is the proper time along the world line of the body's centre of mass, $\vec{x}_a(\eta)$ are the spatial coordinates of the body's centre of mass at the proper time η, $u^\alpha_a(\eta) = \gamma_a(c,\vec{v}_a(t))$ is the four-velocity of the body with $\gamma_a = (1 - v_a^2/c^2)^{-1/2}$, $\vec{v}_a(t)$ is the three-velocity of the body in space, $p^\alpha_a(\eta)$ is the body's linear momentum (in the approximation, which neglects the rotation of the bodies, $p^\alpha_a(\eta) = m_a u^\alpha_a(\eta)$, where m_a is the invariant mass of the a^{th} body), $S^{\mu\nu}_a(\eta)$ is an antisymmetric tensor, which represents the body's spin angular momentum attached to the body's centre of mass (spin-tensor), ∇_γ denotes covariant differentiation with regard to the metric tensor $g_{\mu\nu}$, the parentheses around indices indicate symmetrization, and $g = \det(g_{\mu\nu})$ is the determinant of the metric tensor.

The definition of the spin-tensor $S_a^{\gamma\nu}$ is arbitrary up to the choice of a spin subsidiary condition that is chosen as follows:

$$S_a^{\gamma\nu} u_{a\nu} = 0. \tag{3.8}$$

Because of this subsidiary condition, the antisymmetric tensor $S_a^{\gamma\nu}$ has only three independent components, which can be mapped uniquely onto the spin-vector (intrinsic angular momentum) $S_{a\alpha}$ by means of

$$S_{a\alpha} = \frac{1}{2}\eta_{\alpha\beta\gamma\nu} u_a^\beta S_a^{\gamma\nu}/c, \quad S_a^{\gamma\nu} = \eta^{\alpha\beta\gamma\nu} S_{a\alpha} u_{a\beta}/c, \tag{3.9}$$

with

$$S_a^\rho u_{a\rho} = 0. \tag{3.10}$$

Here, $\eta^{\alpha\beta\gamma\nu}$ is the Levi-Civita tensor in curved space-time related to the Levi-Civita tensor $\epsilon_{\alpha\beta\gamma\nu}$ in Minkowskian space as follows:

$$\eta^{\alpha\beta\gamma\nu} = -(-g)^{-1/2}\epsilon_{\alpha\beta\gamma\nu}, \quad \eta_{\alpha\beta\gamma\nu} = (-g)^{1/2}\epsilon_{\alpha\beta\gamma\nu}, \tag{3.11}$$

where $\epsilon_{0123} = +1$. In what follows we denote the spin-vector in the frame comoving with the body as $\mathcal{S}_a^\alpha = (0, \vec{\mathcal{S}}_a)$. In this frame, the temporal component of the spin-vector vanishes as a consequence of (3.10). After applying a Lorentz transformation from the comoving frame to the underlying inertial frame, we have in the post-Minkowskian approximation

$$S_a^0 = \gamma_a\left(\frac{\vec{v}_a \cdot \vec{\mathcal{S}}_a}{c}\right), \quad S_a^i = \mathcal{S}_a^i + \frac{\gamma_a - 1}{v_a^2}\left(\vec{v}_a \cdot \vec{\mathcal{S}}_a\right) v_a^i, \tag{3.12}$$

where $\gamma_a \equiv (1 - v_a^2/c^2)^{-1/2}$ and $\vec{v}_a = (v_a^i)$ is the velocity of the a^{th} mass with respect to the frame at rest.

3.1.2 The metric perturbation $h^{\mu\nu}_{(1)}$

In order to facilitate the computation of the metric perturbation $h^{\mu\nu}_{(1)}$, we split it into two parts $h^{\mu\nu}_{M(1)}$ and $h^{\mu\nu}_{S(1)}$ (see [8]) that are linearly independent in the first post-Minkowskian approximation, that is

$$h^{\mu\nu}_{(1)}(t, \vec{x}) = h^{\mu\nu}_{M(1)}(t, \vec{x}) + h^{\mu\nu}_{S(1)}(t, \vec{x}). \tag{3.13}$$

Thus, the solution for each part can be found from the Einstein field equations (3.2) with the corresponding energy-momentum tensor.

Solution for the point-mass part

After substituting $h^{\mu\nu}_{(1)}(t,\vec{x})$ by (3.13) and the parts of the energy-momentum tensor given by (3.6) and (3.7) into equation (3.2), we obtain for the point-mass part

$$\Box h^{\mu\nu}_{M(1)}(t,\vec{x}) = -16\pi\frac{G}{c^4}\mathcal{T}^{\mu\nu}_M(t,\vec{x}), \tag{3.14}$$

where

$$\mathcal{T}^{\mu\nu}_M(t,\vec{x}) = \sum_{a=1}^{N} m_a c\int_{-\infty}^{\infty} d\eta\,\delta(x^0 - x^0_a(\eta))\delta^{(3)}(\vec{x}-\vec{x}_a(\eta))\Big[u^\mu_a(\eta)u^\nu_a(\eta) + \frac{1}{2}\eta^{\mu\nu}c^2\Big]. \tag{3.15}$$

The field equations (3.14) are integrated by using the retarded flat propagator $D_r(x,x')$ which is given by

$$D_r(x,x') = \frac{\theta(x^0 - x^{0\,\prime})}{4\pi r}\delta(x^0 - x^{0\,\prime} - r), \tag{3.16}$$

where $r = |\vec{x}-\vec{x}\,'|$.

We finally get

$$h^{\mu\nu}_{M(1)}(t,\vec{x}) = 4\frac{G}{c^4}\sum_{a=1}^{N} m_a\sqrt{1-\frac{v_a^2(s_a)}{c^2}}\left[\frac{u^\mu_a(s_a)u^\nu_a(s_a)+\frac{1}{2}\eta^{\mu\nu}c^2}{r_a(s_a)-(1/c)(\vec{v}_a(s_a)\cdot\vec{r}_a(s_a))}\right], \tag{3.17}$$

where $\vec{r}_a(s_a)$ is given by $\vec{r}_a(s_a) = \vec{x}-\vec{x}_a(s_a)$ and $r_a(s_a)$ is the Euclidean norm of $\vec{r}_a(s_a)$. In the equation above s_a denotes the retarded time $s_a = s_a(t,\vec{x})$ for the a^{th} body which is a solution of the light-cone equation

$$s_a + \frac{1}{c}r_a(s_a) = t. \tag{3.18}$$

Solution for the spin part

From (3.2), (3.7) and (3.13), we find that the field equations for the spin part are

$$\Box h^{\mu\nu}_{S(1)}(t,\vec{x}) = -16\pi\frac{G}{c^4}\mathcal{T}^{\mu\nu}_S(t,\vec{x}), \tag{3.19}$$

where

$$\mathcal{T}^{\mu\nu}_S(t,\vec{x}) = -\sum_{a=1}^{N} c\frac{\partial}{\partial x^\gamma_a}\int_{-\infty}^{\infty} d\eta\, S^{\gamma(\mu}_a(\eta)u^{\beta)}_a(\eta)\delta(x^0 - x^0_a(\eta))\delta^{(3)}(\vec{x}-\vec{x}_a(\eta)). \tag{3.20}$$

Integration of (3.19) with the help of the flat retarded propagator (3.16) leads to[1]:

$$h^{\mu\nu}_{S(1)}(t,\vec{x}) = -4\frac{G}{c^4}\sum_{a=1}^{N}\frac{\partial}{\partial x^\gamma_a}\left\{\sqrt{1-\frac{v_a^2(s_a)}{c^2}}\left[\frac{S^{\gamma(\mu}(s_a)u^{\beta)}(s_a)}{r_a(s_a)-(1/c)(\vec{v}_a(s_a)\cdot\vec{r}_a(s_a))}\right]\right\}. \tag{3.21}$$

[1] Note that in equation (15) of Ref. [8] the factor $\sqrt{1-\frac{v_a^2(s_a)}{c^2}}$ is missing.

Since in this chapter we are interested only in the effects arising from the linear gravitational field, we can treat the sources as a system of free noninteracting spinning point-like masses, each moving with arbitrary constant velocity with its spin axis pointing in an arbitrary fixed direction. This follows from the Mathisson-Papapetrou equations in the underlying inertial coordinate system.

After performing the differentiation in equation (3.21) we finally arrive at

$$h^{\mu\nu}_{S(1)}(t,\vec{x}) = 4\frac{G}{c^4}\sum_{a=1}^{N}\left[1-\frac{v_a^2(s_a)}{c^2}\right]^{3/2}\left[\frac{r_{a\gamma}S_a^{\gamma(\mu}u_a^{\nu)}}{\left[r_a(s_a)-(1/c)(\vec{v}_a(s_a)\cdot\vec{r}_a(s_a))\right]^3}\right], \tag{3.22}$$

where $\vec{v}_a$ and $S_a^{\gamma\mu}$ are treated as constants and we define $r_a^\alpha = (r_a, \vec{r}_a)$.

3.2 Angle of light deflection

To compute the angle of light deflection, we follow the approximation scheme presented in Section 2.2 of the preceding chapter. For the angle of light deflection linear in G, we first have to compute the linear perturbation $\delta l^i_{(1)}$. The differential equation for the perturbation $\delta l^i_{(1)}$ is (see (2.33)):

$$\begin{aligned}\frac{d\delta l^i_{(1)}(t)}{dt} &= \frac{1}{2}c^2 h^{(1)}_{00,i}(t,\vec{z}) - c\,h^{(1)}_{0i,t}(t,\vec{z}) - c\,l^m_{(0)}\,h^{(1)}_{0i,m}(t,\vec{z}) + c\,l^m_{(0)}\,h^{(1)}_{0m,i}(t,\vec{z})\\ &- l^m_{(0)}\,h^{(1)}_{mi,t}(t,\vec{z}) - l^m_{(0)}\,l^n_{(0)}\,h^{(1)}_{mi,n}(t,\vec{z}) + \frac{1}{2}\,l^m_{(0)}\,l^n_{(0)}\,h^{(1)}_{mn,i}(t,\vec{z}) - l^i_{(0)}\,\frac{1}{2}h^{(1)}_{00,t}(t,\vec{z})\\ &- l^k_{(0)}\,l^i_{(0)}\,h^{(1)}_{00,k}(t,\vec{z}) + l^i_{(0)}\,l^m_{(0)}\,l^p_{(0)}\left(\frac{1}{2}c^{-2}h^{(1)}_{mp,t}(t,\vec{z}) - c^{-1}h^{(1)}_{0p,m}(t,\vec{z})\right),\end{aligned} \tag{3.23}$$

where by $_{,i}$ and $_{,t}$ we denote $\partial/\partial z^i$ and $\partial/\partial t$. In this work we assume that the unperturbed light ray trajectory (i.e. the light ray trajectory in the Minkowski space-time) is given by (2.36).

Since, the metric coefficients are smooth functions of t and $\vec{z}$, we can apply to (3.23) the following rule of differentiation for an arbitrary smooth function $F(t,\vec{z})$ given in [18],

$$\left[\left(\frac{\partial}{\partial z^i} + \frac{l_{(0)i}}{c^2}\frac{\partial}{\partial t}\right)F(t,\vec{z})\right]_{\vec{z}=\vec{z}_{(0)}+\vec{l}_{(0)}(t-t_0)} = \left(P_i^j\frac{\partial}{\partial\xi^j} + \frac{l_{(0)i}}{c^2}\frac{\partial}{\partial\tau}\right)F[\tau,\vec{\xi}+\vec{l}_{(0)}\tau], \tag{3.24}$$

where P_i^j is the projection tensor (2.42). Equation (3.24) states that the differentiation of $F(t,\vec{z})$ with respect to time t and spatial coordinates z^i followed by the substitution $\vec{z} = \vec{z}_{(0)} + \vec{l}_{(0)}(t-t_0)$ is equivalent to performing the substitution of t by τ and $\vec{z}$ by $\vec{z} = \vec{\xi} + \vec{l}_{(0)}\tau$ in $F(t,\vec{z})$ followed by the differentiation with respect to the time τ and the impact parameter ξ^i. Here, it is important to remark that the new variables ξ^i and τ are independent. For this reason, the integration of any function, which can be represented as a time derivative with regard to the parameter τ, is always quite straightforward:

$$\int d\tau \frac{\partial}{\partial\tau} F(\tau, \vec{\xi}) = F(\tau, \vec{\xi}) + C(\vec{\xi}), \tag{3.25}$$

where $C(\vec{\xi})$ is an arbitrary function of the constant impact parameter. Moreover, since the vector ξ^i does not depend on the time τ, the partial derivatives with respect to ξ^i can be taken outside the time integrals when we are computing them along the photon's trajectory, that is

$$\int d\tau \frac{\partial}{\partial\xi^i} F(\tau, \vec{\xi}) = \frac{\partial}{\partial\xi^i} \int d\tau F(\tau, \vec{\xi}). \tag{3.26}$$

As we shall see, the equations for the linear perturbations (3.23) become simpler in terms of the parameters $\vec{\xi}$ and τ. After applying the rule (3.24) to (3.23) we get[2]

$$\begin{aligned} \frac{d\delta l^i_{(1)}(\tau)}{d\tau} &= \frac{1}{2} l^\alpha_{(0)} l^\beta_{(0)} \hat{\partial}_i h^{(1)}_{\alpha\beta}(\tau, \vec{z}(\tau)) - \hat{\partial}_\tau \Big[l_{(0)\alpha} h^{(1)\alpha i}(\tau, \vec{z}(\tau)) + \frac{1}{2} l^i_{(0)} h^{(1)}_{00}(\tau, \vec{z}(\tau)) \\ &\quad - \frac{1}{2} l^i_{(0)} \frac{l^m_{(0)} l^n_{(0)}}{c^2} h^{(1)}_{mn}(\tau, \vec{z}(\tau)) \Big], \end{aligned} \tag{3.27}$$

where $\hat{\partial}_i \equiv P^q_i \partial/\partial\xi^q$ and $\vec{z}(\tau)$ is given by (2.36).

It follows from (3.27) and (2.41) that the expression for the angle of light deflection is

$$\alpha^i_{(1)}(\tau) = \frac{1}{2c} \int_{-\infty}^{\tau} d\sigma l^\alpha_{(0)} l^\beta_{(0)} \hat{\partial}_i h^{(1)}_{\alpha\beta}(\tau, \vec{z}(\tau)\,) - \frac{1}{c} P^i_q l_{(0)\alpha} h^{(1)\alpha q}(\tau, \vec{z}(\tau)\,). \tag{3.28}$$

Equation (3.28) gives the angle of light deflection measured by an observer located at a spatial distance $|\vec{z}(\tau)|$ from the origin of the coordinate system when the light source is located at infinity in an asymptotically flat space.

3.3 The light cone equation

In order to compute the integral in (3.28), it is useful to replace in the integrand the time argument σ with the arguments ζ_a, defined by the light-cone equation (3.18), which after substituting $\vec{x}$ with the unperturbed light trajectory (2.36) reads as follows:

$$\sigma + t^* = \zeta_a + |\vec{\xi} + \sigma \vec{l}_{(0)} - \vec{x}_a(\zeta_a)|. \tag{3.29}$$

[2] Equation (3.27) is equivalent to equation (19) in [7].

Differentiation of this equation yields a relationship between differentials of the time variables σ and ζ_a, and the parameters t^*, ξ^i, $l^i_{(0)}$:

$$d\zeta_a(cr_a - \vec{r}_a \cdot \vec{v}_a) = d\sigma(cr_a - \vec{l}_{(0)} \cdot \vec{r}_a) + cr_a dt^* - \vec{r}_a \cdot d\vec{\xi} - \sigma\vec{r}_a \cdot d\vec{l}_{(0)}, \tag{3.30}$$

where the coordinates $\vec{x}_a$ and the velocity $\vec{v}_a$ of the a^{th} mass are taken at the retarded time ζ_a, and the coordinates of the photon $\vec{z}$ are taken at the time $\sigma(\zeta_a)$. From (3.30) we obtain the partial derivative of ζ_a with regard to the parameter ξ^i

$$\frac{\partial\zeta_a}{\partial\xi^i} = -\frac{r^i_a/c}{[r_a - (\vec{r}_a \cdot \vec{v}_a)/c]}, \tag{3.31}$$

and the relationship between the time differentials along the photon's world line which reads

$$d\sigma = d\zeta_a \frac{[r_a - (\vec{r}_a \cdot \vec{v}_a)/c]}{[r_a - (\vec{l}_{(0)} \cdot \vec{r}_a)/c]}. \tag{3.32}$$

If the parameter σ runs from $-\infty$ to $+\infty$, the new parameters ζ_a run from $\zeta_a(-\infty) = -\infty$ to $\zeta_a(+\infty) = t^* + \vec{l}_{(0)} \cdot \vec{x}_a(\zeta_a(+\infty))$ when the motion of each mass is restricted to a bounded domain of space, as in the case for a binary system. For bodies moving along straight lines with constant velocities, the parameter σ also runs from $-\infty$ to $+\infty$, but here the parameters ζ_a run from $-\infty$ to $+\infty$.

3.4 Gravitational lens approximation

In this section we shall derive some important equations, which are valid when we treat the system of arbitrarily moving and spinning masses as a moving gravitational lens.

In what follows it is convenient to introduce the vector (see Fig. 3.1 for more details on the geometry of the lens)

$$\vec{y}_a = \vec{z}(s_a) - \vec{x}_a(s_a), \tag{3.33}$$

where $\vec{z}(s_a)$ is the location of the photon at the retarded time s_a. Since in the event of gravitational lensing, the impact parameter of the light ray is very small in comparison with the distances of the light-deflecting masses to the observer and the source of light, we can assume that the length of the vector $\vec{y}_a$ is small compared to the distances $D = |\vec{z}(t) - \vec{z}(t_0)|$ (distance between the light source and the observer) and $r_a = |\vec{z}(t) - \vec{x}_a(s_a)|$.

From the light-cone equation (3.18) we have

$$s_a = t - \frac{1}{c} r_a. \tag{3.34}$$

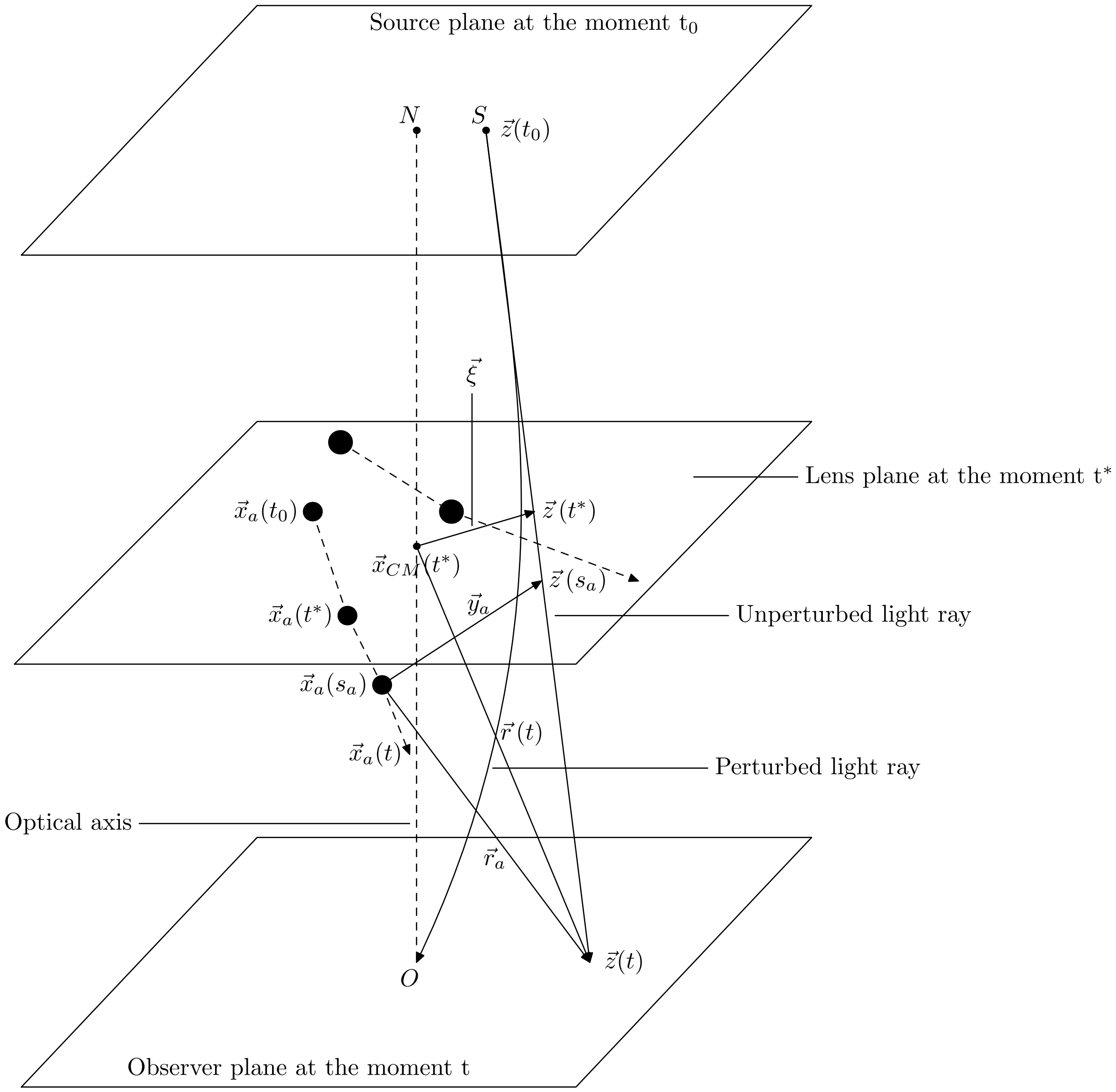

Figure 3.1: Relative configuration of observer O, source of light S, and a moving gravitational lens. The centre of the lens is at $\vec{x}_{CM}(t^*)$, and the line through $\vec{x}_{CM}(t^*)$ and the observer O is the 'optical axis'.

Upon introducing (3.34) into (3.33) and taking into account (2.36) we get

$$\begin{aligned}\vec{y}_a &= \vec{z}(t - \frac{1}{c} r_a) - \vec{x}_a(s_a),\\ &\approx \vec{z}(t) - \frac{1}{c}\vec{l}_{(0)} r_a - \vec{x}_a(s_a),\\ &\approx \vec{r}_a(t, s_a) - \frac{1}{c}\vec{l}_{(0)} r_a,\end{aligned} \tag{3.35}$$

where, as in other parts of the present work, we have $\vec{r}_a(t, s_a) = \vec{z}(t) - \vec{x}_a(s_a)$. From (3.35) it follows that

$$\frac{1}{c}(\vec{l}_{(0)} \cdot \vec{y}_a) = -\frac{d_a^2}{2r_a}, \tag{3.36}$$

where the distance $d_a = |\vec{y}_a|$ is the Euclidean length of $\vec{y}_a$. It is easy to see that the preceding equation can be written as

$$r_a - \frac{1}{c}(\vec{l}_{(0)} \cdot \vec{r}_a(t, s_a)) = \frac{d_a^2}{2r_a}. \tag{3.37}$$

3.5 The angle of light deflection linear in G caused by the point-mass part of the energy-momentum tensor

Upon substituting the value of $h^{M(1)}_{\alpha\beta}$ given by (3.17) into (3.28) and performing the integration with the help of the relationships (3.31) and (3.32), we finally obtain

$$\begin{aligned}\alpha^i_{M(1)}(\tau) = &-2\frac{G}{c^2}\sum_{a=1}^{N} \frac{m_a[1 - \frac{(\vec{e}_{(0)}\cdot\vec{v}_a(s_a))}{c}]^2[r_a(\tau, s_a) + (\vec{e}_{(0)} \cdot \vec{r}_a(\tau, s_a))]P^i_q r^q_a(\tau, s_a)}{\sqrt{1 - \frac{v_a^2(s_a)}{c^2}}[r_a^2(\tau, s_a) - (\vec{e}_{(0)} \cdot \vec{r}_a(\tau, s_a))^2][r_a(\tau, s_a) - \frac{\vec{v}_a(s_a)\cdot\vec{r}_a(\tau,s_a)}{c}]}\\ &+ 4\frac{G}{c^3}\sum_{a=1}^{N} \frac{m_a[1 - \frac{\vec{e}_{(0)}\cdot\vec{v}_a(s_a)}{c}]}{\sqrt{1 - \frac{v_a^2(s_a)}{c^2}}[r_a(\tau, s_a) - \frac{\vec{v}_a(s_a)\cdot\vec{r}_a(\tau,s_a)}{c}]} P^i_q v^q_a(s_a),\end{aligned} \tag{3.38}$$

which is equivalent to equation (68) in [7]. Note that (3.38) and equation (68) in [7] have opposite signs, since our definition of the angle of light deflection in (2.41) has the opposite sign with respect to the definition used by Kopeikin and Schäfer in [7].

For an observer located at infinity, we find

$$\begin{aligned}\alpha^i_{M(1)} &= \lim_{\tau\to\infty} \alpha^i_{M(1)}(\tau)\\ &= -4\frac{G}{c^2}\sum_{a=1}^{N} \frac{m_a\left[1 - \frac{\vec{e}_{(0)}\cdot\vec{v}_a(s_a)}{c}\right]}{\sqrt{1 - \frac{v_a^2(s_a)}{c^2}} R_a(s_a)}\left[\xi^i - P^i_q x^q_a(s_a)\right],\end{aligned} \tag{3.39}$$

where the quantity $R_a(s_a)$ is defined by

$$R_a(s_a) = r_a^2(0, s_a) - (\vec{e}_{(0)} \cdot \vec{x}_a(s_a))^2. \tag{3.40}$$

Here, it is worthwhile to note that the preceding expression for the light deflection angle is equivalent to the expression given by equation (139) in [7].

With the help of (3.18), (3.37) and the relationship for the time of closest approach,

$$\begin{aligned} t^* &= t - \frac{1}{c}(\vec{e}_{(0)} \cdot \vec{z}(t)) \\ &= t - \frac{1}{c}\vec{e}_{(0)} \cdot \vec{r}_a(t, s_a) - \frac{1}{c}\vec{e}_{(0)} \cdot \vec{x}_a(s_a), \end{aligned} \tag{3.41}$$

it is straightforward to show (see also Sec. VII B in [7]) that

$$\begin{aligned} s_a - t^* &= \frac{1}{c}[\vec{e}_{(0)} \cdot \vec{x}_a(s_a) - \frac{d_a^2}{2r_a}] \\ &\simeq \frac{1}{c}\vec{e}_{(0)} \cdot \vec{x}_a(s_a). \end{aligned} \tag{3.42}$$

If the speeds of the masses are small with respect to the speed of light and the retarded times do not differ significantly from the time of closest approach t^*, we are allowed to use the Taylor expansion of the quantity

$$x_a^i(s_a) \simeq x_a^i(t^*) + v_a^i(t^*)(s_a - t^*) + \frac{1}{2}a_a^i(t^*)(s_a - t^*)^2. \tag{3.43}$$

After substituting into (3.42), we find

$$s_a - t^* \simeq \frac{1}{c}(\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) + \frac{1}{c}(\vec{e}_{(0)} \cdot \vec{v}_a(t^*))(s_a - t^*) + \frac{1}{2c}(\vec{e}_{(0)} \cdot \vec{a}_a(t^*))(s_a - t^*)^2. \tag{3.44}$$

Now we solve equation (3.44) iteratively with respect to $(s_a - t^*)$ to obtain

$$s_a - t^* \simeq \frac{1}{c}\vec{e}_{(0)} \cdot \vec{x}_a(t^*) + \frac{1}{c^2}(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))(\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) + \mathcal{O}\Big(\frac{1}{c^3}\Big). \tag{3.45}$$

After performing the Taylor expansion of the expression for the light deflection angle given by (3.39) and taking into account (3.45), we finally obtain

$$
\begin{aligned}
\alpha^i_{M(1)} &= -4\frac{G}{c^2}\sum_{a=1}^{N}\frac{m_a}{R_a}\big[\xi^i - P^i_q x^q_a(t^*)\big] \\
&+ 4\frac{G}{c^3}\sum_{a=1}^{N}\frac{m_a}{R_a}(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))\big[\xi^i - P^i_q x^q_a(t^*)\big] \\
&+ 4\frac{G}{c^3}\sum_{a=1}^{N}\frac{m_a}{R_a}(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))P^i_q v^q_a(t^*) \\
&- 8\frac{G}{c^3}\sum_{a=1}^{N}\frac{m_a}{R_a^2}(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))\Big[\vec{\xi}\cdot\vec{v}_a(t^*) - \vec{x}_a(t^*)\cdot\vec{v}_a(t^*) + (\vec{e}_{(0)}\cdot\vec{x}_a(t^*))(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))\Big] \\
&\times\big[\xi^i - P^i_q x^q_a(t^*)\big] \\
&- 2\frac{G}{c^4}\sum_{a=1}^{N}\frac{m_a}{R_a}v_a^2(t^*)\big[\xi^i - P^i_q x^q_a(t^*)\big] \\
&+ 4\frac{G}{c^4}\sum_{a=1}^{N}\frac{m_a}{R_a^2}v_a^2(t^*)(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2\big[\xi^i - P^i_q x^q_a(t^*)\big] \\
&- 16\frac{G}{c^4}\sum_{a=1}^{N}\frac{m_a}{R_a^3}(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2\Big[\vec{\xi}\cdot\vec{v}_a(t^*) - \vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\Big]^2\big[\xi^i - P^i_q x^q_a(t^*)\big] \\
&- 32\frac{G}{c^4}\sum_{a=1}^{N}\frac{m_a}{R_a^3}(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^3(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))\Big[\vec{\xi}\cdot\vec{v}_a(t^*) - \vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\Big]\big[\xi^i - P^i_q x^q_a(t^*)\big] \\
&- 16\frac{G}{c^4}\sum_{a=1}^{N}\frac{m_a}{R_a^3}(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^4(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))^2\big[\xi^i - P^i_q x^q_a(t^*)\big] \\
&+ 8\frac{G}{c^4}\sum_{a=1}^{N}\frac{m_a}{R_a^2}(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2\Big[\vec{\xi}\cdot\vec{v}_a(t^*) - \vec{x}_a(t^*)\cdot\vec{v}_a(t^*) \\
&+ (\vec{e}_{(0)}\cdot\vec{x}_a(t^*))(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))\Big]P^i_q v^q_a(t^*) \\
&- 4\frac{G}{c^4}\sum_{a=1}^{N}\frac{m_a}{R_a^2}(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))^2\big[\xi^i - P^i_q x^q_a(t^*)\big] \\
&- 4\frac{G}{c^4}\sum_{a=1}^{N}\frac{m_a}{R_a^2}(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2\Big[\vec{\xi}\cdot\vec{a}_a(t^*) - \vec{x}_a(t^*)\cdot\vec{a}_a(t^*) + (\vec{e}_{(0)}\cdot\vec{x}_a(t^*))(\vec{e}_{(0)}\cdot\vec{a}_a(t^*))\Big] \\
&\times\big[\xi^i - P^i_q x^q_a(t^*)\big] \\
&+ 2\frac{G}{c^4}\sum_{a=1}^{N}\frac{m_a}{R_a}(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2 P^i_q a^q_a(t^*) \\
&+ 4\frac{G}{c^4}\sum_{a=1}^{N}\frac{m_a}{R_a}(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))(\vec{e}_{(0)}\cdot\vec{a}_a(t^*))\big[\xi^i - P^i_q x^q_a(t^*)\big].
\end{aligned}
\tag{3.46}
$$

3.6 The angle of light deflection linear in G caused by the spin part of the energy-momentum tensor

After inserting the value of $h^{S(1)}_{\alpha\beta}$ given by (3.21) as well as (3.22) into (3.28) and evaluating the integral with the help of the relationships (3.31) and (3.32), we arrive at

$$\begin{aligned}
\alpha^i_{S(1)}(\tau) = & -2\frac{G}{c^4}\sum_{a=1}^{n}\left[1-\frac{\vec{e}_{(0)}\cdot\vec{v}_a(s_a)}{c}\right]\left\{\frac{\left[1-\frac{v_a^2(s_a)}{c^2}\right]}{\left[r_a(\tau,s_a)-\frac{\vec{r}_a(\tau,s_a)\cdot\vec{v}_a(s_a)}{c}\right]^3}\right.\\
& \times\frac{l_{(0)\alpha}r_{a\beta}S_a^{\alpha\beta}P^i_p r^p_a(\tau,s_a)}{\left[r_a(\tau,s_a)-\vec{e}_{(0)}\cdot\vec{r}_a(\tau,s_a)\right]}P^i_q r^q_a(\tau,s_a)\\
& +\frac{\left[1-\frac{\vec{e}_{(0)}\cdot\vec{v}_a(s_a)}{c}\right]l_{(0)\alpha}r_{a\beta}S_a^{\alpha\beta}}{\left[r_a(\tau,s_a)-\frac{\vec{r}_a(\tau,s_a)\cdot\vec{v}_a(s_a)}{c}\right]^2\left[r_a(\tau,s_a)-\vec{e}_{(0)}\cdot\vec{r}_a(\tau,s_a)\right]^2}P^i_q r^q_a(\tau,s_a)\\
& -\frac{l_{(0)\alpha}r_{a\beta}S_a^{\alpha\beta}}{\left[r_a(\tau,s_a)-\frac{\vec{r}_a(\tau,s_a)\cdot\vec{v}_a(s_a)}{c}\right]^2\left[r_a(\tau,s_a)-\vec{e}_{(0)}\cdot\vec{r}_a(\tau,s_a)\right]}\frac{1}{c}P^i_q v^q_a(s_a)\\
& \left.-\frac{P^i_q l_{(0)\alpha}S_a^{\alpha q}}{\left[r_a(\tau,s_a)-\frac{\vec{r}_a(\tau,s_a)\cdot\vec{v}_a(s_a)}{c}\right]\left[r_a(\tau,s_a)-\vec{e}_{(0)}\cdot\vec{r}_a(\tau,s_a)\right]}\right\}\\
& +2\frac{G}{c^3}\left[1-\frac{v_a^2(s_a)}{c^2}\right]\left\{\frac{\left[1-\frac{\vec{e}_{(0)}\cdot\vec{v}_a(s_a)}{c}\right]P^i_q r_{a\gamma}S_a^{\gamma q}}{\left[r_a(\tau,s_a)-\frac{\vec{r}_a(\tau,s_a)\cdot\vec{v}_a(s_a)}{c}\right]^3}\right.\\
& \left.+\frac{1}{c^2}\frac{P^i_q l_{(0)\alpha}r_{a\gamma}S_a^{\alpha\gamma}v^q_a(s_a)}{\left[r_a(\tau,s_a)-\frac{\vec{r}_a(\tau,s_a)\cdot\vec{v}_a(s_a)}{c}\right]^3}\right\},
\end{aligned}\tag{3.47}$$

where $r^\alpha_a = (r_a, \vec{r}_a)$.

For an observer located at infinity, (3.47) becomes

$$\alpha^i_{S(1)} = -4\frac{G}{c^4}\sum_{a=1}^{N}\left\{2\frac{l_{(0)\alpha}r_{a\beta}S_a^{\alpha\beta}}{d_a^4}\left[\xi^i - P^i_q x^q_a(s_a)\right]-\frac{P^i_q l_{(0)\alpha}S_a^{\alpha q}}{d_a^2}\right\},\tag{3.48}$$

where we have neglected all residual terms of order $\mathcal{O}(d_a/r_a)$, since in this case $d_a/r_a = 0$. In order to obtain the analytic expression for the angle of light deflection valid for a system of spinning masses having arbitrarily high velocities $\vec{v}_a$ and spin $\vec{\mathcal{S}}_a$, we have to substitute into the preceding equation the expressions given in (A.3) and (A.4). After performing the substitution we get

$$\alpha^i_{S(1)} = -8\frac{G}{c^4}\sum_{a=1}^{N}\frac{\gamma_a}{d_a^4}\Bigg\{\vec{\mathcal{S}}_a\cdot(\vec{l}_{(0)}\times\vec{r}_a)+\vec{\mathcal{S}}_a\cdot\Big[(\vec{r}_a-\frac{r_a\vec{l}_{(0)}}{c})\times\vec{v}_a\Big] + \frac{(1-\gamma_a)}{\gamma_a v_a^2}(\vec{v}_a\cdot\vec{\mathcal{S}}_a)(\vec{l}_{(0)}\times\vec{r}_a)\cdot\vec{v}_a\Bigg\}[\xi^i-P^i_q x^q_a] + 4\frac{G}{c^4}\sum_{a=1}^{N}\frac{\gamma_a}{d_a^2}\Bigg\{P^i_q(\vec{v}_a\times\vec{\mathcal{S}}_a)-(\vec{l}_{(0)}\times\vec{\mathcal{S}}_a)^i-\frac{(1-\gamma_a)}{\gamma_a v_a^2}(\vec{v}_a\cdot\vec{\mathcal{S}}_a)(\vec{l}_{(0)}\times\vec{v}_a)^i\Bigg\}, \quad (3.49)$$

where the quantities $\vec{\mathcal{S}}_a$, $\vec{v}_a$, d_a and x^i_a are evaluated at the retarded time s_a. In the event of slow motion, the Taylor expansion of (3.49) with respect to the parameter v/c yields

$$\alpha^i_{S(1)} = \sum_{a=1}^{N}\alpha^i_{aS}, \quad (3.50)$$

where

$$\alpha^i_{aS} = -8\frac{G}{c^4}\frac{\vec{\mathcal{S}}_a\cdot(\vec{l}_{(0)}\times\vec{r}_a)}{d_a^4}[\xi^i-P^i_q x^q_a]-4\frac{G}{c^4}\frac{(\vec{l}_{(0)}\times\vec{\mathcal{S}}_a)^i}{d_a^2}. \quad (3.51)$$

The angle of light deflection α^i_{aS} can also be written as the gradient of the potential ψ_{aS}:

$$\alpha^i_{S(1)} = -4\frac{\partial\psi_{aS}}{\partial\chi^i_a}, \qquad \psi_{aS} = \frac{G}{c^3}(\vec{e}_0\times\vec{\mathcal{S}}_a)^q\frac{\partial d_a}{\partial\chi^q_a}, \quad (3.52)$$

where $\chi^i_a = \xi^i - P^i_q x^q_a(s_a)$ and $d_a = |\vec{\chi}_a|$.

3.7 Linear light deflection in the far zone gravitational field

In order to compute the part of the linear light deflection arising from the quadrupole moment of a system of arbitrarily moving and spinning masses it is convenient to work with the expression for the light deflection caused by the point-mass part of the energy-momentum tensor given by equation (139) in [7],

$$\alpha^i_{M(1)} = -4\frac{G}{c^2}\sum_{a=1}^{N}\frac{m_a[1-\frac{\vec{e}_{(0)}\cdot\vec{v}_a(s_a)}{c}]}{\sqrt{1-\frac{v_a^2(s_a)}{c^2}}}\frac{\xi^i-\xi^i_a(s_a)}{|\vec{\xi}-\vec{\xi}_a(s_a)|^2}, \quad (3.53)$$

which results from (3.39) after substituting $R_a(s_a)$ by its expression given by (A.7).

In this work we assume that the impact parameter is always larger than the distance $|\vec{\xi}_a(s_a)|$. Upon performing the Taylor expansion of the right hand side of (3.53) with respect to $\xi_a^i(s_a)$ and $v_a(s_a)/c$, one can prove (see [7]) that the angle of light deflection caused by the point-mass part of the energy-momentum tensor is represented in the form

$$\alpha^i_{M(1)} = -4\hat{\partial}_i \psi_M, \tag{3.54}$$

where the potential ψ_M is given by

$$\begin{aligned}\psi_M = \frac{G}{c^2}\Bigg\{ &\sum_{a=1}^{N} m_a - \frac{1}{c}\vec{e}_{(0)} \cdot \sum_{a=1}^{N} m_a \vec{v}_a(s_a) - \sum_{a=1}^{N} m_a x_a^j(s_a)\hat{\partial}_j \\ &+ \frac{1}{c}\vec{e}_{(0)} \cdot \sum_{a=1}^{N} m_a \vec{v}_a(s_a) x_a^j(s_a)\hat{\partial}_j + \frac{1}{2}\sum_{a=1}^{N} m_a x_a^p(s_a) x_a^q(s_a)\hat{\partial}_{pq}\Bigg\} \ln|\vec{\xi}\,| + \ldots,\end{aligned} \tag{3.55}$$

and the ellipsis denotes residual terms of higher order.

The potential ψ_M is the, so-called, point-mass part of the gravitational lens potential [19].

If we treat the system of N point-like masses as an isolated system, the multipole moments are defined in the Newtonian approximation by

$$\begin{aligned}\mathcal{M} = \sum_{a=1}^{N} m_a, \quad & \mathcal{I}^i(t) = \sum_{a=1}^{N} m_a x_a^i(t) \\ \mathcal{J}^i(t) = \sum_{a=1}^{N} m_a (\vec{x}_a(t) \times \vec{v}_a(t))^i \quad & \mathcal{I}^{ij}(t) = \sum_{a=1}^{N} m_a \left(x_a^i(t) x_a^j(t) - \frac{1}{3}|\vec{x}_a(t)|^2 \delta^{ij} \right),\end{aligned} \tag{3.56}$$

where the multiplication symbol denotes the usual Euclidean cross product and, coordinates as well as velocities of all point-masses are taken at one and the same instant of time t. In the rest of this section we assume that the velocities of the point-like masses are small with regard to the velocity of light and that the origin of the coordinate frame is located at the barycentre of the system. This means that

$$\mathcal{I}^i(t) = \sum_{a=1}^{N} m_a x_a^i(t) = 0 \qquad \text{and} \qquad \dot{\mathcal{I}}^i(t) = \sum_{a=1}^{N} m_a v_a^i(t) = 0. \tag{3.57}$$

After expanding all terms in (3.55) with respect to the time t^*, noting that the second projective derivative $\hat{\partial}_{pq} \ln|\vec{\xi}\,|$ is traceless, and taking into account (3.42) and (3.43), the centre of mass conditions (3.57), the definitions of multipole moments (3.56) and the the vector equality

$$x_a^j(\vec{e}_{(0)} \cdot \vec{v}_a) - v_a^j(\vec{e}_{(0)} \cdot \vec{x}_a) = [\vec{e}_{(0)} \times (\vec{x}_a \times \vec{v}_a)]^j, \tag{3.58}$$

we find out that, up to the required order, the potential ψ_M reads

$$\psi_M = \frac{G}{c^2}\left\{ \sum_{a=1}^{N} m_a + \frac{1}{c}\epsilon_{jpq} e^p_{(0)} \mathcal{J}^q(t^*)\hat{\partial}_j + \frac{1}{2}\mathcal{I}^{pq}(t^*)\hat{\partial}_{pq} \right\} \ln|\vec{\xi}\,|, \tag{3.59}$$

where ϵ_{jpq} is the fully antisymmetric Levi-Civita symbol.

The gravitational lens potential (3.59) is equal to the lens potential given by equation (168) in [18].

In order to get the total lens potential, we have to add to (3.59) the lens potential arising from the spin of the masses. It follows from (3.52), in the case that the impact parameter is larger than the distance $|\vec{\xi}_a(s_a)|$, that the lens potential is given by

$$\psi_S = \frac{G}{c^3}[\epsilon_{jpq} e^p_{(0)} S^q(t^*)\hat{\partial}_j], \tag{3.60}$$

where

$$S^q(t^*) = \sum_{a=1}^{N} \mathcal{S}^q_a(t^*). \tag{3.61}$$

Since the effect on the light propagation arising from the wave zone or far zone gravitational field is caused by the quadrupole moment of the deflector, to obtain the corresponding angle of light deflection we have to introduce into (3.54) only the quadrupole term of the potential ψ_M,

$$\alpha^i_{(1)\mathrm{Quad}} = -2\frac{G}{c^2}\mathcal{I}^{pq}(t^*)\hat{\partial}_{ipq} \ln|\vec{\xi}\,|. \tag{3.62}$$

After evaluating the projective derivatives $\hat{\partial}_{ipq}$, we finally get

$$\begin{aligned}\alpha^i_{(1)\mathrm{Quad}} = &-4\frac{G}{c^2}\sum_{a=1}^{N} m_a\left[4(\vec{x}_a(t^*)\cdot\vec{e}_\xi)^2 - |\vec{x}_a(t^*)|^2\right]\frac{e^i_\xi}{\xi^3} \\ &+ 8\frac{G}{c^2}\sum_{a=1}^{N} m_a(\vec{x}_a(t^*)\cdot\vec{e}_\xi)\frac{1}{\xi^3}P^i_q x^q_a(t^*),\end{aligned} \tag{3.63}$$

where $\xi = |\vec{\xi}\,|$.

The equation above shows that the angle of light deflection caused by the quadrupole moment of the source of the gravitational field falls off as the inverse cube of the impact parameter ξ. Taking into account this property of strong suppression of the influence of gravitational waves on light propagation, we conclude that light deflection in the linear gravitational field of a system of arbitrarily moving and spinning masses is mainly determined by the near-zone gravitational field.

4 The post-linear gravitational field of two bounded masses

In the computation of the metric generated by a system of two bounded point-like masses we distinguish between 3 zones [20, 21]: the near-zone, the intermediate-zone and the far-zone or wave-zone. In Chapter 3 as well as in Refs [18, 22] it was shown that leading order terms for the effect of light deflection in the case of a small impact parameter ξ (i.e. an impact parameter small with respect to the distance between the deflector and the observer) depend neither on the radiative part ($\sim 1/\xi$) of the gravitational field nor on the intermediate ($\sim 1/\xi^2$) zone terms. The main effect rather comes from the near zone ($\sim 1/\xi^3$) terms. Taking into account this property of strong suppression of the influence of gravitational waves on light propagation, we can assume in the present work that light deflection in the post-linear gravitational field of two point-like masses is mainly determined by the near-zone metric. This chapter is devoted to the computation of the post-linear metric in harmonic coordinates in the near-zone of a system of two bounded point-like masses. It is based on papers by L. Blanchet [23, 24] and a paper by L. Blanchet *et al.* [25].

4.1 Einstein's Field Equations

The field equations of general relativity form a system of ten second-order partial differential equations that are fulfilled by the space-time metric $g_{\mu\nu}$,

$$G^{\mu\nu}[g, \partial g, \partial^2 g] = \kappa T^{\mu\nu}[g], \tag{4.1}$$

where the Einstein curvature tensor $G^{\mu\nu} \equiv R^{\mu\nu} - \frac{1}{2} R g^{\mu\nu}$ is generated, through the gravitational coupling $\kappa = 8\pi G/c^4$, by the stress-energy tensor $T^{\mu\nu}$. Four of these ten equations control through the contracted Bianchi identity the evolution of the matter system,

$$G^{\mu\nu}{}_{;\nu} \equiv 0 \Longrightarrow T^{\mu\nu}{}_{;\nu} = 0. \tag{4.2}$$

The space-time geometry is constrained by the six remaining equations, which place six independent constraints on the ten components of the metric $g_{\mu\nu}$, leaving four of them to be fixed by a choice of the coordinate system.

In the present work we shall solve Einstein's field equations in harmonic, or de Donder coordinates in order to compute the gravitational field for two bounded point-like masses.

We define, as a basic variable, the gravitational amplitude

$$\bar{h}^{\mu\nu} = \sqrt{|g|}g^{\mu\nu} - \eta^{\mu\nu}, \tag{4.3}$$

with $g^{\mu\nu}$ and g being the inverse and the determinant of the covariant metric $g_{\mu\nu}$. The absolute value of g is given in terms of a series expansion in the field variable $\bar{h}^{\mu\nu}$, i.e.

$$|g| = 1 + \bar{h} + \frac{1}{2}\left(\bar{h}^2 - \bar{h}_{\sigma\rho}\bar{h}^{\sigma\rho}\right) + \mathcal{O}(\bar{h}^3), \tag{4.4}$$

where $\bar{h}_{\sigma\rho} = \eta_{\sigma\alpha}\eta_{\rho\beta}\bar{h}^{\alpha\beta}$ and $\bar{h} = \eta_{\alpha\beta}\bar{h}^{\alpha\beta}$. By $\eta^{\mu\nu} = \eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$, as before, we denote the Minkowskian metric.

The harmonic coordinate condition, which accounts exactly for the four equations (4.2) corresponding to the conservation of the matter tensor, reads

$$\partial_\nu \bar{h}^{\mu\nu} = 0. \tag{4.5}$$

Under this condition the Einstein field equations (4.1) take the form

$$\Box\bar{h}^{\mu\nu} = \frac{16\pi G}{c^4}|g|T^{\mu\nu} + \Lambda^{\mu\nu}, \tag{4.6}$$

where $\Lambda^{\mu\nu}$ is the gravitational source term. Here, $\Box = \eta^{\mu\nu}\partial_\mu\partial_\nu$, as in the preceding chapter, is the flat d'Alembertian operator.

By means of the integral of the retarded potentials given by

$$\Box^{-1}_{\mathcal{R}} f(\vec{x},t) = -\frac{1}{4\pi}\int d^3x' \frac{f(\vec{x}\,',t-|\vec{x}-\vec{x}\,'|/c)}{|\vec{x}-\vec{x}\,'|}, \tag{4.7}$$

and under the condition of no incoming radiation, the Einstein field equations (4.6) can be written equivalently in the form of the integro-differential equations

$$\bar{h}^{\mu\nu} = \Box^{-1}_{\mathcal{R}}\left[\frac{16\pi G}{c^4}|g|T^{\mu\nu} + \Lambda^{\mu\nu}\right]. \tag{4.8}$$

The gravitational source term $\Lambda^{\mu\nu}$ is related to the Landau-Lifschitz pseudo-tensor through equation

$$\Lambda^{\mu\nu} = \frac{16\pi G}{c^4}|g|t^{\mu\nu}_{LL} + \partial_\rho\bar{h}^{\mu\sigma}\partial_\sigma\bar{h}^{\nu\rho} - \bar{h}^{\rho\sigma}\partial_{\rho\sigma}\bar{h}^{\mu\nu}. \tag{4.9}$$

Equation (4.9) can be expanded as an infinite non-linear series in h and its first and second space-time derivatives; in this work we need only retain the non-linear terms quadratic in h^2 (i.e. in G^2),

$$\Lambda^{\mu\nu} = N^{\mu\nu}(\bar{h}, \bar{h}) + \mathcal{O}(\bar{h}^3), \tag{4.10}$$

where the quadratic non-linearity reads

$$\begin{aligned} N^{\mu\nu} &= -\bar{h}^{\rho\sigma}\partial_{\rho\sigma}\bar{h}^{\mu\nu} + \frac{1}{2}\partial^{\mu}\bar{h}_{\rho\sigma}\partial^{\nu}\bar{h}^{\rho\sigma} - \frac{1}{4}\partial^{\mu}\bar{h}\,\partial^{\nu}\bar{h} + \partial_{\sigma}\bar{h}^{\mu\rho}(\partial^{\sigma}\bar{h}^{\nu}_{\rho} + \partial_{\rho}\bar{h}^{\nu\sigma}) \\ &- 2\partial^{(\mu}\bar{h}_{\rho\sigma}\partial^{\rho}\bar{h}^{\nu)\sigma} + \eta^{\mu\nu}\big[-\frac{1}{4}\partial_{\tau}\bar{h}_{\rho\sigma}\partial^{\tau}\bar{h}^{\rho\sigma} + \frac{1}{8}\partial_{\rho}\bar{h}\,\partial^{\rho}\bar{h} + \frac{1}{2}\partial_{\rho}\bar{h}_{\sigma\tau}\partial^{\sigma}\bar{h}^{\rho\tau}\big]. \end{aligned} \tag{4.11}$$

In the preceding expression, all indices are lowered and raised with the Minkowski metric $\eta_{\mu\nu}$; $\bar{h} = \eta^{\mu\nu}\bar{h}_{\mu\nu}$; the parentheses around indices, as before, indicate symmetrization.

4.2 Solution of Einstein's equations in the near zone

The near zone of the source is defined as the domain $D_i = \{(\vec{x}, t) \mid |\vec{x}| < r_i\}$ in which the radius r_i is adjusted so that

1. $r_i > a$, where a is the radius of a sphere which totally encloses the source, and
2. $r_i \ll \lambda\!\!\!^{-}$, where $\lambda\!\!\!^{-} = \lambda/2\pi \sim ac/v$ is a characteristic reduced wavelength of the emitted gravitational radiation, and v is a typical internal velocity in the source.

The definition of D_i assumes in particular that $a \ll \lambda\!\!\!^{-}$, or equivalently $\varepsilon \ll 1$ where $\varepsilon \sim v/c$ is a small "post-Newtonian" parameter appropriate to the description of slowly moving sources. We shall also assume that the source is self-gravitating so that $GM/(a\,c^2) \sim \varepsilon^2$, where M is the total mass of the source and that the internal stresses are such that $T^{ij}/T^{00} \sim \varepsilon^2$.

In the domain D_i we can solve Einstein's equations (4.6) using the harmonic coordinate condition (4.5) by formally taking the limit $\varepsilon \to 0$.

4.2.1 The retarded potentials and the general 2PN metric

We describe the matter source by means of the density of mass σ, of current σ_i, and of the stress σ_{ij} which are defined as functions of the contravariant components of the stress-energy tensor $T^{\alpha\beta}$ by

$$\sigma \equiv \frac{1}{c^2}\big(T^{00} + T^{ii}\big), \tag{4.12}$$

$$\sigma_i \equiv \frac{1}{c}T^{0i}, \tag{4.13}$$

$$\sigma_{ij} \equiv T^{ij}, \tag{4.14}$$

where $T^{ii} = \delta_{ij}T^{ij}$ denotes the spatial trace of $T^{\alpha\beta}$.

From the covariant conservation of the matter stress-energy tensor (i.e. $\nabla_\nu T^{\mu\nu} = 0$) we deduce the equations of continuity and motion, which read to the Newtonian order

$$\partial_t\sigma + \partial_i\sigma_i = \mathcal{O}(\varepsilon^2), \tag{4.15}$$

$$\partial_t\sigma_i + \partial_j\sigma_{ij} = \sigma\partial_i V + \mathcal{O}(\varepsilon^2). \tag{4.16}$$

As in Refs [24] and [25] we introduce retarded potentials generated by the densities σ, σ_i, and σ_{ij}. First, V and V_i are the usual retarded scalar and vector potentials of the mass and current densities σ and σ^i, i.e.,

$$V(\vec{x},t) = G\int \frac{d^3x'}{|\vec{x}-\vec{x}\,'|}\sigma\Big(\vec{x}\,', t - \frac{1}{c}|\vec{x}-\vec{x}\,'|\Big), \tag{4.17}$$

$$V_i(\vec{x},t) = G\int \frac{d^3x'}{|\vec{x}-\vec{x}\,'|}\sigma_i\Big(\vec{x}\,', t - \frac{1}{c}|\vec{x}-\vec{x}\,'|\Big), \tag{4.18}$$

which fulfil the equations $\Box V = -4\pi G\sigma$ and $\Box V_i = -4\pi G\sigma_i$. Secondly, $\hat{W}_{ij}$ is a more complicated retarded tensor potential defined by

$$\hat{W}_{ij}(\vec{x},t) = G\int \frac{d^3x'}{|\vec{x}-\vec{x}\,'|}\Big[\sigma_{ij} - \delta_{ij}\sigma_{kk} + \frac{1}{4\pi G}\partial_i V\partial_j V\Big]\Big(\vec{x}\,', t - \frac{1}{c}|\vec{x}-\vec{x}\,'|\Big). \tag{4.19}$$

Furthermore, we shall often consider the trace of the potential $\hat{W}_{ij}$; i.e.

$$\hat{W}_{ii}(\vec{x},t) = G\int \frac{d^3x'}{|\vec{x}-\vec{x}\,'|}\Big[-2\sigma_{ii} + \frac{1}{4\pi G}\partial_i V\partial_i V\Big]. \tag{4.20}$$

From the Newtonian equations of continuity (4.15) and motion (4.16) we deduce that the potentials V, V_i, $\hat{W}_{ij}$, and $\hat{W}_{ii}$ satisfy the conservations laws

$$\partial_t V + \partial_i V_i = \mathcal{O}(\varepsilon^2), \tag{4.21}$$

$$\partial_t V_i + \partial_j\Big\{\hat{W}_{ij} - \frac{1}{2}\delta_{ij}\hat{W}_{kk}\Big\} = \mathcal{O}(\varepsilon^2). \tag{4.22}$$

We now proceed to solve Einstein's equations (4.5) and (4.6) with an accuracy corresponding to the second post-Newtonian order. After introducing the lowest-order results $\bar{h}^{00} = -4V/c^2 + \mathcal{O}(\varepsilon^4)$, $\bar{h}^{0i} = \mathcal{O}(\varepsilon^3)$, and $\bar{h}^{ij} = \mathcal{O}(\varepsilon^4)$ into the right-hand-side of (4.6) with the explicit expression (4.11), we get the equations

$$\Box\bar{h}^{00} = \frac{16\pi G}{c^4}\left(1 + \frac{4V}{c^2}\right)T^{00} - \frac{14}{c^4}\partial_k V\partial_k V + \mathcal{O}(\varepsilon^6), \tag{4.23}$$

$$\Box\bar{h}^{0i} = \frac{16\pi G}{c^4}T^{0i} + \mathcal{O}(\varepsilon^5), \tag{4.24}$$

$$\Box\bar{h}^{ij} = \frac{16\pi G}{c^4}T^{ij} + \frac{4}{c^4}\left\{\partial_i V\partial_j V - \frac{1}{2}\delta_{ij}\partial_k V\partial_k V\right\} + \mathcal{O}(\varepsilon^6). \tag{4.25}$$

These equations can be straightforwardly solved by means of the potentials V, V_i, $\hat{W}_{ij}$ and $\hat{W}_{ii}$ that are given by equations (4.17)–(4.20). The result is

$$\bar{h}^{00} = -\frac{4}{c^2}V - \frac{2}{c^4}(\hat{W}_{kk} + 4V^2) + \mathcal{O}(\varepsilon^6), \tag{4.26}$$

$$\bar{h}^{0i} = -\frac{4}{c^3}V_i + \mathcal{O}(\varepsilon^5), \tag{4.27}$$

$$\bar{h}^{ij} = -\frac{4}{c^4}\left[\hat{W}_{ij} - \frac{1}{2}\delta_{ij}\hat{W}_{kk}\right] + \mathcal{O}(\varepsilon^6). \tag{4.28}$$

Since the potentials satisfy the conservations laws (4.21) and (4.22), it is easy to see that the corresponding field quantities (4.26), (4.27) and (4.28) satisfy the approximate harmonic gauge condition

$$\partial_0\bar{h}^{00} + \partial_i\bar{h}^{0i} = \mathcal{O}(\varepsilon^5), \tag{4.29}$$

$$\partial_0\bar{h}^{0i} + \partial_j\bar{h}^{ij} = \mathcal{O}(\varepsilon^6). \tag{4.30}$$

On inserting the coefficients $\bar{h}^{\alpha\beta}$ given by (4.26)–(4.28) into (4.3) and (4.4) and computing the inverse of $g^{\mu\nu}$ we finally obtain,

$$g_{00} = -1 + \frac{2}{c^2}V - \frac{2}{c^4}V^2 + \mathcal{O}(\varepsilon^6), \tag{4.31}$$

$$g_{0i} = -\frac{4}{c^3}V_i + \mathcal{O}(\varepsilon^5), \tag{4.32}$$

$$g_{ij} = \delta_{ij}\left(1 + \frac{2}{c^2}V + \frac{2}{c^4}V^2\right) + \frac{4}{c^4}\hat{W}_{ij} + \mathcal{O}(\varepsilon^6). \tag{4.33}$$

In what follows we are going to compute the retarded potentials V, V_i, $\hat{W}_{ij}$ and $\hat{W}_{kk}$ for a system of two bounded point-like masses.

4.2.2 Application to a system of two bounded point-like masses

For a system of two bounded point-like masses (i.e. a point-mass binary) we use the matter stress-energy tensor

$$T^{\mu\nu} = \mu_1(t) v_1^\mu v_1^\nu \delta(\vec{x} - \vec{x}_1(t)) + (1 \leftrightarrow 2), \tag{4.34}$$

where the symbol $(1 \leftrightarrow 2)$ refers to the preceding term but with the labels 1 and 2 exchanged; δ denotes the three-dimensional Dirac distribution; the trajectories of the two masses (in harmonic coordinates) are denoted by $\vec{x}_1(t)$ and $\vec{x}_2(t)$; the two coordinate velocities are $\vec{v}_1(t) = d\vec{x}_1(t)/dt$, $\vec{v}_2(t) = d\vec{x}_2(t)/dt$ and $v_1^\mu \equiv (c, \vec{v}_1)$, $v_2^\mu \equiv (c, \vec{v}_2)$; $\mu_1(t)$ represents an effective time-dependent mass of body 1 defined by

$$\mu_1(t) = \left(\frac{m_1}{\sqrt{g g_{\rho\sigma} \frac{v_1^\rho v_1^\sigma}{c^2}}} \right)_1, \tag{4.35}$$

where m_1 is the (constant) Schwarzschild mass, with $g_{\rho\sigma}$ the metric and g its determinant. After introducing into equations (4.12)–(4.14) the matter stress-energy tensor (4.34) we find

$$\sigma = \tilde{\mu}_1(t) \delta(\vec{x} - \vec{x}_1(t)) + (1 \leftrightarrow 2), \tag{4.36}$$

$$\sigma_i = \mu_1(t) v_1^i(t) \delta(\vec{x} - \vec{x}_1(t)) + (1 \leftrightarrow 2), \tag{4.37}$$

$$\sigma_{ij} = \mu_1(t) v_1^i(t) v_1^j(t) \delta(\vec{x} - \vec{x}_1(t)) + (1 \leftrightarrow 2). \tag{4.38}$$

Here, the quantity $\tilde{\mu}_1(t)$ is given by

$$\tilde{\mu}_1(t) = \mu_1(t) \left[1 + \frac{v_1^2(t)}{c^2} \right], \tag{4.39}$$

where $v_1^2(t) = \vec{v}_1(t)^2$. At the Newtonian order the quantities $\mu_1(t)$ and $\tilde{\mu}_1(t)$ reduce to the Schwarzschild mass: $\mu_1(t) = m_1 + \mathcal{O}(\varepsilon^2)$ and $\tilde{\mu}_1(t) = m_1 + \mathcal{O}(\varepsilon^2)$.

Since the stress-energy tensor for a point-like mass depends on the values of the metric coefficients at the very location of the masses and the metric coefficients there become infinite, we must supplement the model of the stress-energy tensor (4.34) by a regularization procedure in order to remove the infinite self-field of the point-like sources. The choice of one or another regularization procedure represents (*a priori*) an integral part of the choice of physical model for describing the point-like masses. In the present work as in the paper of Blanchet *et al.* [25] we shall use the Hadamard regularization based on the finite part of functions admitting a special ("tempered") type of singularity.

Hadamard's "partie finie" regularization

Let F be a real valued function defined in a neighbourhood of a point $\vec{x}_0 \in \mathbb{R}$, excluding this point. At $\vec{x}_0$ the function F is assumed to be singular. We consider the family of auxiliary functions $f_{\vec{n}}(\epsilon) := F(\vec{x}_0 + \epsilon\vec{n})$, labelled by the unit vectors $\vec{n}$. We expand $f_{\vec{n}}$ as a Laurent series around $\epsilon = 0$,

$$f_{\vec{n}}(\epsilon) = \sum_{m=-N}^{\infty} a_m(\vec{n})\epsilon^m, \tag{4.40}$$

where the coefficients a_m depend on the unit vector $\vec{n}$. The regularized value of the function F at $\vec{x}_0$ is defined as the coefficient of ϵ^0 in the expansion (4.40) averaged over all directions:

$$F_{\text{reg}}(\vec{x}_0) := \frac{1}{4\pi} \oint d\Omega a_0(\vec{n}). \tag{4.41}$$

We use the formula (4.41) to give a sense to the spatial integral of the product of F and the Dirac delta function. It means that we define

$$\int d^3x F(\vec{x})\delta(\vec{x} - \vec{x}_a) := F_{\text{reg}}(\vec{x}_a). \tag{4.42}$$

More details about Hadamard's regularization can be found in [26].

We shall compute the 2PN metric in the form known as order-reduced, by which we mean that in the final result all accelerations are replaced by explicit functions of the positions by means of the Newtonian equations of motion:

$$\frac{dv_1^i}{dt} = -\frac{Gm_2}{r_{12}^2} n_{12}^i, \tag{4.43}$$

$$\frac{dv_2^i}{dt} = \frac{Gm_1}{r_{12}^2} n_{12}^i, \tag{4.44}$$

where $r_{12} = |\vec{x}_1(t) - \vec{x}_2(t)|$ and $\vec{n}_{12} = (\vec{x}_1(t) - \vec{x}_2(t))/r_{12}$.

4.2.3 Computation of the potentials V, V_i and $\hat{W}_{ij}$

The potentials V and V_i are generated by the compact-supported source densities σ and σ_i. Similarly $\hat{W}_{ij}$ consists of a part generated by a compactly supported source, but also a part whose source is given by quadratic products of the spatial derivative of the potential V. We denote the compact part of $\hat{W}_{ij}$ by $\hat{W}_{ij}^{(C)}$ and the quadratic part by $\hat{W}_{ij}^{(\partial V \partial V)}$. So we can write the potential $\hat{W}_{ij}$ as

$$\hat{W}_{ij} = \hat{W}_{ij}^{(C)} + \hat{W}_{ij}^{(\partial V \partial V)}, \tag{4.45}$$

where $\hat{W}_{ij}^{(C)}$ and $\hat{W}_{ij}^{(\partial V \partial V)}$ are given by

$$\hat{W}_{ij}^{(C)} = \Box_{\mathcal{R}}^{-1}\{-4\pi G(\sigma_{ij} - \delta_{ij}\sigma_{kk})\}, \tag{4.46}$$
$$\hat{W}_{ij}^{(\partial V \partial V)} = \Box_{\mathcal{R}}^{-1}\{-\partial_i V \partial_j V\}. \tag{4.47}$$

Compact Parts of Potentials

First, we compute the compactly supported potentials V, V_i, and the compactly supported parts of $\hat{W}_{ij}$ for a system of two bounded point-like masses described by the stress-energy tensor (4.34) and the regularization (4.41). For our computation of the post-linear light deflection, we only need to compute V up to the 1PN order and the potentials V_i and $\hat{W}_{ij}$ up to the Newtonian order.

By performing the Taylor expansion up to the 1PN order of the retardation inside the integral (4.17) and using the mass density in the form (4.37), we obtain

$$V = G\left\{\frac{\tilde{\mu}_1}{r_1} - \frac{1}{c}\partial_t(\tilde{\mu}_1) + \frac{1}{2c^2}\partial_t^2(\tilde{\mu}_1 r_1)\right\} + \mathcal{O}(\varepsilon^3) + (1 \leftrightarrow 2). \tag{4.48}$$

We start by computing $\tilde{\mu}_1$ up to the 1PN order. After substituting in (4.35) the metric coefficients by their values (4.31)–(4.33), $\tilde{\mu}_1$ becomes

$$\tilde{\mu}_1 = m_1\left\{1 + \frac{1}{c^2}\left[-(V)_1 + \frac{3}{2}v_1^2\right]\right\} + \mathcal{O}(\varepsilon^4), \tag{4.49}$$

where the potential V is to be evaluated at the location of mass 1 by means of the rule (4.41). Application of this rule to the Newtonian part of V given by

$$V = \frac{Gm_1}{r_1} + \mathcal{O}(\varepsilon^2) + (1 \leftrightarrow 2), \tag{4.50}$$

leads to

$$(V)_1 = \frac{Gm_2}{r_{12}} + \mathcal{O}(\varepsilon^2) + (1 \leftrightarrow 2), \tag{4.51}$$

where $r_{12} = |\vec{x}_1 - \vec{x}_2|$. After substituting in (4.49) the quantity $(V)_1$ by its expression (4.51) we get

$$\tilde{\mu}_1 = m_1\left\{1 + \frac{1}{c^2}\left[-\frac{Gm_2}{r_{12}} + \frac{3}{2}v_1^2\right]\right\} + \mathcal{O}(\varepsilon^4). \tag{4.52}$$

To obtain the final expression for V we have to introduce into (4.48) the effective mass (4.52) and compute the time derivatives. The explicit expression for V is given in Appendix B. Notice, that we do not need to compute the time derivative of the effective mass $\tilde{\mu}$ since it starts at 1PN and we are interested only in V up to the 1PN order.

The potential V_i and the compact potential $\hat{W}_{ij}^{(C)}$ to the Newtonian order are computed in the same manner. As an example we give

$$\hat{W}_{ij}^{(C)} = \frac{Gm_1}{r_1}(v_1^i v_1^j - \delta^{ij} v_1^2) + \mathcal{O}(\varepsilon) + (1 \leftrightarrow 2). \tag{4.53}$$

The explicit expressions for the potentials V and V_i to the order required in the computation of the metric to the conservative 2PN order are given in Appendix B.

Quadratic Part of the Potential $\hat{W}_{ij}$

To compute the quadratic part of the potential $\hat{W}_{ij}$ given by equation (4.47), we have first to work out the sources using (4.50). After computing $\partial_i V \partial_j V$ we obtain

$$\begin{aligned}\partial_i V \partial_j V = {} & \frac{G^2 m_1^2}{8}(\partial^2_{1ij} + \delta^{ij}\Delta_1)\left(\frac{1}{r_1^2}\right) \\ & + G^2 m_1 m_2 \partial_{1i}\partial_{2j}\left(\frac{1}{r_1 r_2}\right) + \mathcal{O}(\varepsilon^2) + (1 \leftrightarrow 2),\end{aligned} \tag{4.54}$$

where $\partial^2_{1ij} \equiv \partial^2/\partial x_1^i \partial x_1^j$, $\partial_{1i}\partial_{2j} \equiv \partial^2/\partial x_1^i \partial x_2^j$ and $\Delta_1 \equiv \partial^2/\partial x_1^i \partial x_1^i$.

In order to compute $\hat{W}_{ij}^{(\partial V \partial V)}$ to the Newtonian order we first perform the Taylor expansion of the retardation in (4.47) to that order. This yields

$$\begin{aligned}\hat{W}_{ij}^{(\partial V \partial V)} &= \Box_{\mathcal{R}}^{-1}\{-\partial_i V \partial_j V\} \\ &= \Delta^{-1}\{-\partial_i V \partial_j V\} + \mathcal{O}(\varepsilon),\end{aligned} \tag{4.55}$$

where the source $\partial_i V \partial_j V$ is given by (4.54).

The Poisson integral of the self-terms can be readily deduced from $\Delta(\ln r_1) = 1/r_1^2$, while the Poisson integral of the interaction terms is obtained by solving the elementary Poisson equation

$$\Delta g = \frac{1}{r_1 r_2}. \tag{4.56}$$

A regular solution of the preceding equation is

$$g = \ln S, \tag{4.57}$$

where $S \equiv r_1 + r_2 + r_{12}$.

On inserting the source term given by (4.54) into (4.55) and computing the Poisson Integrals we obtain,

$$\begin{aligned}\hat{W}_{ij}^{(\partial V \partial V)} = {} & -\frac{G^2 m_1^2}{8}\left\{\partial^2_{ij}(\ln r_1) + \delta^{ij}\frac{1}{r_1^2}\right\} - G^2 m_1 m_2 \partial_{1i}\partial_{2j} g \\ & + \mathcal{O}(\varepsilon) + (1 \leftrightarrow 2).\end{aligned} \tag{4.58}$$

After summing up equations (4.53) and (4.58) and computing the derivatives $\partial_{1i}\partial_{2j}g$ we finally get the expression for the potential $\hat{W}_{ij}$, which is given in an explicit form in Appendix B.

4.3 The metric

Upon introducing the potentials V, V_i and $\hat{W}_{ij}$ given by equations (B.1)–(B.3) into (4.31), (4.32) and (4.33), we obtain the conservative 2PN harmonic coordinate metric generated by two bounded point-like masses as a function of the coordinate position $\vec{z}$ and of the coordinate positions and velocities of the masses $\vec{x}_a(t), \vec{v}_a(t)$ with $a = 1, 2$. The post-linear metric for two bounded point-like masses (to the 2PN-order) reads

$$
\begin{aligned}
h_{00}^{(2)} &= \frac{1}{c^4}\Bigg\{ -2\frac{G^2 m_1^2}{r_1^2} + G^2 m_1 m_2 \left(-\frac{2}{r_1 r_2} - \frac{r_1}{2r_{12}^3} + \frac{r_1^2}{2r_2 r_{12}^3} - \frac{5}{2r_2 r_{12}} \right) \Bigg\} \\
&\quad + \frac{1}{c^4}(1 \leftrightarrow 2) && (4.59) \\
h_{pq}^{(2)} &= \frac{1}{c^4}\Bigg\{ \delta^{pq}\Big[\frac{G^2 m_1^2}{r_1^2} + G^2 m_1 m_2 \Big(\frac{2}{r_1 r_2} - \frac{r_1}{2r_{12}^3} + \frac{r_1^2}{2r_2 r_{12}^3} - \frac{5}{2r_1 r_{12}} + \frac{4}{r_{12} S} \Big) \Big] \\
&\quad + \frac{G^2 m_1^2}{r_1^2} n_1^p n_1^q - 4G^2 m_1 m_2 n_{12}^p n_{12}^q \left(\frac{1}{S^2} + \frac{1}{r_{12} S} \right) + \frac{4G^2 m_1 m_2}{S^2} \left(n_1^{(p} n_2^{q)} + 2n_1^{(p} n_{12}^{q)} \right) \Bigg\} \\
&\quad + \frac{1}{c^4}(1 \leftrightarrow 2), && (4.60)
\end{aligned}
$$

where $r_1 = |\vec{z} - \vec{x}_1(t)|$, $r_2 = |\vec{z} - \vec{x}_2(t)|$ and $r_{12} = |\vec{x}_1(t) - \vec{x}_2(t)|$. The vectors n_1^p, n_2^p and n_{12}^p are unit vectors defined by $n_1^p = r_1^p / r_1$, $n_2^p = r_2^p / r_2$ and $n_{12}^p = r_{12}^p / r_{12}$.

In our computations we also need a part of the linear gravitational field of two accelerating point-like masses. The part that is relevant to our calculation is given by

$$h_{00}^{(1)} = 2\frac{G}{c^2} \sum_{a=1}^{2} \frac{m_a}{r_a} + \frac{G}{c^4} \sum_{a=1}^{2} \frac{m_a}{r_a} \Big[- (\vec{n}_a \cdot \vec{v}_a)^2 + 4v_a^2 \Big] \tag{4.61}$$

$$h_{0p}^{(1)} = -4\frac{G}{c^3} \sum_{a=1}^{2} \frac{m_a}{r_a} v_a^p \tag{4.62}$$

$$h_{pq}^{(1)} = 2\frac{G}{c^2} \sum_{a=1}^{2} \frac{m_a}{r_a} \delta^{pq} + \frac{G}{c^4} \sum_{a=1}^{2} \frac{m_a}{r_a} \Big[- (\vec{n}_a \cdot \vec{v}_a)^2 \delta^{pq} + 4v_a^p v_a^q \Big], \tag{4.63}$$

where v_a^p denotes the velocity of the mass m_a.

Here, it is worthwhile to point out that the parts of the linear gravitational field in $h_{00}^{(1)}$ and $h_{pq}^{(1)}$ that contain the accelerations of the masses were introduced into the part of the gravitational field quadratic in G after substituting the accelerations by explicit functionals of the coordinate positions of the masses by means of the Newtonian equations of motion.

4.4 The barycentric coordinate system

We use a harmonic coordinate system in which the 1PN-centre of mass is at rest at the origin. Using the 1PN-accurate centre of mass theorem of Ref. [27], we can express the individual centre of mass frame positions of the two masses in terms of the relative position $\vec{r}_{12} \equiv \vec{x}_1 - \vec{x}_2$ and the relative velocity $\vec{v}_{12} \equiv \vec{v}_1 - \vec{v}_2$
as

$$\vec{x}_1 = \Big[X_2 + \frac{1}{c^2}\epsilon_{\rm 1PN}\Big]\vec{r}_{12}, \tag{4.64}$$

$$\vec{x}_2 = \Big[-X_1 + \frac{1}{c^2}\epsilon_{\rm 1PN}\Big]\vec{r}_{12}, \tag{4.65}$$

where X_1, X_2 and ϵ_{1PN} are given by

$$X_1 \equiv \frac{m_1}{M}, \tag{4.66}$$

$$X_2 \equiv \frac{m_2}{M}, \tag{4.67}$$

$$\epsilon_{\rm 1PN} = \frac{\nu(m_1 - m_2)}{2M}\Big[v_{12}^2 - \frac{GM}{r_{12}}\Big]. \tag{4.68}$$

Here, we have defined

$$M \equiv m_1 + m_2, \;\; v_{12} = |\vec{v}_{12}| \tag{4.69}$$

and

$$\nu \equiv \frac{m_1 m_2}{M^2}. \tag{4.70}$$

5 Light deflection in the post-linear gravitational field of bounded point-like masses

In this chapter, light deflection in the post-linear gravitational field of a system of two bounded point-like masses is treated. Both the light source and the observer are assumed to be located at infinity in an asymptotically flat space. The equations of light propagation are explicitly integrated to the second order in G/c^2. Some of the integrals are evaluated by making use of an expansion in powers of the ratio of the relative separation distance to the impact parameter r_{12}/ξ. The correction terms arising from the effect of the motion of the masses on light propagation, the linear perturbation of the light ray trajectory and the shift of the 1PN centre of mass with respect to the Newtonian centre of mass are computed. It is shown that the expression obtained in this chapter for the angle of light deflection is fully equivalent to the expression obtained by Kopeikin and Schäfer in [7] up to the order given there. This chapter along with the associated appendices B, C and D, are based on a paper by the author [10].

5.1 Light deflection in the linear gravitational field of two bounded point-like masses

In this section we compute a part of the perturbation term $\delta l^i_{(1)}(\tau)$ and the corresponding angle of light deflection for an observer located at infinity in an asymptotically flat space.

After substituting in (2.38) the linear metric coefficients by their values (4.61), (4.62) and (4.63), we obtain

$$\delta l^i_{(1)}(\tau) = -2G\sum_{a=1}^{2} m_a \int_{-\infty}^{\tau} d\sigma\, \frac{1}{r_a^3}\big[z^i - x^i_a(t)\big]|_{(\rightarrow)}$$
$$+ 4\frac{G}{c}\sum_{a=1}^{2} m_a \int_{-\infty}^{\tau} d\sigma\, \frac{1}{r_a^3}(\vec{e}_{(0)}\cdot\vec{v}_a(t))\big[z^i - x^i_a(t)\big]|_{(\rightarrow)}$$
$$- 2\frac{G}{c^2}\sum_{a=1}^{2} m_a \int_{-\infty}^{\tau} d\sigma\, \frac{1}{r_a^3}\left[v_a^2(t) + (\vec{e}_{(0)}\cdot\vec{v}_a(t))^2\right]\big[z^i - x^i_a(t)\big]|_{(\rightarrow)}$$

$$+ 3\frac{G}{c^2}\sum_{a=1}^{2} m_a \int_{-\infty}^{\tau} d\sigma \, \frac{1}{r_a^5} (\vec{r}_a \cdot \vec{v}_a(t))^2 \big[z^i - x_a^i(t)\big]|_{(\rightarrow)}$$
$$- 4\frac{G}{c^2}\sum_{a=1}^{2} m_a \int_{-\infty}^{\tau} d\sigma \, \frac{1}{r_a^3} (\vec{e}_{(0)} \cdot \vec{v}_a(t)) \Big[c\,\sigma - (\vec{e}_{(0)} \cdot \vec{x}_a(t))\Big] l_{(0)}^i|_{(\rightarrow)}$$
$$- 2\frac{G}{c^2}\sum_{a=1}^{2} m_a \int_{-\infty}^{\tau} d\sigma \, \frac{1}{r_a^3} \Bigg\{ (\vec{e}_{(0)} \cdot \vec{v}_a(t)) c\,\sigma + \Big[\vec{\xi} \cdot \vec{v}_a(t) - \vec{x}_a(t) \cdot \vec{v}_a(t)\Big] \Bigg\} v_a^i(t)|_{(\rightarrow)}$$
$$+ 4\frac{G}{c^2}\sum_{a=1}^{2} \frac{m_a}{r_a} v_a^i(t) - 4\frac{G}{c^2}\sum_{a=1}^{2} \frac{m_a}{r_a} l_{(0)}^i - 4\frac{G}{c^3}\sum_{a=1}^{2} \frac{m_a}{r_a} (\vec{e}_{(0)} \cdot \vec{v}_a(t)) v_a^i(t)$$
$$+ 2\frac{G}{c^4}\sum_{a=1}^{2} m_a \Bigg\{ \frac{(\vec{r}_a \cdot \vec{v}_a(t))^2}{r_a^3} - 2\frac{1}{r_a} v_a^2(t) \Bigg\} l_{(0)}^i, \tag{5.1}$$

where $\vec{r}_a = \vec{z} - \vec{x}_a(t)$ and $r_a = |\vec{r}_a|$.

Because the linear metric coefficients are functions of the positions and velocities of the masses $\vec{x}_a(t)$ and $\vec{v}_a(t)$ respectively, the expression for $\delta l_{(1)}^i(\tau)$ given in (5.1) is a function of these quantities. This means that we have to take into account the motion of the masses when we are going to compute the integrals in (5.1). Considering that the influence of the gravitational field on light propagation is strongest near the barycentre of the binary and that the velocities of the masses are small with respect to the velocity of light, we are allowed to make the following approximations:

1. We may assume that the linear gravitational field is determined by the positions and velocities of the masses taken at the time of closest approach ($t = t^*$) of the unperturbed light ray to the barycentre of the binary (i.e. to the origin of the asymptotically flat harmonic coordinate system). The expression, resulting from (5.1) after setting $t = t^*$ for the positions and velocities, is denoted by $\delta l_{(1)\mathrm{I}}^i(\tau)$;

2. We treat the effect of the motion of the masses on light propagation as a correction to the expression of $\delta l_{(1)\mathrm{I}}^i(\tau)$, which we denote by $\delta l_{(1)\mathrm{II}}^i(\tau)$. We shall compute this correction in Section 5.3.

Consequently, the corresponding angle of light deflection reads

$$\alpha_{(1)}^i(\tau) = \frac{1}{c} P_q^i \big[\delta l_{(1)\mathrm{I}}^q(\tau) + \delta l_{(1)\mathrm{II}}^q(\tau)\big], \tag{5.2}$$

where P_q^i is given by (2.42).

Here, it is important to remark that in order to obtain the total linear light deflection we have to add to (5.2) terms arising from the 1PN-corrections in the positions of the masses, which we shall compute in Section 5.5. Since these terms are proportional to

v_{12}^2/c^2, it is easy to see by virtue of the virial theorem that they are of the same order as the terms in G^2/c^4.

After fixing the values of the quantities $\vec{x}_a(t)$ and $\vec{v}_a(t)$ to $\vec{x}_a(t^*)$ and $\vec{v}_a(t^*)$ in (5.1), we find

$$
\begin{aligned}
\delta l^i_{(1)\mathrm{I}}(\tau) &= -2G\sum_{a=1}^{2} m_a \int_{-\infty}^{\tau} d\sigma\, \frac{1}{r_a^3}\big[z^i - x^i_a(t^*)\big]|_{(\rightarrow)} \\
&+ 4\frac{G}{c}\sum_{a=1}^{2} m_a \int_{-\infty}^{\tau} d\sigma\, \frac{1}{r_a^3}(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))\big[z^i - x^i_a(t^*)\big]|_{(\rightarrow)} \\
&- 2\frac{G}{c^2}\sum_{a=1}^{2} m_a \int_{-\infty}^{\tau} d\sigma\, \frac{1}{r_a^3}\Big[v_a^2(t^*) + (\vec{e}_{(0)}\cdot\vec{v}_a(t^*))^2\Big]\big[z^i - x^i_a(t^*)\big]|_{(\rightarrow)} \\
&+ 3\frac{G}{c^2}\sum_{a=1}^{2} m_a \int_{-\infty}^{\tau} d\sigma\, \frac{1}{r_a^5}(\vec{r}_a\cdot\vec{v}_a(t^*))^2\big[z^i - x^i_a(t^*)\big]|_{(\rightarrow)} \\
&- 4\frac{G}{c^2}\sum_{a=1}^{2} m_a \int_{-\infty}^{\tau} d\sigma\, \frac{1}{r_a^3}(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))\Big[c\,\sigma - (\vec{e}_{(0)}\cdot\vec{x}_a(t^*))\Big] l^i_{(0)}|_{(\rightarrow)} \\
&- 2\frac{G}{c^2}\sum_{a=1}^{2} m_a \int_{-\infty}^{\tau} d\sigma\, \frac{1}{r_a^3}\Big\{(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))c\,\sigma + \Big[\vec{\xi}\cdot\vec{v}_a(t^*) - \vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\Big]\Big\} v^i_a(t^*)|_{(\rightarrow)} \\
&+ 4\frac{G}{c^2}\sum_{a=1}^{2}\frac{m_a}{r_a}v^i_a(t^*) - 4\frac{G}{c^2}\sum_{a=1}^{2}\frac{m_a}{r_a}l^i_{(0)} - 4\frac{G}{c^3}\sum_{a=1}^{2}\frac{m_a}{r_a}(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))v^i_a(t^*) \\
&+ 2\frac{G}{c^4}\sum_{a=1}^{2} m_a\Big\{\frac{(\vec{r}_a\cdot\vec{v}_a(t^*))^2}{r_a^3} - 2\frac{1}{r_a}v_a^2(t^*)\Big\} l^i_{(0)}.
\end{aligned}
\tag{5.3}
$$

Evaluation of the integrals in (5.3) leads to

$$
\begin{aligned}
\delta l^i_{(1)\mathrm{I}}(\tau) = &-2\frac{G}{c}\sum_{a=1}^{2} m_a B_a\big[\xi^i - x^i_a(t^*)\big] - \frac{G}{c^2}\sum_{a=1}^{2} m_a\Big\{2A_a + \frac{4}{r_a}\Big\} l^i_{(0)} \\
&+ 4\frac{G}{c^2}\sum_{a=1}^{2} m_a(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))B_a\big[\xi^i - x^i_a(t^*)\big] + 4\frac{G}{c^2}\sum_{a=1}^{2}\frac{m_a}{r_a}v^i_a(t^*) \\
&+ 4\frac{G}{c^3}\sum_{a=1}^{2} m_a(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))\Big\{A_a + \frac{1}{r_a}\Big\} l^i_{(0)} \\
&+ \frac{G}{c^3}\sum_{a=1}^{2} m_a(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))^2\Big\{3F_{a2} - 2B_a\Big\}\big[\xi^i - x^i_a(t^*)\big] \\
&+ 6\frac{G}{c^3}\sum_{a=1}^{2} m_a(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))\Big[\vec{\xi}\cdot\vec{v}_a(t^*) - \vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\Big] F_{a3}\big[\xi^i - x^i_a(t^*)\big]
\end{aligned}
$$

$$
\begin{aligned}
& - 2\frac{G}{c^3}\sum_{a=1}^{2} m_a v_a^2(t^*) B_a \big[\xi^i - x_a^i(t^*)\big] \\
& + 3\frac{G}{c^3}\sum_{a=1}^{2} m_a \Big[\vec{\xi}\cdot\vec{v}_a(t^*) - \vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\Big]^2 F_{a4}\big[\xi^i - x_a^i(t^*)\big] \\
& - \frac{G}{c^3}\sum_{a=1}^{2} m_a \Bigg\{ (\vec{e}_{(0)}\cdot\vec{v}_a(t^*))\Big[2A_a + \frac{4}{r_a}\Big] \\
& + 2\Big[\vec{\xi}\cdot\vec{v}_a(t^*) - \vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\Big] B_a \Bigg\} v_a^i(t^*) \\
& + 6\frac{G}{c^4}\sum_{a=1}^{2} m_a (\vec{e}_{(0)}\cdot\vec{v}_a(t^*))\Big[\vec{\xi}\cdot\vec{v}_a(t^*) - \vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\Big] F_{a2} l_{(0)}^i \\
& + \frac{G}{c^4}\sum_{a=1}^{2} m_a (\vec{e}_{(0)}\cdot\vec{v}_a(t^*))^2 \Big\{3F_{a1} - 2A_a\Big\} l_{(0)}^i \\
& - \frac{G}{c^4}\sum_{a=1}^{2} m_a v_a^2(t^*) \Big\{2A_a + \frac{4}{r_a}\Big\} l_{(0)}^i + 2\frac{G}{c^4}\sum_{a=1}^{2} \frac{m_a}{r_a^3} (\vec{r}\cdot\vec{v}_a(t^*))^2 l_{(0)}^i \\
& + 3\frac{G}{c^4}\sum_{a=1}^{2} m_a \Big[\vec{\xi}\cdot\vec{v}_a(t^*) - \vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\Big]^2 F_{a3} l_{(0)}^i ,
\end{aligned}
\tag{5.4}
$$

where the functions A_a, B_a, F_{a1}, F_{a2}, F_{a3}, and F_{a4} are given by

$$
A_a = \frac{1}{r_a\, R_a}\Big[- r_a^2(0,t^*) + (\vec{e}_{(0)}\cdot\vec{x}_a(t^*))\big(c\,\tau + r_a\big)\Big], \tag{5.5}
$$

$$
B_a = \frac{1}{r_a\, R_a}\Big[- (\vec{e}_{(0)}\cdot\vec{x}_a(t^*)) + c\,\tau + r_a\Big], \tag{5.6}
$$

$$
\begin{aligned}
F_{a1} = & \frac{1}{3\, r_a^3\, R_a^2}\Bigg\{ - r_a^2(0,t^*)\Bigg[2\, r_a^4(0,t^*) - (\vec{e}_{(0)}\cdot\vec{x}_a(t^*))\Big(3\, r_a^2(0,t^*) - (\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2\Big)\, r_a\Bigg] \\
& + 2\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))\Bigg[3\, r^4(0,t^*) - (\vec{e}_{(0)}\cdot\vec{x}_a(t^*))\Big(3\, r_a^2(0,t^*) - (\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2\Big)\, r_a\Bigg] c\,\tau \\
& - \Bigg[3\, r_a^4(0,t^*) + 3\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2 r_a^2(0,t^*) \\
& - (\vec{e}_{(0)}\cdot\vec{x}_a(t^*))\Big(3\, r_a^2(0,t^*) - (\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2\Big)\, r_a\Bigg] c^2\tau^2 \\
& + (\vec{e}_{(0)}\cdot\vec{x}_a(t^*))\Bigg[3\, r_a^2(0,t^*) - (\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2\Bigg] c^3\tau^3\Bigg\},
\end{aligned}
\tag{5.7}
$$

$$F_{a2} = \frac{1}{3\,r_a^3\,R_a^2}\Bigg\{ -2\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))\,r_a^4(0,t^*) + \Big[r_a^2(0,t^*) + (\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2\Big]\,r_a^2(0,t^*)\,r_a$$
$$-2\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))\Big[\big(r_a^2(0,t^*) + (\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2\big)\,r_a - 3\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))\,r_a^2(0,t^*)\Big]c\,\tau$$
$$+\Big[r_a^2(0,t^*) + (\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2\Big]\Big[r_a - 3\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))\Big]c^2\tau^2$$
$$+\Big[r_a^2(0,t^*) + (\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2\Big]c^3\tau^3\Bigg\}, \tag{5.8}$$

$$F_{a3} = \frac{1}{3\,r_a^3\,R_a^2}\Bigg\{ -r_a^2(0,t^*)\Big[r_a^2(0,t^*) + (\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2 - 2\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))\,r_a\Big]$$
$$+(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))\Big[3\,r_a^2(0,t^*) + 3\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2 - 4(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))\,r_a\Big]c\,\tau$$
$$+2\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))\Big[r_a - 3\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))\Big]c^2\tau^2 + 2\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))c^3\tau^3\Bigg\}, \tag{5.9}$$

$$F_{a4} = \frac{1}{3\,r_a^3\,R_a^2}\Bigg\{(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))\Big[-3\,r_a^2(0,t^*) + (\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2\Big] + 2\,r_a^2(0,t^*)\,r_a$$
$$+\Big[3\,r_a^2(0,t^*) + 3\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2 - 4\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))r_a\Big]c\,\tau$$
$$+2\,\Big[r_a - 3\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))\Big]c^2\tau^2 + 2\,c^3\tau^3\Bigg\}. \tag{5.10}$$

Here, the subscript a labels the masses and r_a is the distance between the position of the photon along its unperturbed trajectory and the position of the mass m_a at the time t^*. Explicitly, the distance r_a is given by

$$r_a = r_a(\tau,t^*) = \Big[c^2\tau^2 + \xi^2 - 2\,c\,\tau\vec{e}_{(0)}\cdot\vec{x}_a(t^*) - 2\,\vec{\xi}\cdot\vec{x}_a(t^*) + x_a^2(t^*)\Big]^{1/2}. \tag{5.11}$$

It follows from the expression for r_a that $r_a(0,t^*)$ is the distance between the point of closest approach of the unperturbed light ray to the origin of the coordinate system and the position of the mass m_a at the time t^*. The quantity R_a appearing in equations (5.5)–(5.10) is defined by

$$R_a = r_a^2(0,t^*) - (\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2. \tag{5.12}$$

Finally, in order to get the expression for the angle of light deflection for an observer located at infinity, we introduce the expression for $\delta l^i_{(1)\mathrm{I}}(\tau)$ given by (5.4) into (2.41) and compute the limit $\tau \to \infty$. The resulting angle of light deflection reads

$$
\begin{aligned}
\alpha^i_{(1)\mathrm{I}} &= \lim_{\tau\to\infty}\left[\frac{1}{c}P^i_q\delta l^q_{(1)\mathrm{I}}(\tau)\right] \\
&= -4\frac{G}{c^2}\sum_{a=1}^{2}\frac{m_a}{R_a}\left[\xi^i - P^i_q x^q_a(t^*)\right] \\
&+ 8\frac{G}{c^3}\sum_{a=1}^{2}\frac{m_a}{R_a}(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))\left[\xi^i - P^i_q x^q_a(t^*)\right] \\
&- 4\frac{G}{c^4}\sum_{a=1}^{2}\frac{m_a}{R_a}v^2_a(t^*)\left[\xi^i - P^i_q x^q_a(t^*)\right] - 4\frac{G}{c^4}\sum_{a=1}^{2}\frac{m_a}{R_a}(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))^2\left[\xi^i - P^i_q x^q_a(t^*)\right] \\
&- 4\frac{G}{c^4}\sum_{a=1}^{2}\frac{m_a}{R_a}\Big\{(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))(\vec{e}_{(0)}\cdot\vec{v}_a(t^*)) + \vec{\xi}\cdot\vec{v}_a(t^*) - \vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\Big\}P^i_q v^q_a(t^*) \\
&+ 2\frac{G}{c^4}\sum_{a=1}^{2}\frac{m_a}{R_a}(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))^2\left[\xi^i - P^i_q x^q_a(t^*)\right] \\
&+ 4\frac{G}{c^4}\sum_{a=1}^{2}\frac{m_a}{R^2_a}(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))^2\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2\left[\xi^i - P^i_q x^q_a(t^*)\right] \\
&+ 8\frac{G}{c^4}\sum_{a=1}^{2}\frac{m_a}{R^2_a}(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))\Big[\vec{\xi}\cdot\vec{v}_a(t^*) - \vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\Big]\left[\xi^i - P^i_q x^i_a(t^*)\right] \\
&+ 4\frac{G}{c^4}\sum_{a=1}^{2}\frac{m_a}{R^2_a}\Big[(\vec{\xi}\cdot\vec{v}_a(t^*))^2 - 2(\vec{\xi}\cdot\vec{v}_a(t^*)(\vec{x}_a(t^*)\cdot\vec{v}_a(t^*)) + (\vec{x}_a(t^*)\cdot\vec{v}_a(t^*))^2\Big] \\
&\left[\xi^i - P^i_q x^q_a(t^*)\right].
\end{aligned}
\tag{5.13}
$$

5.2 The post-linear light deflection in the post-linear gravitational field of two bounded point-like masses

In this section we present our computations for light deflection in the post-linear gravitational field of two bounded point-like masses. In our computations we assume that both the light source and the observer are at infinity in an asymptotically flat space so that the effects of $h^{(1)}_{\mu\nu}$ and $h^{(2)}_{\mu\nu}$ near the light source and near the observer are negligible. We take into account only the terms of order G^2/c^4. From equations (2.39), (2.40) and (2.41) we see that $\alpha^i_{(2)}$ is a function of the post-linear metric coefficients $h^{(2)}_{\mu\nu}$ and of the linear metric coefficients $h^{(1)}_{\mu\nu}$. To facilitate the computations, we separate the light deflection terms that are functions of the post-linear metric coefficients from the terms that are functions of the linear metric coefficients and the perturbations of the first order in G of the vector tangent to the unperturbed light ray. First we compute the terms of

$\alpha^i_{(2)}$ that are functions of the post-linear metric coefficients, which we denote by $\alpha^i_{(2)\mathrm{I}}$.

5.2.1 The post-linear light deflection terms that depend on the metric coefficients quadratic in G

It follows from equations (2.39) and (2.41) that a part of the post-linear light deflection is given by:

$$\alpha^i_{(2)\mathrm{I}} = \frac{1}{c} P^i_q \left[\frac{1}{2} c^2 \int_{-\infty}^{\infty} d\tau h^{(2)}_{00,q} \,|_{(\rightarrow)} + \int_{-\infty}^{\infty} d\tau \left[\frac{1}{2} h^{(2)}_{mn,q} - h^{(2)}_{qm,n} \right] l^m_{(0)} l^n_{(0)} \,|_{(\rightarrow)} \right]. \tag{5.14}$$

Upon introducing the post-linear metric (4.59) and (4.60) into (5.14) we obtain integrals whose integrands are functions of the distances r_1, r_2, S and their inverses. Through the distances r_1, r_2 and S, the resulting integrals from (5.14) are functions of the positions of the masses $\vec{x}_a(t)$.

For the same reason as in the case of the linear light deflection, we are here allowed to fix the values of the positions of the masses $\vec{x}_a(t)$ to their values at the time t^* before performing the integration. The resulting integrals are given explicitly in Appendix C.

To evaluate the integrals that cannot be represented by elementary functions, we resort as usual to a series expansion of the integrands. To perform the series expansion we consider the integrands as functions of the distances r_1, r_2 and S. Then, we expand these functions in a Taylor series about the origin of the coordinate system $\vec{x}_1 = \vec{x}_2 = 0$ to the second order. We only need to perform the Taylor expansion up to second order, since with an expansion to this order we obtain a result which is sufficiently accurate for the applications that we shall consider in this work.

The positions of the masses in the centre of mass frame defined in Section 4.4 are given by equations (4.64) and (4.65). Here, we do not need to take into account the 1PN-corrections in the positions of the masses, because if we introduced these into (5.14) we would obtain terms of higher order than G^2/c^4.

In Section 5.5 we shall compute the post-linear light deflection terms resulting from the introduction of the 1PN-corrections in the positions of the masses into the equation for linear light deflection.

Also, we do not need to consider here the correction terms arising from introducing the motions of the masses into (5.14), because these terms are of higher order than G^2/c^4. The correction to the post-linear light deflection arising from introducing the motion of the masses into the expression for the linear perturbation is denoted by $\alpha^i_{(2)\mathrm{III}}$ and we shall compute it in Section 5.3.

5.2.2 The post-linear light deflection terms that depend on the metric coefficients linear in G

The post-linear light deflection terms that are functions of the linear metric coefficients and the linear perturbations $\delta\vec{l}_{(1)}(\tau)$ we denote by $\alpha^i_{(2)\mathrm{II}}$. It follows from equations (2.40) and (2.41) that the resulting expression for the post-linear light deflection $\alpha^i_{(2)\mathrm{II}}$ reads

$$\begin{aligned}
\alpha^i_{(2)\mathrm{II}} = \frac{1}{c}P^i_q \Bigg[& -\frac{1}{2}c^2 \int_{-\infty}^{\infty} d\tau\, h^{(1)qm} h^{(1)}_{00,m}|_{(\rightarrow)} \\
& + \int_{-\infty}^{\infty} d\tau \left[h^{(1)qp}\big(h^{(1)}_{mp,n} - \frac{1}{2}h^{(1)}_{mn,p}\big)\right] l^m_{(0)} l^n_{(0)}|_{(\rightarrow)} \\
& + c\int_{-\infty}^{\infty} d\tau \left[h^{(1)}_{0m,q} - h^{(1)}_{0q,m}\right] \delta l^m_{(1)}(\tau)|_{(\rightarrow)} \\
& + \int_{-\infty}^{\infty} d\tau \left[h^{(1)}_{mn,q}\delta l^m_{(1)}(\tau) l^n_{(0)} - h^{(1)}_{mq,n}\delta l^m_{(1)}(\tau) l^n_{(0)} - h^{(1)}_{mq,n} l^m_{(0)} \delta l^n_{(1)}(\tau)\right]|_{(\rightarrow)} \\
& - \int_{-\infty}^{\infty} d\tau\, h^{(1)}_{00,k} l^k_{(0)} \delta l^q_{(1)}(\tau)|_{(\rightarrow)} \\
& - \frac{1}{c}\int_{-\infty}^{\infty} d\tau\, h^{(1)}_{0p,m} l^m_{(0)} l^p_{(0)} \delta l^q_{(1)}(\tau)|_{(\rightarrow)} \Bigg].
\end{aligned} \tag{5.15}$$

To compute $\alpha^i_{(2)\mathrm{II}}$, we introduce the expression for the perturbation $\delta l^i_{(1)}(\tau)$ given by (5.35) and the metric functions (4.61), (4.63) and (4.62) into the expression for $\alpha^i_{(2)\mathrm{II}}$. Here, we may use the same approximations as before, i.e. we can fix the values of the positions and velocities of the masses to their values at the time t^* before performing the integrals. The resulting integrals are given in Appendix D. As explained in the preceding section, with the help of a Taylor expansion of the integrands we can evaluate the integrals, which cannot be represented by elementary functions.

5.3 Light deflection and the motion of the masses

In this section we compute the correction terms to the linear and post-linear light deflection arising from the effect of the motion of the masses on light propagation. The correction terms to the linear and post-linear light deflection can be found by means of Taylor expansions of the linear perturbation (5.1) in which the coefficients depend on the sources' coordinates x^i_a and their successive derivatives with respect to t, namely

$$\frac{dx^i_a}{dt} = v^i_a(t), \quad \frac{d^2x^i_a}{dt^2} = \frac{dv^i_a}{dt} = a^i_a(t), ...,$$

taken at the time t^*.

5.3.1 The linear light deflection and the motion of the masses

The correction terms to the linear perturbation arising from the Taylor expansion of (5.1) are

$$
\begin{aligned}
\delta l^i_{(1)\mathrm{II}}(\tau) &= G\sum_{a=1}^{2} m_a \int_{-\infty}^{\tau} d\sigma \Bigg\{\Bigg[-\frac{6}{r_a^5}(\vec{r}_a\cdot\vec{v}_a(t^*))r_a^i + \frac{2}{r_a^3}v_a^i(t^*)\Bigg]\sigma \\
&+ \Bigg[-\frac{15}{r_a^7}(\vec{r}_a\cdot\vec{v}_a(t^*))^2 r_a^i + \frac{6}{r_a^5}(\vec{r}_a\cdot\vec{v}_a(t^*))v_a^i(t^*) + \frac{3}{r_a^5}v_a^2(t^*)r_a^i\Bigg]\sigma^2\Bigg\}_{|(\rightarrow)} \\
&+ \frac{G}{c}\sum_{a=1}^{2} m_a(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))\int_{-\infty}^{\tau} d\sigma \Bigg\{\Bigg[\frac{12}{r_a^5}(\vec{r}_a\cdot\vec{v}_a(t^*))r_a^i - \frac{4}{r_a^3}v_a^i(t^*)\Bigg]\sigma \\
&+ \Bigg[\frac{30}{r_a^7}(\vec{r}_a\cdot\vec{v}_a(t^*))^2 r_a^i - \frac{12}{r_a^5}(\vec{r}_a\cdot\vec{v}_a(t^*))v_a^i(t^*) - \frac{6}{r_a^5}v_a^2(t^*)r_a^i\Bigg]\sigma^2\Bigg\}_{|(\rightarrow)} \\
&+ \frac{G}{c^2}\sum_{a=1}^{2} m_a \int_{-\infty}^{\tau} d\sigma \Bigg\{\Big[v^2(t^*) + (\vec{e}_{(0)}\cdot\vec{v}(t^*))^2\Big]\Bigg[\Bigg(-\frac{6}{r_a^5}(\vec{r}_a\cdot\vec{v}_a(t^*))r_a^i \\
&+ \frac{2}{r_a^3}v_a^i(t^*)\Bigg)\sigma + \Bigg(-\frac{15}{r_a^7}(\vec{r}_a\cdot\vec{v}_a(t^*))^2 r_a^i + \frac{6}{r_a^5}(\vec{r}_a\cdot\vec{v}_a(t^*))v_a^i(t^*) + \frac{3}{r_a^5}v_a^2(t^*)r_a^i\Bigg)\sigma^2\Bigg] \\
&+ \Bigg[\Bigg(\frac{15}{r_a^7}(\vec{r}_a\cdot\vec{v}_a(t^*))^3 r_a^i - \frac{3}{r_a^5}(\vec{r}_a\cdot\vec{v}_a(t^*))^2 v_a^i(t^*) - \frac{6}{r_a^5}(\vec{r}_a\cdot\vec{v}_a(t^*))v_a^2(t^*)r_a^i\Bigg)\sigma \\
&+ \Bigg(\frac{105}{2}\frac{(\vec{r}_a\cdot\vec{v}_a(t^*))^4}{r_a^9}r_a^i - \frac{15}{r_a^7}(\vec{r}_a\cdot\vec{v}_a(t^*))^3 v_a^i(t^*) - \frac{75}{2}\frac{(\vec{r}_a\cdot\vec{v}_a(t^*))^2}{r_a^7}v_a^2(t^*)r_a^i \\
&+ \frac{6}{r_a^5}(\vec{r}_a\cdot\vec{v}_a(t^*))v_a^2(t^*)v_a^i(t^*) + \frac{3}{r_a^5}v_a^4(t^*)r_a^i\Bigg)\sigma^2\Bigg] \\
&- (\vec{e}_{(0)}\cdot\vec{v}_a(t^*))\Big[c\,\sigma - (\vec{e}_{(0)}\cdot\vec{x}_a(t^*))\Big]\Bigg[\frac{12}{r_a^5}(\vec{r}_a\cdot\vec{v}_a(t^*))\sigma \\
&+ \Bigg(\frac{30}{r_a^7}(\vec{r}_a\cdot\vec{v}_a(t^*))^2 - \frac{6}{r_a^5}v_a^2(t^*)\Bigg)\sigma^2\Bigg] l^i_{(0)} \\
&+ (\vec{e}_{(0)}\cdot\vec{v}_a(t^*))^2\Bigg[\frac{4}{r_a^3}\sigma + \frac{12}{r_a^5}(\vec{r}_a\cdot\vec{v}_a(t^*))\sigma^2\Bigg] l^i_{(0)} \\
&+ \Bigg[(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))c\,\sigma + \Big(\vec{\xi}\cdot\vec{v}_a(t^*) - \vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\Big)\Bigg]\Bigg[-\frac{6}{r_a^5}(\vec{r}_a\cdot\vec{v}_a(t^*))\sigma \\
&+ \Bigg(-\frac{15}{r_a^7}(\vec{r}_a\cdot\vec{v}_a(t^*))^2 + \frac{3}{r_a^5}v_a^2(t^*)\Bigg)\sigma^2\Bigg] v_a^i(t^*) \\
&+ v_a^2(t^*)\Bigg[\frac{2}{r_a^3}\sigma + \frac{6}{r_a^5}(\vec{r}_a\cdot\vec{v}_a(t^*))\sigma^2\Bigg] v_a^i(t^*)\Bigg\}_{|(\rightarrow)}
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{G}{c^2}\sum_{a=1}^{2} m_a\Bigg\{\frac{4}{r_a^3}(\vec{r}_a\cdot\vec{v}_a(t^*))\tau+\left[\frac{6}{r_a^5}(\vec{r}_a\cdot\vec{v}_a(t^*))^2-\frac{2}{r_a^3}v_a^2(t^*)\right]\tau^2\Bigg\}\left[v_a^i(t^*)-l_{(0)}^i\right]\\
&+\frac{G}{c^3}\sum_{a=1}^{2} m_a(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))\Bigg\{-\frac{4}{r_a^3}(\vec{r}_a\cdot\vec{v}_a(t^*))\tau+\Bigg[-\frac{6}{r_a^5}(\vec{r}_a\cdot\vec{v}_a(t^*))^2\\
&+\frac{2}{r_a^3}v_a^2(t^*)\Bigg]\tau^2\Bigg\}v_a^i(t^*)\\
&+\frac{G}{c^4}\sum_{a=1}^{2} m_a\Bigg\{\left[\frac{6}{r_a^5}(\vec{r}_a\cdot\vec{v}_a(t^*))^3-\frac{4}{r_a^3}(\vec{r}_a\cdot\vec{v}_a(t^*))v_a^2(t^*)\right]\tau\\
&+\left[\frac{15}{r_a^7}(\vec{r}_a\cdot\vec{v}_a(t^*))^4-\frac{15}{r_a^5}(\vec{r}_a\cdot\vec{v}_a(t^*))^2v_a^2(t^*)+\frac{2}{r_a^3}v_a^4(t^*)\right]\tau^2\\
&-4v_a^2(t^*)\left[\frac{(\vec{r}_a\cdot\vec{v}_a(t^*))}{r_a^3}\tau+\left(\frac{3}{2}\frac{(\vec{r}_a\cdot\vec{v}_a(t^*))^2}{r_a^5}-\frac{1}{2}\frac{v_a^2(t^*)}{r_a^3}\right)\tau^2\right.\Bigg\}l_{(0)}^i
\end{aligned}
\tag{5.16}
$$

Considering that in the present work we compute the post-linear light deflection up to the order G^2/c^4, we must retain the linear light deflection terms up to the order G/c^4. Notice that the linear terms of the order G/c^4 are of the same order as the post-linear terms of the order G^2/c^4 since for a system of bounded point-like masses, the virial theorem applies (i.e. $v_a^2 \sim G/d$) and, considering that the terms of the order G/c^4 are also terms in v_a^2, it is easy to see that these terms are of the same order as the post-linear terms of the order G^2/c^4. Of course, we get linear light deflection terms from the perturbation $\delta l^i_{(1)\mathrm{II}}(\tau)$ too. To obtain the perturbation $\delta l^i_{(1)\mathrm{II}}(\tau)$, we have to evaluate the integrals in the expression above. Taking into account that we are only interested in the angle of light deflection to the order G/c^4, we need only retain the terms of the order G/c^2 and G/c^3, since the expression for the light deflection angle (see equation (5.21)) contains a further factor $1/c$. After performing the integration of the expression above and retaining only terms of the order G/c^2 and G/c^3, we finally find

$$
\begin{aligned}
\delta l^i_{(1)\mathrm{II}}(\tau)&=-6\frac{G}{c^2}\sum_{a=1}^{2} m_a\left[(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))F_{a2}+\left[\vec{\xi}\cdot\vec{v}_a(t^*)-\vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\right]F_{a3}\right][\xi^i-x_a^i(t^*)]\\
&-4\frac{G}{c^2}\sum_{a=1}^{2}\frac{m_a}{r_a^3}\left[(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))c\,\tau+\left[\vec{\xi}\cdot\vec{v}_a(t^*)-\vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\right]\right]\tau l_{(0)}^i\\
&+2\frac{G}{c^2}\sum_{a=1}^{2} m_a\left\{A_a+\frac{2}{r_a^3}\left[(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))c\,\tau+\left[\vec{\xi}\cdot\vec{v}_a(t^*)-\vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\right]\right]\tau\right\}v_a^i(t^*)\\
&+4\frac{G}{c^2}\sum_{a=1}^{2}\frac{m_a}{r_a^3}\left[(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))c\,\tau+\left[\vec{\xi}\cdot\vec{v}_a(t^*)-\vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\right]\right]\tau v_a^i(t^*)
\end{aligned}
$$

$$
\begin{aligned}
&+12\frac{G}{c^3}\sum_{a=1}^{2} m_a(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))\Big[(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))F_{a2}+\Big[\vec{\xi}\cdot\vec{v}_a(t^*)-\vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\Big]F_{a3}\Big]\left[\xi^i-x_a^i(t^*)\right]\\
&-\frac{G}{c^3}\sum_{a=1}^{2} m_a\Big[15(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))^2G_{a2}+30(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))\Big[\vec{\xi}\cdot\vec{v}_a(t^*)-\vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\Big]G_{a3}\\
&+15\Big[\vec{\xi}\cdot\vec{v}_a(t^*)-\vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\Big]^2G_{a4}-3v_a^2(t^*)F_{a2}\Big]\left[\xi^i-x_a^i(t^*)\right]\\
&-6\frac{G}{c^3}\sum_{a=1}^{2} m_a\Big[(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))F_{a1}+\Big[\vec{\xi}\cdot\vec{v}_a(t^*)-\vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\Big]F_{a2}\Big]l_{(0)}^i\\
&+6\frac{G}{c^3}\sum_{a=1}^{2} m_a\Big[(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))F_{a1}+\Big[\vec{\xi}\cdot\vec{v}_a(t^*)-\vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\Big]F_{a2}\Big]v_a^i(t^*)\\
&-4\frac{G}{c^3}\sum_{a=1}^{2} m_a(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))\left\{A_a+\frac{1}{r_a^3}\Big[(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))c\tau+\Big[\vec{\xi}\cdot\vec{v}_a(t^*)-\vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\Big]\Big]\tau\right\}v_a^i(t^*),
\end{aligned}
\tag{5.17}
$$

where the functions A_a, F_{a1}, F_{a2}, F_{a3} are given in Section 5.1 by equations (5.5)–(5.10). The functions G_{a2}, G_{a3} and G_{a4} are defined by

$$
\begin{aligned}
G_{a2} &= \frac{1}{15\,r_a^5\,R_a^3}\Big\{\Big[3\,r_a^8(0,t^*)\,r_a-8\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))\,r_a^8(0,t^*)\\
&+6\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2\,r_a^6(0,t^*)\,r_a-(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^4\,r_a^4(0,t^*)\,r_a\Big]\\
&-\Big[12\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))\,r_a^6(0,t^*)\,r_a-40\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2\,r_a^6(0,t^*)\\
&+24\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^3\,r_a^4(0,t^*)\,r_a-4\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^5\,r_a^2(0,t^*)\,r_a\Big]\,c\,\tau\\
&-\Big[-6\,r_a^6(0,t^*)\,r_a+20\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))\,r_a^6(0,t^*)-24\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2\,r_a^4(0,t^*)\,r_a\\
&+60\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^3\,r_a^4(0,t^*)-22\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^4\,r_a^2(0,t^*)\,r_a+4\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^6\,r_a\Big]\,c^2\tau^2\\
&-\Big[12\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))\,r_a^4(0,t^*)\,r_a-60\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2\,r_a^4(0,t^*)\\
&+24\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^3\,r_a^2(0,t^*)\,r_a-20\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^4\,r_a^2(0,t^*)-4\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^5\,r_a\Big]\,c^3\tau^3\\
&-\Big[-3\,r_a^4(0,t^*)\,r_a+15\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))\,r_a^4(0,t^*)-6\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2\,r_a^2(0,t^*)\,r_a\\
&+30\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^3\,r_a^2(0,t^*)+(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^4\,r_a-5\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^5\Big]\,c^4\tau^4
\end{aligned}
$$

$$- \left[- 3\, r_a^4(0,t^*) - 6\, (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2\, r_a^2(0,t^*) + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^4 \right] c^5 \tau^5 \Bigg\}, \tag{5.18}$$

$$G_{a3} = \frac{1}{15\, r_a^5\, R_a^3} \Bigg\{ - 2\, r_a^4(0,t^*) \Big[r_a^4(0,t^*) - (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^3\, r_a$$
$$+ 3\, (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))\, r_a^2(0,t^*) \Big[- r_a + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) \Big] \Big]$$
$$+ 2\, (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))\, r_a^2(0,t^*) \Big[5\, r_a^4(0,t^*) - 4\, (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^3\, r_a$$
$$+ 3\, (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))\, r_a^2(0,t^*) \Big[- 4\, r_a + 5\, (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) \Big] \Big]\, c\, \tau$$
$$- \Big[5\, r_a^6(0,t^*) - 8\, (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^5\, r_a + 6\, (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))\, r_a^4(0,t^*) \Big[- 2\, r_a$$
$$+ 5\, (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) \Big] + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^3\, r_a^6(0,t^*) \Big[- 28\, r_a + 45\, (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) \Big] \Big]\, c^2 \tau^2$$
$$+ (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) \Big[3\, r_a^2(0,t^*) + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 \Big] \Big[5\, r_a^2(0,t^*) + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) \Big[- 8\, r_a$$
$$+ 15\, (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) \Big] \Big]\, c^3 \tau^3$$
$$- (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) \Big[- r_a + 5\, (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) \Big] \Big[3\, r_a^2(0,t^*) + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 \Big]\, c^4 \tau^4$$
$$+ 2\, (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) \Big[3\, r_a^2(0,t^*) + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 \Big]\, c^5 \tau^5 \Bigg\}, \tag{5.19}$$

$$G_{a4} = \frac{1}{15\, r_a^5\, R_a^3} \Bigg\{ 2\, r_a^4(0,t^*) \Big[r_a^2(0,t^*) \Big[r_a - 3\, (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) \Big] + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 \Big[3\, r_a$$
$$- (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) \Big] \Big]$$
$$+ 2\, (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))\, r_a^2(0,t^*) \Big[(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 \Big[- 12\, r_a + 5\, (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) \Big]$$
$$+ r_a^2(0,t^*) \Big[- 4\, r_a + 15\, (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) \Big] \Big]\, c\, \tau$$
$$+ \Big[r_a^2(0,t^*) + 3\, (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 \Big] \Big[r_a^2(0,t^*) \Big[4\, r_a - 15\, (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) \Big]$$
$$+ (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 \Big[8\, r_a - 5\, (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) \Big] \Big]\, c^2 \tau^2$$

$$+\left[r_a^2(0,t^*)+3\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2\right]\left[5\,r_a^2(0,t^*)+(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))\,\Big[-8\,r_a$$
$$+15\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))\Big]\right]c^3\tau^3$$
$$+2\left[r_a-5\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))\right]\left[r_a^2(0,t^*)+3\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2\right]c^4\tau^4$$
$$+2\left[r_a^2(0,t^*)+3\,(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2\right]c^5\tau^5\Bigg\}. \tag{5.20}$$

After introducing the perturbation $\delta l^i_{(1)\mathrm{II}}(\tau)$ into (2.41) and computing the limit for $\tau\to\infty$, we obtain

$$\begin{aligned}
\alpha^i_{(1)\mathrm{II}} &= \lim_{\tau\to\infty}\left[\frac{1}{c}P^i_q\delta l^q_{(1)\mathrm{II}}(\tau)\right]\\
&= -4\frac{G}{c^3}\sum_{a=1}^{2}\frac{m_a}{R_a}(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))\left[\xi^i-P^i_qx^q_a(t^*)\right]\\
&- 8\frac{G}{c^3}\sum_{a=1}^{2}\frac{m_a}{R_a^2}(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))\Big[(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))+\vec{\xi}\cdot\vec{v}_a(t^*)-\vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\Big]\\
&\times\left[\xi^i-P^i_qx^q_a(t^*)\right]\\
&+4\frac{G}{c^3}\sum_{a=1}^{2}\frac{m_a}{R_a}(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))P^i_qv^q_a(t^*)\\
&+2\frac{G}{c^4}\sum_{a=1}^{2}\frac{m_a}{R_a}v_a^2(t^*)\left[\xi^i-P^i_qx^q_a(t^*)\right]\\
&+4\frac{G}{c^4}\sum_{a=1}^{2}\frac{m_a}{R_a^2}v_a^2(t^*)(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2\left[\xi^i-P^i_qx^q_a(t^*)\right]\\
&+2\frac{G}{c^4}\sum_{a=1}^{2}\frac{m_a}{R_a}(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))^2\left[\xi^i-P^i_qx^q_a(t^*)\right]\\
&-8\frac{G}{c^4}\sum_{a=1}^{2}\frac{m_a}{R_a^2}(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))^2\left[\xi^i-P^i_qx^q_a(t^*)\right]\\
&-16\frac{G}{c^4}\sum_{a=1}^{2}\frac{m_a}{R_a^3}(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^4(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))^2\left[\xi^i-P^i_qx^q_a(t^*)\right]\\
&-8\frac{G}{c^4}\sum_{a=1}^{2}\frac{m_a}{R_a^2}(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))\Big[\vec{\xi}\cdot\vec{v}_a(t^*)-\vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\Big]\left[\xi^i-P^i_qx^q_a(t^*)\right]\\
&-32\frac{G}{c^4}\sum_{a=1}^{2}\frac{m_a}{R_a^3}(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^3(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))\Big[\vec{\xi}\cdot\vec{v}_a(t^*)-\vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\Big]\left[\xi^i-P^i_qx^q_a(t^*)\right]
\end{aligned}$$

$$
\begin{aligned}
& - 4\frac{G}{c^4}\sum_{a=1}^{2}\frac{m_a}{R_a^2}\Big[\vec{\xi}\cdot\vec{v}_a(t^*) - \vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\Big]^2\big[\xi^i - P^i_q x^q_a(t^*)\big] \\
& - 16\frac{G}{c^4}\sum_{a=1}^{2}\frac{m_a}{R_a^3}(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2\Big[\vec{\xi}\cdot\vec{v}_a(t^*) - \vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\Big]^2\big[\xi^i - P^i_q x^q_a(t^*)\big] \\
& + 4\frac{G}{c^4}\sum_{a=1}^{2}\frac{m_a}{R_a}(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))P^i_q v^q_a(t^*) \\
& + 8\frac{G}{c^4}\sum_{a=1}^{2}\frac{m_a}{R_a^2}(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2\Bigg\{\Big[\vec{\xi}\cdot\vec{v}_a(t^*) - \vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\Big] \\
& + (\vec{e}_{(0)}\cdot\vec{x}_a(t^*))(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))\Bigg\}P^i_q v^q_a(t^*) \\
& + 4\frac{G}{c^4}\sum_{a=1}^{2}\frac{m_a}{R_a}\Big[\vec{\xi}\cdot\vec{v}_a(t^*) - \vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\Big]P^i_q v^q_a(t^*),
\end{aligned} \tag{5.21}
$$

where the quantity R_a is defined by (5.12).

5.3.2 The post-linear light deflection and the motion of the masses

The correction terms to the post-linear light deflection to the order G^2/c^4 resulting from the Taylor expansion of (5.1) are given by

$$
\begin{aligned}
\alpha^i_{(2)\mathrm{III}} = & -3\frac{G}{c}\sum_{a=1}^{2}m_a\int_{-\infty}^{\infty}d\tau\,\frac{\tau^2}{r_a^5}\Big[c\tau(\vec{e}_{(0)}\cdot\vec{a}_a(t^*))+\vec{\xi}\cdot\vec{a}_a(t^*)-\vec{x}_a(t^*)\cdot\vec{a}_a(t^*)\Big]\big[\xi^i-P^i_q x^q_a(t^*)\big] \\
& + \frac{G}{c}\sum_{a=1}^{2}m_a\int_{-\infty}^{\infty}d\tau\,\frac{\tau^2}{r_a^3}P^i_q a^q_a(t^*) \\
& + 4\frac{G}{c^2}\sum_{a=1}^{2}m_a\int_{-\infty}^{\infty}d\tau\,\frac{\tau}{r_a^3}(\vec{e}_{(0)}\cdot\vec{a}_a(t^*))\big[\xi^i-P^i_q x^q_a(t^*)\big].
\end{aligned} \tag{5.22}
$$

In the preceding equation the second integral diverges. Therefore we resort to a Taylor expansion of its integrand about the origin of the coordinate system $\vec{x}_a = 0$ up to second order. Then only the first term of the Taylor expansion is a divergent integral and it is given by

$$
\frac{G}{c}\sum_{a=1}^{2}m_a\int_{-\infty}^{\infty}d\tau\tau^2\frac{1}{\big[c^2\tau^2+\xi^2\big]^{3/2}}P^i_q a^q_a(t^*).
$$

Because in this case we do not need to take into account the 1PN-corrections in the positions of the masses we can assume that the origin of the coordinate system, which is located at the 1PN-centre of mass, coincides with the position of the Newtonian centre of mass. Taking into account the consequence of the Newtonian centre of mass theorem

$$\sum_{a=1}^{2} m_a \vec{a}_a = 0,$$

it is easy to see that the divergent integral vanishes. After performing the integration with respect to the parameter τ we find:

$$\begin{aligned}
\alpha^i_{(2)\mathrm{III}} = {} & 2\frac{G}{c^4}\sum_{a=1}^{2}\frac{m_a}{R_a}(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))(\vec{e}_{(0)}\cdot\vec{a}_a(t^*))\left[\xi^i - P^i_q x^q_a(t^*)\right] \\
& - 4\frac{G}{c^4}\sum_{a=1}^{2}\frac{m_a}{R_a^2}(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^3(\vec{e}_{(0)}\cdot\vec{a}_a(t^*))\left[\xi^i - P^i_q x^q_a(t^*)\right] \\
& - 2\frac{G}{c^4}\sum_{a=1}^{2}\frac{m_a}{R_a}\left[\vec{\xi}\cdot\vec{a}_a(t^*) - \vec{x}_a(t^*)\cdot\vec{a}_a(t^*)\right]\left[\xi^i - P^i_q x^q_a(t^*)\right] \\
& - 4\frac{G}{c^4}\sum_{a=1}^{2}\frac{m_a}{R_a^2}(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2\left[\vec{\xi}\cdot\vec{a}_a(t^*) - \vec{x}_a(t^*)\cdot\vec{a}_a(t^*)\right]\left[\xi^i - P^i_q x^q_a(t^*)\right] \\
& + \frac{G}{c^4}\sum_{a=1}^{2} m_a\left\{2\frac{\vec{\xi}\cdot\vec{x}_a(t^*)}{\xi^2} - \frac{x_a^2(t^*)}{\xi^2} + 3\frac{(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2}{\xi^2} + 2\frac{(\vec{\xi}\cdot\vec{x}_a(t^*))^2}{\xi^4}\right\}P^i_q a^q_a(t^*),
\end{aligned} \tag{5.23}$$

where the quantity R_a is defined by (5.12). Note that in (5.22) two of the three integrals were exactly integrated, so that the resulting expression (5.23) is a combination of exact terms with a term which is represented as an expansion in powers of $x_a(t^*)/\xi$. In view of further applications and for the sake of uniformity we perform the Taylor expansion of the exact terms about the origin of the coordinate system $\vec{x}_a = 0$ up to second order and obtain

$$\begin{aligned}
\alpha^i_{(2)\mathrm{III}} = {} & 2\frac{G}{c^4}\sum_{a=1}^{2} m_a(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))(\vec{e}_{(0)}\cdot\vec{a}_a(t^*))\left\{\frac{1}{\xi^2}+2\frac{(\vec{\xi}\cdot\vec{x}_a(t^*))}{\xi^4}-\frac{x_a^2(t^*)}{\xi^4}+\frac{(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2}{\xi^4}\right. \\
& \left. + 4\frac{(\vec{\xi}\cdot\vec{x}_a(t^*))^2}{\xi^6}\right\}\left[\xi^i - P^i_q x^q_a(t^*)\right] \\
& - 4\frac{G}{c^4}\sum_{a=1}^{2} m_a(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^3(\vec{e}_{(0)}\cdot\vec{a}_a(t^*))\left\{\frac{1}{\xi^4}+4\frac{\vec{\xi}\cdot\vec{x}_a(t^*)}{\xi^6}-2\frac{x_a^2(t^*)}{\xi^6}+2\frac{(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2}{\xi^6}\right. \\
& \left. +12\frac{(\vec{\xi}\cdot\vec{x}_a(t^*))^2}{\xi^8}\right\}\left[\xi^i - P^i_q x^q_a(t^*)\right]
\end{aligned}$$

$$
\begin{aligned}
&- 2\frac{G}{c^4}\sum_{a=1}^{2} m_a\Big[\vec{\xi}\cdot\vec{a}_a(t^*) - \vec{x}_a(t^*)\cdot\vec{a}_a(t^*)\Big]\Bigg\{\frac{1}{\xi^2} + 2\frac{(\vec{\xi}\cdot\vec{x}_a(t^*))}{\xi^4} - \frac{x_a^2(t^*)}{\xi^4} + \frac{(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2}{\xi^4}\\
&+ 4\frac{(\vec{\xi}\cdot\vec{x}_a(t^*))^2}{\xi^6}\Bigg\}\big[\xi^i - P^i_q x^q_a(t^*)\big]\\
&- 4\frac{G}{c^4}\sum_{a=1}^{2} m_a(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2\Big[\vec{\xi}\cdot\vec{a}_a(t^*) - \vec{x}_a(t^*)\cdot\vec{a}_a(t^*)\Big]\Bigg\{\frac{1}{\xi^4} + 4\frac{\vec{\xi}\cdot\vec{x}_a(t^*)}{\xi^6} - 2\frac{x_a^2(t^*)}{\xi^6}\\
&+ 2\frac{(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2}{\xi^6} + 12\frac{(\vec{\xi}\cdot\vec{x}_a(t^*))^2}{\xi^8}\Bigg\}\big[\xi^i - P^i_q x^q_a(t^*)\big]\\
&+ \frac{G}{c^4}\sum_{a=1}^{2} m_a\Bigg\{2\frac{\vec{\xi}\cdot\vec{x}_a(t^*)}{\xi^2} - \frac{x_a^2(t^*)}{\xi^2} + 3\frac{(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2}{\xi^2} + 2\frac{(\vec{\xi}\cdot\vec{x}_a(t^*))^2}{\xi^4}\Bigg\}P^i_q a^q_a(t^*). \qquad (5.24)
\end{aligned}
$$

To replace the accelerations in the preceding equation by functions of the positions we shall use the Newtonian equations of motion. After replacing the accelerations in (5.24) and expressing the positions of the masses by their centre-of-mass-frame coordinates without considering the 1PN-corrections we find,

$$
\begin{aligned}
\alpha^i_{(2)\mathrm{III}} = \frac{G^2 m_1 m_2}{c^4 r_{12}^2}\Bigg\{ & 2\,(\vec{e}_\xi\cdot\vec{n}_{12}) - 2\,X_2\Big[1 - 2\,(\vec{e}_\xi\cdot\vec{n}_{12})^2 - 3\,(\vec{e}_{(0)}\cdot\vec{n}_{12})^2\Big]\left(\frac{r_{12}}{\xi}\right)\\
& - 6\,X_2^2\,(\vec{e}_\xi\cdot\vec{n}_{12})\Big[1 - \frac{4}{3}\,(\vec{e}_\xi\cdot\vec{n}_{12})^2 - 3\,(\vec{e}_{(0)}\cdot\vec{n}_{12})^2\Big]\left(\frac{r_{12}}{\xi}\right)^2\\
& + 2\,X_2^3\,\Big[1 - 4\,(\vec{e}_\xi\cdot\vec{n}_{12})^2 - 3\,(\vec{e}_{(0)}\cdot\vec{n}_{12})^2\Big]\left(\frac{r_{12}}{\xi}\right)^3 + \mathcal{O}\left[\left(\frac{r_{12}}{\xi}\right)^4\right]\Bigg\}e^i_\xi\\
& - \frac{G^2 m_1 m_2}{c^4 r_{12}^2}\Bigg\{2X_2(\vec{e}_\xi\cdot\vec{n}_{12})\left(\frac{r_{12}}{\xi}\right) - 2X_2^2\Big[1 - 2(\vec{e}_\xi\cdot\vec{n}_{12})^2 - 3(\vec{e}_{(0)}\cdot\vec{n}_{12})^2\Big]\left(\frac{r_{12}}{\xi}\right)^2\\
& - 6\,X_2^3\,(\vec{e}_\xi\cdot\vec{n}_{12})\Big[1 - \frac{4}{3}\,(\vec{e}_\xi\cdot\vec{n}_{12})^2 - 3\,(\vec{e}_{(0)}\cdot\vec{n}_{12})^2\Big]\left(\frac{r_{12}}{\xi}\right)^3\\
& + 2\,X_2^3\,\Big[1 - 4\,(\vec{e}_\xi\cdot\vec{n}_{12})^2 - 3\,(\vec{e}_{(0)}\cdot\vec{n}_{12})^2\Big]\left(\frac{r_{12}}{\xi}\right)^4 + \mathcal{O}\left[\left(\frac{r_{12}}{\xi}\right)^5\right]\Bigg\}P^i_q n^q_{12}\\
& + \frac{G^2 m_1 m_2}{c^4 r_{12}^2}\Bigg\{ - 2\,X_2\,(\vec{e}_\xi\cdot\vec{n}_{12})\left(\frac{r_{12}}{\xi}\right) + X_2^2\,\Big[1 - 2\,(\vec{e}_\xi\cdot\vec{n}_{12})^2\\
& - 3\,(\vec{e}_{(0)}\cdot\vec{n}_{12})^2\Big]\left(\frac{r_{12}}{\xi}\right)^2 + 2\,X_2^3\,(\vec{e}_\xi\cdot\vec{n}_{12})\Big[1 - \frac{4}{3}(\vec{e}_\xi\cdot\vec{n}_{12})^2\\
& - 3\,(\vec{e}_{(0)}\cdot\vec{n}_{12})^2\Big]\left(\frac{r_{12}}{\xi}\right)^3 + \mathcal{O}\left[\left(\frac{r_{12}}{\xi}\right)^4\right]\Bigg\}P^i_q n^q_{12}
\end{aligned}
$$

$$- 8\,\frac{G^2 m_1 m_2}{c^4 r_{12}^2} X_2 (\vec{e}_{(0)} \cdot \vec{n}_{12})^2 \Bigg\{ \left(\frac{r_{12}}{\xi}\right) + 2\,X_2\,(\vec{e}_\xi \cdot \vec{n}_{12}) \left(\frac{r_{12}}{\xi}\right)^2 + \mathcal{O}\left[\left(\frac{r_{12}}{\xi}\right)^3\right] \Bigg\} e_\xi^i$$
$$+ 8\,\frac{G^2 m_1 m_2}{c^4 r_{12}^2} X_2^2 (\vec{e}_{(0)} \cdot \vec{n}_{12})^2 \Bigg\{ \left(\frac{r_{12}}{\xi}\right)^2 + 2\,X_2\,(\vec{e}_\xi \cdot \vec{n}_{12}) \left(\frac{r_{12}}{\xi}\right)^3$$
$$+ \mathcal{O}\left[\left(\frac{r_{12}}{\xi}\right)^4\right] \Bigg\} P_q^i n_{12}^q + (1 \leftrightarrow 2), \tag{5.25}$$

where the quantities $\vec{n}_{12}$, $\vec{v}_{12}$ and r_{12} are taken at the time t^*.

5.4 The post-linear light deflection and the perturbed light ray trajectory

If we introduce into the equations for the linear perturbations $\delta l^i_{(1)\mathrm{I}}(\tau)$ and $\delta l^i_{(1)\mathrm{II}}(\tau)$ the expression for the perturbed light ray trajectory, we get additional post-linear light deflection terms. To compute these terms, we first have to find the expression for the perturbation of the photon's trajectory that is linear in G. This perturbation is obtained by integrating the expression for the total linear perturbation (5.35) with respect to the parameter τ. Considering that in this work we compute the post-linear light deflection to the order G^2/c^4, we do not need to retain in the expression resulting from the integration of (5.35) the terms of the order G/c^4, since these terms are related to the post-linear light deflection terms of higher order than G^2/c^4. After performing the integration of (5.35) with respect to τ and retaining only terms of the order G/c^2 and G/c^3 we obtain

$$\delta z^i_{(1)}(\tau) = -2\frac{G}{c^2}\sum_{a=1}^{2} m_a \mathcal{B}_a \left[\xi^i - x^i_a(t^*)\right] - 2\frac{G}{c^3}\sum_{a=1}^{2} m_a (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) \mathcal{B}_a l^i_{(0)}$$
$$- 2\frac{G}{c^3}\sum_{a=1}^{2} m_a \ln\left[\frac{c\tau - \vec{e}_{(0)} \cdot \vec{x}_a(t^*) + r_a}{r_a(0,t^*) - \vec{e}_{(0)} \cdot \vec{x}_a(t^*)}\right] l^i_{(0)} + 4\frac{G}{c^3}\sum_{a=1}^{2} m_a (\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) \mathcal{B}_a \left[\xi^i - x^i_a(t^*)\right]$$
$$+ 2\frac{G}{c^3}\sum_{a=1}^{2} m_a (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) \mathcal{B}_a v^i_a(t^*) + 2\frac{G}{c^3}\sum_{a=1}^{2} m_a \ln\left[\frac{c\tau - \vec{e}_{(0)} \cdot \vec{x}_a(t^*) + r_a}{r_a(0,t^*) - \vec{e}_{(0)} \cdot \vec{x}_a(t^*)}\right] v^i_a(t^*)$$
$$- 6\frac{G}{c^3}\sum_{a=1}^{2} m_a \Big[(\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) \mathcal{F}_{a2} + (\vec{r}_a(0,t^*) \cdot \vec{v}_a(t^*)) \mathcal{F}_{a3}\Big] \left[\xi^i - x^i_a(t^*)\right] + \mathcal{O}\left(\frac{G}{c^4}\right), \tag{5.26}$$

where the functions $\mathcal{B}_a$, $\mathcal{F}_{a2}$ and $\mathcal{F}_{a3}$ are given by

$$
\begin{aligned}
\mathcal{B}_a &= \frac{1}{R_a}\Big[c\tau + r_a\Big],\\
\mathcal{F}_{a2} &= \frac{1}{3r_a R_a^3}\Bigg\{(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2\Big[3r_a^4(0,t^*) + r_a^2 R_a + (\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2 r_a^2(0,t^*)\Big]\\
&\quad + r_a^2(0,t^*)\Big[r_a^4(0,t^*) + r_a^2 R_a - 5(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2 r_a^2(0,t^*)\Big] + \Big[-2(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))r_a^4(0,t^*)\\
&\quad + \Big(r_a^2(0,t^*) + (\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2\Big) r_a R_a + 2(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^3\Big(2r_a^2(0,t^*) - (\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2\Big)\Big]c\tau\Bigg\},\\
\mathcal{F}_{a3} &= \frac{1}{3r_a R_a^2}\Bigg\{2(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))r_a^2(0,t^*) - \Big[R_a - 2(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))r_a + 4(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2\Big]c\tau\\
&\quad + 2(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))c^2\tau^2\Bigg\}.
\end{aligned}
\tag{5.27}
$$

As our integration constant we have chosen in (5.26)

$$
\begin{aligned}
K^i &= 2\frac{G}{c^3}\sum_{a=1}^{2} m_a \ln\Big[2\Big(r_a(0,t^*) - \vec{e}_{(0)}\cdot\vec{x}_a(t^*)\Big)\Big] l^i_{(0)}\\
&\quad - 2\frac{G}{c^3}\sum_{a=1}^{2} m_a \ln\Big[2\Big(r_a(0,t^*) - \vec{e}_{(0)}\cdot\vec{x}_a(t^*)\Big)\Big] v^i_a(t^*)\\
&\quad + \mathcal{O}\left(\frac{G}{c^4}\right),
\end{aligned}
$$

because with this integration constant we recover from our expression for the post-linear light deflection the correct expression for the post-linear light deflection in the event that the value of one of the masses is equal to zero (i.e. the Epstein-Shapiro post-linear light deflection).

It follows from (5.2) that the expression for the linear light deflection for an observer located at infinity reads

$$
\alpha^i_{(1)} = \lim_{\tau\to\infty}\left\{\frac{1}{c}P^i_q\big[\delta l^q_{(1)\mathrm{I}}(\tau) + \delta l^q_{(1)\mathrm{II}}(\tau)\big]\right\}, \tag{5.28}
$$

where the perturbations $\delta l^q_{(1)\mathrm{I}}(\tau)$ and $\delta l^q_{(1)\mathrm{II}}(\tau)$ are given by (5.3) and (5.16). Upon inserting the perturbation $\delta\vec{z}_{(1)}$ into the equations for $\delta l^q_{(1)\mathrm{I}}(\tau)$ and $\delta l^q_{(1)\mathrm{II}}(\tau)$, we get a perturbed linear light deflection. Because the perturbation $\delta\vec{z}_{(1)}$ is a small quantity compared to $\vec{z}(\tau)_{\mathrm{unpert.}}$, we can resort to a Taylor expansion of the perturbed linear light deflection about $\delta\vec{z}_{(1)} = 0$ in order to get the terms of the perturbed linear light deflection that are quadratic in G. We denote these terms by $\alpha^i_{(2)\mathrm{IV}}$ and they are given by

$$
\begin{aligned}
\alpha^i_{(2)\mathrm{IV}} = \lim_{\tau\to\infty}&\left\{\frac{1}{c}P^i_q\left[\left(\frac{\partial\delta l^{q(\mathrm{Pert.})}_{(1)\mathrm{I}}}{\partial\delta z^m_{(1)}}\right)_{\delta\vec{z}_{(1)}=0}\delta z^m_{(1)}+\left(\frac{\partial\delta l^{q(\mathrm{Pert.})}_{(1)\mathrm{II}}}{\partial\delta z^m_{(1)}}\right)_{\delta\vec{z}_{(1)}=0}\delta z^m_{(1)}\right]\right\}\\
=&\,\frac{G}{c}\sum_{a=1}^{2}m_a\int_{-\infty}^{\infty}d\tau\left\{\left[\frac{6}{r_a^5}(\vec{r}_a\cdot\delta\vec{z}_{(1)})-6\left[\frac{1}{r_a^5}(\vec{v}_a(t^*)\cdot\delta\vec{z}_{(1)})\right.\right.\right.\\
&-\left.\frac{5}{r_a^7}(\vec{r}_a\cdot\vec{v}_a(t^*))(\vec{r}_a\cdot\delta\vec{z}_{(1)})\right]\tau-15\left[\frac{2}{r_a^7}(\vec{r}_a\cdot\vec{v}_a(t^*))(\vec{v}_a(t^*)\cdot\delta\vec{z}_{(1)})\right.\\
&+\left.\left.\frac{1}{r_a^7}v_a^2(t^*)(\vec{r}_a\cdot\delta\vec{z}_{(1)})-\frac{7}{r_a^9}(\vec{r}_a\cdot\vec{v}_a(t^*))^2(\vec{r}_a\cdot\delta\vec{z}_{(1)})\right]\tau^2\right]P^i_q r^q_a\\
&-\left[\frac{2}{r_a^3}+\frac{6}{r_a^5}(\vec{r}_a\cdot\vec{v}_a(t^*))\tau-3\left[\frac{1}{r_a^5}v_a^2(t^*)-\frac{5}{r_a^7}(\vec{r}_a\cdot\vec{v}_a(t^*))^2\right]\tau^2\right]P^i_q\delta z^q_{(1)}\\
&-\left.\left[\frac{6}{r_a^5}(\vec{r}_a\cdot\delta\vec{z}_{(1)})\tau-3\left[\frac{2}{r_a^5}(\vec{v}_a\cdot\delta\vec{z}_{(1)})-\frac{10}{r_a^7}(\vec{r}_a\cdot\vec{v}_a(t^*))(\vec{r}_a\cdot\delta\vec{z}_{(1)})\right]\tau^2\right]P^i_q v^q_a(t^*)\right\}_{|(\rightarrow)}\\
&+\frac{G}{c^2}\sum_{a=1}^{2}m_a(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))\int_{-\infty}^{\infty}d\tau\left\{-6\left[\frac{2}{r_a^5}(\vec{r}_a\cdot\delta\vec{z}_{(1)})-2\left[\frac{1}{r_a^5}(\vec{v}_a(t^*)\cdot\delta\vec{z}_{(1)})\right.\right.\right.\\
&-\left.\frac{5}{r_a^7}(\vec{r}_a\cdot\vec{v}_a(t^*))(\vec{r}_a\cdot\delta\vec{z}_{(1)})\right]\tau-5\left[\frac{1}{r_a^7}v_a^2(t^*)(\vec{r}_a\cdot\delta\vec{z}_{(1)})+\frac{2}{r_a^7}(\vec{r}_a\cdot\vec{v}_a(t^*))(\vec{v}_a(t^*)\cdot\delta\vec{z}_{(1)})\right.\\
&-\left.\left.\frac{7}{r_a^9}(\vec{r}_a\cdot\vec{v}_a(t^*))^2(\vec{r}_a\cdot\delta\vec{z}_{(1)})\right]\tau^2\right]P^i_q r^q_a+2\left[\frac{2}{r_a^3}+\frac{6}{r_a^5}(\vec{r}_a\cdot\vec{v}_a(t^*))\tau\right.\\
&-\left.3\left[\frac{1}{r_a^5}v_a^2(t^*)-\frac{5}{r_a^7}(\vec{r}_a\cdot\vec{v}_a(t^*))^2\right]\tau^2\right]P^i_q\delta z^q_{(1)}+12\left[\frac{1}{r_a^5}(\vec{r}_a\cdot\delta\vec{z}_{(1)})\tau\right.\\
&-\left.\left.\left[\frac{1}{r_a^5}(\vec{v}_a(t^*)\cdot\delta\vec{z}_{(1)})-\frac{5}{r_a^7}(\vec{r}_a\cdot\vec{v}_a(t^*))(\vec{r}_a\cdot\delta\vec{z}_{(1)})\right]\tau^2\right]P^i_q v^q_a(t^*)\right\}_{|(\rightarrow)}+\frac{G}{c^3}\left\{\ldots\right\}.
\end{aligned}
\tag{5.29}
$$

In the equation above we need not write out the expression $G/c^3\{\ldots\}$ explicitly since it contributes only terms of order greater than G^2/c^4. After substituting the perturbation (5.26) into the preceding equation, we obtain the integrals for the post-linear light deflection $\alpha^i_{(2)\mathrm{IV}}$. Here, we take into account only the integrals of the order G^2/c^4. These integrals are given in an explicit form in Appendix E.

5.5 Light deflection and the centre of mass

In this section we compute the corrections to the linear and the post-linear light deflection resulting from the introduction of the 1PN-corrections to the positions of the masses in the equations for the linear perturbations $\delta l^q_{(1)\mathrm{I}}(\tau)$ and $\delta l^q_{(1)\mathrm{II}}(\tau)$ given by (5.3) and (5.16). It follows from equations (4.64) and (4.65) that the 1PN-corrections in the positions are

$$\delta\vec{x}_1 = \delta\vec{x}_2 = \frac{1}{c^2}\left[\frac{\nu(m_1-m_2)}{2M}\left[v_{12}^2 - \frac{GM}{r_{12}}\right]\right]\vec{r}_{12}. \tag{5.30}$$

From (5.30), it is easy to see that the corrections vanish when $m_1 = m_2$. The corrections also vanish for the case of circular orbits. After introducing the 1PN-corrections into the equations for the linear perturbations $\delta l^q_{(1)\mathrm{I}}(\tau)$ and $\delta l^q_{(1)\mathrm{II}}(\tau)$, we obtain the expression for the perturbed linear light deflection. Because the corrections $\delta\vec{x}_a$ are small quantities compared to $\vec{x}_a$, we can resort to a Taylor expansion of the perturbed linear light deflection about $\delta\vec{x}_a = 0$ in order to find the correction terms for the linear and post-linear light deflection. We denote these terms by $\tilde{\alpha}^i_{(1)(2)}$ and they are given by

$$\begin{aligned}
\tilde{\alpha}^i_{(1)(2)} &= \lim_{\tau\to\infty}\left\{\frac{1}{c}P^i_q\left[\left(\frac{\partial\delta l^{q(\mathrm{Pert.})}_{(1)\mathrm{I}}}{\partial\delta x^m_a}\right)_{\delta\vec{x}_a=0}\delta x^m_a + \left(\frac{\partial\delta l^{q(\mathrm{Pert.})}_{(1)\mathrm{II}}}{\partial\delta x^m_a}\right)_{\delta\vec{x}_a=0}\delta x^m_a\right]\right\}\\
&= \frac{G}{c}\sum_{a=1}^{2} m_a\int_{-\infty}^{\infty}d\tau\left\{\left[\left[-\frac{6}{r_a^5}(\vec{r}_a\cdot\delta\vec{x}_a) + 6\left[\frac{1}{r_a^5}(\vec{v}_a(t^*)\cdot\delta\vec{x}_a)\right.\right.\right.\right.\\
&\quad\left.\left. - \frac{5}{r_a^7}(\vec{r}_a\cdot\vec{v}_a(t^*))(\vec{r}_a\cdot\delta\vec{x}_a)\right]\tau + 15\left[\frac{2}{r_a^7}(\vec{r}_a\cdot\vec{v}_a(t^*))(\vec{v}_a(t^*)\cdot\delta\vec{x}_a)\right.\right.\\
&\quad\left.\left. + \frac{1}{r_a^7}v_a^2(t^*)(\vec{r}_a\cdot\delta\vec{x}_a) - \frac{7}{r_a^9}(\vec{r}_a\cdot\vec{v}_a(t^*))^2(\vec{r}_a\cdot\delta\vec{x}_a)\right]\tau^2\right]P^i_q r^q_a\\
&\quad + \left[\frac{2}{r_a^3} + \frac{6}{r_a^5}(\vec{r}_a\cdot\vec{v}_a(t^*))\tau - 3\left[\frac{1}{r_a^5}v_a^2(t^*) - \frac{5}{r_a^7}(\vec{r}_a\cdot\vec{v}_a(t^*))^2\right]\tau^2\right]P^i_q\delta x^q_a\\
&\quad \left. + \left[\frac{6}{r_a^5}(\vec{r}_a\cdot\delta\vec{x}_a)\tau - 3\left[\frac{2}{r_a^5}(\vec{v}_a\cdot\delta\vec{x}_a) - \frac{10}{r_a^7}(\vec{r}_a\cdot\vec{v}_a(t^*))(\vec{r}_a\cdot\delta\vec{x}_a)\right]\tau^2\right]P^i_q v^q_a(t^*)\right\}_{|(\to)}\\
&\quad + \frac{G}{c^2}\sum_{a=1}^{2} m_a(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))\int_{-\infty}^{\infty}d\tau\left\{6\left[\frac{2}{r_a^5}(\vec{r}_a\cdot\delta\vec{x}_a) - 2\left[\frac{1}{r_a^5}(\vec{v}_a(t^*)\cdot\delta\vec{x}_a)\right.\right.\right.\\
&\quad\left. - \frac{5}{r_a^7}(\vec{r}_a\cdot\vec{v}_a(t^*))(\vec{r}_a\cdot\delta\vec{x}_a)\right]\tau - 5\left[\frac{1}{r_a^7}v_a^2(t^*)(\vec{r}_a\cdot\delta\vec{x}_a) + \frac{2}{r_a^7}(\vec{r}_a\cdot\vec{v}_a(t^*))(\vec{v}_a(t^*)\cdot\delta\vec{x}_a)\right.\\
&\quad\left.\left. - \frac{7}{r_a^9}(\vec{r}_a\cdot\vec{v}_a(t^*))^2(\vec{r}_a\cdot\delta\vec{x}_a)\right]\tau^2\right]P^i_q r^q_a - 2\left[\frac{2}{r_a^3} + \frac{6}{r_a^5}(\vec{r}_a\cdot\vec{v}_a(t^*))\tau\right.\\
&\quad\left. - 3\left[\frac{1}{r_a^5}v_a^2(t^*) - \frac{5}{r_a^7}(\vec{r}_a\cdot\vec{v}_a(t^*))^2\right]\tau^2\right]P^i_q\delta x^q_a - 12\left[\frac{1}{r_a^5}(\vec{r}_a\cdot\delta\vec{x}_a)\tau\right.\\
&\quad\left.\left. - \left[\frac{1}{r_a^5}(\vec{v}_a(t^*)\cdot\delta\vec{x}_a) - \frac{5}{r_a^7}(\vec{r}_a\cdot\vec{v}_a(t^*))(\vec{r}_a\cdot\delta\vec{x}_a)\right]\tau^2\right]P^i_q v^q_a(t^*)\right\}_{|(\to)} + \frac{G}{c^3}\{\ldots\}.
\end{aligned}\tag{5.31}$$

For the same reason as in equation (5.29), we do not need to write out the expression $G/c^3\{...\}$ explicitly in (5.31). After substituting the 1PN-corrections (5.30) into the equation above and taking into account only the terms of the orders G/c^4 and G^2/c^4 we find,

$$\begin{aligned}
\tilde{\alpha}^i_{(1)(2)} = &-6\,\frac{G}{c^3}\sum_{a=2}^{2} m_a\left[\frac{\nu(m_1-m_2)}{2M}\left[v_{12}^2(t^*)-\frac{GM}{r_{12}(t^*)}\right]\right] \\
&\int_{-\infty}^{\infty} d\tau\,\frac{1}{r_a^5}\left[c\,\tau\vec{e}_{(0)}\cdot\vec{r}_{12}(t^*)-\vec{x}_a\cdot\vec{r}_{12}(t^*)\right]\left[\xi^i-P^i_q x^q_a(t^*)\right] \\
&+2\,\frac{G}{c^3}\sum_{a=2}^{2} m_a\left[\frac{\nu(m_1-m_2)}{2M}\left[v_{12}^2(t^*)-\frac{GM}{r_{12}(t^*)}\right]\right]\int_{-\infty}^{\infty} d\tau\,\frac{1}{r_a^3}P^i_q r^q_{12}(t^*)+(1\leftrightarrow 2).
\end{aligned} \tag{5.32}$$

Here, we have already replaced the photon trajectory by its unperturbed approximation $\vec{z}(\tau)_{\text{unpert.}}=\tau\,\vec{l}_{(0)}+\vec{\xi}$.

After performing the integration we obtain,

$$\begin{aligned}
\tilde{\alpha}^i_{(1)(2)} = {}&8\frac{G}{c^4}\left[\frac{\nu(m_1-m_2)}{2M}\left[v_{12}^2(t^*)-\frac{GM}{r_{12}(t^*)}\right]\right]\sum_{a=1}^{2}\frac{m_a}{R_a^2}\Big[\left(\vec{x}_a(t^*)\cdot\vec{r}_{12}(t^*)\right) \\
&-\left(\vec{e}_{(0)}\cdot\vec{x}_a(t^*)\right)\left(\vec{e}_{(0)}\cdot\vec{r}_{12}(t^*)\right)\Big]\left[\xi^i-P^i_q x^q_a(t^*)\right] \\
&+4\frac{G}{c^4}\left[\frac{\nu(m_1-m_2)}{2M}\left[v_{12}^2(t^*)-\frac{GM}{r_{12}(t^*)}\right]\right]\sum_{a=1}^{2}\frac{m_a}{R_a}P^i_q r^q_{12}(t^*),
\end{aligned} \tag{5.33}$$

where the quantity R_a is defined by (5.12). Finally, considering further applications we express the positions of the masses by their centre of mass coordinates and expand the preceding expression about the origin of the coordinate system to the second order in r_{12}/c to obtain,

$$\begin{aligned}
\tilde{\alpha}^i_{(1)(2)} = {}&8X_2\frac{G}{c^4}m_1\left[\frac{\nu(m_1-m_2)}{2M}\left[v_{12}^2-\frac{GM}{r_{12}}\right]\right]\left\{\left[1-(\vec{e}_{(0)}\cdot\vec{n}_{12})^2\right]\left(\frac{r_{12}}{\xi}\right)^2+\mathcal{O}\left[\left(\frac{r_{12}}{\xi}\right)^3\right]\right\}\frac{e^i}{\xi} \\
&+4\frac{G}{c^4}m_1\left[\frac{\nu(m_1-m_2)}{2M}\left[v_{12}^2-\frac{GM}{r_{12}}\right]\right]\left\{\left(\frac{r_{12}}{\xi}\right)+2X_2(\vec{e}_\xi\cdot\vec{n}_{12})\left(\frac{r_{12}}{\xi}\right)^2\right. \\
&\left.+\mathcal{O}\left[\left(\frac{r_{12}}{\xi}\right)^3\right]\right\}\frac{1}{\xi}P^i_q n^q_{12}+(1\leftrightarrow 2).
\end{aligned} \tag{5.34}$$

As in the preceding section, the quantities $\vec{n}_{12}$, $\vec{v}_{12}$ and r_{12} are taken at the time t^*.

5.6 The total linear perturbation and the linear light deflection in the gravitational field of two bounded masses

To obtain the total linear perturbation $\delta l^i_{(1)}(\tau)$, we have to sum up the expressions for the linear perturbations $\delta l^i_{(1)\mathrm{I}}(\tau)$ and $\delta l^i_{(1)\mathrm{II}}(\tau)$ given by equations (5.4) and (5.17). The resulting expression is

$$
\begin{aligned}
&\delta l^i_{(1)}(\tau) = -2\frac{G}{c}\sum_{a=1}^{2} m_a B_a\left[\xi^i - x^i_a(t^*)\right] \\
&-\frac{G}{c^2}\sum_{a=1}^{2} m_a\left\{2A_a + \frac{4}{r_a} + \frac{4}{r_a^3}\left[(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))c\,\tau^2 + \left[\vec{\xi}\cdot\vec{v}_a(t^*) - \vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\right]\tau\right]\right\} l^i_{(0)} \\
&+\frac{G}{c^2}\sum_{a=1}^{2} m_a\left\{4(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))B_a - 6(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))F_{a2} - 6\left[\vec{\xi}\cdot\vec{v}_a(t^*) - \vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\right]F_{a3}\right\} \\
&\times\left[\xi^i - x^i_a(t^*)\right] \\
&+2\frac{G}{c^2}\sum_{a=1}^{2} m_a\left\{A_a + \frac{2}{r_a} + \frac{2}{r_a^3}\left[(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))c\,\tau^2 + \left[\vec{\xi}\cdot\vec{v}_a(t^*) - \vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\right]\tau\right]\right\} v^i_a(t^*) \\
&+\frac{G}{c^3}\sum_{a=1}^{2} m_a\left\{4(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))\left[A_a + \frac{1}{r_a}\right] - 6\left[(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))F_{a1}\right.\right. \\
&\left.\left.+\left[\vec{\xi}\cdot\vec{v}_a(t^*) - \vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\right]F_{a2}\right]\right\} l^i_{(0)} \\
&+\frac{G}{c^3}\sum_{a=1}^{2} m_a\left\{(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))^2\left[15F_{a2} - 2B_a - 15G_{a2}\right]\right. \\
&+(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))\left[\vec{\xi}\cdot\vec{v}_a(t^*) - \vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\right]\left[18F_{a3} - 30G_{a3}\right] \\
&\left.+\left[\vec{\xi}\cdot\vec{v}_a(t^*) - \vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\right]^2\left[3F_{a4} - 15G_{a4}\right] + v_a^2(t^*)\left[3F_{a2} - 2B_a\right]\right\}\left[\xi^i - x^i_a(t^*)\right] \\
&+\frac{G}{c^3}\sum_{a=1}^{2} m_a\left\{(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))\left[6F_{a1} - 6A_a - \frac{4}{r_a} - \frac{4}{r_a^3}(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))c\,\tau^2\right.\right. \\
&\left.\left.-\frac{4}{r_a^3}\left[\vec{\xi}\cdot\vec{v}_a(t^*) - \vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\right]\tau\right] + \left[\vec{\xi}\cdot\vec{v}_a(t^*) - \vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\right]\left[6F_{a2} - 2B_a\right]\right\} v^i_a(t^*) \\
&+\mathcal{O}\left(\frac{G}{c^4}\right). \qquad (5.35)
\end{aligned}
$$

In the expression above we need only retain the terms of the order G/c^2 and G/c^3, since the terms of the order G/c^4 are related to linear and post-linear light deflection terms of order higher than the terms which we compute in this work. After introducing the perturbation $\delta l^i_{(1)}(\tau)$ into (2.41) and computing the limit for $\tau \to \infty$, we find

$$
\begin{aligned}
\alpha^i_{(1)} &= \lim_{\tau\to\infty}\left[\frac{1}{c}P^i_q\delta l^q_{(1)}(\tau)\right] \\
&= -4\frac{G}{c^2}\sum_{a}^{2}\frac{m_a}{R_a}\big[\xi^i - P^i_q x^q_a(t^*)\big] \\
&+ 4\frac{G}{c^3}\sum_{a=1}^{2}\frac{m_a}{R_a}(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))\big[\xi^i - P^i_q x^q_a(t^*)\big] \\
&+ 4\frac{G}{c^3}\sum_{a=1}^{2}\frac{m_a}{R_a}(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))P^i_q v^q_a(t^*) \\
&- 8\frac{G}{c^3}\sum_{a=1}^{2}\frac{m_a}{R_a^2}(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))\Big[\vec{\xi}\cdot\vec{v}_a(t^*) - \vec{x}_a(t^*)\cdot\vec{v}_a(t^*) + (\vec{e}_{(0)}\cdot\vec{x}_a(t^*))(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))\Big] \\
&\times\big[\xi^i - P^i_q x^q_a(t^*)\big] \\
&- 2\frac{G}{c^4}\sum_{a=1}^{2}\frac{m_a}{R_a}v_a^2(t^*)\big[\xi^i - P^i_q x^q_a(t^*)\big] \\
&+ 4\frac{G}{c^4}\sum_{a=1}^{2}\frac{m_a}{R_a^2}v_a^2(t^*)(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2\big[\xi^i - P^i_q x^q_a(t^*)\big] \\
&- 16\frac{G}{c^4}\sum_{a=1}^{2}\frac{m_a}{R_a^3}(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2\Big[\vec{\xi}\cdot\vec{v}_a(t^*) - \vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\Big]^2 \\
&\times\big[\xi^i - P^i_q x^q_a(t^*)\big] \\
&- 32\,\frac{G}{c^4}\sum_{a=1}^{2}\frac{m_a}{R_a^3}(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^3(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))\Big[\vec{\xi}\cdot\vec{v}_a(t^*) - \vec{x}_a(t^*)\cdot\vec{v}_a(t^*)\Big]\big[\xi^i - P^i_q x^q_a(t^*)\big] \\
&- 16\frac{G}{c^4}\sum_{a=1}^{2}\frac{m_a}{R_a^3}(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^4(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))^2\big[\xi^i - P^i_q x^q_a(t^*)\big] \\
&+ 8\frac{G}{c^4}\sum_{a=1}^{2}\frac{m_a}{R_a^2}(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2\Big[\vec{\xi}\cdot\vec{v}_a(t^*) - \vec{x}_a(t^*)\cdot\vec{v}_a(t^*) + (\vec{e}_{(0)}\cdot\vec{x}_a(t^*))(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))\Big]P^i_q v^q_a(t^*) \\
&- 4\frac{G}{c^4}\sum_{a=1}^{2}\frac{m_a}{R_a^2}(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2(\vec{e}_{(0)}\cdot\vec{v}_a(t^*))^2\big[\xi^i - P^i_q x^q_a(t^*)\big],
\end{aligned}
\tag{5.36}
$$

where R_a is defined by (5.12).

5.6.1 Comparison of $\alpha^i_{(1)}$ with the angle of light deflection obtained by Kopeikin and Schäfer

It is easy to see that the sum of the expression above with equations (5.23) and (F.4) is equal to (3.46) for the case $N = 2$. In Chapter 3 it was shown that (3.46) follows from the expression for the light deflection computed by Kopeikin and Schäfer in [7] in the event that the velocities of the masses are small with respect to the velocity of light and the retarded times are close to the time of closest approach of the unperturbed light ray to the origin of the coordinate system. Note, that to obtain the term in $P^i_a a^q_a(t^*)$ of (3.46), it is better to sum up the corresponding terms in equations (5.22) and (F.3) before performing the integration in order to remove the formal divergences. After summing up and performing the integration of these terms we get:

$$\begin{aligned}
&\frac{G}{c}\sum_{a=1}^{2} m_a \int_{-\infty}^{\infty} d\tau \, \frac{\tau^2}{r_a^3} P^i_q a^q_a(t^*) - \frac{G}{c^3}\sum_{a=1}^{2} m_a \int_{-\infty}^{\infty} d\tau \, \frac{1}{r_a} P^i_q a^q_a(t^*) \\
&= \frac{G}{c^3}\sum_{a=1}^{2} m_a \left\{ \int_{-\infty}^{\infty} d\tau \, \frac{1}{r_a^3}\left[c^2\tau^2 - r_a^2\right] P^i_q a^q_a(t^*) \right\} \\
&= -2\frac{G}{c^4}\sum_{a=1}^{2} m_a P^i_q a^q_a(t^*) + 2\frac{G}{c^4}\sum_{a=1}^{2} \frac{m_a}{R_a} (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 P^i_q a^q_a(t^*),
\end{aligned} \tag{5.37}$$

where the first term on the last line vanishes as consequence of the Newtonian centre of mass theorem.

5.6.2 The linear light deflection in terms of the centre-of-mass-frame coordinates

Considering that the expression for the post-linear light deflection computed in this work is given in terms of the centre-of-mass-frame coordinates and as an expansion in powers of r_{12}/ξ, we must perform the expansion of the expression above about the origin of the coordinate system $\vec{x}_a = 0$ and express the positions of the masses in terms of their centre of mass coordinates. After expressing the positions of the masses by their centre-of-mass-frame coordinates without considering the 1PN-corrections and expanding the expression (5.36) about the origin of the coordinate system to the third order in r_{12}/ξ we finally obtain,

$$\alpha^i_{(1)} = \frac{G m_1}{c^2\,\xi} \left\{ \left[-4 + 4X_2 \frac{1}{c} (\vec{e}_{(0)} \cdot \vec{v}_{12}) - 2X_2^2 \frac{v_{12}^2}{c^2} \right] \right.$$

$$+\Big[-8X_2(\vec{e}_\xi\cdot\vec{n}_{12})+8X_2^2(\vec{e}_\xi\cdot\vec{n}_{12})\frac{1}{c}(\vec{e}_{(0)}\cdot\vec{v}_{12})-8X_2^2\frac{1}{c}(\vec{e}_\xi\cdot\vec{v}_{12})(\vec{e}_{(0)}\cdot\vec{n}_{12})$$
$$-4X_2^3(\vec{e}_\xi\cdot\vec{n}_{12})\frac{v_{12}^2}{c^2}\Big]\left(\frac{r_{12}}{\xi}\right)$$
$$+\Big[4X_2^2-16X_2^2(\vec{e}_\xi\cdot\vec{n}_{12})^2-4X_2^2(\vec{e}_{(0)}\cdot\vec{n}_{12})^2-4X_2^3\frac{1}{c}(\vec{e}_{(0)}\cdot\vec{v}_{12})$$
$$+16X_2^3(\vec{e}_\xi\cdot\vec{n}_{12})^2\frac{1}{c}(\vec{e}_{(0)}\cdot\vec{v}_{12})-32X_2^3\frac{1}{c}(\vec{e}_\xi\cdot\vec{v}_{12})(\vec{e}_\xi\cdot\vec{n}_{12})(\vec{e}_{(0)}\cdot\vec{n}_{12})$$
$$-4X_2^3\frac{1}{c}(\vec{e}_{(0)}\cdot\vec{v}_{12})(\vec{e}_{(0)}\cdot\vec{n}_{12})^2+8X_2^3(\vec{e}_{(0)}\cdot\vec{n}_{12})\frac{1}{c}(\vec{n}_{12}\cdot\vec{v}_{12})$$
$$+2X_2^4\frac{v_{12}^2}{c^2}-8X_2^4(\vec{e}_\xi\cdot\vec{n}_{12})^2\frac{v_{12}^2}{c^2}+2X_2^4(\vec{e}_{(0)}\cdot\vec{n}_{12})^2\frac{v_{12}^2}{c^2}$$
$$-16X_2^4\frac{1}{c^2}(\vec{e}_\xi\cdot\vec{v}_{12})^2(\vec{e}_{(0)}\cdot\vec{n}_{12})^2-4X_2^4\frac{1}{c^2}(\vec{e}_{(0)}\cdot\vec{v}_{12})^2(\vec{e}_{(0)}\cdot\vec{n}_{12})^2\Big]\left(\frac{r_{12}}{\xi}\right)^2$$
$$+\Big[16X_2^4\frac{1}{c}(\vec{e}_\xi\cdot\vec{v}_{12})(\vec{e}_{(0)}\cdot\vec{n}_{12})-96X_2^4\frac{1}{c}(\vec{e}_\xi\cdot\vec{v}_{12})(\vec{e}_\xi\cdot\vec{n}_{12})^2(\vec{e}_{(0)}\cdot\vec{n}_{12})$$
$$-32X_2^4(\vec{e}_\xi\cdot\vec{n}_{12})\frac{1}{c}(\vec{e}_{(0)}\cdot\vec{v}_{12})(\vec{e}_{(0)}\cdot\vec{n}_{12})^2-16X_2^4\frac{1}{c}(\vec{e}_\xi\cdot\vec{v}_{12})(\vec{e}_{(0)}\cdot\vec{n}_{12})^3$$
$$+32X_2^4(\vec{e}_\xi\cdot\vec{n}_{12})(\vec{e}_{(0)}\cdot\vec{n}_{12})\frac{1}{c}(\vec{n}_{12}\cdot\vec{v}_{12})+16X_2^5(\vec{e}_\xi\cdot\vec{n}_{12})(\vec{e}_{(0)}\cdot\vec{n}_{12})^2\frac{v_{12}^2}{c^2}$$
$$-96X_2^5\frac{1}{c^2}(\vec{e}_\xi\cdot\vec{v}_{12})^2\,(\vec{e}_\xi\cdot\vec{n}_{12})(\vec{e}_{(0)}\cdot\vec{n}_{12})^2$$
$$-16X_2^5(\vec{e}_\xi\cdot\vec{n}_{12})\frac{1}{c^2}(\vec{e}_{(0)}\cdot\vec{v}_{12})^2(\vec{e}_{(0)}\cdot\vec{n}_{12})^2$$
$$-32X_2^5\frac{1}{c^2}(\vec{e}_\xi\cdot\vec{v}_{12})(\vec{e}_{(0)}\cdot\vec{v}_{12})(\vec{e}_{(0)}\cdot\vec{n}_{12})^3$$
$$+32X_2^5\frac{1}{c^2}(\vec{e}_\xi\cdot\vec{v}_{12})(\vec{e}_{(0)}\cdot\vec{n}_{12})^2(\vec{n}_{12}\cdot\vec{v}_{12})\Big]\left(\frac{r_{12}}{\xi}\right)^3+\mathcal{O}\left[\left(\frac{r_{12}}{\xi}\right)^4\right]\Bigg\}e_\xi^i$$
$$+\frac{Gm_1}{c^2\,\xi}\Bigg\{\Big[4X_2-4X_2^2\frac{1}{c}(\vec{e}_{(0)}\cdot\vec{v}_{12})+2X_2^3\frac{v_{12}^2}{c^2}\Big]\left(\frac{r_{12}}{\xi}\right)$$
$$+\Big[8X_2^2(\vec{e}_\xi\cdot\vec{n}_{12})-8X_2^3(\vec{e}_\xi\cdot\vec{n}_{12})\frac{1}{c}(\vec{e}_{(0)}\cdot\vec{v}_{12})+8X_2^3\frac{1}{c}(\vec{e}_\xi\cdot\vec{v}_{12})(\vec{e}_{(0)}\cdot\vec{n}_{12})$$
$$+4X_2^4(\vec{e}_\xi\cdot\vec{n}_{12})\frac{v_{12}^2}{c^2}\Big]\left(\frac{r_{12}}{\xi}\right)^2+\Big[-4X_2^3+16X_2^3(\vec{e}_\xi\cdot\vec{n}_{12})^2+4X_2^3(\vec{e}_{(0)}\cdot\vec{n}_{12})^2$$
$$+4X_2^4\frac{1}{c}(\vec{e}_{(0)}\cdot\vec{v}_{12})-16X_2^4(\vec{e}_\xi\cdot\vec{n}_{12})^2\frac{1}{c}(\vec{e}_{(0)}\cdot\vec{v}_{12})$$
$$+32X_2^4\frac{1}{c}(\vec{e}_\xi\cdot\vec{v}_{12})(\vec{e}_\xi\cdot\vec{n}_{12})(\vec{e}_{(0)}\cdot\vec{n}_{12})+4X_2^4\frac{1}{c}(\vec{e}_{(0)}\cdot\vec{v}_{12})(\vec{e}_{(0)}\cdot\vec{n}_{12})^2$$
$$-8X_2^4(\vec{e}_{(0)}\cdot\vec{n}_{12})\frac{1}{c}(\vec{n}_{12}\cdot\vec{v}_{12})-2X_2^5\frac{v_{12}^2}{c^2}+8X_2^5(\vec{e}_\xi\cdot\vec{n}_{12})^2\frac{v_{12}^2}{c^2}$$
$$-2X_2^5(\vec{e}_{(0)}\cdot\vec{n}_{12})^2\frac{v_{12}^2}{c^2}+16X_2^5\frac{1}{c^2}(\vec{e}_\xi\cdot\vec{v}_{12})^2(\vec{e}_{(0)}\cdot\vec{n}_{12})^2$$
$$+4X_2^5\frac{1}{c^2}(\vec{e}_{(0)}\cdot\vec{v}_{12})^2(\vec{e}_{(0)}\cdot\vec{n}_{12})^2\Big]\left(\frac{r_{12}}{\xi}\right)^3+\mathcal{O}\left[\left(\frac{r_{12}}{\xi}\right)^4\right]\Bigg\}P_q^i n_{12}^q$$

$$+ \frac{G m_1}{c^2\,\xi}\Bigg\{4X_2^2(\vec{e}_{(0)}\cdot\vec{n}_{12})\left(\frac{r_{12}}{\xi}\right) + \Bigg[8X_2^3(\vec{e}_\xi\cdot\vec{n}_{12})(\vec{e}_{(0)}\cdot\vec{n}_{12})$$
$$+ 8X_2^4\frac{1}{c}(\vec{e}_\xi\cdot\vec{v}_{12})(\vec{e}_{(0)}\cdot\vec{n}_{12})^2\Bigg]\left(\frac{r_{12}}{\xi}\right)^2$$
$$+ \Bigg[-4X_2^4(\vec{e}_{(0)}\cdot\vec{n}_{12}) + 16X_2^4(\vec{e}_\xi\cdot\vec{n}_{12})^2(\vec{e}_{(0)}\cdot\vec{n}_{12}) + 4X_2^4(\vec{e}_{(0)}\cdot\vec{n}_{12})^3$$
$$+ 32X_2^5\frac{1}{c}(\vec{e}_\xi\cdot\vec{v}_{12})(\vec{e}_\xi\cdot\vec{n}_{12})(\vec{e}_{(0)}\cdot\vec{n}_{12})^2 + 8X_2^5\frac{1}{c}(\vec{e}_{(0)}\cdot\vec{v}_{12})(\vec{e}_{(0)}\cdot\vec{n}_{12})^3$$
$$- 8X_2^5(\vec{e}_{(0)}\cdot\vec{n}_{12})^2\frac{1}{c}(\vec{n}_{12}\cdot\vec{v}_{12})\Bigg]\left(\frac{r_{12}}{\xi}\right)^3 + \mathcal{O}\left[\left(\frac{r_{12}}{\xi}\right)^4\right]\Bigg\}\left(\frac{v_{12}}{c}\right)P^i_q e^q_{v_{12}} + (1\leftrightarrow 2), \quad (5.38)$$

where the quantities $\vec{n}_{12}$, $\vec{v}_{12}$ and r_{12} are taken at the time t^*. Notice that in (5.38) we chose the order of the expansion in an arbitrary manner in order to show the structure of the terms belonging to the linear light deflection. In concrete applications we have to choose the order of the expansion of the linear light deflection in accordance with the accuracy reached by the post-linear light deflection.

As we pointed out in Section 5.1, the total linear light deflection is obtained by adding to (5.38) the correction terms arising from the part of $\tilde{\alpha}^i_{(1)(2)}$ (i.e. of (5.34)) that is linear in G.

5.7 The post-linear light deflection

The final expression for the post-linear light deflection in the gravitational field of two bounded masses is obtained by summing up the parts of the light deflection, which are given in the preceding sections and in Appendices A to C. The final expression for the angle of light deflection quadratic in G to the first order in r_{12}/ξ, in which the positions of the masses are expressed in the centre-of-mass-frame coordinates, is given by

$$\alpha^i_{(2)} = \Bigg\{\frac{G^2 m_1^2}{c^4\,\xi^2}\left[-\frac{15}{4}\pi - X_2\left(8(\vec{e}_{(0)}\cdot\vec{n}_{12}) + \frac{45}{4}\pi(\vec{e}_\xi\cdot\vec{n}_{12})\right)\left(\frac{r_{12}}{\xi}\right) + \mathcal{O}\left[\left(\frac{r_{12}}{\xi}\right)^2\right]\right]$$
$$+ \frac{G^2 m_2^2}{c^4\,\xi^2}\left[-\frac{15}{4}\pi + X_1\left(8(\vec{e}_{(0)}\cdot\vec{n}_{12}) + \frac{45}{4}\pi(\vec{e}_\xi\cdot\vec{n}_{12})\right)\left(\frac{r_{12}}{\xi}\right) + \mathcal{O}\left[\left(\frac{r_{12}}{\xi}\right)^2\right]\right]\Bigg\}e^i_\xi$$
$$+ 4\frac{G^2 m_1 m_2}{c^4\,\xi\,r_{12}}e^i_\xi$$
$$+ \frac{G^2 m_1 m_2}{c^4\,\xi^2}\Bigg\{-\frac{15}{2}\pi - 4(X_1 - X_2)(\vec{e}_\xi\cdot\vec{n}_{12}) + \Bigg[-\frac{2}{3} - (X_1 - X_2)^2 - 3(X_1^2 + X_2^2)$$

$$
\begin{aligned}
&+2(X_1^3+X_2^3)+8(X_1-X_2)(\vec{e}_{(0)}\cdot\vec{n}_{12})+\frac{45}{4}\pi(X_1-X_2)(\vec{e}_\xi\cdot\vec{n}_{12})\\
&+\Bigg[\frac{2}{3}+\frac{5}{3}(X_1^2+X_2^2)-6(X_1^3+X_2^3)+3(X_1^4+X_2^4)+\frac{4}{3}X_1X_2-2X_1^2X_2^2\\
&+2X_1X_2(X_1^2+X_2^2)\Bigg](\vec{e}_{(0)}\cdot\vec{n}_{12})^2+\left[\frac{44}{3}-8X_1X_2(3+X_1X_2)+4(X_1^4+X_2^4)\right](\vec{e}_\xi\cdot\vec{n}_{12})^2\\
&+\frac{16}{3}\left[X_1^2+X_2^2-X_1X_2-3(X_1^3+X_2^3)\right](\vec{e}_{(0)}\cdot\vec{n}_{12})^2(\vec{e}_\xi\cdot\vec{n}_{12})^2-16(X_1^3+X_2^3)(\vec{e}_\xi\cdot\vec{n}_{12})^4\\
&+\frac{2}{3}\left[2(X_1^2+X_2^2)-2X_1X_2-3(X_1^3+X_2^3)\right](\vec{e}_{(0)}\cdot\vec{n}_{12})^4\Bigg]\left(\frac{r_{12}}{\xi}\right)+\left[\mathcal{O}\left(\frac{r_{12}}{\xi}\right)^2\right]\Bigg\}e_\xi^i\\
&+\frac{G^2m_1^2}{c^4\,\xi^2}\Bigg[\frac{15}{4}\pi X_2\left(\frac{r_{12}}{\xi}\right)+X_2^2\left(8(\vec{e}_{(0)}\cdot\vec{n}_{12})+\frac{45}{4}\pi(\vec{e}_\xi\cdot\vec{n}_{12})\right)\left(\frac{r_{12}}{\xi}\right)^2\\
&+\mathcal{O}\left[\left(\frac{r_{12}}{\xi}\right)^3\right]\Bigg]P_q^i n_{12}^q\\
&+\frac{G^2m_2^2}{c^4\,\xi^2}\Bigg[-\frac{15}{4}\pi X_1\left(\frac{r_{12}}{\xi}\right)+X_1^2\left(8(\vec{e}_{(0)}\cdot\vec{n}_{12})+\frac{45}{4}\pi(\vec{e}_\xi\cdot\vec{n}_{12})\right)\left(\frac{r_{12}}{\xi}\right)^2\\
&+\mathcal{O}\left[\left(\frac{r_{12}}{\xi}\right)^3\right]\Bigg]P_q^i n_{12}^q\\
&+\frac{G^2m_1m_2}{c^4\,\xi^2}\Bigg\{2(X_1-X_2)+\Bigg[-\frac{15}{4}\pi(X_1-X_2)\\
&+\left[-\frac{22}{3}+4X_1X_2(3+X_1X_2)+4(X_1^3+X_2^3)-2(X_1^4+X_2^4)\right](\vec{e}_\xi\cdot\vec{n}_{12})\\
&-\frac{8}{3}\left[1-3X_1X_2\right](\vec{e}_\xi\cdot\vec{n}_{12})(\vec{e}_{(0)}\cdot\vec{n}_{12})^2\Bigg]\left(\frac{r_{12}}{\xi}\right)+\mathcal{O}\left[\left(\frac{r_{12}}{\xi}\right)^2\right]\Bigg\}P_q^i n_{12}^q+\alpha^i_{(2)\mathrm{V}},
\end{aligned}
\tag{5.39}
$$

where $\alpha^i_{(2)\mathrm{V}}$ is the part of $\tilde{\alpha}^i_{(1)(2)}$ (i.e. of (5.34)), that is quadratic in G. Here, it should be pointed out that some terms which belong to the post-linear light deflection (5.39) (e.g. the term $4\frac{G^2m_1m_2}{c^4\xi r_{12}}e_\xi^i$) are related to terms in v_{12}^2 of the linear light deflection (5.38) through the virial theorem. In Section 6.2, we shall group these terms together before computing the limit $r_{12}\to 0$ in equations (5.38) and (5.39) in order to remove the formal divergences. As in the case of the linear light deflection, the order of the expansion in (5.39) was chosen in an arbitrary manner in order to show the structure of the terms belonging to the post-linear light deflection. In concrete applications the order of the expansion of the post-linear light deflection is to be chosen in accordance with the accuracy reached by the linear light deflection.

6 Results

In this chapter we shall apply the formulae for the linear and post-linear light deflection given in Sections 5.6 and 5.7 to some special cases in order to study the important features of the derived results.

6.1 The value of one of the two masses is equal to zero

If in equations (5.38) and (5.39) we put the value of one of the masses equal to zero (e.g. $m_1 = M$, $m_2 = 0$), we obtain expressions for the light deflection angle for a static point-like mass,

$$\alpha^i_{(1)(\mathrm{E})} = -4\,\frac{G M_{\mathrm{ADM}}}{c^2\,\xi}\,e^i_\xi, \tag{6.1}$$

$$\alpha^i_{(2)(\mathrm{E-S})} = -\frac{15}{4}\,\pi\,\frac{G^2 M^2_{\mathrm{ADM}}}{c^4\,\xi^2}\,e^i_\xi, \tag{6.2}$$

where in this case the ADM mass M_{ADM} is equal to the mass M. The deflection angle linear in G is the well known "Einstein angle" (see [1]). The light deflection angle (6.2) is the post-post-Newtonian light deflection for a point-like mass, which was obtained for the first time by Epstein and Shapiro and by other authors in 1980 and 1982 (see [28, 29, 30, 31]).

6.2 The light deflection when $r_{12} \longrightarrow 0$

In this subsection we are going to compute the limit of the expression for the linear and post-linear angle of light deflection (equations (5.38) and (5.39)) in the event that the distance r_{12} between the components of the binary goes towards zero (i.e. $r_{12} \rightarrow 0$). As we explained at the end of the preceding chapter, we have to group together the terms of (5.34), (5.38) and (5.39) in an appropriate manner in order to remove the formal divergences. By inspection, it is clear that the remaining terms in (5.34), (5.38) and

(5.39) are given by

$$\alpha^i_{(1)} = \frac{Gm_1}{c^2\,\xi}\Bigg\{\bigg[-4+4\,X_2\,\frac{1}{c}\,(\vec{e}_{(0)}\cdot\vec{v}_{12})-2\,X_2^2\,\frac{v_{12}^2}{c^2}\bigg]-4\,X_2^3\,(\vec{e}_\xi\cdot\vec{n}_{12})\,\frac{v_{12}^2}{c^2}\left(\frac{r_{12}}{\xi}\right)\Bigg\}e^i_\xi$$
$$+\frac{Gm_2}{c^2\,\xi}\Bigg\{\bigg[-4-4\,X_1\,\frac{1}{c}\,(\vec{e}_{(0)}\cdot\vec{v}_{12})-2\,X_1^2\,\frac{v_{12}^2}{c^2}\bigg]+4\,X_1^3\,(\vec{e}_\xi\cdot\vec{n}_{12})\,\frac{v_{12}^2}{c^2}\left(\frac{r_{12}}{\xi}\right)\Bigg\}e^i_\xi$$
$$+\left\{2\,\frac{Gm_1}{c^2\,\xi}\,X_2^3-2\,\frac{Gm_2}{c^2\,\xi}\,X_1^3\right\}\frac{v_{12}^2}{c^2}\left(\frac{r_{12}}{\xi}\right)P^i_q n^q_{12}, \tag{6.3}$$

$$\tilde{\alpha}^i_{(1)(2)} = \left[\frac{\nu(m_1-m_2)}{2M}\left[v_{12}^2-\frac{GM}{r_{12}}\right]\right]\left\{4\,\frac{Gm_1}{c^4\,\xi}+4\,\frac{Gm_2}{c^4\,\xi}\right\}\left(\frac{r_{12}}{\xi}\right)P^i_q n^q_{12}, \tag{6.4}$$

$$\alpha^i_{(2)} = \Bigg\{-\frac{15}{4}\,\pi\,\frac{G^2m_1^2}{c^4\,\xi^2}-\frac{15}{4}\,\pi\,\frac{G^2m_2^2}{c^4\,\xi^2}+4\,\frac{G^2m_1m_2}{c^4\,\xi\,r_{12}}$$
$$+\frac{G^2m_1m_2}{c^4\,\xi^2}\bigg[-\frac{15}{2}\,\pi-4\,(X_1-X_2)\,(\vec{e}_\xi\cdot\vec{n}_{12})\bigg]\Bigg\}e^i_\xi$$
$$+2\,\frac{G^2m_1m_2}{c^4\,\xi^2}\,(X_1-X_2)P^i_q n^q_{12}. \tag{6.5}$$

After grouping together the terms that are related through the virial theorem in (6.3), (6.4) and (6.5) we get

$$\alpha^i_{(1)} = \Bigg\{\bigg[-4\,\frac{G}{c^2\,\xi}(m_1+m_2)-2\,\frac{G}{c^2\,\xi}\Big[m_1\,X_2^2+m_2\,X_1^2\Big]\frac{v_{12}^2}{c^2}+4\,\frac{G^2m_1m_2}{c^4\,\xi\,r_{12}}\bigg]$$
$$+\bigg[-4\,\frac{Gm_1}{c^2\,\xi}\,X_2^3\,\frac{v_{12}^2}{c^2}+4\,\frac{Gm_2}{c^2\,\xi}\,X_1^3\,\frac{v_{12}^2}{c^2}-4\,\frac{G^2m_1m_2}{c^4\,\xi\,r_{12}}(X_1-X_2)\bigg](\vec{e}_\xi\cdot\vec{n}_{12})\left(\frac{r_{12}}{\xi}\right)\Bigg\}e^i_\xi$$
$$+2\,\frac{G}{c^2\,\xi}\left\{\Big[m_1\,X_2^3-m_2\,X_1^3\Big]+\frac{\nu(m_1-m_2)}{M}(m_1+m_2)\right\}\frac{v_{12}^2}{c^2}\left(\frac{r_{12}}{\xi}\right)P^i_q n^q_{12}, \tag{6.6}$$
$$\alpha^i_{(2)} = -\frac{15}{4}\,\pi\,\frac{G^2}{c^4\,\xi^2}(m_1+m_2)^2 e^i_\xi$$
$$+2\left\{\frac{G^2m_1m_2}{c^4\,\xi\,r_{12}}(X_1-X_2)-\frac{\nu(m_1-m_2)}{M}\frac{GM}{r_{12}}\left[\frac{Gm_1}{c^4\,\xi}+\frac{Gm_2}{c^4\,\xi}\right]\right\}\left(\frac{r_{12}}{\xi}\right)P^i_q n^q_{12}. \tag{6.7}$$

After simplifying (6.6) and (6.7) we finally obtain

$$\alpha^i_{(1)} = -4\,\frac{GM_{\mathrm{ADM}}}{c^2\,\xi}e^i_\xi+4\,\frac{Gm_1m_2}{c^4\,\xi\,M^2}(m_1-m_2)\left[v_{12}^2-\frac{GM}{r_{12}}\right](\vec{e}_\xi\cdot\vec{n}_{12})\left(\frac{r_{12}}{\xi}\right)e^i_\xi, \tag{6.8}$$
$$\alpha^i_{(2)} = -\frac{15}{4}\,\pi\frac{G^2M^2}{c^4\,\xi^2}e^i_\xi, \tag{6.9}$$

where

$$M_{\rm ADM} = M + \frac{1}{2}\,\mu\,\frac{v_{12}^2}{c^2} - \frac{G m_1 m_2}{c^2\,r_{12}} \tag{6.10}$$

is the ADM mass of the system with $\mu = m_1 m_2/M$ and $M = m_1 + m_2$. In equations (6.3)–(6.8), the quantities $\vec{n}_{12}$, v_{12} and r_{12} are taken at the time t^*. Note that the second term in (6.8) goes to zero when $r_{12} \to 0$ since the expression in brackets remains finite. At the end, we recover the Einstein-angle (i.e. equation (6.1)) with the ADM mass as given by (6.10) as well as the Epstein-Shapiro angle (i.e. equation (6.2)).

6.3 The values of the two masses are equal and the light ray is originally orthogonal to the orbital plane of the binary

In this case we choose $M/2$ for the value of the masses in equations (5.38) and (5.39) and assume that the light ray is originally propagating orthogonal to the orbital plane of the two bounded masses (i.e. $\vec{e}_{(0)} \cdot \vec{n}_{12} = 0$, $\vec{e}_{(0)} \cdot \vec{v}_{12} = 0$). After introducing the ADM mass in the resulting expression for the angle of light deflection linear and quadratic in G and rearranging the terms, we finally find

$$\begin{aligned}
\alpha^i_{(1)\perp} = \frac{G M_{\rm ADM}}{c^2\,\xi}\Bigg\{ &-4 + \left[1 - 4\,(\vec{e}_\xi \cdot \vec{n}_{12})^2\right]\left(\frac{r_{12}}{\xi}\right)^2 + \left[-\frac{1}{4} + 3\,(\vec{e}_\xi \cdot \vec{n}_{12})^2\right. \\
&\left. - 4\,(\vec{e}_\xi \cdot \vec{n}_{12})^4\right]\left(\frac{r_{12}}{\xi}\right)^4 + \left[\frac{1}{16} - \frac{3}{2}\,(\vec{e}_\xi \cdot \vec{n}_{12})^2 + 5\,(\vec{e}_\xi \cdot \vec{n}_{12})^4\right. \\
&\left. - 4\,(\vec{e}_\xi \cdot \vec{n}_{12})^6\right]\left(\frac{r_{12}}{\xi}\right)^6 + \left[-\frac{1}{64} + \frac{5}{8}\,(\vec{e}_\xi \cdot \vec{n}_{12})^2 - \frac{15}{4}\,(\vec{e}_\xi \cdot \vec{n}_{12})^4\right. \\
&\left. + 7\,(\vec{e}_\xi \cdot \vec{n}_{12})^6 - 4\,(\vec{e}_\xi \cdot \vec{n}_{12})^8\right]\left(\frac{r_{12}}{\xi}\right)^8 + \left[\frac{1}{256} - \frac{15}{64}\,(\vec{e}_\xi \cdot \vec{n}_{12})^2\right. \\
&\left. + \frac{35}{16}\,(\vec{e}_\xi \cdot \vec{n}_{12})^4 - 7\,(\vec{e}_\xi \cdot \vec{n}_{12})^6 + 9\,(\vec{e}_\xi \cdot \vec{n}_{12})^8 - 4\,(\vec{e}_\xi \cdot \vec{n}_{12})^{10}\right]\left(\frac{r_{12}}{\xi}\right)^{10} \\
&+ \left[-\frac{1}{1024} + \frac{21}{256}\,(\vec{e}_\xi \cdot \vec{n}_{12})^2 - \frac{35}{32}\,(\vec{e}_\xi \cdot \vec{n}_{12})^4 + \frac{21}{4}\,(\vec{e}_\xi \cdot \vec{n}_{12})^6\right. \\
&\left. - \frac{45}{4}\,(\vec{e}_\xi \cdot \vec{n}_{12})^8 + 11\,(\vec{e}_\xi \cdot \vec{n}_{12})^{10} - 4\,(\vec{e}_\xi \cdot \vec{n}_{12})^{12}\right]\left(\frac{r_{12}}{\xi}\right)^{12} \\
&+ \mathcal{O}\left[\left(\frac{r_{12}}{\xi}\right)^{14}\right]\Bigg\}\,e^i_\xi
\end{aligned}$$

$$
\begin{aligned}
&+\frac{GM_{\mathrm{ADM}}}{c^2\,\xi}\,(\vec{e}_\xi\cdot\vec{n}_{12})\left\{2\left(\frac{r_{12}}{\xi}\right)^2+\left[-1+2\,(\vec{e}_\xi\cdot\vec{n}_{12})^2\right]\left(\frac{r_{12}}{\xi}\right)^4\right.\\
&+\left[\frac{3}{8}-2\,(\vec{e}_\xi\cdot\vec{n}_{12})^2+2\,(\vec{e}_\xi\cdot\vec{n}_{12})^4\right]\left(\frac{r_{12}}{\xi}\right)^6+\left[-\frac{1}{8}+\frac{5}{4}\,(\vec{e}_\xi\cdot\vec{n}_{12})^2\right.\\
&\left.-3\,(\vec{e}_\xi\cdot\vec{n}_{12})^4+2\,(\vec{e}_\xi\cdot\vec{n}_{12})^6\right]\left(\frac{r_{12}}{\xi}\right)^8+\left[\frac{5}{128}-\frac{5}{8}\,(\vec{e}_\xi\cdot\vec{n}_{12})^2\right.\\
&\left.+\frac{21}{8}\,(\vec{e}_\xi\cdot\vec{n}_{12})^4-4\,(\vec{e}_\xi\cdot\vec{n}_{12})^6+2\,(\vec{e}_\xi\cdot\vec{n}_{12})^8\right]\left(\frac{r_{12}}{\xi}\right)^{10}\\
&+\left[-\frac{3}{256}+\frac{35}{128}\,(\vec{e}_\xi\cdot\vec{n}_{12})^2-\frac{7}{4}\,(\vec{e}_\xi\cdot\vec{n}_{12})^4+\frac{9}{2}\,(\vec{e}_\xi\cdot\vec{n}_{12})^6-5\,(\vec{e}_\xi\cdot\vec{n}_{12})^8\right.\\
&\left.\left.+2\,(\vec{e}_\xi\cdot\vec{n}_{12})^{10}\right]\left(\frac{r_{12}}{\xi}\right)^{12}+\mathcal{O}\left[\left(\frac{r_{12}}{\xi}\right)^{14}\right]\right\}P^i_q n^q_{12}
\end{aligned}
\tag{6.11}
$$

and

$$
\begin{aligned}
\alpha^i_{(2)\perp}=\;&\frac{G^2M^2_{\mathrm{ADM}}}{c^4\,\xi^2}\left\{-\frac{15}{4}\,\pi-\frac{1}{6}\left[1-7\,(\vec{e}_\xi\cdot\vec{n}_{12})^2+6\,(\vec{e}_\xi\cdot\vec{n}_{12})^4\right]\left(\frac{r_{12}}{\xi}\right)\right.\\
&\left.+\frac{75}{256}\,\pi\left[10-31\,(\vec{e}_\xi\cdot\vec{n}_{12})^2\right]\left(\frac{r_{12}}{\xi}\right)^2+\mathcal{O}\left[\left(\frac{r_{12}}{\xi}\right)^3\right]\right\}e^i_\xi\\
&+\frac{G^2M^2_{\mathrm{ADM}}}{c^4\,\xi^2}\,(\vec{e}_\xi\cdot\vec{n}_{12})\left\{-\frac{1}{3}\left(\frac{r_{12}}{\xi}\right)+\frac{465}{128}\,\pi\left(\frac{r_{12}}{\xi}\right)^2+\mathcal{O}\left[\left(\frac{r_{12}}{\xi}\right)^3\right]\right\}P^i_q n^q_{12}
\end{aligned}
\tag{6.12}
$$

where in this case the ADM mass is given by

$$
M_{\mathrm{ADM}}=M\left[1+\frac{1}{4}\left(\frac{v^2_{12}}{2c^2}-\frac{GM}{c^2r_{12}}\right)\right].
\tag{6.13}
$$

The expression for the linear light deflection (6.11) was expanded to the order $(r_{12}/\xi)^{12}$ in order to reach the accuracy of the post-linear light deflection (6.12). Note that an expansion to the order $(r_{12}/\xi)^{12}$ in the linear part of the light deflection which is of the same accuracy as an expansion to the second order in r_{12}/ξ in the post-linear part implies that $r_{12}/\xi\sim(GM_{\mathrm{ADM}}/c^2\xi)^{1/10}$. In equations (6.11)–(6.13), the quantities $\vec{n}_{12}$, v_{12} and r_{12} are taken at the time t^*. In this case, the correction arising from the shift of the 1PN-centre of mass with respect to the Newtonian centre of mass (see (5.34)) vanishes.

6.4 The values of the two masses are equal and the light ray is originally parallel to the orbital plane of the binary

Again, we choose the value of the masses equal to $M/2$ in equations (5.38) and (5.39) and assume that the light ray is originally propagating parallel to the orbital plane of the binary (i.e. $\vec{e}_\xi \cdot \vec{n}_{12} = 0$). After introducing the ADM mass as given by (6.13) and rearranging the terms, we finally find:

$$\begin{aligned}
\alpha^i_{(1)\|} &= \frac{G M_{\mathrm{ADM}}}{c^2\,\xi} \Bigg\{ -4 + \Big[1 - (\vec{e}_{(0)} \cdot \vec{n}_{12})^2\Big] \left(\frac{r_{12}}{\xi}\right)^2 + \Big[-\frac{1}{4} + \frac{1}{2}\,(\vec{e}_{(0)} \cdot \vec{n}_{12})^2 \\
&\quad - \frac{1}{4}\,(\vec{e}_{(0)} \cdot \vec{n}_{12})^4\Big] \left(\frac{r_{12}}{\xi}\right)^4 + \Big[\frac{1}{16} - \frac{3}{16}\,(\vec{e}_{(0)} \cdot \vec{n}_{12})^2 + \frac{3}{16}\,(\vec{e}_{(0)} \cdot \vec{n}_{12})^4 \\
&\quad - \frac{1}{16}\,(\vec{e}_{(0)} \cdot \vec{n}_{12})^6\Big] \left(\frac{r_{12}}{\xi}\right)^6 + \Big[-\frac{1}{64} + \frac{1}{16}\,(\vec{e}_{(0)} \cdot \vec{n}_{12})^2 - \frac{3}{32}\,(\vec{e}_{(0)} \cdot \vec{n}_{12})^4 \\
&\quad + \frac{1}{16}\,(\vec{e}_{(0)} \cdot \vec{n}_{12})^6 - \frac{1}{64}\,(\vec{e}_{(0)} \cdot \vec{n}_{12})^8\Big] \left(\frac{r_{12}}{\xi}\right)^8 + \Big[\frac{1}{256} - \frac{5}{256}\,(\vec{e}_{(0)} \cdot \vec{n}_{12})^2 \\
&\quad + \frac{5}{128}\,(\vec{e}_{(0)} \cdot \vec{n}_{12})^4 - \frac{5}{128}\,(\vec{e}_{(0)} \cdot \vec{n}_{12})^6 + \frac{5}{256}\,(\vec{e}_{(0)} \cdot \vec{n}_{12})^8 \\
&\quad - \frac{1}{256}\,(\vec{e}_{(0)} \cdot \vec{n}_{12})^{10}\Big] \left(\frac{r_{12}}{\xi}\right)^{10} + \Big[-\frac{1}{1024} + \frac{3}{512}\,(\vec{e}_{(0)} \cdot \vec{n}_{12})^2 \\
&\quad - \frac{15}{1024}\,(\vec{e}_{(0)} \cdot \vec{n}_{12})^4 + \frac{5}{256}\,(\vec{e}_{(0)} \cdot \vec{n}_{12})^6 - \frac{15}{1024}\,(\vec{e}_{(0)} \cdot \vec{n}_{12})^8 \\
&\quad + \frac{3}{512}\,(\vec{e}_{(0)} \cdot \vec{n}_{12})^{10} - \frac{1}{1024}\,(\vec{e}_{(0)} \cdot \vec{n}_{12})^{12}\Big] \left(\frac{r_{12}}{\xi}\right)^{12} + \mathcal{O}\left[\left(\frac{r_{12}}{\xi}\right)^{14}\right] \Bigg\} e^i_\xi \\
&\quad + \frac{G M_{\mathrm{ADM}}}{c^3\,\xi} \Bigg\{ -(\vec{e}_{(0)} \cdot \vec{v}_{12}) \left(\frac{r_{12}}{\xi}\right) + \Big[\frac{1}{4}\,(\vec{e}_{(0)} \cdot \vec{v}_{12}) \Big[1 + (\vec{e}_{(0)} \cdot \vec{n}_{12})^2\Big] \\
&\quad - \frac{1}{2}\,(\vec{n}_{12} \cdot \vec{v}_{12})\,(\vec{e}_{(0)} \cdot \vec{n}_{12})\Big] \left(\frac{r_{12}}{\xi}\right)^3 + \Big[-\frac{1}{16}\,(\vec{e}_{(0)} \cdot \vec{v}_{12}) \Big[1 + 2\,(\vec{e}_{(0)} \cdot \vec{n}_{12})^2 \\
&\quad - 3\,(\vec{e}_{(0)} \cdot \vec{n}_{12})^4\Big] + \frac{1}{4}\,(\vec{n}_{12} \cdot \vec{v}_{12})\,(\vec{e}_{(0)} \cdot \vec{n}_{12}) \Big[1 - (\vec{e}_{(0)} \cdot \vec{n}_{12})^2\Big]\Big] \left(\frac{r_{12}}{\xi}\right)^5 \\
&\quad + \Big[\frac{1}{64}\,(\vec{e}_{(0)} \cdot \vec{v}_{12}) \Big[1 + 3\,(\vec{e}_{(0)} \cdot \vec{n}_{12})^2 - 9\,(\vec{e}_{(0)} \cdot \vec{n}_{12})^4 + 5\,(\vec{e}_{(0)} \cdot \vec{n}_{12})^6\Big] \\
&\quad - \frac{3}{32}\,(\vec{n}_{12} \cdot \vec{v}_{12})\,(\vec{e}_{(0)} \cdot \vec{n}_{12}) \Big[1 - 2\,(\vec{e}_{(0)} \cdot \vec{n}_{12})^2 + (\vec{e}_{(0)} \cdot \vec{n}_{12})^4\Big]\Big] \left(\frac{r_{12}}{\xi}\right)^7 \\
&\quad + \mathcal{O}\left[\left(\frac{r_{12}}{\xi}\right)^9\right] \Bigg\} P^i_q n^q_{12}
\end{aligned}$$

$$+\frac{GM_{\mathrm{ADM}}}{c^3\,\xi}\,(\vec{e}_{(0)}\cdot\vec{n}_{12})\left\{\frac{1}{2}\left(\frac{r_{12}}{\xi}\right)-\frac{1}{8}\left[1-(\vec{e}_{(0)}\cdot\vec{n}_{12})^2\right]\left(\frac{r_{12}}{\xi}\right)^3\right.$$
$$+\frac{1}{32}\left[1-2\,(\vec{e}_{(0)}\cdot\vec{n}_{12})^2+(\vec{e}_{(0)}\cdot\vec{n}_{12})^4\right]\left(\frac{r_{12}}{\xi}\right)^5$$
$$\left.-\frac{1}{128}\left[1-3\,(\vec{e}_{(0)}\cdot\vec{n}_{12})^2+3\,(\vec{e}_{(0)}\cdot\vec{n}_{12})^4-(\vec{e}_{(0)}\cdot\vec{n}_{12})^6\right]\left(\frac{r_{12}}{\xi}\right)^7+\mathcal{O}\left[\left(\frac{r_{12}}{\xi}\right)^9\right]\right\}P^i_q v^q_{12}$$
$$+\frac{GM_{\mathrm{ADM}}}{c^4\,\xi}(\vec{e}_{(0)}\cdot\vec{n}_{12})^2\left\{\frac{1}{4}\left[v^2_{12}-(\vec{e}_{(0)}\cdot\vec{v}_{12})^2\right]\left(\frac{r_{12}}{\xi}\right)^2+\mathcal{O}\left[\left(\frac{r_{12}}{\xi}\right)^4\right]\right\}e^i_\xi \tag{6.14}$$

and

$$\alpha^i_{(2)\|}=\frac{G^2M^2_{\mathrm{ADM}}}{c^4\,\xi^2}\left\{-\frac{15}{4}\pi-\frac{1}{24}\left[4+(\vec{e}_{(0)}\cdot\vec{n}_{12})^2+(\vec{e}_{(0)}\cdot\vec{n}_{12})^4\right]\left(\frac{r_{12}}{\xi}\right)\right.$$
$$\left.+\frac{3}{256}\,\pi\left[250-797\,(\vec{e}_{(0)}\cdot\vec{n}_{12})^2\right]\left(\frac{r_{12}}{\xi}\right)^2+\mathcal{O}\left[\left(\frac{r_{12}}{\xi}\right)^3\right]\right\}e^i_\xi$$
$$+\frac{G^2M^2_{\mathrm{ADM}}}{c^4\,\xi^2}\left\{4\,(\vec{e}_{(0)}\cdot\vec{n}_{12})\left(\frac{r_{12}}{\xi}\right)^2+\mathcal{O}\left[\left(\frac{r_{12}}{\xi}\right)^3\right]\right\}P^i_q n^q_{12} \tag{6.15}$$

where, the quantities $\vec{n}_{12}$, $\vec{v}_{12}$ and r_{12} are taken at the time t^*.

In (6.14) the components e^i_ξ, $P^i_q n^q_{12}$ and $P^i_q v^q_{12}$ of the linear light deflection were expanded to the order $(r_{12}/\xi)^{12}$, $(r_{12}/\xi)^7$ and $(r_{12}/\xi)^7$ respectively in order to reach the accuracy of the post-linear light deflection (6.15). As in the preceding subsection here the correction arising from the shift of the 1PN-centre of mass with respect to the Newtonian centre of mass (see equation (5.34)) vanishes.

6.5 Light deflection in the post-linear gravitational field of the double Pulsar PSR J0737-3039

Finally, we apply the formulae for the angle of light deflection (6.1)–(6.15) to the double pulsar PSR J0737-3039. The parameters of the pulsar PSR J0737-3039 (e.g. see [32]) are given in Table 6.1.

We compute the angle of light deflection for the cases when the distance between the two stars r_{12} is maximal and minimal. In our computations we assume that the masses of the binary's components are equal, i.e. that the mass ratio R is equal to 1. For the impact parameter, we choose $\xi = 5\,r_{12}$. In order to compute the angle of light deflection we have first to calculate r_{12} and v_{12}. Note that we only use Newtonian relations, since

Pulsar	PSR J0737-3039A	PSR J0737-3039B
Pulse Period P (ms)	22.69937855615(6)	2773.4607474(4)
Orbital period T (day)	0.102251563(1)	
Eccentricity e	0.087779(5)	
Total mass $m_A + m_B$ $(M_\odot)$	2.588(3)	
Mass ratio $R \equiv m_A/m_B$	1.069(6)	

Table 6.1: The parameters of PSR J0737-3039.

the uncertainties in observational data, although small, are nonetheless greater than the corrections that post-Newtonian corrections would yield.

To compute r_{12} we have to calculate the semi-major axis of the elliptical orbit by means of the following equation (because of the low accuracy of the observational data, Newtonian relations are sufficient),

$$a = \sqrt[3]{\frac{G(m_1 + m_2)T^2}{4\,\pi^2}}, \tag{6.16}$$

where a denotes the semi-major axis and T the orbital period. The preceding equation follows from Kepler's third law (e.g. see [33]). The relationships between the distances $r_{12\mathrm{max}}$, $r_{12\mathrm{min}}$ and the semi-major axis a are given by,

$$r_{12\mathrm{max}} = a(1 + e), \tag{6.17}$$

$$r_{12\mathrm{min}} = a(1 - e). \tag{6.18}$$

We obtain the corresponding velocities to $r_{12\mathrm{max}}$ and $r_{12\mathrm{min}}$ from equations

$$v_{12\mathrm{min}} = 8\,\pi^3 \sqrt{\frac{G(m_1 + m_2)}{a}\frac{(1 - e)}{(1 + e)}}, \tag{6.19}$$

and

$$v_{12\mathrm{max}} = 8\,\pi^3 \sqrt{\frac{G(m_1 + m_2)}{a}\frac{(1 + e)}{(1 - e)}} \tag{6.20}$$

respectively.

After introducing the parameters of PSR J0737-3039 into the preceding equations and into the formulae for the light deflection given in this chapter, we find the results presented in Table 6.2.

Table 6.2 shows the angles of light deflection computed by means of formulae (6.12) and (6.15) corresponding to the Epstein-Shapiro angles of $\alpha^i_{(E-S)} = -1.55 \cdot 10^{-6}\, e^i_\xi$ arcsec

	$r_{12\max} = 9.56 \cdot 10^{10}$ cm $v_{12\min} = 5.72 \cdot 10^{7}$ cm/s $\xi = 5\, r_{12\max}$	$r_{12\min} = 8.02 \cdot 10^{10}$ cm $v_{12\max} = 6.83 \cdot 10^{7}$ cm/s $\xi = 5\, r_{12\min}$
$\alpha^i_{(1)(\mathrm{E})}$	$-0.659\, e^i_\xi$	$-0.785\, e^i_\xi$
$\alpha^i_{(2)(\mathrm{E-S})}$	$-1.55 \cdot 10^{-6}\, e^i_\xi$	$-2.20 \cdot 10^{-6}\, e^i_\xi$
$\alpha^i_{(1)\perp}$	$-0.679\, e^i_\xi + 0.013\, P^i_q n^q_{12}$	$-0.809\, e^i_\xi + 0.016\, P^i_q n^q_{12}$
$\alpha^i_{(2)\perp}$	$-1.65 \cdot 10^{-6}\, e^i_\xi + 5 \cdot 10^{-8}\, P^i_q n^q_{12}$	$-2.35 \cdot 10^{-6}\, e^i_\xi + 7 \cdot 10^{-8}\, P^i_q n^q_{12}$
$\alpha^i_{(1)\parallel}$	$-0.659\, e^i_\xi$	$-0.785\, e^i_\xi$
$\alpha^i_{(2)\parallel}$	$-1.66 \cdot 10^{-6}\, e^i_\xi + 2 \cdot 10^{-8}\, P^i_q n^q_{12}$	$-2.36 \cdot 10^{-6}\, e^i_\xi + 3 \cdot 10^{-8}\, P^i_q n^q_{12}$

Table 6.2: The angles of light deflection linear and quadratic in G are given in arcsec. We denote by $\alpha^i_{(1)(\mathrm{E})}$ and $\alpha^i_{(2)(\mathrm{E-S})}$ the Einstein angle and the Epstein-Shapiro angle. For the light ray originally orthogonal to the orbital plane we assume that $\vec{e}_\xi \cdot \vec{n}_{12} = 1$. For the light ray originally parallel to the orbital plane we assume that $\vec{e}_{(0)} \cdot \vec{n}_{12} = 1$.

and $\alpha^i_{(E-S)} = -2.20 \cdot 10^{-6}\, e^i_\xi$ arcsec. If we define the corrections to the Epstein-Shapiro angle that we calculated in this paper by $\delta\alpha^i_{(2)\perp,\parallel} = \alpha^i_{(2)\perp,\parallel} - \alpha^i_{(E-S)}$, we find:

1. light ray originally orthogonal to the orbital plane
   $\delta\alpha^i_{(2)\perp} = (-1.0 \cdot 10^{-7}\, e^i_\xi + 5 \cdot 10^{-8}\, P^i_q n^q_{12})$ arcsec for $\xi = 5\, r_{12\max}$,
   $\delta\alpha^i_{(2)\perp} = (-1.5 \cdot 10^{-7}\, e^i_\xi + 7 \cdot 10^{-8}\, P^i_q n^q_{12})$ arcsec for $\xi = 5\, r_{12\min}$.

2. light ray originally parallel to the orbital plane
   $\delta\alpha^i_{(2)\parallel} = (-1.1 \cdot 10^{-7}\, e^i_\xi + 2 \cdot 10^{-8}\, P^i_q n^q_{12})$ arcsec for $\xi = 5\, r_{12\max}$,
   $\delta\alpha^i_{(2)\parallel} = (-1.6 \cdot 10^{-7}\, e^i_\xi + 3 \cdot 10^{-8}\, P^i_q n^q_{12})$ arcsec for $\xi = 5\, r_{12\min}$.

From the values of $\delta\alpha^i_{(2)\perp}$ and $\delta\alpha^i_{(2)\parallel}$ we see that the corrections to the Epstein-Shapiro light deflection angle are slightly smaller for the case when the light ray is originally orthogonal to the orbital plane. Details related to the measurement of the corrections to the Epstein-Shapiro angle will be discussed in the next chapter.

Corrections arising from the spins of the components of PSR J0737-3039

The double Pulsar PSR J0737-3039 is a highly relativistic, double neutron star system, the components of which are the 23 ms pulsar J0737-3039A and the 2.8 s pulsar J0737-

3039B (see Table 6.1 and Ref. [32]). In this subsection we compute the corrections to the angle of light deflection arising from the spins of the neutron stars by means of equation

$$\alpha^i_{S(1)} = -8\frac{G}{c^3}\sum_{a=1}^{2}\frac{\vec{\mathcal{S}}_a\cdot(\vec{e}_{(0)}\times\vec{e}_\xi)}{|\vec{\xi}|^2}e^i_\xi - 4\frac{G}{c^3}\sum_{a=1}^{2}\frac{(\vec{e}_{(0)}\times\vec{\mathcal{S}}_a)}{|\vec{\xi}|^2}, \tag{6.21}$$

which results from (3.50) and (3.51) for the case when $|\vec{\xi}_a| < |\vec{\xi}|$. Before computing the corrections, we have to calculate the value of the spin for each neutron star by applying the formula

$$\vec{\mathcal{S}}_a = I_a(m_a)\vec{\omega}_a, \tag{6.22}$$

where $\vec{\omega}_a$ and $I_a(m_a)$ are the angular velocity and the function giving the moment of inertia of the star, respectively. The angular velocity we determine from the pulse period of the pulsar. Because of our present insufficient knowledge of the equation of state of condensed matter, the function $I_a(m_a)$ is not known with precision. According to [34] and [35] the value of $I_a(m_a)$ lies somewhere between $1.14\cdot 10^{45}$ g cm^2 and $1.84\cdot 10^{45}$ g cm^2 for $m_a \sim 1.4M_\odot$. As in the other parts of this chapter, we assume here that the masses of the two components of PSR 0737-3039 are equal to 1.29 $M_\odot$. Also we assume that both neutron stars have a moment of inertia equal to $1.5\cdot 10^{45}$ g cm^2.

After introducing the parameters of PSR J0737-3039 and the chosen value of the moment of inertia into (6.21) and (6.22) we find the results given in Table 6.3.

		$r_{12\max} = 9.56\cdot 10^{10}$ cm $v_{12\min} = 5.72\cdot 10^7$ cm/s $\xi = 5\,r_{12\max}$	$r_{12\min} = 8.02\cdot 10^{10}$ cm $v_{12\max} = 6.83\cdot 10^7$ cm/s $\xi = 5\,r_{12\min}$
$\vec{e}_S = (\vec{e}_{(0)}\times\vec{e}_\xi)$	$\alpha^i_{S(1)\perp}$	$-7.47\cdot 10^{-9}\,e^i_\xi$	$-1.06\cdot 10^{-8}\,e^i_\xi$
$\vec{e}_S = \vec{e}_\xi$	$\alpha^i_{S(1)\perp}$	$-3.74\cdot 10^{-9}\,(\vec{e}_{(0)}\times\vec{e}_\xi)^i$	$-5.32\cdot 10^{-9}\,(\vec{e}_{(0)}\times\vec{e}_\xi)^i$
$\vec{e}_S = \vec{e}_{(0)}$	$\alpha^i_{S(1)\perp}$	0	0
$(*)\vec{e}_S = (\vec{e}_{(0)}\times\vec{e}_\xi)$	$\alpha^i_{S(1)\parallel}$	$-7.47\cdot 10^{-9}\,e^i_\xi$	$-1.06\cdot 10^{-8}\,e^i_\xi$
$\vec{e}_S = \vec{e}_\xi$	$\alpha^i_{S(1)\parallel}$	$-3.74\cdot 10^{-9}\,(\vec{e}_{(0)}\times\vec{e}_\xi)^i$	$-5.32\cdot 10^{-9}\,(\vec{e}_{(0)}\times\vec{e}_\xi)^i$
$\vec{e}_S = \vec{e}_{(0)}$	$\alpha^i_{S(1)\parallel}$	0	0

Table 6.3: The angles of light deflection are given in arcsec. By $\vec{e}_S$ we denote the unit vector in the direction of the spin vector $\vec{\mathcal{S}} = \vec{\mathcal{S}}_1 + \vec{\mathcal{S}}_2$.

From the values of Table 6.3 we conclude that the corrections to the angle of light deflection coming from the spins of the components of PSR J0737-3039 lie between 10^{-8}

and 10^{-9} arcsec. Taking into account that PSR J0737-3039 is observed nearly edge-on with an inclination angle i of about 87° (see [32]), it is easy to see that in Table 6.3 the case marked with $(*)$ is closer to reality.

7 Discussion and Conclusions

The angle of light deflection in the post-linear gravitational field of two bounded point-like masses has been computed to the second order in G/c^2. Both the light source and the observer were assumed to be located at infinity in an asymptotically flat space.

The light deflection linear in G has been exactly computed. It was shown that the expression obtained for the linear light deflection is fully equivalent to the expression given by Kopeikin and Schäfer in [7] in the event that the velocities of the masses are small with respect to the velocity of light and the retarded times in the expression of Kopeikin and Schäfer are close to the time of closest approach of the unperturbed light ray to the origin of the coordinate system.

To evaluate the integrals related to the light deflection quadratic in G, which could not be integrated by means of elementary functions, we resorted to a series expansion of the integrands. For this reason the resulting expressions for the angle of light deflection quadratic in G are only valid for the case when the distance between the two masses r_{12} is smaller than the impact parameter ξ (i.e. $r_{12}/\xi < 1$). The final result is given as a power series in r_{12}/ξ.

The expression for the angle of light deflection in terms of the ADM mass to the order G^2/c^4 including a power expansion to the second order in r_{12}/ξ, in which $r_{12}/\xi \sim (GM_{\mathrm{ADM}}/c^2\xi)^{1/10}$ is being assumed, is given in an explicit form for a binary with equal masses in the event that the light ray is originally orthogonal to the orbital plane of the binary as well as for a binary with equal masses in the event that the light ray is originally parallel to the orbital plane of the binary. For a light ray originally propagating orthogonal to the orbital plane of a binary with equal masses, the deflection angle takes a particularly simple form.

In the case when one of the masses is equal to zero, we obtain the "Einstein angle" and the "Shapiro-Epstein light deflection angle", as we do when $r_{12} \to 0$.

Application of the derived formulae for the deflection angle to the double pulsar PSR J0737-3039 has shown that the corrections to the "Einstein angle" are of the order 10^{-2} arcsec for the case when $r_{12}/\xi = 0.2$, see Table 6.2. The corrections to the "Epstein-Shapiro light deflection angle" lie between 10^{-7} and 10^{-8} arcsec, see Table 6.2. The corrections arising from the spins of the neutron stars lie between 10^{-8} and 10^{-9} arcsec, see Table 6.3.

We conclude that the corrections to the "Epstein-Shapiro light deflection angle" are beyond the sensitivity of the current astronomical interferometers. Nevertheless, taking into account that the interferometer for the planned mission LATOR [36, 37] will be able to measure light deflection angles of the order 10^{-8} arcsec, we believe that the corrections to the "Epstein-Shapiro light deflection angle" computed in the present work could well be measured by space-borne interferometers in the foreseeable future.

Finally, it should be pointed out that a further development of the method presented in this thesis can be used in the derivation of a more accurate formula for the timing of binary pulsars (see Refs [38], [39] and [40]) as well as in the computation of effects arising from moving gravitational lenses (see Refs [7] and [41]).

A Auxiliary algebraic relationships

In this appendix we give several algebraic relationships which are useful in the computation of observable effects. After inserting the expressions for S_a^0 and S_a^i given by equation (3.12) into (3.9), we get

$$S_a^{i0} = \gamma_a \Big(\frac{\vec{v}_a \times \vec{\mathcal{S}}_a}{c}\Big)^i, \tag{A.1}$$

$$S_a^{ij} = \gamma_a \epsilon_{ikm} \mathcal{S}_a^m + \frac{(1-\gamma_a)}{v_a^2}(\vec{v}_a \cdot \vec{\mathcal{S}}_a)\epsilon_{ikm} v_a^m, \tag{A.2}$$

$$\begin{aligned} l_{(0)\alpha} r_{a\beta} S_a^{\alpha\beta} &= \gamma_a \Big[\vec{\mathcal{S}}_a \cdot (\vec{l}_{(0)} \times \vec{r}_a) + \vec{\mathcal{S}}_a \cdot \big[(\vec{r}_a - \frac{r_a \vec{l}_{(0)}}{c}) \times \vec{v}_a\big] \\ &+ \frac{(1-\gamma_a)}{v_a^2}(\vec{v}_a \cdot \vec{\mathcal{S}}_a)\vec{v}_a \cdot (\vec{l}_{(0)} \times \vec{v}_a)\Big], \end{aligned} \tag{A.3}$$

$$l_{(0)\alpha} S_a^{\alpha m} = \gamma_a \Big[(\vec{v}_a \times \vec{\mathcal{S}}_a)^m - (\vec{l}_{(0)} \times \vec{\mathcal{S}}_a)^m - \frac{(1-\gamma_a)}{\gamma_a v_a^2}(\vec{v}_a \cdot \vec{\mathcal{S}}_a)(\vec{l}_{(0)} \times \vec{v}_a)\Big], \tag{A.4}$$

where $\gamma_a = (1 - v_a^2/c^2)^{-1/2}$ and $\epsilon_{ikm} = \epsilon_{0ikm}$.

The quantity $R_a(s_a)$ is defined by

$$R_a(s_a) = r_a^2(0, s_a) - (\vec{e}_{(0)} \cdot \vec{x}_a(s_a))^2, \tag{A.5}$$

where $r_a(0, s_a) = |\vec{z}(0) - \vec{x}_a(s_a)|$. Since $x_a^i(s_a)$ can be written as

$$\begin{aligned} x_a^i(s_a) &= (\vec{e}_{(0)} \cdot \vec{x}_a(s_a))e_{(0)}^i + P_q^i x_a^q(s_a), \\ &= (\vec{e}_{(0)} \cdot \vec{x}_a(s_a))e_{(0)}^i + \xi_a^i(s_a), \end{aligned} \tag{A.6}$$

where $P_q^i = \delta_q^i - e_{(0)}^i e_{(0)q}$ and $\xi_a^i(s_a) \equiv P_q^i x_a^q(s_a)$, it follows after inserting (A.6) into (A.5) that

$$\begin{aligned} R_a(s_a) &= \Big[\xi^2 - 2\vec{\xi} \cdot \vec{\xi}_a(s_a) + \xi_a^2(s_a)\Big], \\ &= |\vec{\xi} - \vec{\xi}_a(s_a)|^2. \end{aligned} \tag{A.7}$$

B The Potentials V, V_i and $\hat{W}_{ij}$

The explicit expressions for the potentials V, V_i and $\hat{W}_{ij}$ to the orders relevant for the computation of the post-linear light deflection presented in this work are

$$V = \frac{Gm_1}{r_1} + \frac{Gm_1}{c^2}\left[-\frac{(\vec{n}\cdot\vec{v}_1)^2}{2r_1} + \frac{2v_1^2}{r_1} + Gm_2\left(-\frac{r_1}{4r_{12}^3} - \frac{5}{4r_1r_{12}} + \frac{r_2^2}{4r_1r_{12}^3}\right)\right]$$
$$+\mathcal{O}(\varepsilon^3) + 1 \leftrightarrow 2, \tag{B.1}$$
$$V_i = \frac{Gm_1v_1^i}{r_1} + \mathcal{O}(\varepsilon^2) + 1 \leftrightarrow 2, \tag{B.2}$$
$$\hat{W}_{ij} = \delta^{ij}\left(-\frac{Gm_1v_1^2}{r_1} - \frac{G^2m_1^2}{4r_1^2} + \frac{Gm_1v_1^iv_1^i}{r_1}\right) + \frac{G^2m_1^2n_1^in_1^j}{4r_1^2}$$
$$+G^2m_1m_2\left\{\frac{1}{S^2}\left(n_1^{(i}n_2^{j)} + 2n_1^{(i}n_{12}^{j)}\right) - n_{12}^in_{12}^j\left(\frac{1}{S^2} + \frac{1}{r_{12}S}\right)\right\} + \mathcal{O}(\varepsilon) + (1 \leftrightarrow 2), \tag{B.3}$$

where the distance S is defined by

$$S = r_1 + r_2 + r_{12}. \tag{B.4}$$

After applying to the preceding potentials (B.1), (B.2) and (B.3) the Hadamard regularization procedure in order to calculate their values at the location of the point-mass 1, we get

$$(V)_1 = \frac{Gm_2}{r_{12}}\left\{1 + \frac{1}{c^2}\left[-\frac{3}{2}\frac{Gm_1}{r_{12}} + 2v_2^2 - \frac{1}{2}(\vec{n}_{12}\cdot\vec{v}_2)^2\right]\right\} + \mathcal{O}(\varepsilon^3), \tag{B.5}$$
$$(V_i)_1 = \frac{Gm_2}{r_{12}} + \mathcal{O}(\varepsilon^2), \tag{B.6}$$
$$(\hat{W}_{ij})_1 = \frac{Gm_2}{r_{12}}\left\{v_2^iv_2^j - \delta^{ij}v_2^2 + \frac{Gm_1}{r_{12}}[-2n_{12}^in_{12}^j + \delta^{ij}]\right.$$
$$\left. + 4\frac{Gm_1}{4r_{12}}[n_{12}^in_{12}^j - \delta^{ij}]\right\} + \mathcal{O}(\varepsilon), \tag{B.7}$$
$$(\partial_iV)_1 = -\frac{Gm_2}{r_{12}^2}n_{12}^i + \mathcal{O}(\varepsilon^2). \tag{B.8}$$

C The post-linear light deflection $\alpha^i_{(2)\mathrm{I}}$

In the integrals that are given in this appendix as well as in Appendices D and E, we already replaced the photon trajectory by its unperturbed approximation $\vec{z}(\tau) = \tau\,\vec{l}_{(0)} + \vec{\xi}$, where $\vec{l}_{(0)}$ is given by $\vec{l}_{(0)} = c\,\vec{e}_{(0)}$. The distances r_1 and r_2 are given by

$$\begin{aligned} r_a &= |\vec{z}(\tau) - \vec{x}_a(t^*)| \\ &= \left[z^2(\tau) - 2\vec{z}(\tau)\cdot\vec{x}_a(t^*) + x_a^2(t^*)\right]^{1/2} \\ &= \left[c^2\tau^2 + \xi^2 - 2\,c\,\tau\,\vec{e}_{(0)}\cdot\vec{x}_a(t^*) - 2\,\vec{\xi}\cdot\vec{x}_a(t^*) + x_a^2(t^*)\right]^{1/2}, \end{aligned} \tag{C.1}$$

with $a = 1$ and $a = 2$ for the distances r_1 and r_2. The distance S is defined by (B.4). Here, $r_{12} = |\vec{x}_1(t^*) - \vec{x}_2(t^*)|$ is the distance between the two masses m_1 and m_2 at the time t^* and the unit vector $\vec{n}_{12}$ reads

$$\vec{n}_{12} = \frac{1}{r_{12}}\left[\vec{x}_1(t^*) - \vec{x}_2(t^*)\right]. \tag{C.2}$$

The positions of the masses in the centre of mass frame without considering the 1PN-corrections are given by

$$\vec{x}_1 = X_2\,\vec{r}_{12}(t^*) \tag{C.3}$$

and

$$\vec{x}_2 = -X_1\,\vec{r}_{12}(t^*). \tag{C.4}$$

The integrals resulting from the introduction of the post-linear metric coefficients (4.59) and (4.60) into the expression for $\alpha^i_{(2)\mathrm{I}}$ given by (5.14) are:

$$\begin{aligned}
\alpha^i_{(2)\mathrm{I}}(\vec{\xi}\,) = {} & 2\,\frac{G^2 m_1^2}{c^3}\int\limits_{-\infty}^{\infty} d\tau\,\frac{1}{r_1^6}\left[c\,\tau - X_2\,\vec{e}_{(0)}\cdot\vec{r}_{12}(t^*)\right]^2\left[\xi^i - X_2\,P^i_q r^q_{12}(t^*)\right] \\
& - 4\,\frac{G^2 m_1 m_2}{c^3}\int\limits_{-\infty}^{\infty} d\tau\,\frac{1}{r_{12} r_1 S^2}\left[\xi^i - X_2\,P^i_q r^q_{12}(t^*)\right] \\
& + \frac{G^2 m_1 m_2}{c^3}\int\limits_{-\infty}^{\infty} d\tau\,\frac{1}{r_{12}^3}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]\left[\xi^i - X_2\,P^i_q r^q_{12}(t^*)\right] \\
& + \frac{1}{2}\,\frac{G^2 m_1 m_2}{c^3}\int\limits_{-\infty}^{\infty} d\tau\,\frac{1}{r_{12}^3}\left[\frac{1}{r_1} - \frac{r_2^2}{r_1^3}\right]\left[\xi^i - X_2\,P^i_q r^q_{12}(t^*)\right] \\
& + 16\,\frac{G^2 m_1 m_2}{c^3}\int\limits_{-\infty}^{\infty} d\tau\,\frac{1}{r_1 r_2 S^3}\left(\vec{e}_{(0)}\cdot\vec{n}_{12}\right)\left[c\,\tau + X_1\,\vec{e}_{(0)}\cdot\vec{r}_{12}(t^*)\right] \\
& \times\left[\xi^i - X_2\,P^i_q r^q_{12}(t^*)\right] \\
& + 8\,\frac{G^2 m_1 m_2}{c^3}\int\limits_{-\infty}^{\infty} d\tau\,\frac{1}{r_1 S^3}\left(\vec{e}_{(0)}\cdot\vec{n}_{12}\right)^2\left[\xi^i - X_2\,P^i_q r^q_{12}(t^*)\right] \\
& + 4\,\frac{G^2 m_1 m_2}{c^3}\int\limits_{-\infty}^{\infty} d\tau\,\frac{1}{r_{12} r_1 S^2}\left(\vec{e}_{(0)}\cdot\vec{n}_{12}\right)^2\left[\xi^i - X_2 P^i_q r^q_{12}(t^*)\right] \\
& + \frac{5}{2}\,\frac{G^2 m_1 m_2}{c^3}\int\limits_{-\infty}^{\infty} d\tau\,\frac{1}{r_{12} r_1^3}\left[\xi^i - X_2\,P^i_q r^q_{12}(t^*)\right] \\
& - 16\,\frac{G^2 m_1 m_2}{c^3}\int\limits_{-\infty}^{\infty} d\tau\,\frac{1}{r_{12} r_1 S^3}\left(\vec{e}_{(0)}\cdot\vec{n}_{12}\right)\left[c\,\tau - X_2\,\vec{e}_{(0)}\cdot\vec{r}_{12}(t^*)\right]P^i_q r^q_{12}(t^*) \\
& - 8\,\frac{G^2 m_1 m_2}{c^3}\int\limits_{-\infty}^{\infty} d\tau\,\frac{1}{r_{12}^2 r_1 S^2}\left(\vec{e}_{(0)}\cdot\vec{n}_{12}\right)\left[c\,\tau - X_2\,\vec{e}_{(0)}\cdot\vec{r}_{12}(t^*)\right]P^i_q r^q_{12}(t^*) \\
& - 4\,\frac{G^2 m_1 m_2}{c^3}\int\limits_{-\infty}^{\infty} d\tau\,\frac{1}{r_1 r_2 S^2}\left[\xi^i - X_2\,P^i_q r^q_{12}(t^*)\right] \\
& + 4\,\frac{G^2 m_1 m_2}{c^3}\int\limits_{-\infty}^{\infty} d\tau\,\frac{1}{r_1 r_2^3 S^2}\left[c\,\tau + X_1\,\vec{e}_{(0)}\cdot\vec{r}_{12}(t^*)\right]^2\left[\xi^i - X_2\,P^i_q r^q_{12}(t^*)\right] \\
& + 8\,\frac{G^2 m_1 m_2}{c^3}\int\limits_{-\infty}^{\infty} d\tau\,\frac{1}{r_1 r_2^2 S^3}\left[c\,\tau + X_1\,\vec{e}_{(0)}\cdot\vec{r}_{12}(t^*)\right]^2\left[\xi^i - X_2\,P^i_q r^q_{12}(t^*)\right] \\
& - 4\,\frac{G^2 m_1 m_2}{c^3}\int\limits_{-\infty}^{\infty} d\tau\left[\frac{1}{r_{12} r_1 S^2} - \frac{1}{r_{12} r_1^3 S^2}\left[c\,\tau - X_2\,\vec{e}_{(0)}\cdot\vec{r}_{12}(t^*)\right]^2\right]P^i_q r^q_{12}(t^*) \\
& + 8\frac{G^2 m_1 m_2}{c^3}\int\limits_{-\infty}^{\infty} d\tau\,\frac{1}{r_{12} r_1^2 S^3}\left[c\tau - X_2\vec{e}_{(0)}\cdot\vec{r}_{12}(t^*)\right]^2 P^i_q r^q_{12}(t^*) \\
& + 8\,\frac{G^2 m_1 m_2}{c^3}\int\limits_{-\infty}^{\infty} d\tau\,\frac{1}{r_{12} r_1 r_2 S^3}\left[c\,\tau - X_2\,\vec{e}_{(0)}\cdot\vec{r}_{12}(t^*)\right] \\
& \times\left[c\,\tau + X_1\,\vec{e}_{(0)}\cdot\vec{r}_{12}(t^*)\right]P^i_q r^q_{12}(t^*) \\
& + (1 \leftrightarrow 2).
\end{aligned} \tag{C.5}$$

D The post-linear light deflection $\alpha^i_{(2)\mathrm{II}}$

After inserting the perturbation (5.35) and the metric coefficients (4.61) and (4.62) into (5.15), we find

$$
\begin{aligned}
\alpha^i_{(2)\mathrm{II}}(\vec{\xi}\,) &= 12\,\frac{G^2 m_1^2}{c^3} \int_{-\infty}^{\infty} d\tau\, \frac{1}{r_1^4} \left[\xi^i - X_2\, P^i_q r^q_{12}(t^*)\right] \\
&+ 12\,\frac{G^2 m_1 m_2}{c^3} \int_{-\infty}^{\infty} d\tau\, \frac{1}{r_1^3 r_2} \left[\xi^i - X_2\, P^i_q r^q_{12}(t^*)\right] \\
&+ 4\,\frac{G^2 m_1^2}{c^3} \int_{-\infty}^{\infty} d\tau\, \frac{1}{r_1^3}\, A_1 \left[\xi^i - X_2\, P^i_q r^q_{12}(t^*)\right] \\
&+ 4\,\frac{G^2 m_1 m_2}{c^3} \int_{-\infty}^{\infty} d\tau\, \frac{1}{r_1^3}\, A_2 \left[\xi^i - X_2\, P^i_q r^q_{12}(t^*)\right] \\
&- 4\,\frac{G^2 m_1^2}{c^3} \int_{-\infty}^{\infty} d\tau\, \frac{1}{r_1^3}\, B_1 \left[\vec{e}_{(0)} \cdot (X_2\, \vec{r}_{12}(t^*))\right] \left[\xi^i - X_2\, P^i_q r^q_{12}(t^*)\right] \\
&+ 4\,\frac{G^2 m_1 m_2}{c^3} \int_{-\infty}^{\infty} d\tau\, \frac{1}{r_1^3}\, B_2 \left[\vec{e}_{(0)} \cdot (X_1 \vec{r}_{12}(t^*))\right] \left[\xi^i - X_2\, P^i_q r^q_{12}(t^*)\right] \\
&- 8\,\frac{G^2 m_1^2}{c^3} \int_{-\infty}^{\infty} d\tau\, \frac{1}{r_1^3}\, B_1 \left[c\,\tau - X_2\, \vec{e}_{(0)} \cdot \vec{r}_{12}(t^*)\right] \left[\xi^i - X_2\, P^i_q r^q_{12}(t^*)\right] \\
&- 8\,\frac{G^2 m_1 m_2}{c^3} \int_{-\infty}^{\infty} d\tau\, \frac{1}{r_2^3}\, B_1 \left[c\,\tau + X_1\, \vec{e}_{(0)} \cdot \vec{r}_{12}(t^*)\right] \left[\xi^i - X_2 P^i_q r^q_{12}(t^*)\right] \\
&+ (1 \leftrightarrow 2),
\end{aligned}
\tag{D.1}
$$

where the functions A_1, A_2, B_1 and B_2 are given in Section 5.1.

E The post-linear light deflection $\alpha^i_{(2)\mathrm{IV}}$

The integrals of the order G^2/c^4 resulting from the introduction of the perturbation (5.26) into (5.29) are

$$
\begin{aligned}
&\alpha^i_{(2)\mathrm{IV}}(\vec{\xi}) = -12\frac{G^2 m_1^2}{c^3} r_1^2(0,t^*) \int_{-\infty}^{\infty} d\tau\, \frac{1}{r_1^5} \mathcal{B}_1 \left[\xi^i - X_2\, P^i_q r^q_{12}(t^*)\right] \\
&- 12\frac{G^2 m_1 m_2}{c^3} \left(\vec{r}_1(0,t^*) \cdot \vec{r}_2(0,t^*)\right) \int_{-\infty}^{\infty} d\tau\, \frac{1}{r_1^5} \mathcal{B}_2 \left[\xi^i - X_2\, P^i_q r^q_{12}(t^*)\right] \\
&+ 12\frac{G^2 m_1^2}{c^3} \left(X_2\, \vec{e}_{(0)} \cdot \vec{r}_{12}(t^*)\right)^2 \int_{-\infty}^{\infty} d\tau\, \frac{1}{r_1^5} \mathcal{B}_1 \left[\xi^i - X_2\, P^i_q r^q_{12}(t^*)\right] \\
&- 12\frac{G^2 m_1 m_2}{c^3} (X_1\, X_2) \left(\vec{e}_{(0)} \cdot \vec{r}_{12}(t^*)\right)^2 \int_{-\infty}^{\infty} d\tau\, \frac{1}{r_1^5} \mathcal{B}_2 \left[\xi^i - X_2\, P^i_q r^q_{12}(t^*)\right] \\
&- 12\frac{G^2 m_1^2}{c^3} \int_{-\infty}^{\infty} d\tau\, \frac{1}{r_1^5} \ln\left[\frac{c\tau - X_2\, \vec{e}_{(0)} \cdot \vec{r}_{12}(t^*) + r_1(\tau,t^*)}{r_1(0,t^*) - X_2\, \vec{e}_{(0)} \cdot \vec{r}_{12}(t^*)}\right] \\
&\times \left[c\tau - X_2\, \vec{e}_{(0)} \cdot \vec{r}_{12}(t^*)\right] \left[\xi^i - X_2\, P^i_q r^q_{12}(t^*)\right] \\
&- 12\frac{G^2 m_1 m_2}{c^3} \int_{-\infty}^{\infty} d\tau\, \frac{1}{r_1^5} \ln\left[\frac{c\tau + X_1\, \vec{e}_{(0)} \cdot \vec{r}_{12}(t^*)) + r_2(\tau,t^*)}{r_2(0,t^*) + X_1\, \vec{e}_{(0)} \cdot \vec{r}_{12}(t^*)}\right] \\
&\times \left[c\tau - X_2\, \vec{e}_{(0)} \cdot \vec{r}_{12}(t^*)\right] \left[\xi^i - X_2\, P^i_q r^q_{12}(t^*)\right] \\
&+ 4\frac{G^2 m_1^2}{c^3} \int_{-\infty}^{\infty} d\tau\, \frac{1}{r_1^3} \mathcal{B}_1 \left[\xi^i - X_2\, P^i_q r^q_{12}(t^*)\right] \\
&+ 4\frac{G^2 m_1 m_2}{c^3} \int_{-\infty}^{\infty} d\tau\, \frac{1}{r_2^3} \mathcal{B}_1 \left[\xi^i - X_2\, P^i_q r^q_{12}(t^*)\right] \\
&+ (1 \leftrightarrow 2),
\end{aligned}
\tag{E.1}
$$

where the functions $\mathcal{B}_1$ and $\mathcal{B}_2$ are given in Section 5.4.

F The linear light deflection terms arising from the terms of $h^{(1)}_{00}$ and $h^{(1)}_{pq}$ that contain the accelerations of the masses

As we mentioned in Chapter 4, the terms of the metric coefficients $h^{(1)}_{00}$ and $h^{(1)}_{pq}$ which contain the accelerations of the masses were introduced into the metric quadratic in G after substituting the accelerations by explicit functions of coordinate positions of the masses by means of the Newtonian equations of motion. To get the light deflection terms arising from these terms in a form suitable for the comparison of our computations with the linear light deflection computed by Kopeikin and Schäfer [7], we compute here the light deflection resulting from these terms before performing the substitution of the accelerations. The terms of $h^{(1)}_{00}$ and $h^{(1)}_{pq}$ which contains the accelerations are given by

$$\begin{aligned}\tilde{h}^{(1)}_{00} &= -\frac{G}{c^4}\sum_{a=1}^{2} m_a(\vec{n}_a\cdot\vec{a}_a) = -\frac{G}{c^4}\sum_{a=1}^{2} m_a\frac{(\vec{r}_a\cdot\vec{a}_a)}{r_a},\\ \tilde{h}^{(1)}_{pq} &= \tilde{h}^{(1)}_{00}\delta_{pq}.\end{aligned}\tag{F.1}$$

From (2.41) it follows that the linear light deflection reads

$$\alpha^i_{(1)} = \lim_{\tau\to\infty}\left\{\frac{1}{c}P^i_q\delta l^q_{(1)}(\tau)\right\},\tag{F.2}$$

where $\delta l^q_{(1)}(\tau)$ is given by (2.38). Introduction of the metric coefficients (F.1) into (F.2) leads to

$$\begin{aligned}\tilde{\alpha}^i_{(1)} &= \frac{G}{c^3}\sum_{a=1}^{2} m_a\int_{-\infty}^{\infty} d\tau\,\frac{1}{r_a^3}\Big[c\tau(\vec{e}_{(0)}\cdot\vec{a}_a(t^*)) + \vec{\xi}\cdot\vec{a}_a(t^*) - \vec{x}_a(t^*)\cdot\vec{a}_a(t^*)\Big]\left[\xi^i - P^i_q x^q_a(t^*)\right]\\ &\quad -\frac{G}{c^3}\sum_{a=1}^{2} m_a\int_{-\infty}^{\infty} d\tau\,\frac{1}{r_a}P^i_q a^q_a(t^*).\end{aligned}\tag{F.3}$$

As in (5.22) the second integral in the preceding equation diverges. After performing the Taylor expansion of the second integrand about the origin of the coordinate system $\vec{x}_a = 0$ up to the second order and taking into account the Newtonian centre of mass theorem we perform the integration of (F.3). As result we find

$$\begin{aligned}
\tilde{\alpha}^{i}_{(1)} &= 2\frac{G}{c^4}\sum_{a=1}^{2}\frac{m_a}{R_a}\left(\vec{e}_{(0)}\cdot\vec{x}_a(t^*)\right)\left(\vec{e}_{(0)}\cdot\vec{a}_a(t^*)\right)\left[\xi^i - P^i_q x^q_a(t^*)\right] \\
&\quad + 2\frac{G}{c^4}\sum_{a=1}^{2}\frac{m_a}{R_a}\left[\vec{\xi}\cdot\vec{a}_a(t^*) - \vec{x}_a(t^*)\cdot\vec{a}_a(t^*)\right]\left[\xi^i - P^i_q x^q_a(t^*)\right] \\
&\quad + \frac{G}{c^4}\sum_{a=1}^{2} m_a\left\{-2\frac{(\vec{\xi}\cdot\vec{x}_a(t^*))}{\xi^2} - \frac{(\vec{e}_{(0)}\cdot\vec{x}_a(t^*))^2}{\xi^2} + \frac{x^2_a(t^*)}{\xi^2} - 2\frac{(\vec{\xi}\cdot\vec{x}_a(t^*))^2}{\xi^4}\right\}P^i_q a^q_a(t^*).
\end{aligned} \tag{F.4}$$

G Calculation of Integrals

As we mentioned in Chapter 5, many integrals related to the post-linear light deflection cannot be represented by elementary functions. To evaluate these integrals we resort as usual to a series expansion of the integrands in order to approximate the non-elementary integrals by a sum of elementary integrals. By way of example we shall show in this appendix how this procedure works. As our exemplary integral we choose the integral

$$\mathcal{I}^i = -4\frac{G^2 m_1 m_2}{c^3}\int_{-\infty}^{\infty} d\tau \frac{1}{r_{12} r_1 S^2}[\xi^i - P^i_q x^q_1(t^*)], \tag{G.1}$$

which is given in Appendix C.

The Taylor Expansion

In order to perform the Taylor expansion of the integrand of integral (G.1), we introduce the variable $y = 1/z(\tau)$ and the unit vector $\vec{n} = \vec{z}(\tau)/z(\tau)$, where $z(\tau) = \sqrt{c^2\tau^2 + \xi^2}$. After introducing y and $\vec{n}$ into the integral (G.1), its integrand becomes

$$f(y) = \frac{1}{r_{12} r_1 S^2} = \frac{y^3}{r_{12} w_1(y)[w_1(y) + w_2(y) + y r_{12}]^2}, \tag{G.2}$$

where

$$w_1(y) = \left[1 - 2\vec{n}\cdot\vec{x}_1(t^*)y + x_1^2(t^*)y^2\right]^{1/2}, \tag{G.3}$$

$$w_2(y) = \left[1 - 2\vec{n}\cdot\vec{x}_2(t^*)y + x_2^2(t^*)y^2\right]^{1/2}. \tag{G.4}$$

Since the first three terms of the Taylor expansion about $y = 0$ are equal to zero, we have to perform the Taylor expansion up to the fifth order in order to get an expansion, which is equivalent to a Taylor expansion of the integrand about the origin of the coordinate system $\vec{x}_1 = \vec{x}_2 = 0$ to the second order. After performing the Taylor expansion of the integrand (G.2), we obtain for the integral (G.1):

$$
\begin{aligned}
\mathcal{I}^i = &-4\frac{G^2 m_1 m_2}{c^3}\Bigg\{\frac{1}{4r_{12}}\int_{-\infty}^{\infty}d\tau\frac{1}{z^3(\tau)} - \frac{1}{4}\int_{-\infty}^{\infty}d\tau\frac{1}{z^4(\tau)}\\
&+\Big[\frac{3}{16}r_{12} + \frac{1}{2}\frac{(\vec{\xi}\cdot\vec{x}_1(t^*))}{r_{12}} - \frac{1}{4}\frac{x_1^2(t^*)}{r_{12}} + \frac{1}{4}\frac{(\vec{\xi}\cdot\vec{x}_2(t^*))}{r_{12}} - \frac{1}{8}\frac{x_2^2(t^*)}{r_{12}}\Big]\int_{-\infty}^{\infty}d\tau\frac{1}{z^5(\tau)}\\
&+\Big[\frac{1}{2}\frac{(\vec{e}_{(0)}\cdot\vec{x}_1(t^*))}{r_{12}} + \frac{1}{4}\frac{(\vec{e}_{(0)}\cdot\vec{x}_2(t^*))}{r_{12}}\Big]\int_{-\infty}^{\infty}d\tau\frac{c\tau}{z^5(\tau)}\\
&-\Big[\frac{5}{8}(\vec{\xi}\cdot\vec{x}_1(t^*)) + \frac{3}{8}(\vec{\xi}\cdot\vec{x}_2(t^*))\Big]\int_{-\infty}^{\infty}d\tau\frac{1}{z^6(\tau)}\\
&-\Big[\frac{5}{8}(\vec{e}_{(0)}\cdot\vec{x}_1(t^*)) + \frac{3}{8}(\vec{e}_{(0)}\cdot\vec{x}_2(t^*))\Big]\int_{-\infty}^{\infty}d\tau\frac{c\tau}{z^6(\tau)}\\
&+\Big[\frac{15}{16}\frac{(\vec{\xi}\cdot\vec{x}_1(t^*))^2}{r_{12}} + \frac{5}{8}\frac{(\vec{\xi}\cdot\vec{x}_1(t^*))(\vec{\xi}\cdot\vec{x}_2(t^*))}{r_{12}} + \frac{5}{16}\frac{(\vec{\xi}\cdot\vec{x}_2(t^*))^2}{r_{12}}\Big]\int_{-\infty}^{\infty}d\tau\frac{1}{z^7(\tau)}\\
&+\Big[\frac{15}{8}\frac{(\vec{e}_{(0)}\cdot\vec{x}_1(t^*))(\vec{\xi}\cdot\vec{x}_1(t^*))}{r_{12}} + \frac{5}{8}\frac{(\vec{e}_{(0)}\cdot\vec{x}_1(t^*))(\vec{\xi}\cdot\vec{x}_2(t^*))}{r_{12}}\\
&+\frac{5}{8}\frac{(\vec{e}_{(0)}\cdot\vec{x}_2(t^*))(\vec{\xi}\cdot\vec{x}_2(t^*))}{r_{12}}\Big]\int_{-\infty}^{\infty}d\tau\frac{c\tau}{z^7(\tau)}\\
&+\Big[\frac{15}{16}\frac{(\vec{e}_{(0)}\cdot\vec{x}_1(t^*))^2}{r_{12}} + \frac{5}{8}\frac{(\vec{e}_{(0)}\cdot\vec{x}_1(t^*))(\vec{e}_{(0)}\cdot\vec{x}_2(t^*))}{r_{12}}\\
&+\frac{5}{16}\frac{(\vec{e}_{(0)}\cdot\vec{x}_2(t^*))^2}{r_{12}}\Big]\int_{-\infty}^{\infty}d\tau\frac{c^2\tau^2}{z^7(\tau)}\Bigg\}\Big[\xi^i - P^i_q x^q_1(t^*)\Big].
\end{aligned}
\tag{G.5}
$$

After computing the integrals we finally get,

$$
\begin{aligned}
\mathcal{I}^i = &\frac{G^2 m_1 m_2}{c^4}\Bigg\{-\frac{2}{r_{12}}\Big(\frac{1}{\xi^2}\Big) + \Big[\frac{\pi}{2} - \frac{8}{3}\frac{(\vec{e}_\xi\cdot\vec{x}_1(t^*))}{r_{12}} + \frac{9}{16}\pi(\vec{e}_\xi\cdot\vec{x}_2(t^*))\\
&-\frac{4}{3}\frac{(\vec{e}_\xi\cdot\vec{x}_2(t^*))}{r_{12}}\Big]\Big(\frac{1}{\xi^3}\Big)\\
&+\Big[\frac{15}{16}\pi(\vec{e}_\xi\cdot\vec{x}_1(t^*)) + \frac{4}{3}\frac{x_1^2(t^*)}{r_{12}} - 4\frac{(\vec{e}_\xi\cdot\vec{x}_1(t^*))^2}{r_{12}} - \frac{(\vec{e}_{(0)}\cdot\vec{x}_1(t^*))^2}{r_{12}} - r_{12}\\
&-\frac{8}{3}\frac{(\vec{e}_\xi\cdot\vec{x}_1(t^*))(\vec{e}_\xi\cdot\vec{x}_2(t^*))}{r_{12}} - \frac{2}{3}\frac{(\vec{e}_{(0)}\cdot\vec{x}_1(t^*))(\vec{e}_{(0)}\cdot\vec{x}_2(t^*))}{r_{12}} + \frac{2}{3}\frac{x_2^2(t^*)}{r_{12}}\\
&-\frac{4}{3}\frac{(\vec{e}_\xi\cdot\vec{x}_2(t^*))^2}{r_{12}} - \frac{1}{3}\frac{(\vec{e}_{(0)}\cdot\vec{x}_2(t^*))^2}{r_{12}}\Big]\Big(\frac{1}{\xi^4}\Big)\Bigg\}\Big[\xi^i - P^i_q x^q_1(t^*)\Big].
\end{aligned}
\tag{G.6}
$$

The Taylor expansion of the integrand (G.2) and the computation of the elementary integrals in (G.5) were performed with the help of Mathematica IV.

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Ich erkläre hiermit ehrenwörtlich, dass ich die vorliegende Arbeit selbstständig, ohne unzulässige Hilfe Dritter und ohne Benutzung anderer als der angegebenen Hilfsmittel und Literatur angefertigt habe. Die aus anderen Quellen direkt oder indirekt übernommenen Daten und Konzepte sind unter Angabe der Quelle gekennzeichnet.

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Die geltende Promotionsordnung der Physikalisch-Astronomischen Fakultät ist mir bekannt.

Ich versichere ehrenwörtlich, dass ich nach bestem Wissen die reine Wahrheit gesagt und nichts verschwiegen habe.

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