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Magnetic field calculation of permanent magnet

INTRODUCTION

The paper presents calculation the magnetic field of permanent magnet, homogeneously magnetized in known direction. Method used in the paper is based on a system of equivalent magnetic dipoles. The results that are obtained using this analytical method are compared with results obtained using program packet FEMLAB. Magnetic field and magnetic flux density distributions of permanent magnet are also shown in the paper.

To determine the magnetic field components in vicinity of permanent magnets, it starts from supposition that magnetization of permanent, $\boldsymbol{M}$, magnet is known. The following methods are useful in practical calculation:

a) Method based on determining distribution of microscopic Ampere's current;

b) Method based on Poisson's and Laplace's equations, determining magnetic scalar potential; and

c) Method based on a system of equivalent magnetic dipoles.

OUTLINE OF THE METHODS

Magnetic field inside and outside the permanent magnet, if magnetization of permanent magnet is known, can be calculated using equivalent system of volume and surface microscopic Ampere's currents, which are determined as

$$\boldsymbol{J}_{\mathrm{a}}(\boldsymbol{r}') = \operatorname{rot} \boldsymbol{M}(\boldsymbol{r}'), \text{ and} \tag{1}$$

$$\boldsymbol{J}_{\mathrm{sa}}(r') = \boldsymbol{M}(\boldsymbol{r}') \times \hat{\boldsymbol{n}}, \tag{2}$$

where $\hat{\boldsymbol{n}}$ is unit vector of outgoing normal (Fig.1).

These currents produce magnetic vector potential:

$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0}{4\pi} \int_V \boldsymbol{J}_{\mathrm{a}}(\boldsymbol{r}') \frac{\mathrm{d}V'}{R} + \frac{\mu_0}{4\pi} \oint_S \boldsymbol{J}_{\mathrm{sa}}(\boldsymbol{r}') \frac{\mathrm{d}S'}{R}, \tag{3}$$

where $R = |\boldsymbol{r} - \boldsymbol{r}'|$.

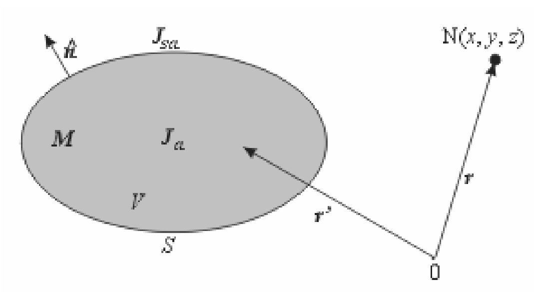

Fig.1 - *Permanent magnet*

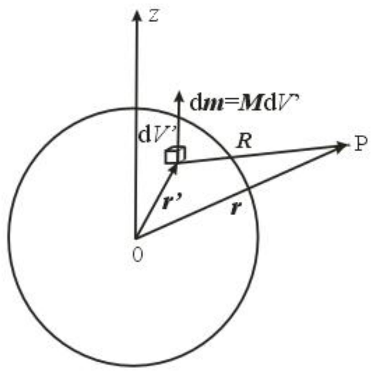

Fig.2 - *Elementary magnetic dipole*

Magnetic flux density is

$$\boldsymbol{B}(\boldsymbol{r}) = \text{rot}\,\boldsymbol{A}(\boldsymbol{r})\,. \tag{4}$$

Inside a permanent magnet, magnetic field can be determined using relation

$$\boldsymbol{H} = \boldsymbol{B}/\mu_0 - \boldsymbol{M}\,. \tag{5}$$

Outside a permanent magnet, magnetic field can be determined using relation

$$\boldsymbol{H} = \boldsymbol{B}/\mu_0\,. \tag{6}$$

The second method is based on determining magnetic scalar potential φ_{m}. Inside a permanent magnet magnetic scalar potential satisfies Poisson's equation

$$\Delta\varphi_{\mathrm{m}} = \text{div}\,\boldsymbol{M}\,. \tag{7}$$

Magnetic field vector can be presented as

$$\boldsymbol{H} = -\text{grad}\,\varphi_{\mathrm{m}}\,. \tag{8}$$

Outside a permanent magnet $\boldsymbol{M} = 0$, because of that magnetic scalar potential, $\varphi_{\mathrm{m}0}$, satisfies Laplace's equation,

$$\Delta\varphi_{\mathrm{m}0} = 0\,, \tag{9}$$

where

$$\boldsymbol{H}_0 = -\text{grad}\,\varphi_{\mathrm{m}0}\,. \tag{10}$$

The third method that is mentioned in the paper for magnetic field calculation is based on superposition of elementary results obtained for elementary magnetic dipoles. Elementary magnetic dipole (Fig.2) has magnetic moment

$$\mathrm{d}\boldsymbol{m} = \boldsymbol{M}\,\mathrm{d}V'\,. \tag{11}$$

This magnetic moment produces, at field point P, elementary magnetic scalar potential

$$\mathrm{d}\varphi_{\mathrm{m}} = \frac{1}{4\pi}\frac{\boldsymbol{R}\,\mathrm{d}\boldsymbol{m}}{R^3} = \frac{1}{4\pi}\frac{\boldsymbol{R}\boldsymbol{M}}{R^3}\mathrm{d}V'\,, \tag{12}$$

where $R = |\boldsymbol{r} - \boldsymbol{r}'|$ is distance from the point where the magnetic field is being calculated to elementary source, and $\boldsymbol{R} = \boldsymbol{r} - \boldsymbol{r}'$.

After integration magnetic scalar potential is obtained as

$$\varphi_{\mathrm{m}} = \frac{1}{4\pi}\int_V \frac{\boldsymbol{R}\,\mathrm{d}\boldsymbol{m}}{R^3} = \frac{1}{4\pi}\int_V \frac{\boldsymbol{R}\boldsymbol{M}}{R^3}\,\mathrm{d}V' . \tag{13}$$

PROBLEM DEFINITION

Permanent magnet that is observed in the paper is homogeneously magnetized in known direction. It consists of three parts that are made of ferromagnetic material. Two ends are magnetized in different direction and the third part is magnetized in angular direction. Dimensions of the permanent magnet are presented in the Fig.3.

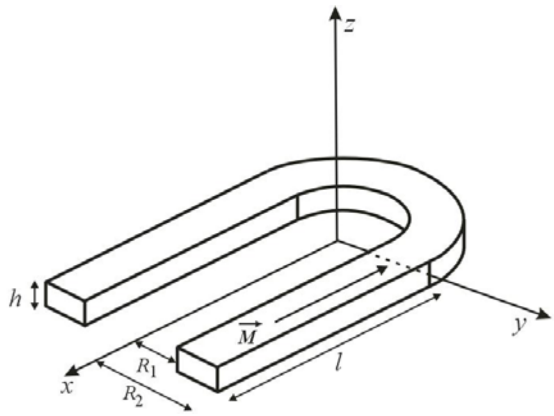

Fig.3 – *Permanent magnet*

Outside the permanent magnet magnetic scalar potential, at field point $\mathrm{P}(x, y, z)$, can be determined using superposition of results obtained for each magnetized part,

$$\varphi_{\mathrm{m}} = \varphi_{\mathrm{m}1} + \varphi_{\mathrm{m}2} + \varphi_{\mathrm{m}3}, \tag{14}$$

where $\varphi_{\mathrm{m}1}$ and $\varphi_{\mathrm{m}2}$ are magnetic scalar potentials of two ends and $\varphi_{\mathrm{m}3}$ is magnetic scalar potentials that originates from the part which is magnetized in angular direction. These magnetic scalar potentials can be determined using the expression (13), where

$$R = |\boldsymbol{r} - \boldsymbol{r}'| = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2} \text{ ,and} \tag{15}$$

$$\boldsymbol{R} = \boldsymbol{r} - \boldsymbol{r}' = (x - x')\hat{\boldsymbol{x}} + (y - y')\hat{\boldsymbol{y}} + (z - z')\hat{\boldsymbol{z}} . \tag{16}$$

The first part of the permanent magnet is homogeneously magnetized in negative direction of x-axes,

$$\boldsymbol{M} = -M\hat{x} . \tag{17}$$

Scalar product is

$$\boldsymbol{RM} = -M(x - x') . \tag{18}$$

Substituting expressions (15) and (18) in (13), magnetic scalar potential $\varphi_{\mathrm{m}1}$ is obtained

$$\varphi_{\mathrm{m}1} = -\frac{M}{4\pi} \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_{R_1}^{R_2} \int_{0}^{l} \frac{x - x'}{\left((x - x')^2 + (y - y')^2 + (z - z')^2\right)^{3/2}} \mathrm{d}x' \mathrm{d}y' \mathrm{d}z' . \tag{19}$$

The second part of the permanent magnet is homogeneously magnetized in positive direction of x-axes,

$$\boldsymbol{M} = M\hat{x} . \tag{20}$$

Scalar product is

$$\boldsymbol{RM} = M(x - x') . \tag{21}$$

Substituting expressions (15) and (21) in (13), magnetic scalar potential $\varphi_{\mathrm{m}2}$ is obtained

$$\varphi_{\mathrm{m}2} = \frac{M}{4\pi} \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_{-R_2}^{-R_1} \int_{0}^{l} \frac{x - x'}{\left((x - x')^2 + (y - y')^2 + (z - z')^2\right)^{3/2}} \mathrm{d}x' \mathrm{d}y' \mathrm{d}z' . \tag{22}$$

The third part of the permanent magnet is homogeneously magnetized in angular direction

$$\boldsymbol{M} = M\hat{\theta} . \tag{23}$$

Relations between coordinates x, y, z and cylindrical coordinates r, θ and z are

$$x = r\cos\theta ,\ y = r\sin\theta \text{ and } z = z . \tag{24}$$

Using these relations, distance from the point where the magnetic field is being calculated to elementary source, can be presented as

$$R = \sqrt{r^2 + r'^2 - 2rr'\cos(\theta - \theta') + (z - z')^2} . \tag{25}$$

As magnetization has only angular component θ, scalar product $\boldsymbol{RM}$ is obtained as

$$\boldsymbol{RM} = \left[(x - x')\hat{\boldsymbol{x}} + (y - y')\hat{\boldsymbol{y}} + (z - z')\hat{\boldsymbol{z}}\right] M\,\hat{\theta} . \tag{26}$$

Relation between unite vectors $\hat{\boldsymbol{x}}$, $\hat{\boldsymbol{y}}$, $\hat{\boldsymbol{z}}$ and $\hat{\theta}$ is

$$\hat{\theta} = \frac{1}{r}\left(\frac{\partial x}{\partial \theta}\hat{\boldsymbol{x}} + \frac{\partial y}{\partial \theta}\hat{\boldsymbol{y}} + \frac{\partial z}{\partial \theta}\hat{\boldsymbol{z}}\right), \tag{27}$$

and the following expressions are also satisfied

$$\hat{\theta}\,\hat{\boldsymbol{x}} = \frac{1}{r}\frac{\partial x}{\partial \theta},\ \hat{\theta}\,\hat{\boldsymbol{y}} = \frac{1}{r}\frac{\partial y}{\partial \theta},\ \hat{\theta}\,\hat{\boldsymbol{z}} = \frac{1}{r}\frac{\partial z}{\partial \theta} . \tag{28}$$

Using relations (17) and (21), the following relations are obtained

$$\hat{\theta}\,\hat{\boldsymbol{x}} = -\sin\theta\,, \hat{\theta}\,\hat{\boldsymbol{y}} = \cos\theta\,, \hat{\theta}\,\hat{\boldsymbol{z}} = 0\,, \tag{29}$$

and scalar product can be presented as

$$\boldsymbol{RM} \; = Mr'\sin(\theta-\theta')\,. \tag{30}$$

Substituting expressions (25) and (30) in (13), magnetic scalar potential is obtained as

$$\varphi_{m3} = \frac{M}{4\pi}\int_{R_1}^{R_2}\int_{-\frac{h}{2}}^{\frac{h}{2}}\int_{0}^{\pi}\frac{r'\sin(\theta-\theta')\,\mathrm{d}r'\mathrm{d}z'\mathrm{d}\theta'}{\left[r^2+r'^2-2rr'\cos(\theta-\theta')+(z-z')^2\right]^{3/2}}\,. \tag{31}$$

The solutions of the integrals presented in the expressions (19), (22) and (31) are very complex. Because of that the expression for magnetic scalar potential (14) is very large and it won't be shown in the paper, but it is used for determining the components of magnetic field.

Magnetic field vector can be expressed as

$$\boldsymbol{H} = -\,\mathrm{grad}\,\varphi_m\,, \tag{32}$$

therefore its components are

$$H_x = -\frac{\partial\varphi_m}{\partial x},\; H_y = -\frac{\partial\varphi_m}{\partial y},\; H_z = -\frac{\partial\varphi_m}{\partial z}\,. \tag{33}$$

NUMERICAL RESULTS

Distribution of magnetic flux density outside the permanent magnet, obtained using the analytical method is presented in the Fig.4, for the following dimension of permanent magnet $R_1/h = 4,\ R_2/h = 6$ and $l/h = 6$.

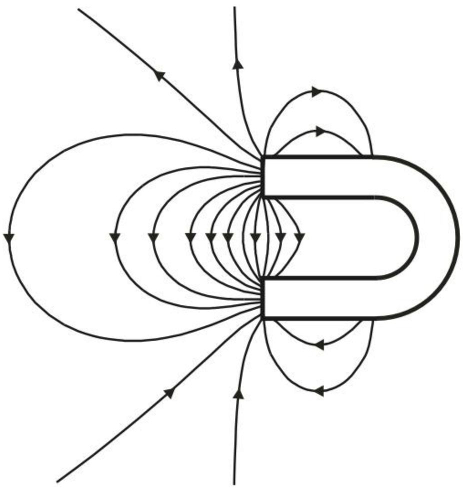

Fig.4 – *Distribution of magnetic flux density*

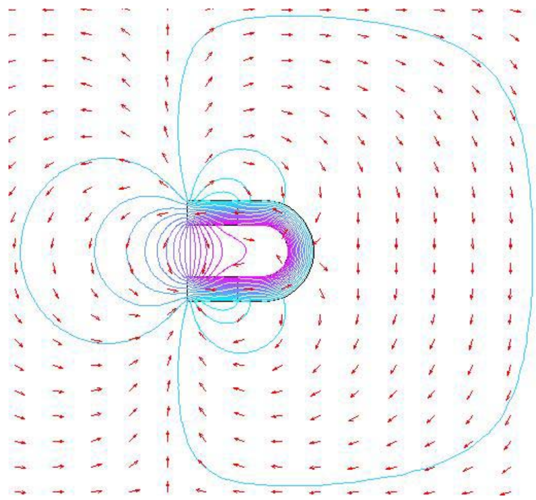

Fig.5 – *Distribution of magnetic flux density (FEMLAB)*

Magnetic field lines have the same form and the same direction as magnetic flux density lines, outside the magnet.

The Fig.5 presents distribution of magnetic flux density (arrow) and magnetic potential (contour) obtained using program packet FEMLAB.

Comparing these figures the excellent agreement between analytical method results and FEMLAB results is evident.

Table I

Normalized magnetic field values along the direction $y = 0, z = 0$

x/l	H/M
0.1	0.436133
0.2	0.206913
0.3	0.124855
0.4	0.081054
0.5	0.053626
0.6	0.035040
0.7	0.021894
0.8	0.012433
0.9	0.005711
1.0	0.001119
1.1	0.001767
1.2	0.003219
1.3	0.003775
1.4	0.003753
1.5	0.003425

Table II

Normalized magnetic field values along the direction $x/l = 7/6, z = 0$

y/l	H/M
0.1	0.003839
0.2	0.006514
0.3	0.010971
0.4	0.018381
0.5	0.031167
0.6	0.052069
0.7	0.074536
0.8	0.084351
0.9	0.080623
1.0	0.063739
1.1	0.040632
1.2	0.023996
1.3	0.014983
1.4	0.010417
1.5	0.008101

In the Table I and Table II, magnetic field values along characteristic direction, for mentioned dimensions of permanent magnet, are presented.

CONCLUSION

Permanent magnet, homogeneously magnetized in known direction is observed in the paper. Method that is used for magnetic field determination is based on superposition of results that are obtained for elementary magnetic dipoles. The tables with magnetic field values, in different points, in vicinity of permanent magnet, are shown. Magnetic flux density distribution of permanent magnet is also presented in the paper. Magnetic field lines have the same form and the same direction as magnetic flux density lines, outside the magnet. Results obtained by analytical method are satisfactory confirmed using program packet FEMLAB.

References:

[1] Textbook of Electromagnetics, Edited by Hermann Uhlmann (Veličković D. M., Chapter 3.2: Magnetostatic field), Technische Universitaet Ilmenau.

[2] A. N. Mladenović, S. R. Aleksić: “Rod shaped permanent magnet”, 48. Conference ETRAN 2004, Čačak, 6th-11th Jun, Proceedings of full papers, pp. 233-236. (In Serbian)

[3] A. N. Mladenović, Slavoljub R. Aleksić: Toroidal shaped permanent magnet with air gap, International Conference on Applied Electromagnetics PES2005, Niš 23rd-25th May 2005, Proceedings of Extended Abstracts, pp. 23-24.

[4] Ana N. Mladenović**,** Slavoljub R. Aleksić:” *Methods for magnetic field calculation*”, 11th International Conference on Electrical Machines, Drives and Power Systems ELMA 2005, Sofia, Bulgaria, 15-16 September 2005, Vol.2, pp. 350-354.

[5] Ana N. Mladenović: “*Toroidal shaped permanent magnet with two air gaps*”, International PhD-Seminar “Numerical Field Computation and Optimization in Electrical Engineering”, Proceedings of Full Papers, Ohrid, Macedonia, 20-25 September 2005, pp.153-158.

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